CHAPTER 199

Analysis of Time Conditions for Hybrid Tidal Models

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Summary

Hybrid models for the investigation of tidal waves in rivers and estuaries are a combination of mathematical and hydraulic models, which are coupled under real-time conditions. The coupling procedure cannot be performed without some time delay which mainly depends on the time needed for the computation of the mathematical model and for the control operations on the instrumentation. An analysis of their influence on the accuracy of a hybrid model is given and experimental results from a feasibility study are presented.

Introduction

During recent years great advances have been made in the field of numerical models. Simple models which start from the assumption of vertically averaged velocities have quite often been applied for the simulation of far-field processes in tidal waters. They are of good economics and high physical reliability and have thus become standard tools for coastal engineers. However, when detailed three-dimensional investigations have to be performed, often mathematical models are less reliable than physical models are. This is due to the fact that only few numerical models of this type [1,2,3] are in use and still only few experience on the choice of physical coefficients is available. Furthermore the computational expense grows considerably. So it is reasonable to stick to physical models for three-dimensional investigations, which should be performed at the largest scale possible. However, a small section of an estuary can be modeled then only. For this the boundary conditions have to be known which often makes a seperate investigation necessary. This can be per-

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formed by setting up a physical model of the whole estuary at a much smaller scale. This technique would be rather expensive. A second approach can be made by using a far-field numerical model. However, now the near-field section has to be included into the computation which starts from the assumption of vertically averaged velocities. A considerable error may result from this simplification, which should not be allowed for. It can be avoided by a new investigation technique, in which the near-field section of the model is realized hydraulically and the outer area of the model, for which the far-field assumptions are valid, is set up numerically. Both models are coupled under real-time conditions and thus integrated into one new formulation, which we call a hybrid model. The three-dimensional physical model interacts dynamically with the two-dimensional numerically simulated part and vice versa. So no special boundary conditions have to be controlled on the physical model part. The only prescribed conditions are those for the outer mathematically simulated part.

The coupling technique between the hydraulic and numerical model follows the strategy of correcting the water-levels and the discharges on the boundary between both models within small fixed time-intervals, thus guaranteeing continuous condition between both models. The practical applicability of hybrid models depends on the computing time needed for the numerical model and on the time needed for performing the control operations on the instrumentation. These time-elements have to be analysed carefully. This will be done by some theoretical considerations and experimentally in a feasibility study which was run for an one-dimensional open channel system.

Hybrid Model

The hybrid model technique will be tested in principle for a rectangular straight channel of about 25 m length. The tank is closed at one end and a sinus-shaped variation of the water-level is controled at the other end (Fig.1).

The channel may be regarded as a hydraulic model of a system. For our investigations the model has to be reformulated in a hybrid way. It is cut into two parts (Fig.1) now, one of which still is a hydraulic model, whereas the other part is simulated numerically on a computer. As with respect to geometry and boundary conditions no changes have been made, the time history of the water-levels must remain the same for any station in the hydraulic model as well as for the hybrid formulation. A comparison between measured values in both model tests gives some idea on the accuracy and reliability of the hybrid approach.

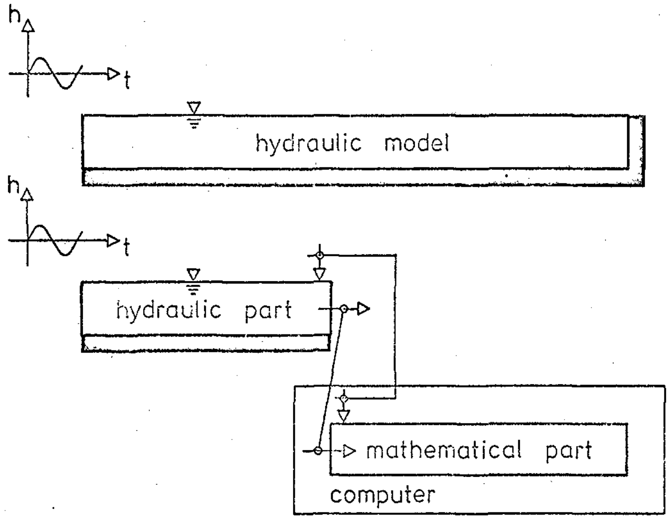


Fig. 1 Feasibility Study

The coupling procedure between the numerical and the physical model part is run on the same computer as the numerical model is (Fig.1). Within fixed time-intervals which we call the coupling intervals Δt, the water-level is measured at the coupling point in the hydraulic model and given as boundary condition to the mathematical model. Now the computation is performed for one time-step, which is equal to the coupling interval. The numerical model is thus updated from the time level of the last measurement to that for which the boundary condition is valid. The computed discharge is then given for control to a pump-system on the hydraulic part (Fig.1). If this procedure works within very small intervals, continuous water-levels and discharge on either side of the coupling point can be expected. Practically, however, the coupling interval cannot be made arbitrary small as within each interval some time must be spent on the computation of the numerical model and some further time for setting the pump-system to the computed discharge values. These time-elements determine not only the frequency of the coupling procedure but they also represent a time-delay between measuring and having the pump set to the computed new position. So they strongly influence the behaviour of the hybrid model which is seen from Fig. 2, in which a comparison is made between water-levels h and discharges q measured in the complete hydraulic model and the corresponding values h^* and q^* in a hybrid for-

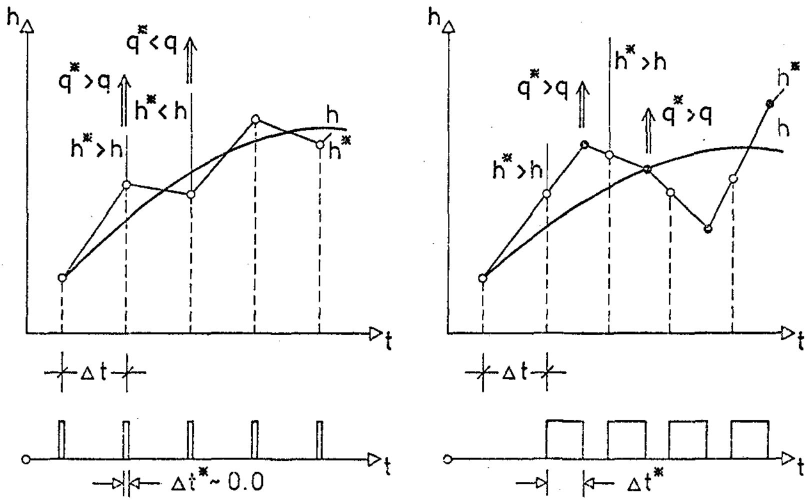


Fig. 2 Coupling Procedure

mulation.

The left side of Fig.2 shows a situation when the delay is assumed to be nearly zero. The coupling procedure starts by measuring a water-level h^* and gives this as boundary condition to the numerical model. This now computes a discharge q^* which will be bigger than the corresponding q in the hydraulic model if h^* was measured higher than h. When now q^* is controled on the pump it causes the water-level to sink for the time Δt of the coupling interval and so the next measured value h^* will be lower than h. A lower water-level h^* now leads to a computed discharge q^* which is smaller than q and thus the water-level starts rising again. This oszillation around the correct value can no longer be guaranteed for when the time-delay is taken into consideration. This is shown on the right side of Fig.2. The water-level keeps rising during all the time needed for the computation of the value q^* for the discharge which, when available, does no longer corresponds to the actual water-level. The computed discharge is too small now and the next water-level will remain too high. But then a rather strong variation of the discharge may follow which will surely induce a considerable error and may even make the system to run out of control. An analytical analysis of such conditions can hardly be given due to the nonlinearity of the describing differential equations. So experiments have to be run in order to find out, up to which delay the hybrid model will give satisfactory answers.

Time Elements

Time delays inherent in the coupling procedure are basically caused by three elements (Fig.3).

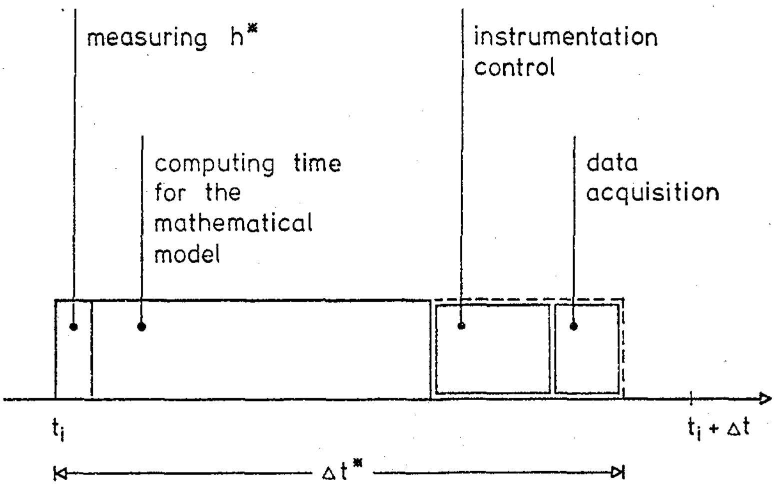


Fig. 3 Time Elements

Within one coupling interval the water-level has to be measured which takes a very short time only. An interrupt is generated by the computer, the sampling performed via the multiplexer and the analog signal from the instrumentation converted into a digital form. A brief subroutine subsequently transforms the measured value into the wanted water-level quantity taking the calibration curve of the instrumentation into account. Performing all these operations takes mostly less than 0.01 sec. So this time element is very small and can be neglected for further considerations. The second step within the coupling routine is the computation of the mathematical model. This may take a considerable amount of time depending on the size of the system and the chosen numerical algorithm for solving the differential equations. These are given by the equation of motion

$$v_{,t} + v\,v_{,x} + g\,h_{,x} + R\,v = J \tag{1}$$

and the equation of mass conservation

$$h_{,t} + h\,v_{,x} + v\,h_{,x} = 0\;. \tag{2}$$

v is the velocity, h the water-level, J the bottom slope and R stands for a parametric term including turbulence and friction. Numerical solutions mostly start from the linearized version of the equations

$$v_{,t} + \bar{v}\,v_{,x} + g\,h_{,x} + R\,v = J \tag{3}$$

$$h_{,t} + \bar{h}\,v_{,x} + \bar{v}\,h_{,x} = 0 \tag{4}$$

The bar-marked quantities are assumed to be known and are chosen identical to the initial conditions for the actual computational time-step.

The numerical solution schemes can be formulated in quite different ways using either a finite difference or a finite element approach. In any case a set of explicit equations is obtained if the problem is formulated explicitly or an equation system will result if an implicit strategy is chosen. With respect to the real-time application in a hybrid model, the computational expense of both formulations has to be analyzed. The solution of the equation system can be performed rather economically by the douple-sweep algorithm [4,5], but explicit schemes are still more economical if a comparison is made on a per step basis. However, these schemes are not unconditionally stable and so often several computational steps are necessary to arrive at a time-level the solution for which is obtained by implicit schemes within only one step. So implicit schemes may become again faster on the coupling interval basis. Furthermore the time-step may be varied arbitrarily without altering space discretizations due to the stability properties of such schemes. These considerations lead to the use of a douple-sweep implicit model for this study.

The third time-element is represented by the control operations on the pump when this is set to the computed new discharge value. This can be performed either under direct computer control or by using additional technical equipment. In any case the computed discharge must first of all be converted into an electrical or mechanical quantity. This is achieved by a simple subroutine refering to the calibration curve of the pump. Then this digital value is given via an output relay to the instrumentation. If this is set to the new position in only one step this would surely be the most time saving way in a real-time application, however, a sudden change in discharge will cause big disturbances near the coupling point and thus falsify the next measurement of the water-level h*. So it is preferable to vary the flow conditions slowly bringing the instrumentation up to the new position by an independent unit being supplied with a clock and a counter or in a more flexible and cheaper way by the computer itself using the interrupt facilities. The gradient, by which the new position is approached, can then be determined by a software solution but then the computer is kept busy and cannot yet start the next coupling interval. So this time must be considered to be the third time-element. However, as the computer calculations are performed much faster than output signals can be given to the instrumentation, this time-element may include additional functions. During intermediate intervals data acquisition routines for the hydraulic model can be run and thus a task be taken into account, for which in many laboratories computers originally have been made available. From this then follows that for hybrid models mostly no additional installations are needed.

Besides this way of performing the control operations, which can be programmed very easily, there is still a more complex alternative. In this again the interrupt facilities of the computer are used, now interrupting the computing process of the numerical model for control and data acquisition. This leads to a more complicated software solution, but allows for neglecting the third time-element in all considerations. So the only element, causing any delay, is the computing time of the numerical model.

Experimental Tests

The test series on the time conditions are run for the system shown by Fig.4.

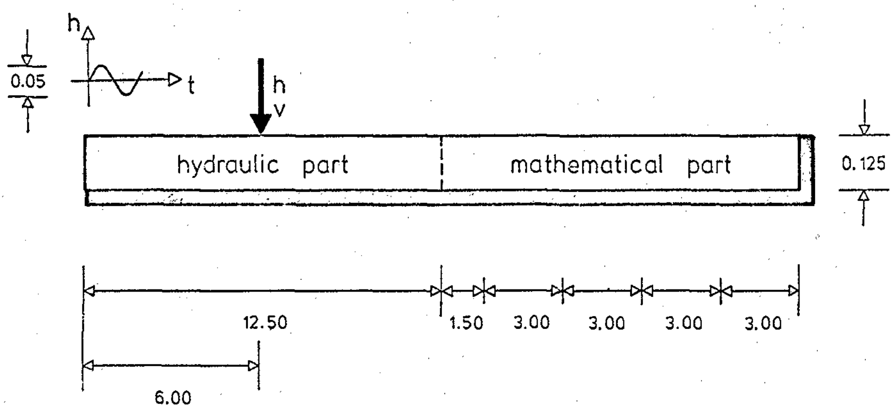


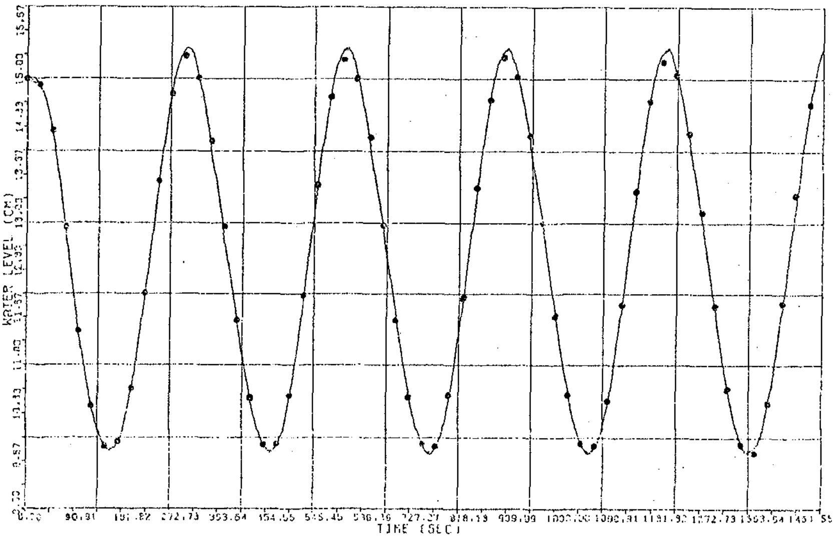
Fig. 4 Hybrid Model

At the left hand side a sinus-shaped variation of the water-level with an amplitude of 2.5 cm and a period of 300 sec is generated. The mean water depth is 12.5 cm. These values correspond to conditions which are often found in estuary studies of shallow water areas. The numerical part of the model is discretized by 5 elements thus giving a computation time of about 0.1 sec per time-step. For comparisons between the original hydraulic and the later hybrid model the time-history of water-levels and velocities is gauged at always the same station (Fig.4) in the hydraulic model part. Any discrepancy in the results can than be interpreted as an error induced by the hybrid realization and by increased or decreased time-elements.

The first test concerns a minimum of delay. The computed new discharge q* is then available after 0.1 sec already, but the control operations on the instrumentation need much more time. So a coupling interval of $\Delta t = 1$ sec is chosen within which the numerical model takes 0.1 sec, the control operations take 0.7 sec and 0.2 sec are left for data acquisition. The results from this test for the hydraulic and hybrid formulation are given by Fig.5. In the upper part a comparison for the water-levels over 5 tidal cycles is made. It shows very good agreement. The same can be stated for the velocities as shown in the lower part of Fig.5. The rather nervous behaviour of these curves must be explained from the instrumentation. A thermo-element based instrument had been used, giving the resultant velocity component at one fixed point only. As no artificial damping on the analog output was made and no averaging performed, turbulent variations are still inherent in the data. The decreasing magnitude of the velocities for the first three periods has to be explained by the fact that the system starts from rest. Quasi stationary conditions are not obtained up to the forth period.

The object of the experiments is to find out up to which size the computing time of the numerical model can be extended. So a further test is run with a delay of 2.1 sec. As the coupling interval is made $\Delta t = 2.0$ sec, all control operations have to interrupt the computation of the numerical model. Due to programming simplification no interrupt is allowed within the first 0.1 sec of a coupling interval, in which all measuring procedures are run. So finally a delay of 2.1 sec results. Figure 6 shows the comparison for the water-levels. In the lower part of the figure a detailed analysis for two periods is made. Refering to the water-levels measured in the original hydraulic model the error for the hybrid formulation can be calculated on a per centage basis. The errors found do not exceed the range of 1%. So the difference between measured water-levels in the hydraulic and hybrid model is of about 1.5 mm at maximum, or when refering to the tidal range of 50 mm, of about 3%. An error of this magnitude, however, seems still to be acceptable as this is within the same range of accuracy in which in many cases also field-data are.

A further extension of the time-delay up to 3.1 sec leads to only slightly increased errors. More interesting to note, however, is the fact, that small local disturbances on the



——— Hydraulic Test • Hybrid Test

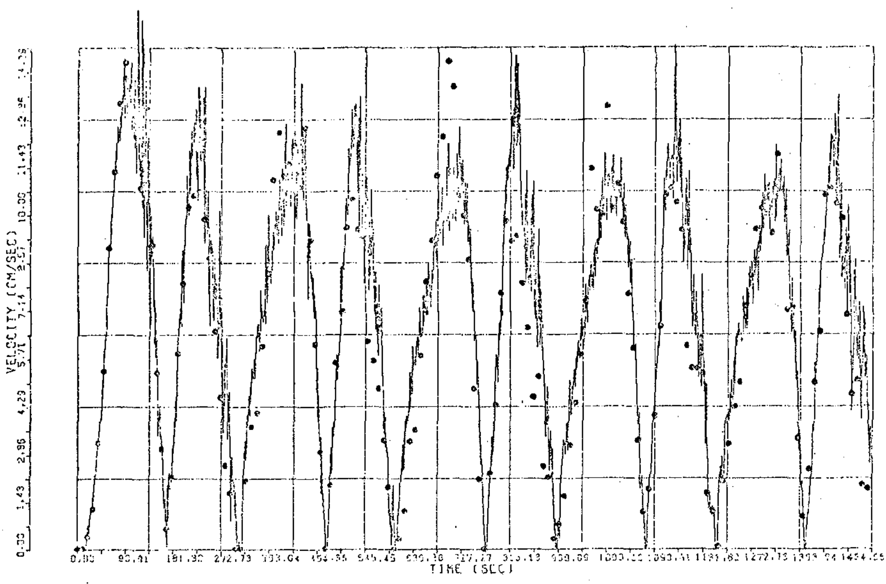


Fig. 5 Comparison of Hybrid and Hydraulic Test, Delay 0.1 sec

measured water-levels at the coupling point, as for instance caused by air bubbles in the water, induce very heavy oszilations of the water-level. Though these mostly disappear after slack water, in general test run at this and further extended time-delays gave no satisfactory agreement with hydraulic model tests.

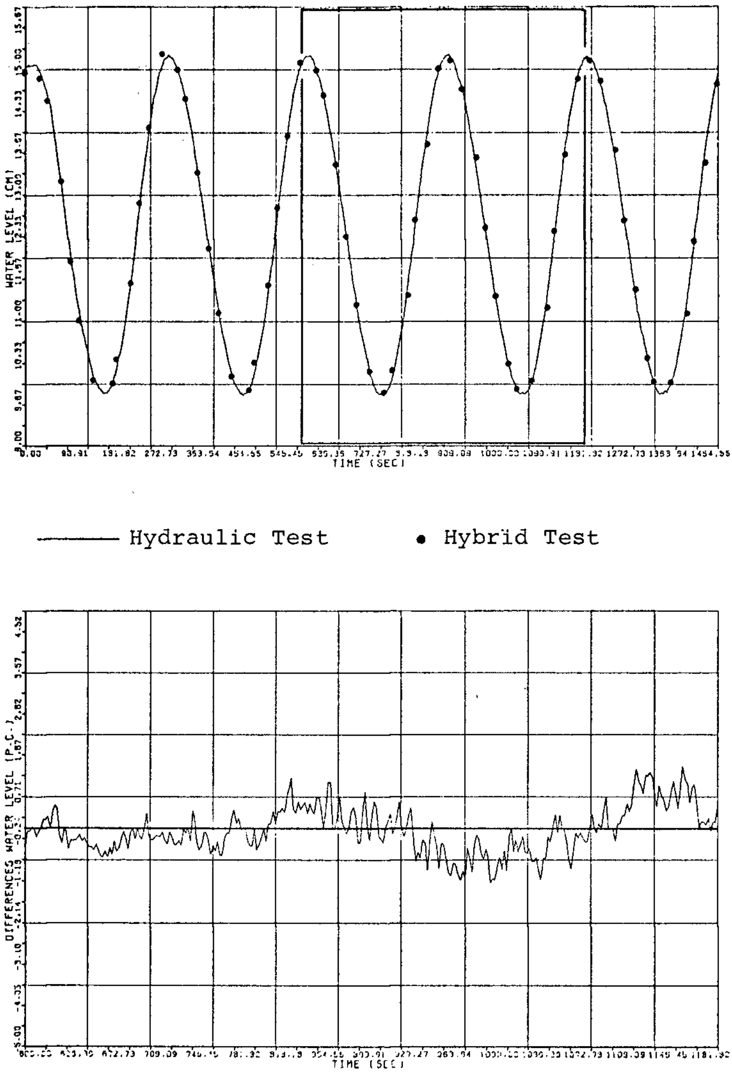


Fig. 6 Comparison of Water-Level, Delay 2.1 sec

Conclusions

A feasibility study was run on the realization of a hybrid model technique for open channel flow situations. An analysis on the time-delay, caused by the computation of the numerical model part, showed that for the system under investigation a delay of about 2 sec lead to an error of about 1.0% for the water-levels. This magnitude seems to be acceptable in practical applications, as it is of the same order, of which in many cases field data are. On the other hand a time of 2 sec allows for the computation of about 100 elements in the used numerical model. In many investigations this number is fully sufficient for a good representation of the far-field area of a system. So these results lead to the conclusion, that the hybrid model technique can readily be applied in practical investigations.

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