CHAPTER 162

ADDED MASSES OF LARGE TANKERS BERTHING TO DOLPHINS

By

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ABSTRACT

The added masses of large tankers berthing to dolphins are studied both theoretically and experimentally. The movements of large vessels in shallow water in the directions normal to their planes of symmetry cause counter-flows of appreciable velocities under the hulls. The inertia of these counter-flows is shown to have an important effect on the added masses of the vessels. A theoretical formula is derived to determine the mass factor of an ocean vessel in shallow water as a function of the ratio Draught/Water-depth, the Froude number of the vessel and the coefficient of head loss of the counter-flow under the hull.

Experiment is made to determine the mass factor. Comparison between the theory and the experiment shows a good agreement.

INTRODUCTION

The size of tankers has remarkably been increasing. For the design of the dolphins to which large tankers are berthed the accurate estimation of added masses of these vessels is essential.

When the depth of water is sufficiently large in comparison with the draught of a vessel, the added mass of the vessel can be estimated by the use of the formulae derived in the case of deep water [1, 2]. However, in consequence of the remarkable increase of the size of tankers, the depth of water in front of dolphins is usually not very large in comparison with the draught of a large tanker, and sometimes the latter ranges to as much as 80 percent, or even more, of the water depth in front of dolphins.

In the case when the clearance between the hull of a vessel and the bottom of water is small the added mass of the vessel is supposed to differ considerably from that in deep water. The purpose of this paper is to determine theoretically and experimentally the added masses of large tankers in shallow water.

THEORETICAL CONSIDERATION

When a large tanker is navigated with a constant speed in a normal

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direction toward dolphins by a tug-boat, it is theoretically supposed, from the consideration based on the equation of continuity of flow of water across the vertical plane in front of the hull of vessel, that the motion of the vessel toward the dolphins causes a counter-flow of an appreciable velocity under the hull bottom.

Figure 1 shows the flow of water caused by the steady movement of a model hull in an experimental flume. The flow was visualized by the use of

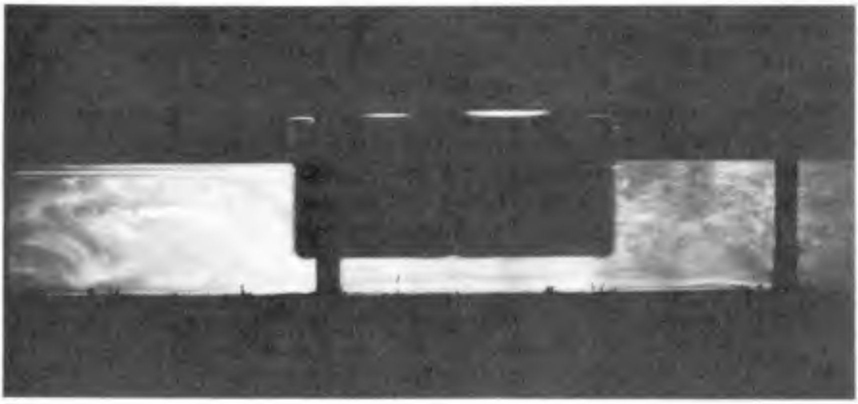

Fig. 1. Flow of water caused by the steady movement of a model hull.

aluminium flakes. The model was pulled with the fine wire, which is seen in the figure. We can clearly notice the counter-flow under the hull. Since the velocity of the counter-flow was appreciable, a vortex filament was induced in front of the hull by the counter-flow, this vortex filament being seen in the figure. The Froude number of the movement of this model hull was 0.0285.

In the steady motion of a vessel (Fig. 2) the equation of continuity for the flow around the vessel may be written as

$$D\ v_o = C\ h'v_o' \tag{1}$$

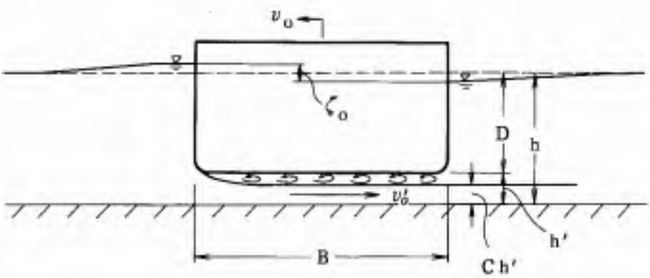

Fig. 2. Steady movement of a vessel and the counter-flow.

where v_0 and v_0' are the steady velocities of the vessel and the counter-flow, respectively, D is the draught of the vessel, C the coefficient of contraction of the counter-flow, and h' the clearance of water under the hull.

When the vessel comes to be in contact with a dolphin, its velocity begins to be decelerated. It is to be noted, however, that even after the instant when the vessel comes to be in contact with the dolphin, the counter-flow under the hull tends to maintain its own velocity owing to its inertia. Consequently, due to the continuity of flow of water, the water level on the dolphin side of the hull goes down forming a negative surge, and the water level on the sea-side goes up forming a positive surge (Fig. 3), until the counter-flow will have eventually been ceased by the negative

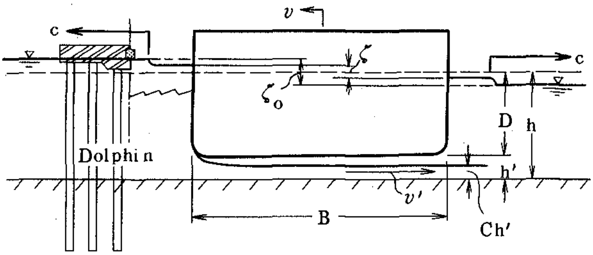

Fig. 3. Negative and positive surges formed in consequence of the deceleration during the movement of a vessel. (ζ decreases as t increases.)

hydraulic gradient, the hydraulic gradient formed by $\zeta < 0$, acting on the counter-flow. Due to the difference of water levels on the both sides of the hull thus brought about, hydrostatic pressure acts on the hull in the direction of the dolphin, hence the force of impact of the vessel is increased. Since this increase of impact of the vessel is caused by the inertia of the counter-flow under the hull of the vessel, we can understand that the inertia of the counter-flow has an important effect on the added mass of the hull.

With all this in mind we make the theoretical analysis, which is shown in the next Chapter.

THEORETICAL ANALYSIS

First, the dynamic equation of the vessel per its unit length is given by

$$\rho D B \dot{v} = - k x - \rho g D \zeta - \tau B \tag{2}$$

Here, ρ is the density of sea-water, B the beam of the vessel, v the instantaneous velocity of the vessel, k the elasticity constant of the system consisting of the mooring structure, intermediate object and the vessel, as a whole, x the total elastic deformation of the same system, g the acceleration of gravity, ζ the instantaneous value of the difference of the water levels on the both sides of the hull, and τ the shearing stress acting on the bottom of the hull. The second term in the right member expresses the hydrostatic pressure acting on the hull owing to the difference of water levels on the both sides of the hull.

The τ is expressed as

$$\tau = \frac{1}{2} \rho \, (v + v')^2 \, C_f \tag{3}$$

where C_f is coefficient of friction and v' is the instantaneous velocity of the counter-flow. We approximate this equation by the following equation.

$$\tau = \beta \, (v + v') \tag{4}$$

Here, β is a parameter.

As for the elastic deformation, x, it is related with the movement of the vessel as

$$\dot{x} = v \tag{5}$$

The equation of continuity of flow across the vertical plane in front of the hull is given by

$$D \, v - C \, h' \, v' = - c \, \frac{1}{2} \, (\zeta_0 - \zeta) \tag{6}$$

in which c is the celerity of surge and ζ_0 is the initial value of ζ. The term in the right member expresses the rate of discharge of water supplied to the counter-flow by the lowering of the water level on the dolphin side of the hull.

The dynamic equation for the counter-flow existing under hull bottom is

$$\rho \, B \, \dot{v}' = \rho \, g \, \zeta - h_f \tag{7}$$

Here, h_f is the head loss of the counter-flow. This h_f can be expressed as

$$h_f = \lambda \, v'^2 / 2g \tag{8}$$

in which λ is a coefficient. The above equation is approximated by the following equation

$$h_f = \alpha \, v' \tag{9}$$

where α is a parameter.

In the six equations, (2), (4), (5), (6), (7) and (9), we have six unknown quantities, v, v', ζ, x, τ and h_f. Therefore, we can solve these equations. Eliminating v, v', ζ, τ and h_f from these six equations we obtain

$$\frac{c \, \rho \, D \, B^2}{g \, (2 \, C \, h' + c \, \alpha)} \, \dddot{x} + \left[\frac{B \, (2 \, \rho \, g \, D^2 + c \, \beta \, B)}{g \, (2 \, C \, h' + c \, \alpha)} + \rho \, D \, B \right] \ddot{x}$$

$$+ \left[\frac{B \, c \, k}{g \, (2 \, C \, h' + c \, \alpha)} + \frac{2 \, \rho \, g \, D^2 + c \, \beta \, B}{c} - \frac{2 \, D \, (2 \, \rho \, g \, C \, D \, h' - c \, \beta \, B)}{c \, (2 \, C \, h' + c \, \alpha)} \right] \dot{x}$$

$$+ k \, x = \rho \, g \, \zeta_0 \left[\frac{2 \, \rho \, g \, C \, D \, h' - c \, \beta \, B}{g \, (2 \, C \, h' + c \, \alpha)} - \rho \, D \right] \tag{10}$$

which is an ordinary differential equation for x of the third order.

As illustrated in Fig. 4, although the time-variation of x is to be quite unsymmetrical with respect to T_1, the time of the maximum deformation of the elastic system, it may approximately be assumed that the variation of x for the range $0 \leqq t \leqq T_1$ is sinusoidal, namely,

$$x \propto \exp\,(i\sigma t) \quad \text{for} \quad 0 \leqq t \leqq T_1$$

Then, (11)

$$\dddot{x} = -\sigma^2 \dot{x} \qquad (12)$$

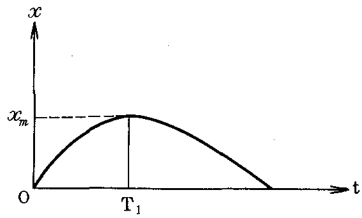

Fig. 4. Time-variation of x.

Therefore, in Eq. (10) the first term having $\dddot{x}$ can be combined with the third term having $\dot{x}$ to give the damping force. Hence, the term which expresses the inertia force of the vessel is the second term having $\ddot{x}$.

Obviously, the two terms within the square blackets of the second term in Eq. (10) may be interpreted as the added mass per unit length of the vessel, M, and the mass per unit length of the vessel, M_o, namely

$$M = B\ (\ 2\ \rho\ g\ D^2 + c\ \beta\ B\ )\ /\ g\ (\ 2\ C\ h' + c\ \alpha\ ) \qquad (13)$$

$$M_o = \rho\ D\ B \qquad (14)$$

In Eq. (13) two parameters, α and β, are present, which parameters are to be expressed in terms of λ and C_f, respectively. The calculation for this is as follows:

As described previously, Eq. (9) is an approximate expression to Eq. (8) (Fig. 5). In order to determine α definitely, let us take α in such a way that the work done against the h_f given by Eq. (9) during the period $0 \sim T_1$ becomes equal to the work done against the h_f given by Eq. (8) during the same period. Therefore,

$$\int_0^{T_1} \lambda \frac{v'^2}{2g} \cdot v' dt = \int_0^{T_1} \alpha\ v' \cdot v' dt \qquad (15)$$

Furthermore, we approximately assume

$$v' = v_o' \sin \frac{\pi}{2T_1}\ t \qquad (16)$$

Then, substitution of Eq. (16) into Eq. (15) yields the following relation.

$$\alpha = \frac{4\lambda}{3\pi} \frac{v_o'}{g} = \frac{8}{3\pi} h_{fo} \qquad (17)$$

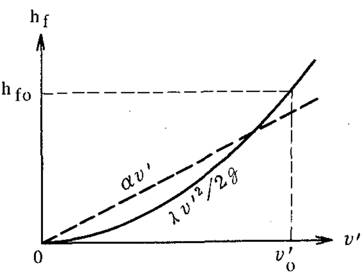

Fig. 5. Approximation of h_f.

in which h_{fo} is the initial value of h_f.

By a similar procedure β can be determined in terms of C_f as

$$\beta = \rho\ C_f\ (v_o + v_o')\ \frac{8}{3\pi} = \frac{8}{3\pi}\ \tau_o \qquad (18)$$

where τ_o is the initial value of τ_o. Substituting the approximate relation,

$v_o' = v_o\, D / C\, h'$, into Eqs. (17) and (18), we obtain the following aquations.

$$\alpha = \frac{4\lambda}{3\pi} \frac{v_o}{g} \frac{D}{C\,h'} \tag{19}$$

$$\beta = \frac{8}{3\pi}\, \rho\, C_f\, v_o \left(1 + \frac{D}{C\,h'} \right) \tag{20}$$

Denoting the water depth by h, the celerity of the surge is given by

$$c = \sqrt{g\,h} \tag{21}$$

Substitution of Eqs. (19), (20) and (21) into Eq. (13) yields the following equation.

$$\frac{M}{M_o} = \frac{D}{C\,h'} \frac{1 + \frac{4}{3\pi} C_f F_r \frac{B\,h}{D^2} \left(1 + \frac{D}{C\,h'} \right)}{1 + \frac{2}{3\pi} \cdot \lambda\, F_r \frac{h}{D} \left(\frac{D}{C\,h'} \right)^2} \tag{22}$$

Here, F_r is the Froude number of the vessel defined by

$$F_r = v_o / \sqrt{g\,h} \tag{23}$$

Since

$$0 (C_f) = 0 (10^{-2}), \quad 0 (F_r) = o (10^{-2}),$$

$$0 (\lambda) = 0 (1), \quad 0 (Bh / D^2) = 0 (1),$$

$$0 (D/h) = 0 (1), \quad 0 (D/C\,h') = 0 (1) \sim 0 (10),$$

the above equation may be approximated as

$$\frac{M}{M_o} = \frac{D}{C\,h'} \Big/ 1 + \frac{2}{3\pi} \lambda\, F_r \frac{h}{D} \left(\frac{D}{C\,h'} \right)^2 \tag{24}$$

This equation is written as the following equations.

$$\frac{M}{M_o} = \frac{C\,h'}{D} \Big/ \left(\frac{C\,h'}{D} \right)^2 + \frac{2}{3\pi} \lambda\, F_r \left(1 + \frac{h'}{D} \right) \tag{25}$$

$$= C \left(\frac{h}{D} - 1 \right) \Big/ C^2 \left(\frac{h}{D} - 1 \right)^2 + \frac{2}{3\pi} \lambda\, F_r \frac{h}{D} \tag{26}$$

The effect of friction on the mass factor is obvious. In the ideal case of no friction, λ disappears, and Eq. (26) is reduced to the following equation.

$$\frac{M}{M_o} = 1 \Big/ C \left(\frac{h}{D} - 1 \right) \tag{27}$$

Accordingly, the value of mass factor tends to infinity as the draught, D, approaches to the water-depth, h.

In actual fluid, however, friction has an important effect on the mass factor. In view of Eq. (8) λ can be assumed for practical purposes as

$$\lambda \sim 1 \tag{28}$$

Equation (26) for the values $\lambda = 1$ and $C = 0.5$ is drawn to scale in Fig. 6.

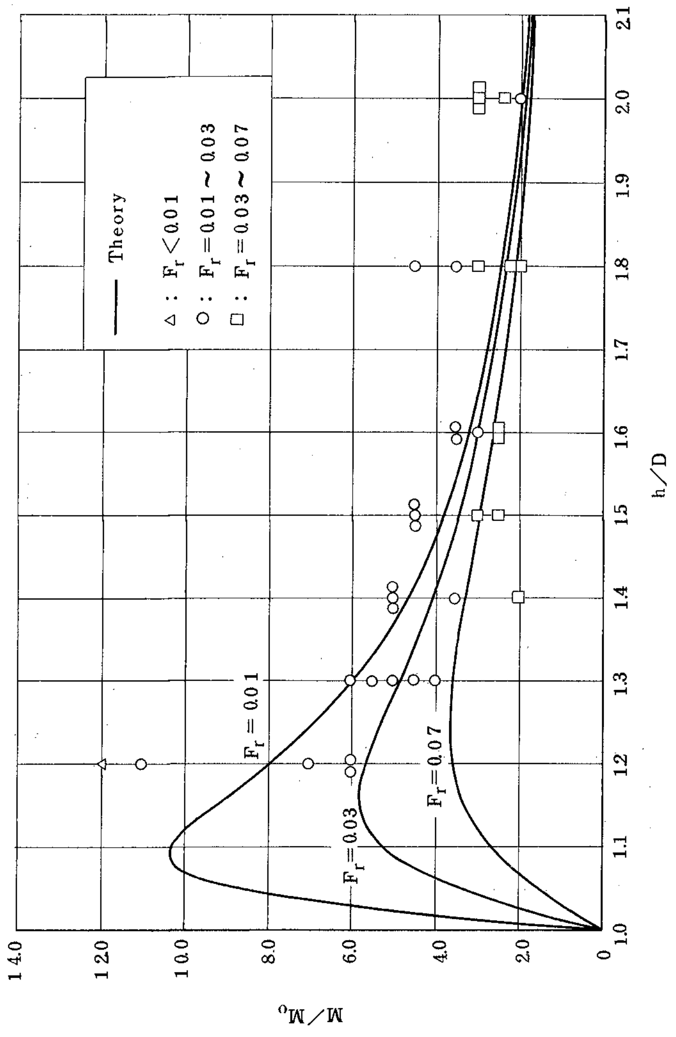

Fig. 6. The ratio Added Mass/Displaced Mass of the large tanker in shallow water.

Owing to the presence of friction the mass factor for a constant value of Froude number has a maximum with respect to the ratio, h/D.

In the calculation for Fig. 6, C has been assumed 0.5. The value of C during deceleration period of the vessel becomes smaller than that during the period of steady motion. In view of this, this value has been taken for C in this analysis.

EXPERIMENT

In order to check the validity of Eq. (26) the authors made laboratory experiment. It was conducted in a two-dimensional flume 30m long, 80cm wide and 90cm high in its side walls. A two-dimensional model of a part hull of a vessel was made (Fig. 7). The model extends to the whole width of the flume, and its dimensions are shown in Fig. 8.

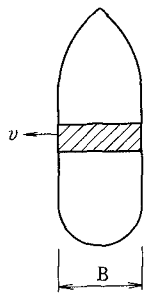

Fig. 7. A part hull.

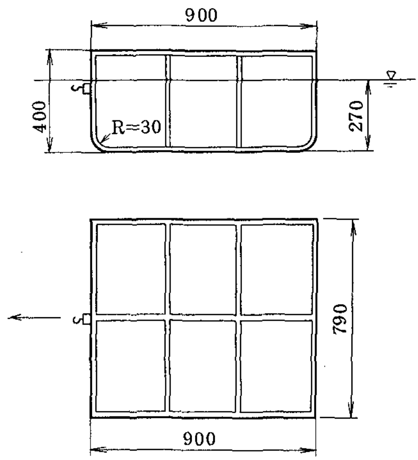

Fig. 8. The dimensions of the model (in mm).

The model hull was steadily pulled by the fine wire having a weight at its other end (Fig. 9). The model makes a steady motion until the weight touches a table installed downwards. During the steady motion the weight, W, is balanced with the sum of the frictional force exerted by fluid, εv^2, and the friction of the side plates of the vessel moving along the side walls, F, namely,

$$W = \varepsilon v^2 + F \tag{29}$$

Since F is the friction of solid bodies, few change of the value of F was observed for various values of v.

At the instant when the weight comes to be in contact with the table, the pulling force of the weight suddenly disappears, and the deceleration

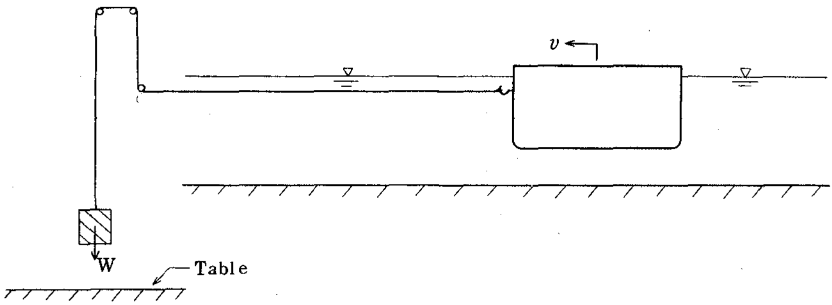

Fig. 9. Device for pulling the model steadily for a while and removing the pulling force suddenly.

of the mocel hull begins. The balance of forces after the pulling force has disappeared is expressed as

$$(M_o + M) \frac{dv}{dt} = - \varepsilon v^2 - F \tag{30}$$

With the initial condition , $v = v_o$ for $t = 0$, the above equation can be integrated to give

$$v = \frac{\sqrt{\frac{\varepsilon}{F}} v_o - \tan \left(\frac{\sqrt{\varepsilon F}}{M_o + M} t \right)}{\frac{\varepsilon}{F} v_o \tan \left(\frac{\sqrt{\varepsilon F}}{M_o + M} t \right) + \sqrt{\frac{\varepsilon}{F}}} \tag{31}$$

Further, this equation is integrated to

$$x = \frac{M_o + M}{\varepsilon} \ln \left\{ \cos \left(\frac{\sqrt{\varepsilon F}}{M_o + M} t \right) + \sqrt{\frac{\varepsilon}{F}} v_o \sin \left(\frac{\sqrt{\varepsilon F}}{M_o + M} t \right) \right\} \tag{32}$$

The decelerating motion of the model hull was recorded by the use of a 8mm motion-picture camera. Measuring the time-variation of the position of the model in this way and also the values of ε and F from preliminary experiment, we calculated the added mass, M, from Eq. (32).

Experiment was made for the range $h/D = 0.2 \sim 1$ and $F_r = 0.007 \sim 0.07$. The result of the experiment is shown in Fig. 6. The agreement between the theory and the experiment is pretty good, as a whole, and validity of the present theory is confirmed.

CONCLUSIONS

The movements of large tankers in shallow water in the directions normal to their planes of symmetry cause counter-flows of appreciable velocities under the hulls. The inertia of these counter-flows has an important effect on the added masses of the vessels.

The mass factor, M/M_o, of large tankers in shallow water can be determined by Eq. (26).

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