CHAPTER 7

AN EVALUATION OF EXTREME WAVE CLIMATE AT KEAHOLE POINT, HAWAII

by

Charles L. Bretschneider[1] and Richard E. Rocheleau[2]

INTRODUCTION AND SUMMARY

An evaluation of extreme wave climate was made for Keahole Point, Hawaii. This evaluation was based on three separate sources of wave data and five methods of statistical evaluations. The three sources of data include (1) wave hindcasts data of 10 severe storms between 1947-1961 by Marine Advisors (1963), (2) wave hindcast data of 11 severe storms between 1947-1965 by U.S. Army Corps of Engineers, Honolulu District, and (3) shipboard wave observations of the U.S. Navy reports. Figure 5 shows SSMO area No. 1 for south and southwest of the islands and SSMO areas Nos. 2 and 3 for north of the islands.

The five methods of statistical extrapolations for extreme events include: (1) Gumbel's (1958) first asymptotic distribution, (2) Weibull distribution (1961), and (3,4,&5) semilog, log normal and normal distributions. The three most widely used distribution functions are: (1) Gumbel's (1958), (2) log normal (see Jasper, 1956), and (3) Weibull (1961), given in order as to the author's preference. The statistical extrapolations for Keahole Point, Hawaii, are given in Table 9 and Figure 9. Only Gumbel's (1958) distribution was applied to the north shore as shown in Table 8.

Based on Gumbel's distribution function, the results of the wave hindcasts statistics on the average (50 year recurrence interval) indicate that (1) the Marine Advisors (1963) wave hindcasts are about 25 percent higher than the U.S. Army Corps of Engineers wave hindcasts and (2) the U.S. Navy SSMO observations and the U.S. Army Corps of Engineers wave hindcasts are in closest agreement.

Shipboard wave observations have always been subjected to questions. However, various authors have correlated instrumentally measured and observed wave observations. A summary of these correlations are given in Table 6. No correction was made to the statistical analysis of the SSMO data.

Explanations are in order for the discrepancies between the three sources of data: (1) Both the U.S. Army Corps of Engineers and Marine Advisors have many years of experience in wave forecasting and hence their data should be reliable, (2) ship captains tend to avoid the worst of any storm, and also they are usually too occupied to make accurate observations

---

[1] Chairman and Professor, Department of Ocean Engineering, University of Hawaii

[2] Ph.D. Graduate Student at University of Delaware, formerly M.S. Graduate Student, University of Hawaii.

during heavy seas, and thus the extreme wave conditions are never experienced in the summaries of SSMO data, (3) it might be assumed that the Corps of Engineers hindcasts are somewhat low because they are in near agreement with the SSMO data which are assumed to be low. Perhaps one might use an average of the two sets of hindcast data, and (4) wave forecasting (and wave hindcasting) by experienced oceanographers is more accurate than visual wave observations. There are a number of experienced oceanographers who can make very accurate wave forecasts. The U.S. Navy Fleet Numerical Weather Central at Monterey, California, has a very excellent computer program that can produce wave forecasts more accurately than shipboard wave observations. There are other commercial computer programs equally calibrated to give the same accuracy.

The usefulness of the wave statistics of Hogben and Lumb (1966) and the U.S. Navy SSMO data should be limited to the similar conditions of ship routes under which the wave observations were taken. These data should never be used for the determination of extremes, since the extreme conditions have been absent. A very good case for the above statement can be made for Cape Horn, South America. Cape Horn is one of the most severe storm areas of the world, and yet the wave statistics of Hogben and Lumb (1966) would indicate the area to be relatively calm when we know from history that this is not true. The point here is that the ships avoid going around the Horn, and instead go either through the Magellan Straits or the Panama Canal. Perhaps an exception to the above is the area in the North Sea, where there is heavy ship traffic and many ships can hardly avoid extreme storm wave conditions in the North Sea.

The problem, of course, is how one can use the visual wave observations since these are very plentiful. First, the visual wave observations are excellent for ships avoiding extreme storm wave conditions. Insofar as obtaining reliable wave statistics from visual wave observations, somebody should determine a correlation between regular enroute shipboard wave observations and the extreme wave conditions that were avoided by the ships captain. Such a correlation is far more important than the correlation equations given in Table 6.

In view of the above discussions, it is concluded that: (1) at present wave hindcasting (now very accurate) is the best method for obtaining extreme wave statistics, since one can go back to 50 or more years of weather maps, and it is the only method that gives both the duration of the storms and simultaneous occurrences of swell from distant storms; (2) presently offshore and coastal engineering structures at Keahole Point should be designed on the basis of Marine Advisors' wave statistics as summarized in Figure 9 and Table 9; (3) a refinement of wave statistics for extremes at Keahole Point can only be obtained by wave hindcasts of severe storms prior to 1947 and after 1965; and, (4) there are available a number of calibrated computer programs for wave forecasting and hindcasting. The University of Hawaii does not yet have a computer program for wave hindcasting, but instead should use one of those already available.

WAVE CLIMATE IN HAWAIIAN WATERS

Waves affecting waters around the Hawaiian islands result from storms in all parts of the Pacific Basin and even in parts of the Indian Ocean. The superposition of distant and locally generated waves together with the influence of the islands result in a complex wave climate. The wave climate for the islands has been classified into four general types characterized by wave height, wave period, and direction of approach. Figure 1, from Moberly and Chamberlain (1964), shows the direction of approach of the four main wave types. Figure 2 shows the generating areas from which waves may approach Keahole Point, Hawaii. A fifth type of waves are those associated with hurricanes. Although hurricanes are less frequent than ordinary storms, hurricane generated waves must be considered for design purposes.

The above types of wave climate are discussed briefly:

(1) Northeast tradewind waves. Northeast tradewind waves are present a large percentage of the time, but dominate from April to November when the tradewinds are present 90 to 95 percent of the time. The tradewinds blow 12 to 15 knots per hour about 50 percent of the time generating waves from 4 to 11 feet in height with periods of 5 to 9 seconds due to the long uninterrupted fetches. These waves approach the island between the north and northeast.

(2) Southern swell. Southern swell approaches from between the southeast and the southwest. It is most frequent from April through October, resulting from severe winter storms in the southern hemisphere. Due to the decay over the distances, these waves usually arrive as low, long-period swell, typically 1 to 6 feet high with periods of 14 to 22 seconds. However, swell substantially larger than 6 feet has been observed along the southern boundary of the islands.

(3) Kona storm waves. These waves approach the islands infrequently with the passage of kona storms which are generally cold-core, low-pressure systems of large radius. During the passage of such systems the tradewinds are replaced by south or southwesterly winds generating waves which arrive from the southeast to west. Although infrequent, the severity of kona storms varies from a light breeze to gale strength. The associated wave climate varies over a wide range. Waves of 10 to 15 feet with periods of 8 to 10 seconds are not uncommon.

(4) North Pacific swell. North Pacific swells, having been generated by severe storms near the Western Aleutians or from mid-latitude low pressure systems, approach the islands from the north. These waves are most frequent from October through May and are responsible for the large surf observed in many areas.

(5) Hurricane waves. Although not included in many discussions of Hawaiian waves, a fifth type of wave is important for design considerations. Large waves produced by passing hurricanes may affect many island locations. These tropical cyclones are characterized by a warm low-pressure core with sustained wind speeds substantially higher than those

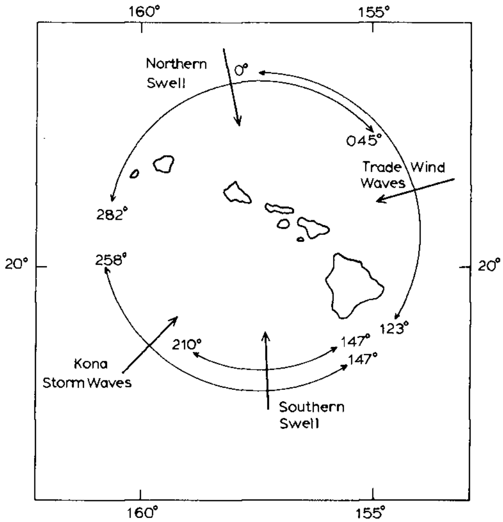

FIG. 1 APPROACH DIRECTION OF FREQUENTLY OCCURRING WAVES IN HAWAIIAN WATERS

After Moberly and Chamberlain (1964)

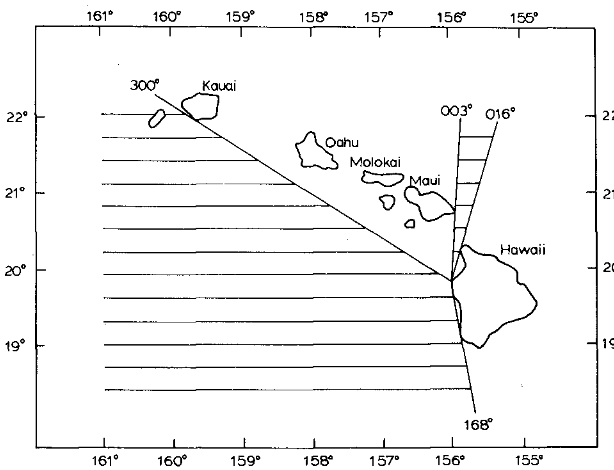

FIG. 2 GENERATION AREAS FROM WHICH WAVES MAY APPROACH KEAHOLE POINT

associated with tropical (kona) storms. Hurricane waves are important for design considerations and are based on the model hurricane for Hawaii as determined by Bretschneider and Tamaye (1976).

SOURCES OF WAVE DATA

Three sources of data have been analyzed to determine long-term wave heights. These are described below.

(1) Marine Advisers: Severe Storm Wave Characteristics in the Hawaiian Islands (1963). The report presents the results of a hindcasting program to determine the wave conditions produced by the 10 most severe storms of the 15-year period from 1947 through 1961 for all Hawaiian waters. These hindcasts were specific for waves approaching either the west coast of Lanai or the west coast of Molokai. Certainly not all the storms analyzed would have affected Keahole Point and even those which did may have had considerably altered wave characteristics at that location. However, these 10 most severe storms, without regard to sheltering effects or direction, were analyzed to provide an upper limit on the severity of the deepwater wave climate expected to occur. The direction of wave approach to the islands is shown in Figure 3. A complete discussion of the storm systems involved may be found in the Marine Advisers' (1963) report. Table 1 gives a summary of these wave hindcasts.

(2) Corps of Engineers: Hindcasts for Harbor Planning (1968. The U.S. Army Corps of Engineers, Honolulu District made hindcasts of 17 storms affecting the Hawaiian Islands from 1947 through 1965. Seven of these which resulted in severe waves at Honokohau Bay are applicable to Keahole Point. In addition, four storms north of the islands which generated large swell from a direction possibly affecting Keahole Point have been included. The inclusion of these storms is based on the location of the generation area relative to the window between Maui and Hawaii rather than on actual observations. Table 2 summarizes these hindcast results specific to Keahole Point. The approach directions of the waves from the 11 storms are presented in Figure 4. Six of the storms shown in Table 2 are also given in Table 1.

(3) U.S. Navy Hydrographic Office: Summary Synoptic Meteorological Observations (SSMO). Figure 5 shows 3 SSMO areas. Area No. 1 is for the windward or north and east areas of the Hawaiian Islands; Area 2 is for the leeward or south and west areas of the Hawaiian Islands; and Area 3 is for the north and west areas of the island of Kauai. SSMO provide monthly and annual summaries of many individual shipboard wave observations over the 8-year period from 1963 to 1971. The waves are classified by observed wave height and wave period, but not by direction. Table 19 of the SSMO reports for the annual summaries are reproduced in Tables 3, 4 and 5 for SSMO areas 1, 2 and 3, respectively. Table 6 represents various equations or relations between measured significant wave heights and shipboard wave observations. It appears that the overall best equation would be equation (d) of Table 6 to convert shipboard wave observations to significant wave height. The observed wave period is essentially the significant wave period, but the relationships given in Table 7 are used to determine f_0^{-1} and $\tilde{\bar{T}}$ of the wave spectrum where

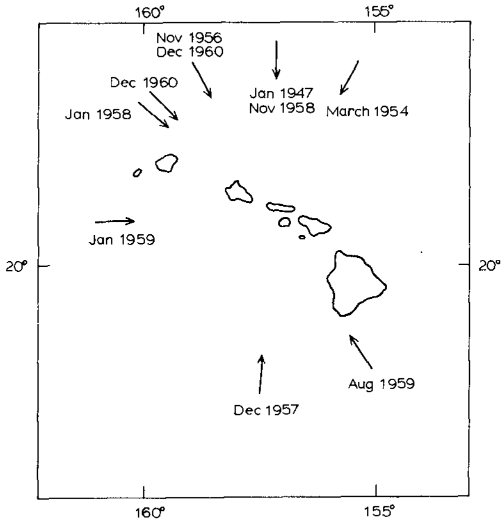

FIG. 3 DIRECTION OF WAVE APPROACH TO THE HAWAIIAN ISLANDS FROM 10 STORMS HINDCAST BY MARINE ADVISERS FROM 1947 THROUGH 1961

TABLE 1

HINDCAST WAVE CHARACTERISTICS FOR TEN STORMS FROM 1947-1961. (FROM MARINE ADVISERS, 1963)

Storm Date	Significant Wave Height (ft.)	Significant Wave Period (seconds)	Direction (°True)
January 3, 1947	15.7	16.4	0
March 6, 1954	25.0	17.2	027
November 27, 1956	12.8	16.8	332
December 2, 1957	32.5	14.5	185
January 12, 1958	27.1	23.5	310
November 22, 1958	12.5	14.6	0
January 18, 1959	7.0	13.5	268
August 6, 1959	16.8	15.9	148
December 11, 1960	18.0	19.6	315
December 20, 1960	13.8	18.0	332

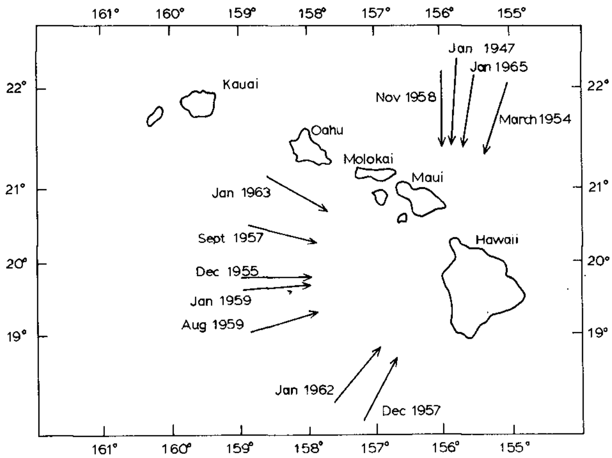

FIG.4 DIRECTION OF WAVE APPROACH TO KEAHOLE POINT FROM 11 STORMS HINDCAST BY THE CORPS OF ENGINEERS FROM 1947 THROUGH 1965

TABLE 2

HINOCAST WAVE CHARACTERISTICS FOR ELEVEN STORMS

EFFECTING KEAHOLE POINT FROM 1947-1965

(Based on hindcasts from the Corps of Engineers, 1968)

Storm Oate	Significant Wave Height (ft.)	Significant Wave Period (seconds)	Direction (°True)
January 3, 1947	14.5	17.3	0D5
March 6, 1954	22.9	17.2	020
December 20, 1955	14.8	11.2	270
September 5, 1957	18.9	21.1	2B6
December 2, 1957	25.5	13.4	210
November 22, 1958	14.6	14.3	357
January 18, 1959	14.0	9.6	267
August 6, 1959	22.5	12.0	255
January 7, 1962	13.6	11.1	222
January 16, 1963	23.0	14.5	300
February 2, 1965	27.0	17.2	010

TABLE 3

SUMMARY OE EIGHT YEARS OF SHIPBOARD OBSERVATIONS WINDWARD OF THE
HAWAIIAN ISLANOS (FROM SUMMARY OF SYNOPTIC METEOROLOGICAL OBSERVATIONS)
TABULATED AS PERCENT FREQUENCY OF OCCURRENCE (SSMO AREA 1)

	Wave Height (feet)													
Period (sec)	<1	1-2	3-4	5-6	7	8-9	10-11	12	13-16	17-19	20-22	23-25	26-32	TOTAL
<6	1.0	8.7	17.9	9.4	3.3	1.3	.3	.1	.1	*	.0	.0	.0	4215
6-7	*	1.3	6.9	10.8	6.0	2.6	1.1	.6	.2	*	*	*	.0	2949
8-9	*	.3	1.6	3.8	4.5	2.3	1.1	.5	.4	.1	*	*	.0	1447
10-11	.0	.1	.4	.9	1.5	1.1	.7	.4	.4	.1	*	*	*	549
12-13	.0	*	.1	.3	.4	.4	.3	.2	.1	*	*	.0	.0	189
>13	.0	*	.0	.1	.1	.1	.1	.1	.1	*	*	*	*	70
INDET	2.4	.8	.8	.7	.3	.2	.1	*	*	.0	.0	.0	.0	528
TOTAL	349	1121	2780	2595	1609	792	354	175	125	26	13	6	2	9947
PCT	3.5	11.3	27.6	26.0	16.2	8.1	3.6	1.8	1.3	.3	.1	.1	*	100.0

TABLE 4

SUMMARY OF EIGHT YEARS OF SHIPBOARD OBSERVATIONS LEEWARD OF THE
HAWAIIAN ISLANOS (from Summary of Synoptic Meteorological Observations)
TABULATED AS PERCENT EREQUENCY OF OCCURRENCE (SSMO AREA 2)

	Wave Height (feet)													
Period (sec)	1	1-2	3-4	5-6	7	8-9	10-11	12	13-16	17-19	20-22	23-25	26-32	TOTAL
6	1.5	12.5	21.2	8.3	3.2	1.0	.3	.1	.1	.0	.0	.0	.0	7978
6-7	.1	2.1	8.1	8.6	6.2	2.2	.9	.3	.2	*	*	*	.0	4707
8-9	*	.4	2.2	2.8	2.8	1.6	1.0	.4	.2	*	*	*	.0	1884
10-11	.0	.1	.5	.7	.7	.6	.4	.2	.2	*	*	*	.0	548
12-13	.0	*	*	.2	.3	.2	.1	*	.1	.0	.0	*	*	163
13	.0	.0	.0	.1	.1	.1	*	*	*	*	*	.0	.0	64
INDET	3.6	1.1	1.1	.6	.2	.1	.1	*	*	.0	.0	.0	.0	1102
TOTAL	852	2696	5501	3476	2218	956	455	152	106	14	9	10	1	16446
PCT	5.3	16.2	33.1	21.2	13.6	5.9	2.8	.9	.7	.1	.1	.1	*	100.0

TABLE 5

SUMMARY OF EIGHT YEARS OF SHIPBOARD OBSERVATIONS FROM

HAWAIIAN ISLANDS (FROM SUMMARY OF SYNOPTIC METEOROLOGICAL OBSERVATIONS - AREA 3)

TABULATED AS PERCENT FREQUENCY OF OCCURRENCE

	Wave Height (feet)													
Period (sec)	<1	1-2	3-4	5-6	7	8-9	10-11	12	13-16	17-19	20-22	23-25	26-32	TOTAL
<6	.7	8.1	13.7	7.8	3.3	1.3	.7	.2	.2	.1	.0	.0	.0	2055
6-7	*	1.2	6.9	9.0	8.5	3.5	1.6	.6	.3	.1	.0	.0	.0	1796
8-9	*	.3	2.1	3.6	4.6	3.2	2.7	.5	.5	.1	.1	*	*	1004
10-11	.0	.1	.7	1.1	1.5	1.5	.9	.4	.6	.1	*	*	.0	389
12-13	.0	*	.1	.2	.5	.3	.3	.2	.3	*	.1	.0	.0	106
>13	.0	.0	.0	.1	.1	.1	.2	.0	.1	*	*	.0	.0	35
INDET	1.8	.3	.9	.6	.5	.2	.1	*	*	.0	.0	.0	.0	255
TOTAL	149	586	1399	1269	1057	570	355	106	106	28	10	3	2	5640
PCT	2.6	10.1	24.5	22.3	18.9	10.3	6.5	2.0	2.0	.5	.2	.1	*	100.0

TABLE 6

CORRELATION BETWEEN INSTRUMENTALLY MEASURED AND OBSERVED WAVE HEIGHTS

(Compiled by Rocheleau, 1977)

Reference	Wave Height (meters)	Correlation Coefficient
Brooks & Jasper (1957)	(a) $H_s = 1.088H_{ob}$	
Cartwright (1964)	(b) $H_s = 1.17H_{ob}$	
(stations I & J)	(c) $H_s = 1.28 + .88H_{ob}$	
Hogben & Lumb (1967)	(d) $H_s = 1.05H_{ob}$	0.86
(stations A,I,J,K)	(e) $H_s = 1.23 + .89H_{ob}$	
Nordenstrom (fit to Hogben & Lumb data)	(f) $H_s = 1.51 + .848H_{ob}$	
Nordenstrom (fit to Cartwright data)	(g) $H_s = 1.78_{ob}^{0.72}$	

*NOTE: Hogben & Lumb use the notation H_{ows} and we use H_{ob}

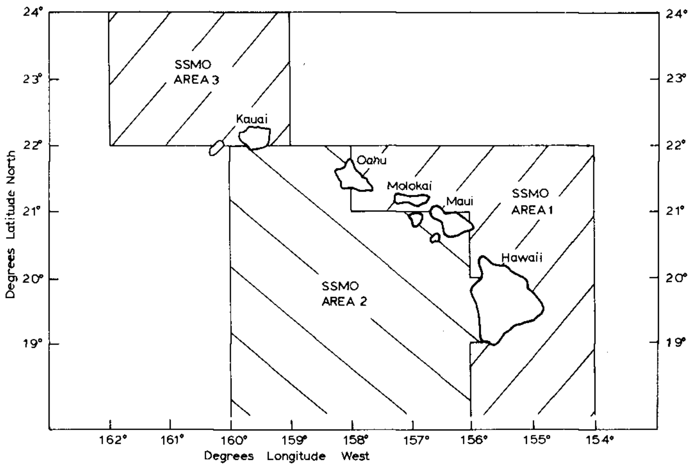

FIG. 5 BOUNDARIES OF THE SYNOPTIC METEOROLOGICAL OBSERVATION AREAS HAWAIIAN SSMO AREAS 1, 2 & 3

TABLE 7

CORRELATION BETWEEN INSTRUMENTALLY MEASURED AND OBSERVED WAVE PERIODS

(After Hogben and Lumb, 1967)*

BEST STRAIGHT LINE (In Seconds)	Root mean square deviation, sec	Correlation Coefficient
(a) $\bar{\tilde{T}} = 4.7 + 0.32\ T_{ob}$	0.88	
(b) $\bar{\tilde{T}} = 5.19 + 0.37\ T_{ob}$	1.12	
(c) $\bar{\tilde{T}} = 4.945 + 0.345\ T_{ob}$**	--	
$f_o^{-1} = 4.1 + 0.76\ T_{ob}$	2.15	
BEST LINE THRU ORIGIN		
(a) $\bar{\tilde{T}} = 0.73\ T_{ob}$	1.20	0.50
(b) $\bar{\tilde{T}} = 0.86\ T_{ob}$	1.40	0.48
(c) $\bar{\tilde{T}} = 0.795\ T_{ob}$**	--	
$f_o^{-1} = 1.12\ T_{ob}$	2.23	0.50

*Hogben and Lumb use the notation $\bar{T}$ for our $\bar{\tilde{T}}$ and T_o for our f_o^{-1}. We use T_{ob} for T_{ows}.

**Equations (c) are the corresponding averages of (a) and (b).

f_o^{-1} is the model period or period of maximum energy density of the frequency spectrum and $\tilde{\tilde{T}}$ is the zero crossing wave period. Figure 2 shows the possible approach directions for Keahole Point. The direction of wave approach is normally essential to determine the affected locations. Figure 5 shows the boundaries of SSMO Area 2, and it has been assumed that any sea state observed in the area could have affected Keahole Point. Similarly, all waves which affect the Point would propagate through the observation area. Unfortunately, there are no data which actually show the observations in SSMO Area 2 to be representative of Keahole Point. However, the SSMO data does give a means by which to make relative comparisons between the various SSMO areas.

STATISTICAL METHODS FOR EXTRAPOLATION TO FUTURE EVENTS

Except for data tabulated as a maximum series there is no widely applicable theoretical basis for determining the underlying probability distribution function or a suitable method of plotting the probabilities of exceedance calculated from the data. The criteria generally used is that data accurately fit by the chosen curve may be extrapolated. It is obviously best if the data are fit by a straight line. Several methods which have yielded such results in the past for wind and wave data and their application are discussed below.

(1) Gumbel's First Asymptotic Distribution. The first asymptote is described by the distribution function

$$F(x) = \exp\{-\exp[-\alpha(x-\mu)]\} \qquad (1)$$

where

x = the variable of interest

α,μ = parameters of the extreme distribution.

A linear reduction is made by introducing the reduced variate

$$y = \alpha(x-\mu) \qquad (2)$$

Substitution of y into equation 1 yields

$$y = -\ell n\{-\ell n F(x)\} \qquad (3)$$

Gumbel has shown that the reduced variate (y) and return period (T) are related by

$$y = -\ell n\ \ell n\left(\frac{T}{T-1}\right) \qquad (4)$$

where

T = the return period in same time interval as the maximum series. For large T, this reduces to

$$y = \ln T - 1/2\,T \tag{5}$$

which has an error of only ± .7% for $T \geq 7$.

Solving equation 5 for T

$$T = \exp(y) + 1/2 \tag{6}$$

$$\approx \exp(y) \text{ for large } T. \tag{7}$$

Substituting $y = \ln T$ into equation 2 yields $x = \ln T/\alpha + \mu$ which lends validity to the use of the semi-logarithmic plot for extreme data with an underlying exponential distribution for return periods greater than seven. Conversely, if the data were fit well by the logarithm of the return period vs x, it should be possible to use Gumbel's first asymptotic distribution for the analysis of these data.

Many techniques have been proposed for the proper selection of the parameters α, μ of the extreme distribution. These often were proposed for calculational convenience. With the proliferation of computers, one of the simplest methods is the use of least squares analysis. The line so determined represents the expected maxima at each return period. The actual values will be dispersed around these expected values.

One significant advantage of the extreme value model is the applicability of confidence bands around the line of expected values. Two sets of confidence bands may be calculated. The first is a function of the reduced variate and slope and is used to determine the goodness of fit of the assumed distribution. The second is a function of the slope only and is used to predict confidence levels of the extreme values.

(2) Weibull Distribution. Weibull in 1961 proposed a simple distribution which has been used for various civil engineering problems. The application of this distribution to the description of wind wave short-term statistics was suggested by Bretschneider (1965). Several sets of wave data, for example, Flatseth and Pederson (1970) and Battjes (1972), have been found to be described by the Weibull distribution. The Weibull distribution function is given by

$$F(X) = 1 - \exp\{-[\tfrac{x-A}{B}]^C\} \tag{8}$$

where

A = the lower limit of the variable x
B = the scale factor
C = the shape factor

When the lower limit A is zero, this is known as a Frechet distribution. Rearranging the terms in equation 8 and taking the logarithm twice yields

$$\ln \ln(1-F(X))^{-1} = C \ln x - C \ln B \tag{9}$$

The appropriate values of B and C are determined from the slope and intercept of the straight line, using the statistical least squares technique.

(3) Semi-log Plot. Another technique which has been used by many authors involves plotting log $\{1-F(X)\}^{-1}$ versus x. If a linear relationship is obtained, extrapolation beyond the observed data should yield an acceptable estimate of the expected wave height for the design return period.

(4) Log Normal Distribution. Following the work of Jasper (1956) and Darbyshire (1956), Draper (1963) concluded that the height and period of the design wave can best be estimated from a Gaussian distribution of the logarithm of the height or period. This technique may be applied to whichever wave height parameter is required (i.e. mean wave height, significant wave height, etc.) However, the data does curve off at the upper limits in some cases.

(5) Normal Distribution. One of the simplest methods of analysis first used in hydraulic studies and later extended for design hurricanes and waves is the method of Beard (1952), which assumes a Gaussian distribution. In this analysis, the cumulative probability and recurrence interval are defined by:

$$P(X \leq x_m) = F(x_m) = 1 - \frac{m+a}{M+b} \qquad (10)$$

$$T_m = [1 - F(x_m)]^{-1}/n \qquad (11)$$

where $F(x_m)$ = the assigned value of the probability distribution function of event m

m = rank of the observed value when ordered by increasing magnitude

M = total number of observed events

T_m = return period in years of event x_m

a, b = arbitrary constants to assure $0<F(x)<1$

n = number of observations/year

The method of Beard (1952) is to plot $F(x_m)$ versus x_m on normal probability paper. The points are then connected by a smooth curve which is extrapolated beyond the region of the observed data. For any return period of interest, the value of the probability distribution function is calculated and the expected value read from the plot.

Design Life and Risk Factors. In the design of an ocean structure the engineer needs to determine the risks involved in using a chosen design wave. The calculated return period is the average expected duration between events of a given magnitude, however, this value provides no indication of when the event may occur. Court (1952) and others have related design life, design return period, and risk in simple probabilistic terms to aid the design engineer. The argument presented is summarized here.

Assuming the annual maxima to be independent, the probability that x_m will not be exceeded in N years is given by

$$[P'(x \leq x_m)]^N \tag{12}$$

The probability of at least one more exceedance in N years is

$$P(X_{max} > x_m) = 1-[P'(X \leq x_m)]^N \tag{13}$$

The return period based on the work of Langbein (1949) and used by Rocheleau (1977) is given by

$$T_y = \{1 - [F(x_m)]^n\}^{-1} \tag{14}$$

Substituting T_y from equation 14 yields

$$P(X_{max} > x_m) = 1-(1-1/T_y)^N \tag{15}$$

N may be considered as the design life with a probability of failure given by $P(X_{max} > x_m)$ for the event with a return period T_y. For convenience the design life is defined in terms of return period by

$$N = T_y/U \tag{16}$$

where

U = a positive value ≥ 1.

For design calculations, the value $P(X_{max} > x_m)$ is considered as the risk (R). Substituting into equation 15 yields

$$R = 1-(1-1/T_y)^{T_y/U} \tag{17}$$

For large T_y this becomes

$$R = 1-e^{-1/U} \tag{18}$$

Solving for the factor U in equation 18,

$$U = -1/\ell n(1-R) \tag{19}$$

Substituting equation 19 into equation 16, yields

$$N = -T_y\ (\ell n(1-R)) \tag{20}$$

Thus for a chosen design life and risk factor, the necessary design return period may be calculated.

SUMMARY OF ANALYSIS

1. Keahole Point, South Shore and SSMO Area 2: Figure 5 will be discussed first, inasmuch as we have available two sets of wave hindcast data for locations around Keahole Point, and also the SSMO data for Area 2. Figures 6 and 7, based on Gumbel's distribution, show the results using the Marine Advisers (1963) wave hindcasts and the U.S. Corps of Engineers (1964) wave hindcast data, respectively. Figure 8 is based on the SSMO data for Area 2, where H, shipboard wave observations, have not been corrected to significant wave height.

Figure 9 shows the comparisons of expected wave heights from various methods of statistical extrapolations for the above three sources of data used in Figures 6, 7 and 8. It is seen that the various methods of statistical analysis give quite a wide variation in results based on the three sources (Figures 9a, 9b, and 9c). Figure 9d shows the variations of the three sets of data for the Gumbel distribution of wave heights.

Table 9 gives the results of statistical evaluations of wave height data (based on Figure 9) for Keahole Point, Hawaii.

2. North Shore of Oahu and SSMO Areas 1 and 3: Wave hindcast data of severe storms (similar to Tables 1 and 2) are not available for the north shores of Hawaii and hence we consider only shipboard wave observations for SSMO Areas 1 and 3. Figures 10 and 11 are the corresponding results based on the Gumbel distribution. We are also interested in the statistics for extreme waves for the north shore of the island of Oahu. SSMO Area 1 is mostly for the island of Hawaii and SSMO Area 3 is mostly for the island of Kauai. Therefore, we have averaged to two sets of data and assumed that this would be generally applicable halfway between Areas 1 and 3, or the north shore of the island of Oahu. Figure 12 shows the results of this averaging.

Table 8 gives a summary of the statistical analysis of the three SSMO areas. Based on shipboard wave observations, it is surprisingly interesting that the statistical analysis results give essentially the same extreme wave heights for all three SSMO areas for corresponding recurrence intervals. The explanation for this can be made, in view of the large SSMO areas surrounding Hawaii (see Figure 5). It would appear that the islands have little effect on the waves over large sections of the SSMO areas where the ship observations are made, because there can be large fetch lengths in any direction away from the islands. Evidently, the southern swells off the south shore of Hawaii either pass around or through the islands from SSMO Area 2 to SSMO Areas 1 and 3. This is probably not completely true, as it is also likely that the southern swell is not important in determining the extremes, and in some cases may even be absent from SSMO reports, particularly when tradewind waves are predominant.

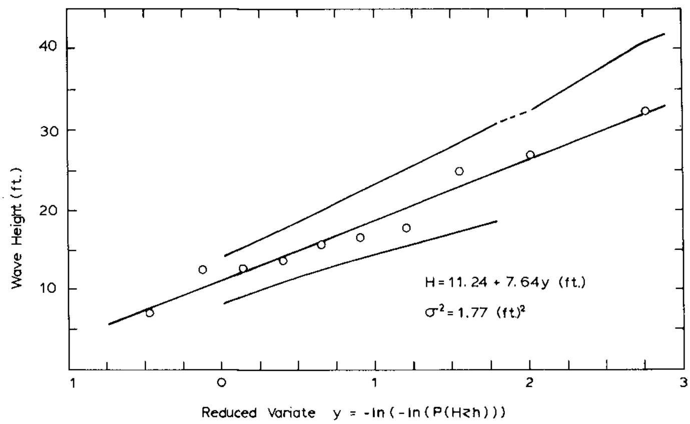

FIG. 6 LINE OF BEST FIT USING GUMBEL'S DISTRIBUTION FOR SIGNIFICANT WAVE HEIGHTS FROM 10 STORMS AFFECTING THE HAWAIIAN ISLANDS FROM 1947 THROUGH 1961 HINDCASTS BY MARINE ADVISORS (1964)

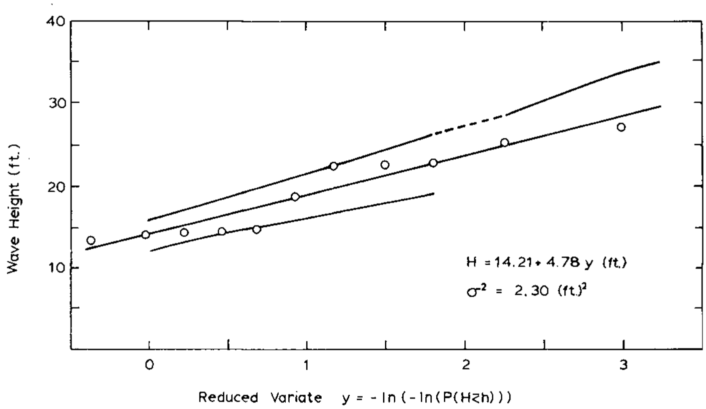

FIG. 7 LINE OF BEST FIT USING GUMBEL'S DISTRIBUTION FOR SIGNIFICANT WAVE HEIGHTS FROM 11 STORMS AFFECTING KEAHOLE POINT FROM 1947 THROUGH 1965 HINDCASTS BY U.S. CORPS ENGINEERS (1968)

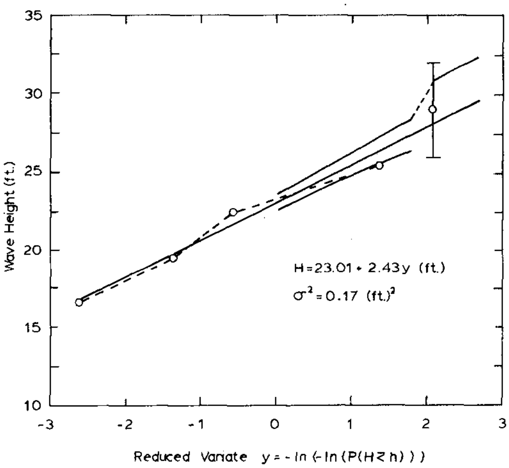

FIG. B LINE OF BEST FIT USING GUMBEL'S DISTRIBUTION FOR EIGHT YEARS OF SHIPBOARD OBSERVATIONS IN SSMO AREA 2

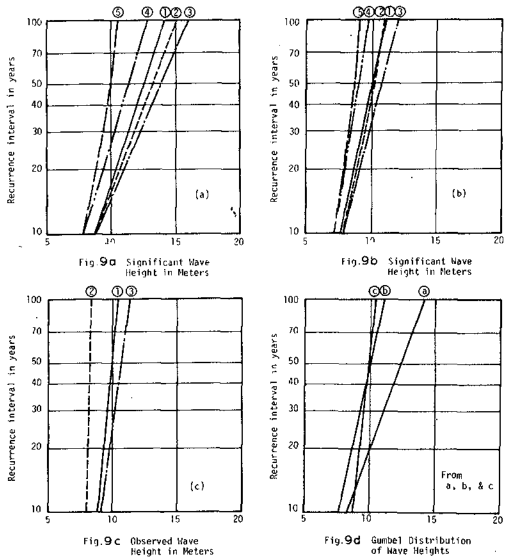

Fig. 9 Comparisons of Expected Wave Heights from Various Methods of Statistical Extrapolations for Three Sources of Data

1 Gumbel's First Distribution
2 Weibull Distribution
3 Semilog Distribution
4 Log Normal Distribution
5 Normal Distribution

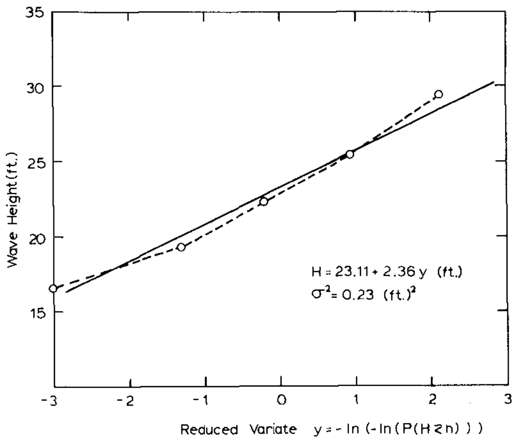

FIG.10 GUMBEL'S FIRST ASYMPTOTIC DISTRIBUTION
SSMO : AREA # 1

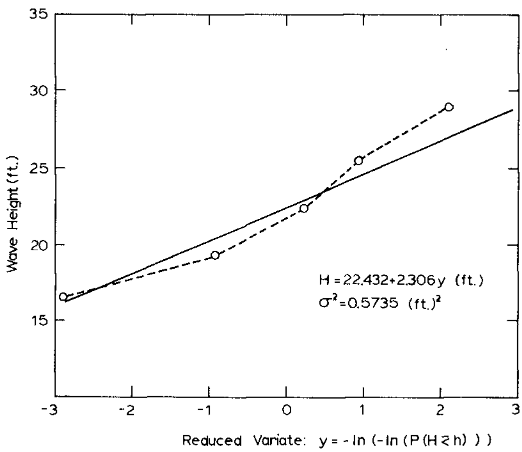

FIG.11 GUMBEL'S FIRST ASYMPTOTIC DISTRIBUTION
SSMO : AREA #3

TABLE 8

RESULTS OF STATISTICAL EVALUATIONS OF SSMO WAVE HEIGHT OATA
FOR SOUTH SHORE ANO NORTH HAWAII BASEO ON GUMBEL'S OISTRIBUTION

Wave Heights in Meters

SSMO Area No.	Recurrence Interval in Years				
	10	25	50	75	100
1	8.7	9.4	9.9	10.2	10.4
2	8.7	9.4	9.9	10.2	10.4
3	8.4	9.1	9.6	9.9	10.1
average 1 and 3	8.8	9.4	9.9	10.1	10.3

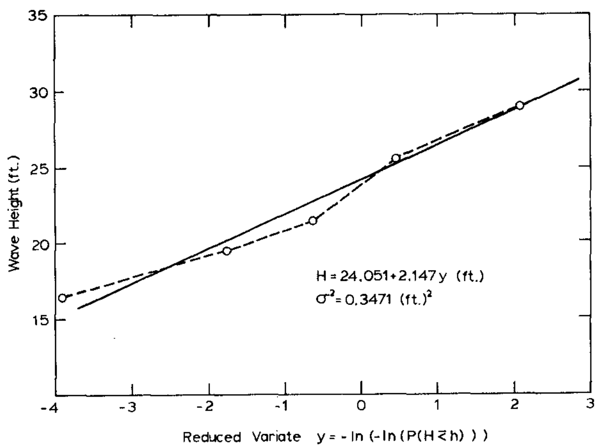

FIG.12 GUMBEL'S FIRST ASYMPTOTIC DISTRIBUTION
SSMO : AREAS #1 & #2

TABLE 9

RESULTS OF STATISTICAL EVALUATIONS OF WAVE HEIGHT DATA FOR KEAHOLE POINT, HAWAII

(a) Based on Significant Wave Hindcasts of 10 Storms (1947-1961) by Marine Advisors (1963)

STATISTICAL METHOD	RECURRENCE INTERVAL IN YEARS				
	10	25	50	75	100
1	8.7	10.9	12.5	13.5	14.1
2	8.8	11.5	13.4	14.4	15.0
3	8.8	11.6	13.8	15.1	16.0
4	7.8	9.8	11.2	12.0	12.7
5	7.8	9.0	9.8	10.2	10.5

(b) Based on Significant Wave Hindcasts of 11 Storms (1947-1965) by U.S. Army Corps of Engineers (1968)

STATISTICAL METHOD	RECURRENCE INTERVAL IN YEARS				
	10	25	50	75	100
1	7.6	8.9	10.0	10.6	11.0
2	7.8	9.1	10.1	10.4	10.7
3	7.8	9.3	10.7	11.5	12.0
4	7.2	8.3	9.0	9.5	9.8
5	7.2	8.1	8.6	8.8	9.0

(c) Based on Shipboard wave observations from U.S. Navy Oceanographic Office SSMO Report for Area 2 (Southern routes of Hawaii, 1963-1971)

STATISTICAL METHOD	RECURRENCE INTERVAL IN YEARS				
	10	25	50	75	100
1	8.7	9.4	9.9	10.2	10.4
2	7.8	8.0	8.1	8.2	8.3
3	9.1	9.9	10.6	11.0	11.3

NOTES: Wave Heights are given in meters.

IN TABLES	NOTATIONS IN FIGURES	TYPE OF DISTRIBUTION
1	———————	Gumbel's First
2	- - - - - - - - -	Weibull
3	— - — - — -	Semilog
4	— -- — -- —	Log Normal
5	—— —— —— ——	Normal

ACKNOWLEDGEMENTS

The material given here represents partial results of research supported by the Sea Grant Office of the National Oceanographic Administration (NOAA) under Sea Grant Contract Nos. 04-6-158-44026 and 04-6-158-44114. Much of the material presented here was taken from the M.S. Plan B paper of Richard Rocheleau (1977) for Keahole Point and SSMO Area 2. Mr. Michael Miller did the analysis of the SSMO data for Areas 1 and 3 and the north shore of Oahu.

REFERENCES

Battjes, J.A. (1972) "Long Term Wave Height Distributions at Seven Stations around the British Isles," Deutsche Hydrographische Zeitschrift.

Beard, R.L. (1952) "Statistical Methods in Hydrology," U.S. Army Office of Chief of Engineers.

Bretschneider, C.L. and E.E. Tamaye (1976) "Hurricane Wind and Wave Forecasting Techniques," Proc. 1976 Coastal Engineering Conference, Honolulu, Ch. 13, pp. 202-237, ASCE, New York.

Court, Arnold (1952) "Some New Statistical Techniques in Geophysics," Advances in Geophysics, Vol. 1.

Darbyshire, J. (1961) "Prediction of Wave Characteristics over the North Atlantic," Journal of the Institute of Navigation, Vol. 14.

Draper, L. (1963) "Derivation of a 'Design Wave' from Instrumental Records of Sea Waves," Proc. of the Institute of Civil Engineers, London, England.

Flatseth, J. and B. Pederson (1970) "Distribution of Wave Height in Different Ocean Areas of the World," Report No. 70-7-S for Research Dept., Det Norske Veritas, Oslo, Norway.

Gumbel, E.J. (1958) Statistics of Extremes, Columbia University Press, New York.

Hogben, N. and F.E. Lumb (1966) Ocean Wave Statistics, Ministry of Technology, National Physical Laboratory, London.

Jasper, N.H. (1956) "Statistical Distribution Patterns of Ocean Waves and Wave Induced Ship Stresses and Motions with Engineering Applications," Transactions, Society of Naval Architects and Marine Engineers, November.

Langbein, W.B. (1949) "Annual Floods and the Partial Duration Series," Transactions, American Geophysical Union, Vol. 30.

Marine Advisers (1964) "Characteristics of Deep-Water Waves in the Oahu Area for a Typical Year," La Jolla, California.

Marine Advisers (1963) "Severe Storm Wave Characteristics in the Hawaiian Islands," La Jolla, California.

Moberly, R. and T. Chamberlain (1964) "Hawaii Beach Systems," Hawaii Institute of Geophysics, Report HIG 64-2.

Munchmeyer, F. and C.L. Bretschneider (1978) "Some Design Criteria for OTEC Installations for Keahole Point, Hawaii," Proc. 16th International Coastal Engineering Conference, Hamburg, Germany.

Rocheleau, R.E. (1977) "Evaluation of Extreme Wind and Wave Climate in Hawaii with Special Application to Keahole Point and the North Shore of Hawaii," M.S. Plan B Paper, Look Lab of Oceanographic Engineering, Department of Ocean Engineering, Technical Report No. 77-41, U. of Hawaii.

U.S. Army Engineer District (1968) "General Design Memorandum-Honokahau Harbor for Light Draft Vessels," Honolulu, Hawaii, January 1968.

U.S. Naval Oceanographic Office, Summary of Synoptic Meteorological Observations, U.S. National Technical Information Service, Springfield, Virginia.

Weibull, W. (1961) "A Statistical Distribution Function of Wide Applicability," Journal of Applied Mechanics.