CHAPTER 190

ENERGY DISSIPATION IN TIDAL ESTUARIES

by

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and

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INTRODUCTION

In this study, the method for damped co-oscillating tides is used to evaluate damping and energy dissipation characteristics for various estuaries of different geometry and depth.

Of special interest are the damping and energy dissipation characteristics of the German tidal rivers such as Elbe, Weser and Ems in comparison with North-American tidal estuaries, since the former are characterized by deep navigation channels and relatively flat wadden areas at their lateral boundaries.

Harleman and Ippen have applied a mathematical model for co-oscillating tides to the Bay of Fundy and the Delaware estuary. This model gives information about the damping behaviour and energy distribution in the tidal estuaries.

Both of the above mentioned estuaries represent special cases since the geometric form of the Bay of Fundy allows it to be approximately represented as a rectangular chanal of constant width and depth, whereas for the Delaware Estuary one can assume a constant depth of water. Partenscky has applied an extended form of this model to the St. Lawrence Estuary.

The method of co-oscillating tides has now also been used in a mathematical model for the German tidal rivers such as Elbe, Weser and Ems. This method takes into account the influence of geometry and depth on the tidal motion and also the damping of the tidal wave due to friction and partial reflection. Data from gauges situated along the estuaries are needed as initial input for the calculation of the damping coefficient and the phase change. The wave amplitude and the time of highwater must be known from these stations.

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DESCRIPTION OF THE GERMAN TIDAL RIVERS ELBE, WESER AND EMS

Fig. 1 shows a plan view of the German tidal rivers Elbe and Weser. Both rivers flow into the German Bay. The third mentioned river, the Ems, is a much smaller tidal river and not a so important navigation channel as Weser or Elbe. It flows more westwards near the boundary to the Netherlands into the Dollart. All three German tidal rivers have relatively flat wadden areas at their lateral boundaries.

The Elbe River is the largest German tidal river, it stretches from the mouth at Cuxhaven to the weir at Geesthacht. The width at the mouth is about 3300 m and the mean water depth is nearly 11 m.

The tidal estuary of the Weser River extends from Bremerhaven to the weir at Hemelingen. That is a distance of about 70 km. The harbour of Bremen is located nearly 5 km seaward from the weir of Hemelingen. 65 km downstream of Bremen at the bend near Bremerhaven the tidal river has a width of 1100 m and a mean water depth of 8 m.

In all three German tidal rivers the semidiurnal component is dominant, hence the tidal period is constant to 12.42 hours.

The cross-sections of the German tidal rivers are bounded upstream by weirs, so that one can assume total reflection of the tidal wave at these points.

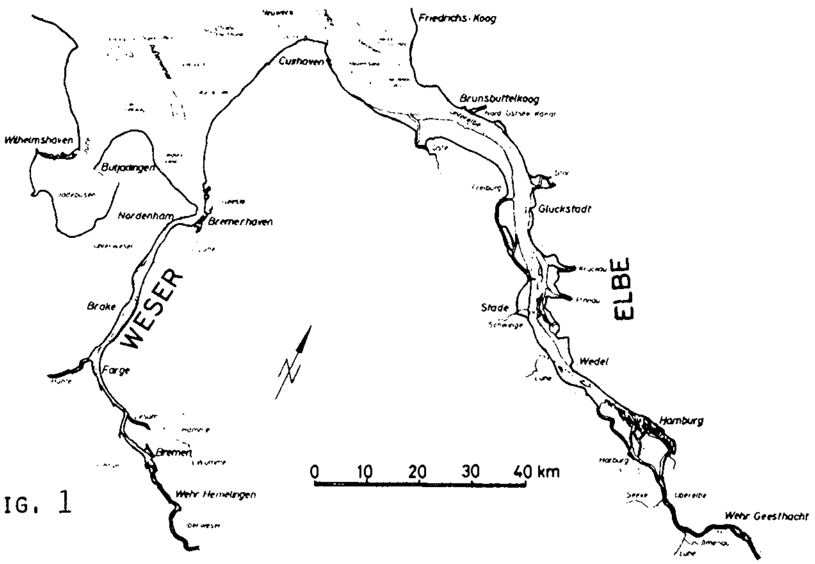

FIG. 1

ENERGY DISSIPATION AND DAMPING PARAMETERS.

Extensive prototype measurements at the Weser river are carried out by the Franzius-Institut to determine the influence of heat rejection of nuclear power plants on water quality parameters such as temperature and oxygen level. In this case, measurements in different cross sections of the river upstream and downstream of the cooling water intake over a stretch of about 4o km are carried out.

In order to examine the mixing process of seawater and freshwater in this area, it is useful to determine the longitudinal salinity distribution by conductivity measurements or chemical analysis. For given boundary conditions such as tidal motion and geometry of the estuary the longitudinal salinity distribution is a function of the freshwater discharge.

The stratification parameter $\beta = G/J$ is a useful parameter for this calculation, where G is the energy transported by the tidal wave with respect to the water mass of the examined area. J is the potential energy that the freshwater gains by increasing salinity.

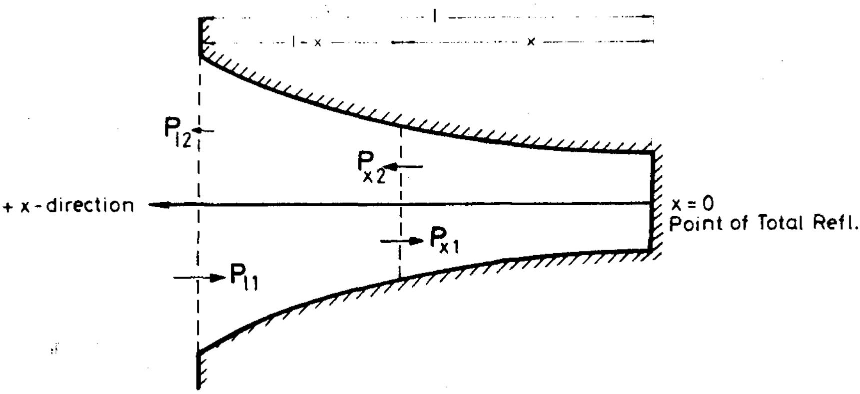

Fig. 2 Energy flux

The energy flux of a tidal wave is given by $P = E_{tot} \cdot c$ with

$$c = L/T = \frac{\sigma}{k}$$

To determine the resulting energy flux for a cross section at a point x, one must split the observed tidal wave into an incoming and a reflected component (Fig. 2).

For a cross section at $x > o$ the resulting energy flux is

$$P_{Rx} = P_{x1} - P_{x2}$$

At the point of total reflection $P_{x1} = P_{x2}$ and there is of course no longitudinal energy transport.

The loss in potential and kinetic energy of the tidal wave between the river mouth and a point between the river mouth and the weir is $\Delta P_x = P_{RL} - P_{Rx}$.

Let ΔP_x be the wave energy transformed into heat by friction and turbulence, then we get the relative loss of energy G_x as the transformed energy related to the water mass in the segment 1 - x.

For the cross section with total reflection we get the energy dissipation $G_o = P_{RL}/M$.

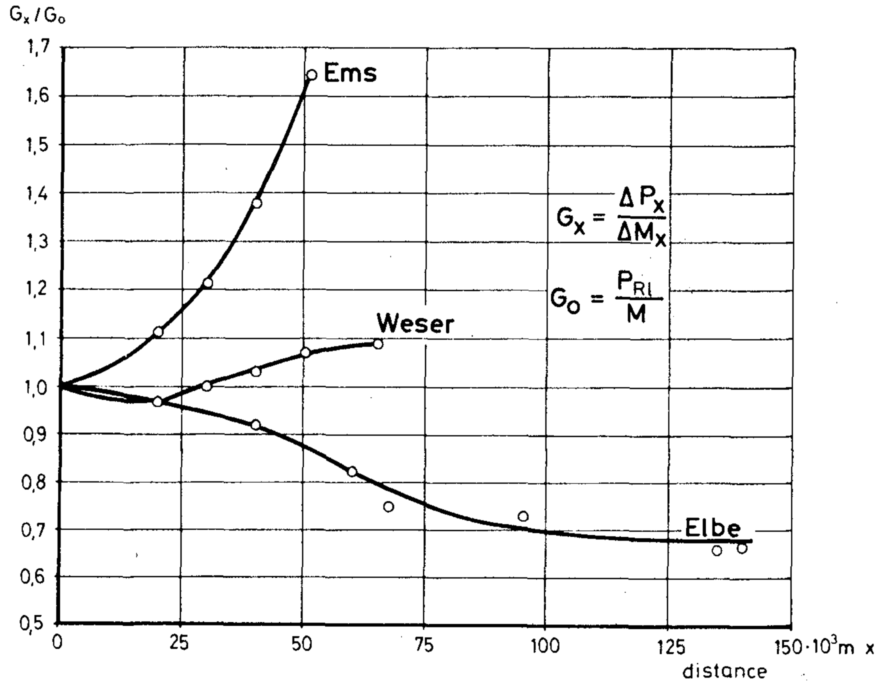

Fig. 3 Energy dissipation

An analysis of the energy dissipation with respect to G_o for all three examined German tidal rivers shows pronounced differences in the plotted curves. At the smallest tidal river, the Ems, the energy ratio G_x/G_o increases towards the mouth to 1,65. For the Weser, which is completely transformed into a navigation channel from the mouth up to the weir, the value is about 1,o. Contrarly to the Ems, the Elbe River shows a decrease in the ratio towards the river mouth to o,65.

To demonstrate these relationships, the resulting energy flux and the corresponding water mass in which the energy flux takes place are shown for the three tidal rivers as functions of distance in the diagramm of Fig. 4.
At x = 1 ,i.e. at the seaward end of the examined tidal river the values of the energy flux and the water mass are assumed to be 1oo %.

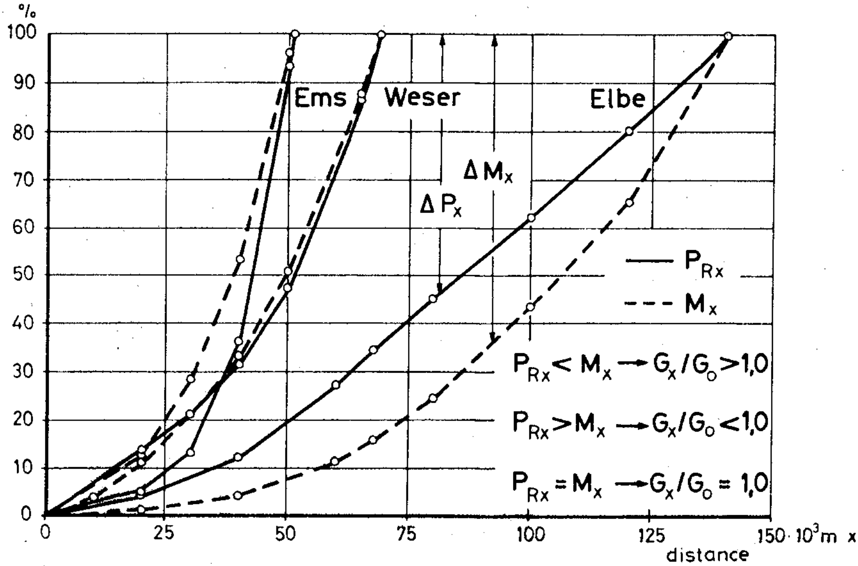

Fig. 4 Energy flux and water mass

With these assumptions, it can be clearly seen from the plots, that

for $P_{Rx} < M_x$ it follows that $G_x/G_o > 1$

$P_{Rx} > M_x$ $G_x/G_o < 1$

$P_{Rx} = M_x$ $G_x/G_o = 1$

For the Weser River, the curve of water mass is almost identical to the curve for the energy flux. Accordingly, the energy ratio G_x/G_o is about 1,o.

The variation is roughly only 1o % in the lower Weser and the energy dissipation can be assumed to be constant in the brackwater zone of the Weser.

The value G_x was used for tests to describe the longitudinal salinity distribution in the brackish area of the lower Weser.

The damping of the tidal wave is characterized by the damping coefficient μ which is important for the energy dissipation.

Referring to HARLEMAN and IPPEN, the equation for the damping coefficient was found by solving the equation for the local time of high water and the equation for the local amplitude of high water.

With application of Green's Law the local amplitude of high water is given by:

$$\eta_{xH} = a_o \left(\frac{b_o}{b_x}\right)^{1/2} \left(\frac{h_o}{h_x}\right)^{1/4} \left[e^{\mu x}\cos(\sigma t_H + kx) + e^{-\mu x}\cos(\sigma t_H - kx) \right]$$

and the corresponding time of high water is then:

$$\sigma t_H = \tan^{-1} (-\tan kx \cdot \tan \mu x)$$

For the damping coefficient follows then:

$$\cosh^2 \mu x = \frac{1}{2} \left[(N+1) + \sqrt{(N+1)^2 - 4\, N \cos^2 \sigma t_H} \right]$$

$$\text{where } N = \frac{\eta^2_{xH}}{\eta^2_{oH}} \cdot \left(\frac{b_o}{b_x}\right)^{-1} \cdot \left(\frac{h_o}{h_x}\right)^{-1/2}$$

The reflected component of the resulting wave can be estimated from the value of μx. The component of the reflected wave is negligible as μx becomes very large. One arrives at an approximation of a progressive tidal wave without reflection.

In the case of no friction and total reflection at the closed end of the estuary, we get $\mu x = 0$ and the phase lag of the entrance of highwater $\sigma t_H = 0$.

In the study of real estuaries one finds values between these limits (Fig. 5).

Since the validity of Green's law is not satisfied in the German tidal rivers, the damping coefficient includes effects of friction, geometry and partial reflection of wave energy from the side walls of the estuary.

In comparison to the Elbe and Ems, the Weser River has the smallest damping coefficient but the local change of this value is greater than for example in the lower part of the Elbe River as shown in Fig. 5.

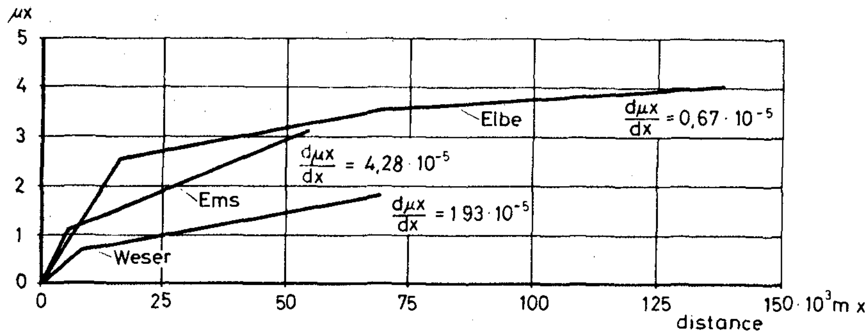

Fig. 5 Damping Parameter

The increase in the energy flux also shows this tendency.

Let the energy flux be 100 % at the river mouth and 0 % at the weir of these three rivers, then the decrease in energy flux from the river mouth to the point of reflection is greatest with the Ems and smallest in the Elbe. The energy flux at the half distance from the mouth to the weir is 10 % for the Ems (x = 26 km), 27 % for the Weser (x = 34 km) and 36 % for the Elbe (x = 70 km) of the initial value at the river mouth.

In general, it can be stated that the greater the change in the damping coefficient, the greater the relative decrease of the energy flux will be.

In comparison with the North American tidal estuaries (such as Bay of Fundy, Delaware Estuary and St. Lawrence) the German tidal rivers Elbe, Weser and Ems are much smaller in size.

The examined estuaries can be classified in three general types:

a) Estuaries of nearly rectangular geometry in plan view having an approximately constant water depth (Example: Bay of Fundy).

b) Estuaries whose widths vary exponentially, whereas the mean water depth remains approximately constant (Examples: Delaware Estuary and Weser Estuary).

c) Estuaries featuring an exponentially varying width and a continous decrease in water depth towards the head of the estuary (Examples: St. Lawrence Estuary and Elbe Estuary).

Comparison of the damping behaviour and energy dissipation has been made in Figure 6 between the three North American and the three German estuaries.

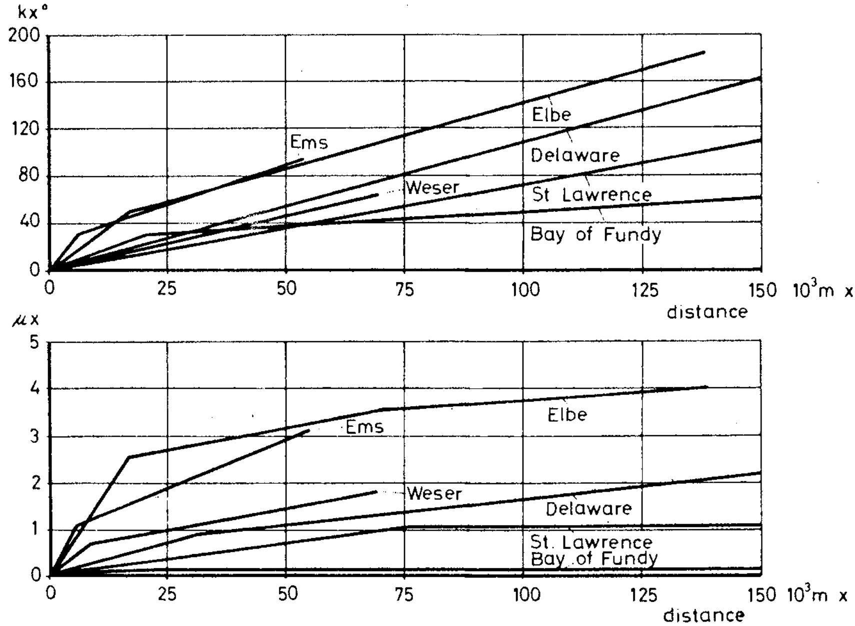

Fig. 6 Phase change and damping parameter

From the diagramm it is evident that the three North American tidal estuaries show smaller damping parameters at points equidistant from the reflection point than German tidal estuaries. This can be readily explained by the fact, that in the North American tidal estuaries due to their size the friction and geometry affect is of smaller importance than in the shallower and narrower German tidal rivers.

Only the mean water depth of the Delaware Estuary is nearly the same as the water depth of the German tidal rivers.

This relationship also can be seen from the plotted damping parameter versus distance x. The comparison is made for a distance of only 150 km seaward the point of total reflection, because all German tidal rivers are shorter.

The Bay of Fundy shows nearly no change of it very small

damping parameter with respect to the distance.
In the St. Lawrence Estuary there is a slope at the first 75 km and then the curve progress horizontal.

Only the Delaware Estuary shows nearly the same change of μx with respect to the distance x as the German tidal rivers Elbe and Weser.

BIBLIOGRAPHY

Partenscky, H.W. and Vincent, R.: Tidal motion in the St. Lawrence Estuary
Extra.9.10.Conference on Coastal Engineering, Tokyo 1966

Partenscky, H.W. and Louchard, L.: The intrusion of salinity into the estuary of the St. Lawrence River.
Paper 101, 11.Conference on Coastal Engineering, London 1968

Partenscky, H.W. and Warmoes, J.C.: Damped co-oscillating tides with negative reflection at the end of the estuary
IAHR, Kyoto 1969

Partenscky, H.W. and Marche, I.C.: Deformation of Tidal Waves in Shallow Estuaries
Conference on Coastal Engineering, Vancouver July 1972.