# CHAPTER 140 

WAVE FORCES ON SUBMERGED OBJECTS<br>by<br>Suphat Vongvisessomjai<br>Asian Institute of Technology<br>Bangkok, Thailand<br>and<br>Richard Silvester<br>Department of Civil Engineering University of Western Australia

## ABSTRACT

Limitations of the Morison equation for computing wave forces on small submerged structures have encouraged the use of dimensionless relationships containing only height, period and water depth. However in dividing the force by the theoretical drag force or inertia force a relationship can be found with the Keulegan parameter ( $U$ im $T / D$ ) over a wide range of conditions and different types of wave. The $U_{i m}$ value can be determined from empirical and theoretical data for all depths and wave steepnesses. The relating coefficients for various dimensions and shapes of submerged object are predictable from potential theory or modified slightly because of viscous and such other forces induced by bottom and free surface boundaries.

For computing wave forces on a submerged object which is large compared to the wave length, the Morison equation is replaced by the Diffraction theory. Criteria for selecting the latter theory are presented.

## INTRODUCTION

Much work has been carried out in recent years on wave forces exerted on submerged structures (1) (2) (3) (4) (5) (6) (7). Work prior to 1974 has been summarised by Hogben (8). The bodies studied have included cylinders, spheres, blocks and other symmetrical shapes. The more complex units being employed as gravity structures in the oil exploration industry (9) are being equated to these simple forms and also tested individually in flumes (10). There is a dire need to further this work and rationalise the whole procedure (8).

The original achievements in this topic were made by Morison et al (11) whose relationship in equation 1, has been used for a number of decades,

$$
\begin{equation*}
F_{T i}=\frac{1}{2} \rho C_{D i} A_{i}\left|U_{i}\right| U_{i}+\rho C_{M i} \forall d U_{i} / d t \tag{1}
\end{equation*}
$$

where the symbols are as listed in the notation at the end of the paper.
Equation 1 , is the addition of a drag and inertial component, whose validity is being questioned on a number of counts. Apart from the necessary assumption that the object must be small compared to the incident wave length (e.g. D/L small), there are a number of limitations to Morison's approach, as follows:
(i) the summation infers the addition of two terms that reach their peak at different times during the wave cycle. The proportions of the drag component to the inertial component vary with the wave conditions making it difficult to assess such a maximum force.
(ii) the velocities and accelerations of water particles must be known in a prototype situation in order to compute the force. These kinematic variables must be calculated from the specified wave characteristics using either linear theory or a more sophisticated approach. It has been pointed out (12) (13) that no theory can presently predict the vertical distribution of water-particle velocities. The linear theory appears to be accurate (14) for $h / L>0.3$ but for shallower depths an empirical approach will be presented later. There is much more work required to completely solve this problem of velocities and accelerations of fluid particles.
(iii) each term on the RHS of equation 1, contains a coefficient, the values of which have ranged widely and wildly in the literature (15). This constant, relating force to wave conditions, is specific to the object shape and dimension. Normally linear theory has been employed to determine velocities and accelerations in flume tests. It may be the inappropriateness of this application in the shallower conditions that has caused such confusion in this matter. On some occasions values from unidirectional flow have been applied but more recently (4) tests with oscillating tunnels have produced consistently good results.
(iv) inspite of the improving quality of coefficient assessment under laboratory conditions, the operative values in complex prototype situations need full-scale verification. Some oil exploration structures are being instrumented for this purposes but some simple full-sized objects need to be installed in the sea and tested, as suggested by Hogben (8).
(v) the summation in equation 1, does not allow for any interaction of the two terms. For example, the influence of acceleration on the drag force in such a way as to produce a maximum value when the velocity is not at its peak.
(vi) once the structure assumes proportions that commence to modify the incident wave the Morison equation is no longer valid. This may be due to the member itself being large or adjacent members influencing the water motions. Whilst theory may account for wave scattering at some limiting condition there is a vast transition range in which the wave/ structure interaction has not been determined.

Because of the difficulties outlined above some workers have recently by-passed the need for computing the velocity and acceleration, and the concomitant coefficients, by relating the maximum force measured to the basic wave variables of height and period in a specified water depth (1) (7) (16). These have been related by dimensionless parameters which appear to follow consistent curves.

Whilst it is proposed to promote the use of velocity terms rather than basic characteristics of waves the case of objects extending through large proportions of the depth needs special attention. Since different levels of the structure, say a vertical cylinder, are receiving differing pressures it is extremely difficult to integrate them into some peak value. Such integration has been suggested (17) using linear theory but as noted already this does not apply for $h / L<0.3$ which is normally the case. In this event it is better to relate a maximum measured force to some dimensionless parameter made up of $\mathrm{H}, \mathrm{T}$ and $h$ (16). This approach has been used by Silvester (13) in his suggested computation of forces on piles, using data and coefficients given by Goda (18).

## VARIABLES EMPLOYING VELOCITY

The Morison equation (1) in nondimensionalized form gives

$$
\begin{align*}
\frac{F_{T i}}{\rho^{\forall}\left(d U_{i} / d t\right)_{m}} & =C_{M i} \sin \sigma t+\frac{C_{D i}}{4 \pi} \frac{U_{i m}{ }^{\top}}{\left(\forall / A_{i}\right)}|\cos \sigma t| \cos \sigma t \\
& \left.=f_{n}\left[C_{M i}, D_{D i}, \frac{U_{i m}}{\left(\forall / A_{i}\right.}\right), \frac{t}{T}\right] \tag{2}
\end{align*}
$$

for an inertia formulation; and,

$$
\begin{align*}
\frac{2 F_{T i}}{\rho A_{i} U_{i m}^{2}} & =4 \pi C_{M i} /\left[\frac{U_{i m}{ }^{\top}}{\left(\forall / A_{i}\right)}\right] \sin \sigma t+C_{D i}|\cos \sigma t| \cos \sigma t  \tag{3}\\
& \left.=f_{n}\left[C_{M i}, C_{D i}, \frac{U_{i m}}{\left(\forall / A_{i}\right.}\right), \frac{t}{T}\right]
\end{align*}
$$

for a drag formulation.
Alternatively, by dimensional analysis factors can be derived which include orbital velocities of water particles. These are

$$
\begin{equation*}
\frac{F_{T i m}}{\rho V\left(d U_{i} / d t\right)_{m}}=f\left[\frac{D}{L}, \frac{U_{i m}^{T}}{D}, \frac{U_{i m}^{D}}{v}, \frac{U_{i m}}{\sqrt{g(h-d)}}\right] \tag{4}
\end{equation*}
$$

for an inertia formulation;
or

$$
\begin{equation*}
\frac{F_{T i m}}{\rho A_{i} U_{i m}^{2}}=f\left[\frac{D}{L}, \frac{U_{i m}^{T}}{D}, \frac{U_{i m}^{T}}{v} ; \frac{U_{i m}}{\sqrt{g(h-d)}}\right] \tag{5}
\end{equation*}
$$

for a drag formulation. Relative roughness can also be introduced as an independent variable, as has been done by Sarpkaya (19).

The inclusion of the orbital velocity term, preferably the maximum value ( $U_{\mathbf{i}}$ ) at the centre line of the body, is preferred over the wave parameters for the following reasons:
(i) the force is directly dependent upon the water motion which varies throughout the water depth in a complicated manner not amenable to any current theory, except linear for h/L> 0.3.
(ii) utilization of the velocity retains a link with other phenomena such as flow separation, vortex generation and wake formation.
(iii) limits of applicability for drag and inertial terms can be determined, in terms of velocity or amplitude of water excursion proportional to dimensions of the object. This is preferable to ratios of object size to wave height alone which does not consider other conditions such as wave steepness and depth ratio.
(iv) inclusion of velocity in the dimensionless parameters permits the assessment of Reynolds number ( $U_{i m} \mathrm{D} / \nu$ ). This permits the assessmed of Reynolds number
might be disregarded for clear water conditions of $20 \times 10^{3}>$ $\mathrm{R}_{\mathrm{D}}>10^{3}$. However, suspended sediment may so alter the apparent kinematic viscosity in prototype conditions that this parameter may have to be considered. Correlations of Reynolds number with force coefficients have met with little success (5) (20) (21). Only at very high Reynolds numbers
$\left(R_{D}>20 \times 10^{3}\right)$ the drag coefficient shows the trend to decrease with increased $R_{D}$, whereas the inertia coefficient exhibits the opposite trend [Sarpkaya, 19].
(v) the Froude number in equations (4) and (5) ( $\left.\frac{U_{i m}}{\sqrt{g(h-d)}}\right)$, which includes the velocity term, will influence the force as the object gets closer to the bottom. The effect of e/D on the inertia coefficient of submerged cylinders has been computed from potential flow theory and verified experimentally (3). As the object gets closer to the surface the Froude number, $\frac{U_{i m}}{\sqrt{g(h-a-e)}}$, or the ratio (h-a-e)/D are sufficiently sensitive to detect surface effect.
(vi) the term $U_{i m} T / D$ in equations (2) - (5) is commonly called the Keulegan parameter. It is the ratio of drag to inertia force, but is equally the ratio of excursion length ( $2 \pi \xi$ ) of the fluid particles in progressive waves to the transverse dimension of the object (D). This has been shown by Keulegan and Carpenter (20) to correlate better with forces averaged over a wave cycle than the other factors in equations (4) and (5). Sarpkaya (4) has displayed a similar correlation for the sphere and cylinder. Even forces on discs (21) can be related to $\mathrm{dU}_{\boldsymbol{i}}, 2$ which is equivalent $\left[\frac{d U_{i}}{d t}\right] D / U_{i m}^{2}$
to $2 \pi /\left(U_{i m} T / 0\right)=1 /(\xi / D)$. There is little doubt (22) that this pardmeter is the dominant variable to which non-dimensional forces should be correlated. Such correlations are illustrated in Fig. 1 for submerged cylinders, where their dependencies are explicitly shown.

## WATER PARTICLE VELOCITIES

Any assessment of forces, be they drag or inertial in character, must rely finally on an accurate determination of velocity or acceleration. Tests by Le Mehauté et al (12) have indicated that no single theory predicts the velocity distribution throughout depth as recorded in flume tests. Silvester (13) has placed eleven theories in order of accuracy, as judged by these experimental results, for motions at the surface, still water level (SWL), and the bed. For the latter two, or the main body of water, the modification of the Airy theory by Goda (18) proved closest to the measured data. For surface velocities the solitary wave theory by Boussinesq, as reported by Munk (23), was nearest to experiment. Overall, Goda's empirical formula was assessed number one and Stokes second order theory least accurate. Reference (13) should be inspected to see the order of other theories.

The report by Goda (18) contains results of an extensive series of
flume tests from which maximum horizontal velocity ( $U_{x m}$ ) at the SWL was listed over a wide range of $h / L$ and $H / h$ ratios. Silvexster (14) plotted $\left(U_{x m}\right)_{S y} / \sqrt{g h}$ against $H L / h^{2}$ and found the points aligned parallel to 1st order theoretical lines of varying $\mathrm{h} / \mathrm{L}$. For constant $\mathrm{HL} / \mathrm{h}^{2}=1$, it was shown that

$$
\begin{equation*}
\frac{\left(u_{x m}\right)_{S W L}}{\sqrt{g h}}=\frac{2}{3} \frac{h}{L} \tag{6}
\end{equation*}
$$

which indicated a complete relationship of

$$
\begin{equation*}
\frac{\left(U_{x m}\right)_{S W L}}{\sqrt{g h}}=\frac{2}{3} \frac{h}{L} \frac{H L}{h^{2}}=\frac{2}{3} \frac{H}{h} \tag{7}
\end{equation*}
$$

Replotting of ( $\left.U_{x m}\right)_{S L L} / \sqrt{g h}$ versus $H / h$ verified equation (7). For greater depths ( Y .e.e., $\mathrm{h} / \mathrm{L}>0.3$ ) the lst order Airy theory matched the experimental data extremely well. This can be expressed, for $H L / h^{2}=1$ as

$$
\begin{equation*}
\frac{\left(U_{X m}\right)_{S W L}}{\sqrt{g h}}=1.255 \frac{H}{h}\left[\frac{h}{L}\right]^{0.5} \tag{8}
\end{equation*}
$$

Horizontal velocities at the bed were also measured from the vertical distribution graphs supplied by Goda (18). These were plotted as percentages of SWL values and were found to follow the linear theory down to $h / L=0.14$. For smaller ratios the experimental points deviated below those predicted by linear theory.

It can be seen from equations (6) and (8) for constant $\mathrm{HL} / \mathrm{h}^{2}$ that $\left(U_{\mathrm{xm}}\right)_{\mathrm{SWL}} / \sqrt{\mathrm{gh}}$ varies only with $\mathrm{h} / \mathrm{L}$, as depicted in Fig. 2. The linear theory is shown for SINL and bed elevations, as also the curves from Goda's experiments (18). It is seen that at small depth ratios these two empirical lines tend towards the hyperbolic theory for SWL and bed velocities as given by Iwagaki and Sakai (25). The break from the theoretical curve occurs at $h / L=0.3$ for SWL values and at $h / L=0.14$ for bed velocities. Note the maximum for the latter at $h / L=0.25$. The two scales of $\mathrm{HL} / \mathrm{h}^{2}$ and $\left(U_{x m}\right)_{\text {SWL }} / \sqrt{\mathrm{gh}}$ are necessary to obviate wave breaking at $H / h=0.78$ approximately.

To obtain percentage values of horizontal velocity at other depths a graph of $\left[U / U_{S W L}\right]_{m} \%$ was graphed versus $h / L$. For $h / L>0.3$ linear theory could be used. For $h / L<0.04$ the hyperbolic theory (25) could assist. For transitional depths the cnoidal theory as modified by Mejlhede (26) could be of assistance. Smooth transitions from deep to shallow water should be expected for any proportional depths, from which percentages of $U_{\text {xm }}$ at SWL as given by equation (7) can be derived. These are contained in the lower parts of Fig. 3 and Table I.

Velocities at levels above SWL are much more dependent on $\mathrm{H} / \mathrm{h}$ than those within the body of water. Goda's modified Airy theory for maximum horizontal velocity at the crest is

$$
\begin{equation*}
\left({ }^{U} \times m\right)_{c}=\frac{\pi H}{T} \sqrt{1+\alpha\left[\frac{H}{h}\right]^{1 / 2}\left[\frac{Z}{h}\right]^{3}} \frac{\cosh 2 \pi}{\sinh 2 \pi ~} \frac{Z / L}{h / L} \tag{9}
\end{equation*}
$$

where $\alpha$ is a factor dependent on $h / L$
$Z$ is height from the bed, such that at the crest

$$
\begin{equation*}
\frac{Z}{h}=1+0.885\left[\frac{H_{h}^{h}}{h}\right] .275 \tag{10}
\end{equation*}
$$

Goda provides graphs for SWL and surface values versus h/L for a range of $\mathrm{H} / \mathrm{h}$. Ratios of $\left[U_{\mathrm{C}} / \mathrm{U}_{\mathrm{SWL}}\right]_{\mathrm{m}}$ were obtained and plotted in Fig. 3, and included in Table I. The theoretical curves so derived have been modified slightly from the experimental trends recorded by Goda (18). The two percentage scales in Fig. 3 to the right and left of $h / L=0.1$ have been used to improve the accuracy of the diagram.

The procedure thus suggested is that $\mathrm{U}_{\mathrm{xm}}$ at SWL be determined by equation (7), which applies for $h / L<0.2$. ${ }^{x m}$ Values at SWL for $h / L>0.2$ and other depths for all depth ratios can be computed by the percentage values.

Once an acceptable value of $U_{x m}$ is available at any particular level maximum horizontal amplitude ${ }^{x m}\left(\xi\right.$ ) and maximum acceleration ( $\mathrm{du}_{\mathrm{x}} / \mathrm{dt}$ ) can be determined from linear theory as follows

$$
\begin{equation*}
\xi=\frac{U_{x m}^{T}}{2 \pi} \text { and }\left[\frac{\mathrm{dU}_{x}}{d t}\right]_{m}=\frac{2 \pi U_{x m}}{T} \tag{11}
\end{equation*}
$$

until suitable modification is suggested from experiment.
Values of vertical maxima of velocity, acceleration and amplitude of water motion, based upon linear theory, are all given by the simple ratio tanh $2 \pi \mathrm{Z} / \mathrm{L}$. Table II provides percentages over a range of $\mathrm{h} / \mathrm{L}$ and $Z / h$, those at the bed being zero.

The values given in the tables and graphs of this paper will no doubt be refined as further experimental data and theoretical analyses become available, but they are believed to be the closest approximation to this data. It is submitted that the large fluctuations in coefficients of drag and inertia derived from flume tests in the past have occurred due to calculation of velocities etc. from linear theory when this was inapplicable for the depth ratios tested. More recent evaluations by water tunnel tests (4) have provided more consistent and realistic results.

APPLICATION OF PARAMETERS
It has been shown above that Reynolds and Froude numbers can be ignored over a wide range of prototype conditions, leaving the predominant parameters of $D / L$ and $U_{i m} T / D=2 \pi \xi / D=2 \pi /\left[\frac{d U_{i}}{d t} D / U_{i m}^{2}\right]$.

It is thus convenient to discuss the application of relevant theories within ranges of $D / L$ and $2 \pi \xi / D$.

When $D / L<0.2$ the diffraction theory gives $C_{M x}=C_{M p}$ as from Figs. 4 to 6 for various shapes of objects. This relative size serves as an upper bound value for selecting the Morison equation. On the contrary when $D / L>0.2$ the incident wave is scattered and the action changes the flow field in the vicinity of the boundary. Though the Morison equation may become unreliable, it does not necessarily call for the application of the diffraction theory because this size factor does not necessarily ensure negligible viscous effects or predominance of inertia. Such condition is gauged from the relative magnitude of the drag to inertia force or the total force to the drag force. This is dictated by the value of the Keulegan parameter above $\left[U_{\mathrm{xm}} T / D=2 \pi \xi / D\right]$ as seen in Fig. 7.

In this figure can be seen the predominance of inertia when $U_{\mathrm{xm}} \mathrm{T} / \mathrm{D}<3$. It is in this region that diffraction theory should be used for force calculation. Between 5 and 12 for this parameter both drag and inertial components make up the total drag. It is to be noted that for the same region the phase difference between $F$ Txm and $\mathrm{U}_{\mathrm{xm}}$ change by $90^{\circ}$ due to the alteration in major force. Between 12 and $25^{x}{ }^{\text {is }}$ a transition region to the predominantly drag force condition. Beyond $U{ }^{T} T / D=25$ the dimensionless force and hence the coefficient $C_{i}$ remaifs essentially constant even though the ratio of drag to inertia increases continually.

In the section (b) of Fig. 7 the actual coefficients $C_{M X}$ and $C_{D X}$ are compared with the potential flow value $C_{M p}$ and the steady flow value $C_{D S}$ respectively, at similar Reynolds numbers ${ }^{27}$ ). For $U_{x m} T / D<3$ it ${ }^{\text {can }}$ be observed that $C_{D x}=C_{D s}=0$ and that $C_{M x} / C_{M p}=1.0$. ${ }^{2 m}$ At the other
 0.75 and $C_{D x} / C_{D s}=1.5$. The major difference in the theoretical and experimental coefficients in the transition zone are to be noted, with maxima or minima at $U_{\mathrm{xm}} T / D=12$ or $\xi / D \div 2$.

## USE OF COEFFICIENTS

By employing only the $U_{i m} T / D$ term on the RHS of equations (2) - (5) as independent variable the ${ }^{\mathrm{jm}}$ force equation can be written as

$$
\begin{equation*}
\frac{2 F_{T i m}}{\rho A_{i} U_{i m}^{2}}=f\left[\frac{U_{i m}^{\top}}{D}\right] \tag{12}
\end{equation*}
$$

This is the ratio of measured force to the theoretical drag force at the same $U_{i m}$ value. Data of several workers listed in Table III have been utilized according to this equation (12) as in Figs. 8, 9, 10a and 10b. The data have been collected mainly for $U_{i m} T / D<3$ and $D / L<0.32$ for
different types of objects. Some have been conducted in flumes (6) (19) (28) (29) (30) (31) (32) (33) whilst others have involved the use of tunnels with oscillating flow (4). The measurement of velocity in the latter is direct, whereas in the former it is computed from the wave characteristics through some theory. As already noted this can introduce some error, the magnitude of which varies with both $H / h$ and $h / L$.

The plots from Figs. 8 to 10 indicate the dimensionless force to vary inversely with $U_{i m} T / D$ for small values of this parameter for all shapes of objects usedmand different wave types. When the relationship in equation (12) is algebraically manipulated it can be written as

$$
\begin{equation*}
\left.\frac{F_{\mathrm{Tim}}}{\rho_{\mathrm{V}}(\mathrm{dU}} / \mathrm{dt}\right)_{\mathrm{m}} \quad=\text { constant } \tag{13}
\end{equation*}
$$

The constant in equation (13) is then the experimental coefficient of inertia ( $C_{M i}$ ) which can be compared with the theoretical potential values $C_{M p}$ as drawn in Figs. (8) - (10) (and also Fig. 1). Thus the relationship between the dimensionless forces, Keulegan parameters, and inertia coefficient from potential flow theory can be fairly established in this range of $U_{i m}{ }^{T / D}<3$; both for the objects laid on the bottom, $e / D=0$, and lifted from the bottom, e/D $\neq 0$.

When the cylinder is half a diameter lifted from the bottom as in Fig. 9, e/D $=0.5$, the inertia coefficient is not deviated much from potential value ( $C_{M p}=2$ for no boundary effect); but due to bottom effect, e/ $D=0, C_{M p}$ is modified from value of 2 to that of about 3.3.

The relative effect of the free surface is distinctly seen in Fig. 5 for shell structure (35). For half cylinder, Table III indicates that with $(h-e-a) / D>1.0$ or (h-e-a)/D $>0.944$ the free surface effect has not come to change the coeeficient values (see Fig. 10 b ). As the object is nearer to the surface, (h-e-a)/D<0.5, change in value of the coefficient is clearly seen (see Fig. 6).

1. Fundamental formulation of Morison equation gives rise to a number of practical limitations mainly confined within the inability to determine accurately the velocity and acceleration of fluid particles.
2. Through interaction of theory and empiricism, means to obtain more accurate velocity is given for all depth ratios and wave steepness.
3. Morison equation is appropriate for computing forces when the size of the submerged object $D / L<0.2$. The equation is replaced by the Diffraction theory when $D / L>0.2$ and $U_{i m}{ }^{\top} / D<3$.
4. Proper choices of forms of non-dimensional forces and independent variables are shown to give good correlations and links with either potential flow or steady flow, over a practical range of dependent variables.
5. Both the non-dimensional inertia and drag formulations are shown to correlate with the Keulegan parameter well for varieties of object shapes and wave types.
6. Effects of bottom and free surface are found to be relatively influential at e/D $<0.5$ and ( $\mathrm{h}-\mathrm{a-e}$ ) $/ \mathrm{D}<0.5$ respectively.

## NOTATION

(i) Variables

A : projected area
a : radius of cylinder or sphere
C : coefficient
D : diameter of the cylinder or sphere
d : submergence depth
d : differential
e : vertical space between object and bed
F : force
g : gravitational acceleration
H : wave height
h : depth of water
L : wave length
$\ell$ : transverse length of object
$R_{D}$ : Reynolds number
$\alpha$ : factor in equation (9)
T : period
t : time
$U$ : fluid velocity at centroid of object
$Z$ : height above bed
$\rho$ : fluid density
$\nu$ : kinematic viscosity
$\sigma$ : wave angular velocity
$\forall$ : displaced volume
$\xi$ : horizontal excursion length from mean position
$\psi$ : phase difference
(ii) Subscripts
c : crest of wave
D : drag
M : inertia
m : maximum
p : potential flow
s : steady state
SWL : still water level
$T$ : total
$x, z, i: \quad h o r i z o n t a l$, vertical and any $i$-directions respectively of forces or coefficients or velocity
$y$ : lateral direction

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TABLE I Percentage of $U_{\mathrm{xm}} / U_{\text {SWL }}$ at Various Depths and Depth Ratios

| h／L | ． 01 | ． 02 | ． 04 | ． 06 | ． 08 | ． 1 | ． 15 | ． 2 | ． 3 | ． 4 | ． 5 | ． 6 | ． 7 | ． 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $h / L_{0}$ | ． 0007 | ． 0025 | ． 01 | ． 022 | ． 037 | ． 056 | ． 11 | ． 17 | ． 286 | ． 395 | ． 498 | ． 599 | ． 7 | ． 8 |
| $\mathrm{H} / \mathrm{h}=0.8$ | 175 | 182 | 190 | 194 | 197 | 200 | 208 | 212 | 229 | 252 | 276 | 305 | 328 | 351 |
| 0.7 | 160 | 164 | 171 | 175 | 177 | 190 | 208 | 212 | 229 | 252 | 276 | 305 | 328 | 351 |
| $\rightarrow 0.6$ | 146 | 145 | 148 | 152 | 156 | 160 | 185 | 212 | $\underline{229}$ | $\underline{252}$ | $\frac{276}{276}$ | 305 | 328 | 351 |
| 出＋ 0.5 | 134 | 133 | 135 | 138 | 142 | 145 | 166 | 186 | $\frac{229}{}$ | 252 | $\frac{276}{276}$ | 305 | 328 | 351 |
| ¢ 40.4 | 126 | 125 | 125 | 125 | 127 | 130 | 141 | 156 | 193 | 246 | 276 | $\frac{305}{305}$ | 328 | 351 |
| $\stackrel{\sim}{\sim}$ | 120 | 118 | 117 | 117 | 118 | 119 | 124 | 133 | 162 | 203 | 251 | $\frac{305}{234}$ | 328 | $\frac{351}{357}$ |
| $n>0.2$ | 116 | 114 | 112 | 110 | 110 | 109 | 112 | 117 | 136 | 161 | 189 | 234 | 281 | $\frac{351}{224}$ |
| 0.1 | 112 | 109 | 106 | 105 | 104 | 103 | 104 | 108 | 119 | 137 | 154 | 178 | 198 | 224 |
| z／h |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1．0（SWL） | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 108 | 120 | 132 | 146 | 156 | 168 |
| 0.75 | 98 | 96 | 93 | 91 | 90 | 88 | 85 | 80 | 74 | 68 | 63 | 59 | 54 | 50 |
| 0.5 | 96 | 92 | 88 | 86 | 84 | 81 | 73 | 65 | 48 | 36 | 28 | 23 | 19 | 16 |
| 0.25 | 94 | 89 | 84 | 82 | 78 | 75 | 66 | 57 | 38 | 24 | 16 | 11 | 8 | 6 |
| 0 （bed） | 92 | 86 | 81 | 77 | 73 | 69 | 60 | 51 | 32 | 19 | 11 | 7 | 4 | 2 |

TABLE II Vertical Maxima of Velocity，Acceleration and Amplitude as Percentage

| ${ }^{\infty}$ ． | 8888 |
| :---: | :---: |
| $\bigcirc$ | 88へ® |
| $\bigcirc$ | 읐ํํ |
| $\bigcirc$ |  |
| $\pm$ |  |
| $\cdots$ |  |
| $\bigcirc$ | かNべN |
| $\stackrel{\square}{\square}$ | ハーすへ |
| － | 号す으응 |
| $\stackrel{\circ}{\circ}$ | ソeNN |
| $\bigcirc$ | ¢Nへのロ |
| O | $\stackrel{\sim}{N}_{\sim}^{\infty} \sim \sim^{\circ}$ |
| \％ | moxm |
| $\bar{\square}$ | べーサN |
| $\pm$ |  |

TABLE III Sources of Data and Experimental Range of Some Parameters for Cylinders and Spheres

| Data Sources | Object | Direction of force/object orientation | Types of waves; <br> $\mathrm{P}=$ progressive <br> $\mathrm{S}=$ standing <br> W=water tunnel <br> generated wave | Figure | Symbol | Relative size D/L | Relative distance from bottom e/D | Relative distance from free surface (h-e-a)/D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Garrison \& Rao(1971) | hemisphere | Hor. | P | 6 | Define in Fig. | 0.0414-0.2387 | 0 | $\left\lvert\, \begin{aligned} & 0.125,0.25, \\ & 0.5,1.0,1.5 \end{aligned}\right.$ |
| Crooke (1955) | $\begin{array}{r} \text { cy1. } \\ \text { cy1. } \\ \text { cy1. } \\ \text { sphere } \end{array}$ | Hor./Hor Ver./Hor Hor./Ver. Hor. | P $p$ P P | 8a | $\pm$ + + + | $\left\|\begin{array}{l} 0.0134-0.0362 \\ 0.0129 \\ 0.0072-0.0365 \\ 0.0083-0.0360 \end{array}\right\|$ | - | - |
| Vongvisessomjai (1973) | cyl. | Hor./Hor. | p | 8b | $\stackrel{1}{4}^{\text {a }}$ | 0.0223-0.3183 | 0.635-1.0 | 1.365-5.5 |
| Keulegan\&Carpenter (1958) <br> Sarpkaya(1975) <br> Sarpkaya(1975) | cyl. <br> cyl. sphere | Hor. /Hor. <br> Hor./Hor. Hor. | $\begin{aligned} & S \\ & W \\ & W \end{aligned}$ | $\begin{aligned} & 8 \mathrm{c} \\ & 8 \mathrm{c} \\ & 8 \mathrm{c} \end{aligned}$ | $\triangle$ $\Delta$ + + | 0.0026-0.0159 | $5.41-$ 34.93 - | 3.28-19.69 |
| Schiller(1971) | cyl. <br> cyl. | Hor. /Hor. Ver./Hor. | P | 9 |  | $\underset{\text { a }}{\text { 0.0280-0.3126 }}$ | $\left\lvert\, \begin{aligned} & 0.0,0.1, \\ & 0.2,0.5 \end{aligned}\right.$ | 1.5-2.5 |
| Shank \& Herbich(1970) | half- cyl. | Hor./Hor. | P | 10b | $\wedge$ | 0.0675-0.2646 | 0 | $\begin{gathered} 0.944,1.5, \\ 2.167 \end{gathered}$ |



Fig. 1 Asymptotes of Dimensionless Wave Forces on Submerged Cylinders

Fig. 2 Variation of velocity parameter with $\mathrm{h} / \mathrm{L}$ for constant values of $\mathrm{HL} / \mathrm{h}^{2}$



Fig. 4 Inertia Coefficients of Vertical Elliptical Cylinders of Different aspect ratios, $b / a$, and angles of athack,
$d$ (Ref. 34 )


Fig. 6 Inertia Coefficient of a Hemisphere Laid on Bottom (See Data Source in Table III)

7a) Plots of total dimensionless force, relative magnitude
of drag and inertia, and phase angle. of drag and inertia, and phase angle.
Fig. 7 Correlation of forces and coefficients with Keulegan parameter



Fig. 9 Relationship between dimensionless forces in horizontal (and vertical) direction, and Keulegan parameters for varying relative distance from floor, $/ 10$, and relative depth $h / 0$ of large object
( $\mathrm{S}_{\mathrm{Be}}$ Onto Soure in trate III)



