

CHAPTER 70

APPLICATION OF A SEDIMENT TRANSPORT MODEL

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Abstract

A mathematical model for sediment transport under waves has been developed from concepts that have been used successfully for unidirectional flow. This model has been combined interactively with numerical models of wave refraction, wave diffraction, longshore currents and circulation currents in order to predict local topographical changes in the vicinity of a cooling water intake basin for a nuclear power station. The sediment model is calibrated using field data of sediment concentration profiles. Verification and adjustments may be made by analysing deep water wave statistics corresponding to periodic beach and hydrographic surveys.

The model can be used to investigate the effects of any wave climate and consequently different layouts of coastal structures can be examined very rapidly. For the particular problem considered it was necessary to optimise the configuration of the breakwaters forming a cooling water intake basin in order to minimise the sediment concentration at the intake, estimate maintenance dredging quantities and investigate extreme events.

Introduction

A mathematical sediment transport model for unidirectional flow has recently been developed (1). This model assumes that sediment is transported in a bed load region, adjacent to the stationary part of the bed, where the grains are supported by inter-particle collisions, and a suspended load region where gravitational forces are overcome by fluid turbulence. Continuity of sediment concentration and velocity between the two regions was assumed and consequently the exact definition of the transition level is not of critical importance as the sediment flux immediately above and below the transition level will be similar. Whilst not attempting a complete physical description, the model relies on the simplest formulations of bed load and suspended load that are required for predictive use. The unidirectional model was tested against the best existing empirical theories and found to give comparable estimates of sediment transport rates.

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It is proposed that under certain conditions it is reasonable to assume that there are some basic physical similarities between the movement of sediment in unidirectional flow and in the combination of waves and currents. The latter case can be treated as a quasi-steady condition with respect to the shear stresses acting on the bed and the dispersion of shear stress due to the orbital wave motion. The wave motion is assumed to act principally in stirring up the sediment while the currents act principally in transporting the sediment. Some concepts from the unidirectional model have been extended in order to develop a model to predict sediment transport rates in the presence of waves and currents.

Attempts have been made to apply the theory to the prediction of local sediment movement and hence topographical changes over a coastal region situated on the S.W. coast of South Africa. Such an application requires the evaluation of wave heights, wave directions and near-shore currents over the region. This is achieved by using a wave refraction model combined with a longshore current model. Comparison of predicted and measured topographical changes can be made to verify and adjust the model parameters.

The model has also been used to optimise the layout of breakwaters forming a cooling water intake basin for a nuclear power station with a particular emphasis on minimising maintenance dredging. This requires the use of a wave diffraction model and a potential-flow model to determine the flow in the vicinity of the basin due to wave induced currents and the proposed cooling water recirculation currents. A physical model of the area has also been constructed and detailed comparisons will be made in due course.

Sediment Transport-Bed Load

The shear stress in excess of the critical shear must be dispersed within the bed load layer. Otherwise successive layers of material would be removed from the bed. This type of argument was substantiated experimentally by Bagnold (2,3) who proposed the relationship

$$\tau_d \approx p \tan \alpha \quad (1)$$

where τ_d is the dispersed shear stress, p is the normal pressure and α is a friction angle which may be approximated by static angle of repose. For the oscillatory case

$$\bar{\tau}_d = \bar{\tau}_w - \tau_c \quad (2)$$

where $\bar{\tau}_w$ is the mean wave shear stress, τ_c is the critical shear stress and generally $\bar{\tau}_w \gg \tau_c$.

The relationship proposed by Jonsson (4 and recently reviewed 5) was used to evaluate the shear stress due to waves. A friction factor is given by

$$\frac{1}{4\sqrt{f_w}} + \log_{10} \frac{1}{4\sqrt{f_w}} = -0.08 + \log_{10} \frac{a_o}{R_N} \quad (3)$$

where a_o is the water particle orbital amplitude at the bed, R_N is a roughness length. f_w is the friction factor defined by

$$\hat{\tau}_w = \frac{1}{2} \rho f_w \hat{u}_o^2 \quad (4)$$

where $\hat{\tau}_w$ is the maximum shear stress acting on the bed, ρ is the fluid density and $\hat{u}_o$ is the maximum particle velocity at the bed. It has been shown by Madsen and Grant (6) that the Shields criterion for incipient sediment motion in uni-directional flow can also be applied to oscillatory flow when the bottom shear stress is evaluated using equations (3) and (4).

If it is assumed that the bed load layer is supported entirely by grain interaction and that the local change in dispersed shear is proportional to the local concentration, a concentration distribution may be implied such as (1)

$$C = C_m \left(\frac{C_e}{C_m} \right)^{y/e} \quad (5)$$

C_e is the concentration at the upper boundary of the bed load layer and C_m is the maximum concentration at the bed (≈ 0.52 to be consistent with grain movement in all directions).

As the excess shear is to be dispersed within the bed load layer, the bed load thickness, assumed to be the height of the transition level, is

$$e = \frac{\bar{\tau}_d \ln(C_e/C_m)}{g(\rho_s - \rho)(C_e - C_m)(\tan \alpha + \tan \theta) \cos \theta} \quad (6)$$

where ρ_s is the sediment density and θ is the slope of the bed.

Hunter (7) has shown that, in the presence of tides, the current friction factor, for the case of a degenerate tidal ellipse, is twice as large in the direction parallel to the wave orbit plane as in the direction normal to the wave orbit plane. This analysis can also be applied to short period gravity waves so that the corresponding components of shear due to a current are

$$\begin{aligned} \bar{\tau}_{cw} &= 4 \rho f_c \hat{u}_o u_{cw} / \pi \\ \bar{\tau}_{cn} &= 2 \rho f_c \hat{u}_o u_{cn} / \pi \end{aligned} \quad (7)$$

where f_c is the current friction factor, u_{cw} and u_{cn} are the currents parallel and normal to the wave direction respectively. The bed load that should be transported by the combination of the wave and current shears can be determined from the thickness (e') required to disperse the resultant shear. However, for the application considered it is assumed that generally $\hat{u}_0 \gg u_c$ and that $e \approx e'$.

Due to the high concentration of sediment within the bed load layer the fluid and sediment mixture has an effective viscosity much greater than the normal fluid viscosity. This is dependent on and increasing with volumetric concentration. It is therefore assumed that the shape of the velocity distribution may be given by

$$\bar{u}_x = \mu_s \frac{du}{dy} \quad (8)$$

where μ_s is the effective viscosity of the fluid and sediment mixture. This is subject to the boundary condition, $u = u_{ce}$ at $y = e$. The bed load transport rate is therefore

$$T_b = \int_0^e c u dy \quad (9)$$

Sediment Transport-Suspended Load

The simplest one dimensional suspended load distribution may be found from

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial y} \left(\epsilon \frac{\partial c}{\partial y} + c \omega \right) \quad (10)$$

where ϵ is a coefficient of eddy viscosity and ω the characteristic fall velocity of the sediment. For the assumed quasi-steady state in oscillatory flow the time dependent term may be omitted. It is also necessary to specify the vertical distribution of eddy viscosity. Johns (8) has developed a turbulent boundary layer model for which a Fourier analysis is carried out for the computation of time means (with respect to turbulent velocity fluctuations) of the distribution of eddy viscosity. He found that according to his model the eddy viscosity distribution is sensibly constant over the whole wave cycle. It is assumed that the distribution takes the form

$$\frac{\epsilon}{\epsilon_e} = \left(\frac{y}{e} \right)^\xi \quad (11)$$

where subscript e refers to values at $y = e$ and ξ is a positive non dimensional constant. Equation (11) should only be applicable to the lower regions of the depth as it cannot satisfy the necessary boundary conditions at the free surface. However, as the sediment concentration diminishes so rapidly away from the bed this inconsistency can be considered to be admissible. The solution of equation (10)

with $\frac{\partial c}{\partial t} = 0$ and equation (11) is

$$\ln \frac{c}{c_e} = \frac{\omega e}{E_e(1-\xi)} \left\{ 1 - \left(\frac{y}{e} \right)^{1-\xi} \right\} \quad (12)$$

when $\xi \neq 1$.

The current velocity distribution is assumed to be

$$u_c = \hat{u}_c \left(\frac{y}{h} \right)^{1/7} + \text{constant} \quad (13)$$

where $\hat{u}_c$ is the maximum current velocity at the free surface and h is the total depth so that the suspended load transport rate is therefore

$$T_s = \int_e^h c u_c dy \quad (14)$$

By combining the bed load and suspended load models the unknown parameters are c_e , E_e and ξ . Using some results from Johns (8) a value for the exponent ξ was chosen to give the best overall fit to the theoretical distributions in the region adjacent to the bed. A suitable value was found to be approximately 0.25 for a fairly wide range of wave conditions.

The remaining two parameters may be evaluated from suitable experimental or field data.

It should be noted that due to equation (7) the resultant shear acting on the bed and hence bed load movement is not necessarily in the same direction as that of the suspended load which must move in the same direction as the current. However, this difference is generally quite small.

Application

An area of coastline some 6 km long on the S.W. coast of South Africa at Duynfontein has been studied. Site investigations (9) include periodic beach and hydrographic surveys, wave rider recordings, radar and sea bed observations of wave direction, current meter and drogue measurements and suspended sediment profiles (10). The foreshore is gently sloping at approximately 1:100 and has sensibly parallel contours. The wave climate is comparatively severe with a predominant wave period between 10 - 11 secs. and wave heights between 1.5 - 2.0 m but with frequent storms in excess of 4.0 m.

The beach and hydrographic survey data were interpolated onto regular grids so that successive surveys could be compared to give volume changes over the area. Figure 1 shows a typical plot of accumulative volume change that has occurred along the coast, calculated to a distance offshore of 1400 m, for the period February to June 1974 during which there has been general accretion. The broken line and full line show the

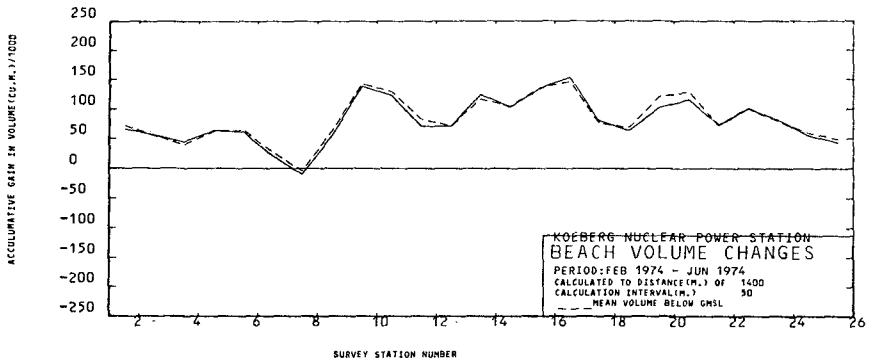


Figure 1. Beach Volume Changes Between Survey Lines , Feb.1974 - Jun. 1974

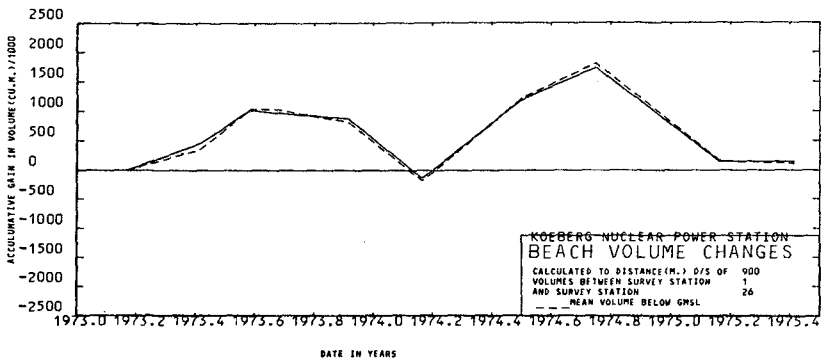


Figure 2. Total Beach Volume Changes

measured volume changes below geodetic mean sea level (GMSL) and below the beach survey station levels respectively, indicating that volume changes above GMSL are generally negligible. The survey stations are shown in Figure 5. Successive volume changes for each survey period could then be plotted to produce a time history of the beach with respect to accumulative volume changes for a common area. Figure 2 shows the results of this analysis of prototype data covering a period of two years and it can be seen that, although there is considerable seasonal variation, the net annual sediment movement is approximately zero.

Due to the wave conditions sediment sampling was extremely difficult. However, concentration profiles both inside and outside the surf zone and for a reasonable range of wave heights and periods were obtained. For each profile it is possible to evaluate a value for the coefficient of eddy viscosity at some arbitrary distance above the bed from equation (11). These values were found to be close enough to take an average for all of the profiles, although some trends with the ratio of wave height to depth can be detected. By combining the bed load and suspended load models and assuming continuity of sediment flux at the transition level it is possible to determine a reference sediment concentration at this level consistent with each set of field data. These values have been plotted against a dimensionless parameter $(\tau_w / g(\zeta - c)T\omega)$ where T is the wave period) and it can be seen from Figure 3 that a clearly defined relationship exists. Consequently using this relationship the sediment transport model can be used predictively given wave heights, periods, directions, current magnitudes and directions.

A wave refraction model is used to evaluate wave directions and heights over the region. The wave ray tracking method, developed by Abernethy and Gilbert (11), is used. In this method the wave speed is assumed to vary linearly over each triangular element so that the wave ray follows a circular arc across each element. Refraction coefficients are simultaneously calculated using the wave intensity equation of Munk and Arthur (12). The grid spacing is chosen according to the local topography and some initial topographical smoothing is applied as discussed by Coudert and Raichlen (13). The wave heights and directions are interpolated onto a regular sediment transport grid which is a sub-area within the refraction grid and, of necessity, completely covered by wave rays. The relative positions and dimensions of the grids are shown in Figure 4. The offshore topography for the area considered is uniform such that it is possible to assume that the sea bed is plane between the refraction grid boundary and "deep water" and consequently starting conditions for each wave ray can easily be established. A typical wave refraction plot is shown in Figure 5 together with the positions of instrumentation and field measurements.

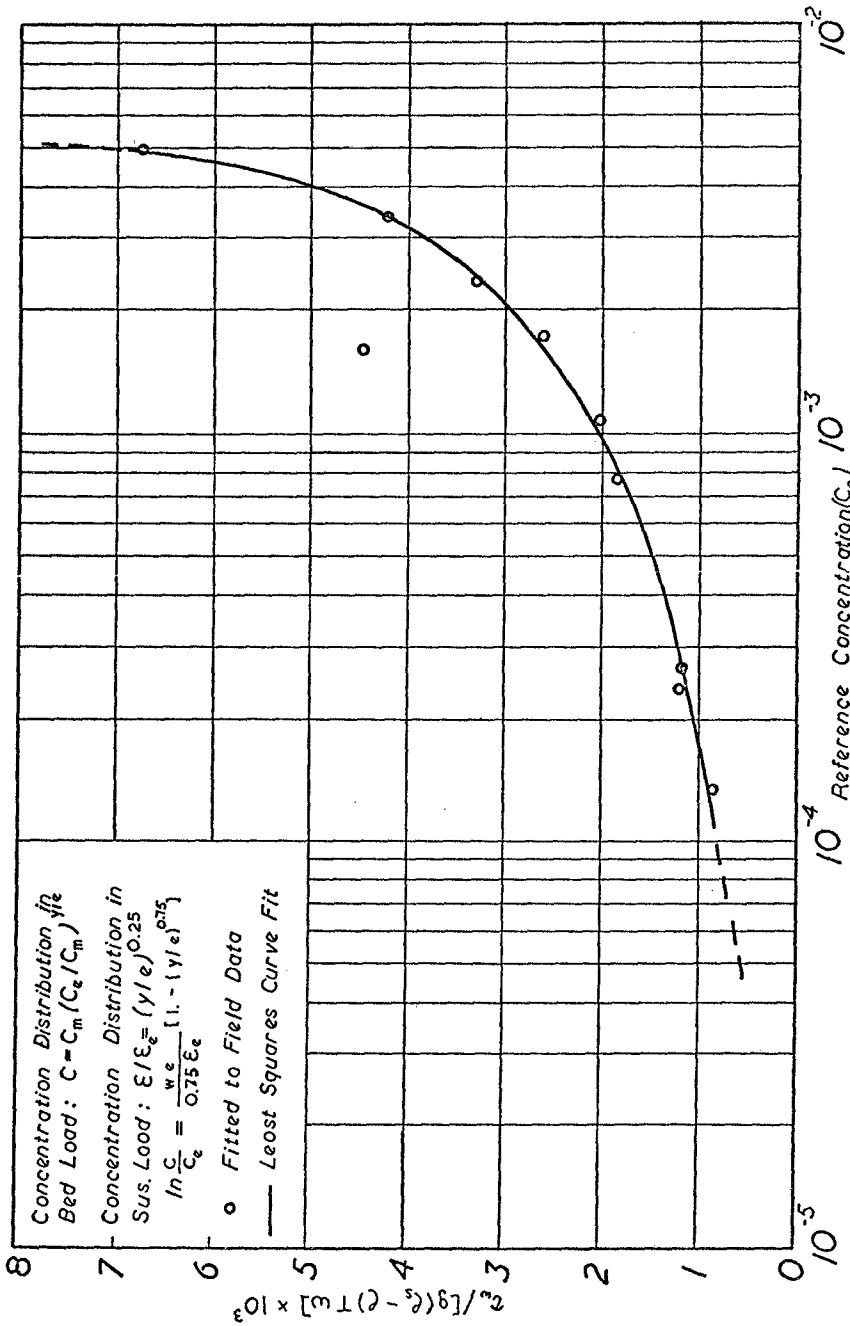


Figure 3. Reference Concentration Fitted to Field Data

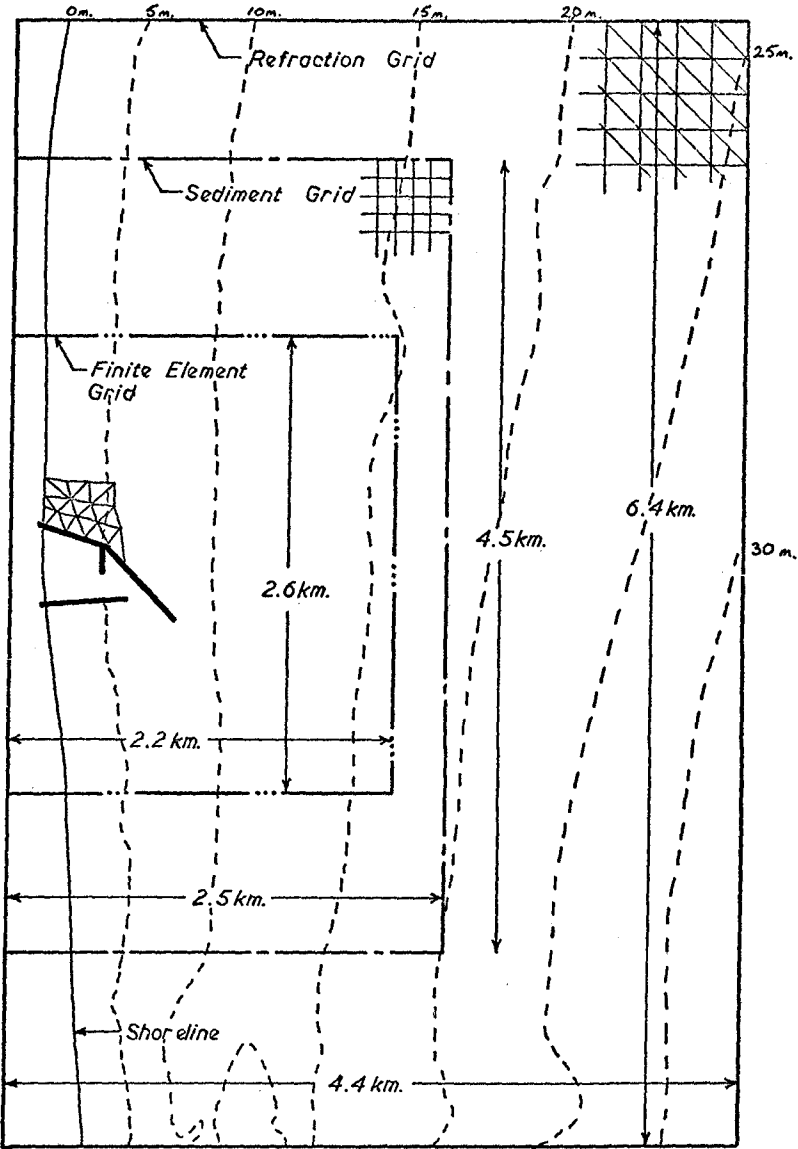


Figure 4. Interrelationship of the Numerical Schemes

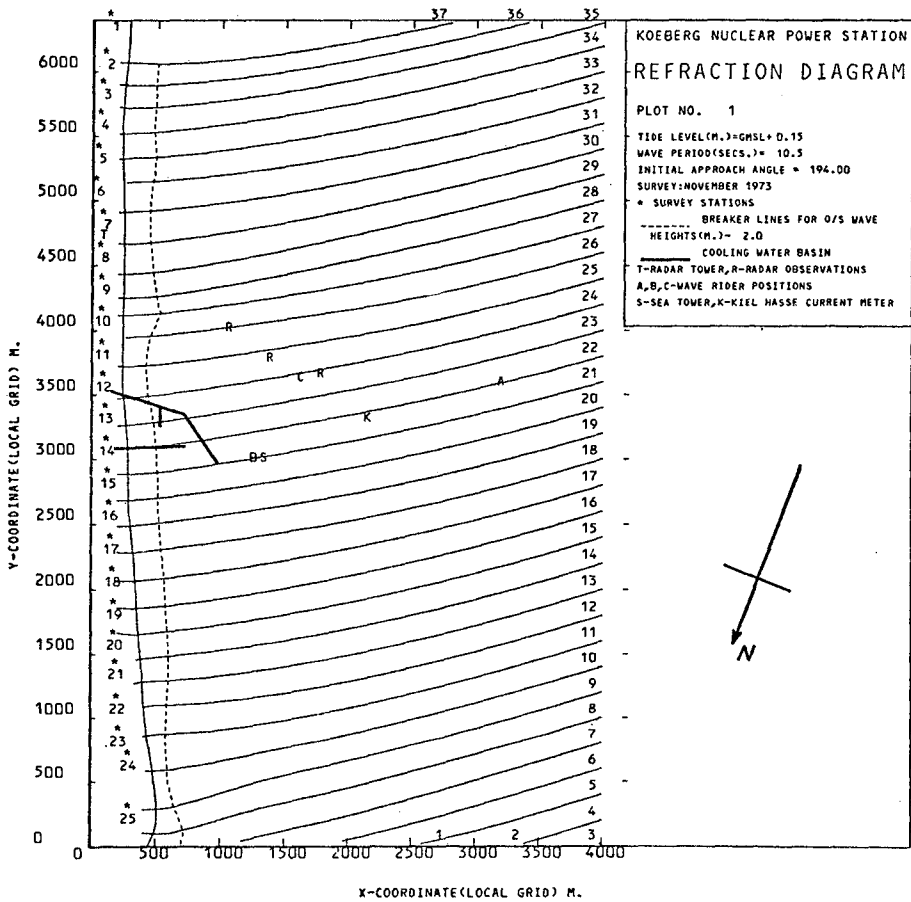


Figure 5. Wave Refraction Diagram

There are very many published wave breaking criteria. However, for this application a relationship of the form

$$d_b = H_b / K_b \quad (15)$$

is used, where H_b is the breaker height, d_b is the breaker depth and K_b a constant coefficient estimated to be 0.7 from physical model tests and field observations.

The longshore current model after Longuet-Higgins (14,15) is then used to calculate wave induced currents inside and outside the surf zone. The foreshore is represented by a series of overlapping planes such that the longshore current profile is calculated at regular sections along the beach. These current vectors are then interpolated onto the discrete points forming the regular sediment transport grid.

Given any combination of offshore wave period, direction and height, the refraction and longshore current models are used to give the required parameters at each point in the mesh and by applying the sediment transport model at these points, predictions of local topographical changes can be made.

The wave data corresponding to each survey period was separately analysed into subsets of wave period, direction and height. These were used as input data and attempts to reproduce historic events, with respect to depth changes and hence volume changes, were made. Initially the model used the wave data for a complete survey period, of approximately three months duration, without intermediate depth adjustments. Consequently the sediment transport for each wave condition was assumed to be accumulative regardless of the order of events. This approach was found to give unrealistically large depth changes. Obviously refinements were necessary and the wave data was reduced to several incremental periods of approximately two weeks duration. Corresponding intermediate depth corrections to the region at the end of each of these periods were made. By comparing the historic and predicted topographical changes for several survey periods it is possible to optimise some of the unknown model parameters. These are

- (i) the current friction factor used in the calculation of longshore currents and reflected by differences in the magnitude of predicted and measured volume changes and
- (ii) the coefficient of horizontal mixing which determines longshore current velocity profiles.

It is intended that a nuclear power station should be sited at Duynfontein and the breakwater formed intake basin is required to protect the intakes from unacceptable wave attack, alleviate associated cooling water recirculation problems and minimise the intrusion of sediment into the intakes.

The proposed layout of the intake basin as shown in Figure 5 consists of a leading breakwater arm giving shelter from the predominant wave direction. This allowed the use of a wave diffraction numerical model, assuming a semi-infinite breakwater, to define wave heights and directions around the entrance and within the intake basin.

The cooling water outfall is to be situated on the beach between 200 and 500 metres from the root of the southern breakwater with the intake located inside the basin. A finite element potential flow numerical model is used to calculate the currents due to the combination of longshore currents and cooling water recirculation currents in the vicinity of the basin. The finite element mesh, consisting of six noded triangular elements, was chosen to cover a sub-area of the sediment transport grid as shown in Figure 4. The offshore boundary was located at a distance where currents are assumed to be negligible and the boundaries normal to the beach are located at distances where the longshore currents would not be influenced by the presence of the coastal structure. The remaining inshore boundary was located along an idealized line corresponding to mean sea level. The longshore currents outside the finite element mesh are calculated as previously described. The current profile at the edge of the finite element mesh is then used as a boundary condition for the potential flow model together with the cooling water intake and outfall flows. The resulting current velocities and modified wave heights are interpolated onto the regular sediment mesh and estimates of local topographical changes and rates of sediment accumulation inside the basin can be made.

A framework for the sequence of operations for the analysis of field data, the overall mathematical model and the interaction between each activity is shown in Figure 6. The model can deal with many options allowing for numerous sets of wave data to be used successively to reproduce historic events or hypothetical sequences. Figures 7(a) and 7(b) show the current vectors (broken lines) and wave height vectors (full lines) for two examples of typical waves approaching the shoreline from the northern and southern sectors respectively. The generated longshore currents for the examples are in opposite directions and different current patterns around the basin are clearly shown.

The model may also be used to investigate equilibrium seabed conditions by repeating the same wave data and updating the depth grid at each iteration. An example is shown in Figures 8(a) and 8(b) for a cooling water basin with a shortened outer breakwater arm. Figure 8(a) is a contour plot for November 1973 and Figure 8(b) shows the contours after running a 2.0 m wave of 10.0 seconds period from a direction of S.W. for 20 days in 5 day steps. It should be noted that these contour plots are produced by an automatic computer plotting program and there is some difficulty with the inclusion of discontinuities such as the breakwater arms. Consequently the contours cannot terminate on the breakwaters. However, it can be seen

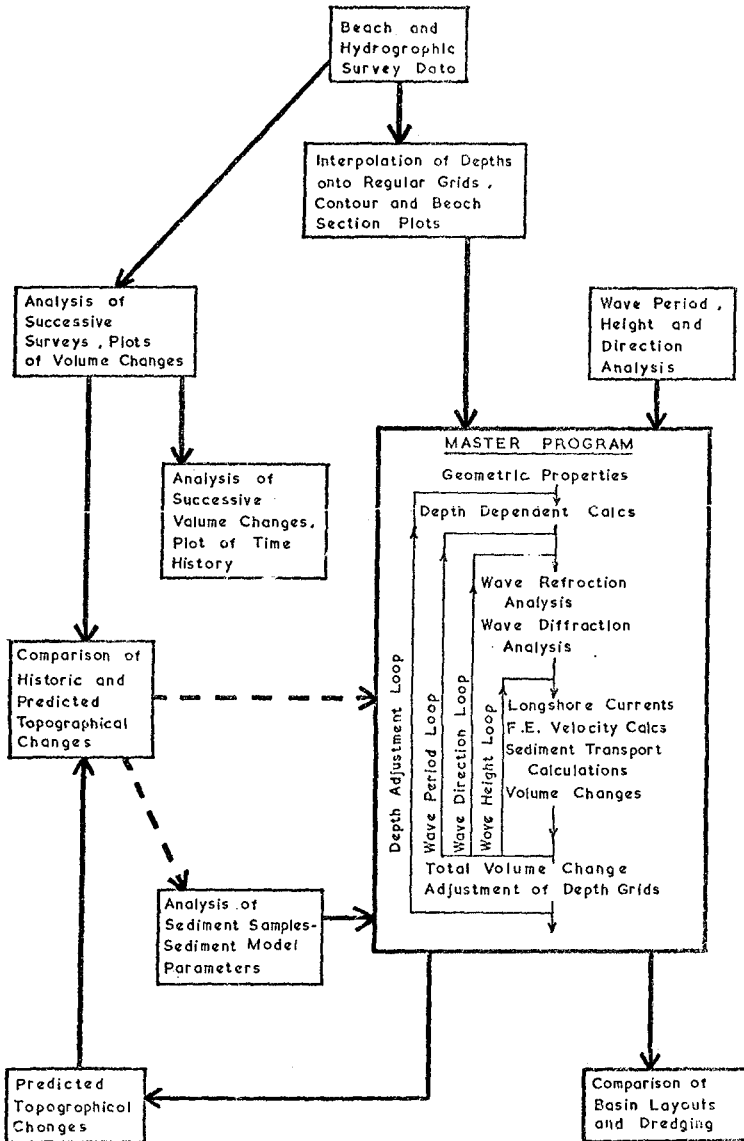
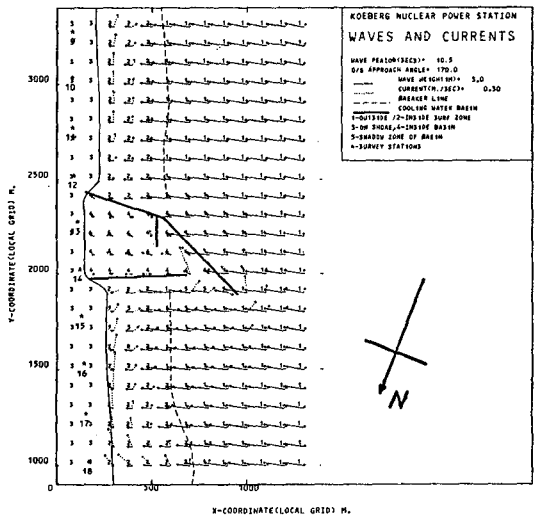
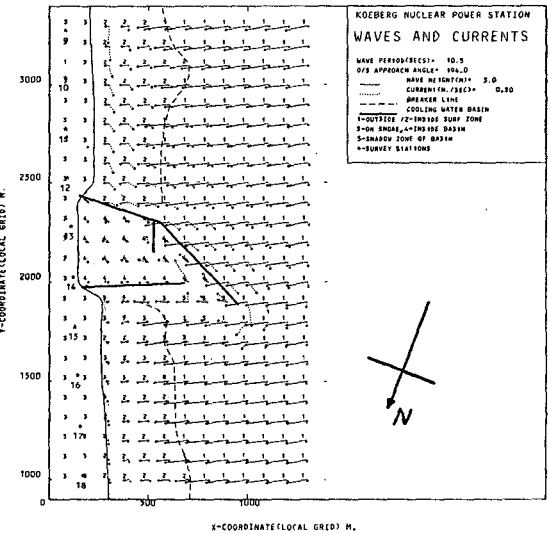


Figure 6. Framework for Sediment Transport Model

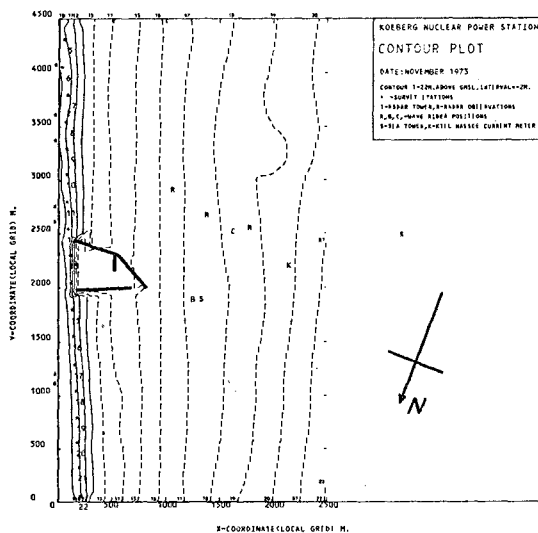


(a) Offshore Wave Direction - WSW

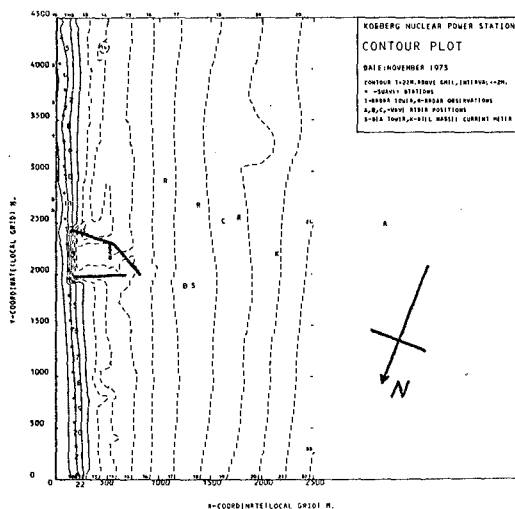


(b) Offshore Wave Direction - SW

Figure 7. Predicted Wave and Current Fields



(a) November 1973 (measured)



(b) November 1973 + 20 days (predicted), wave height=2.0m, period=10.0secs., direction=SW, time step=5 days

Figure 8. Contour Plots

that a bar is shown to be formed in the entrance to the basin, as would be expected. A bar and depression is also shown on the southern side of the southern breakwater indicating the influence of the cooling water outfall as well as some erosion to the north of the intake basin.

Another part of the intake basin investigation was to optimise the distance offshore and hence the depth at which the basin entrance should be situated. The results of this exercise are shown in Figure 9 where the predicted annual maintenance dredging quantities are plotted against entrance depth. The wave data used was based on the one year of complete records available and the range of values indicated by the hatched area represents upper and lower bounds for different assumptions with regard to the wave breaking criteria and wave data analysis. It can be seen that the general trend is for a large increase in predicted dredging quantities as the basin entrance depth decreases below 6 m, a negligible difference of dredging quantity between 6 m and 7 m entrance depth and a decrease in the dredging quantity as the depth is increased beyond 8 m. The shape of this curve is thought to result from the larger waves, possessing the larger sediment transport potential, breaking seaward of the basin entrance and losing much of their energy. The wave height reduces so that a basin entrance located in an intermediate water depth is to a certain extent protected from the larger sediment transporting waves. However, as the depth is further decreased the effect of waves on the bed becomes more pronounced and results in increases to the sediment transport rates. The comparative cost of maintenance dredging and capital cost of construction must be balanced when choosing the optimum depth for the basin entrance. The results of this type of analysis will depend very much on the wave climate for the area considered and in this case the design depth for the entrance was chosen to be 6 m.

Preliminary comparisons between the current patterns and wave heights measured in the physical model show a large measure of agreement. However, more detailed analyses will be carried out.

Conclusions

A mathematical model for sediment transport under waves has been developed from concepts that have been used successfully for unidirectional flow. The model has been applied to the problem of optimising the layout of a cooling water intake basin for a power station in order to minimise both sediment concentrations at the intake and maintenance dredging. This required the simultaneous development of numerical models for wave refraction, longshore currents, wave diffraction and cooling water recirculation as well as the sediment transport model. The longshore current model assumes plane beach sections and does not take account of variations in alongshore wave breaker height and is therefore capable of improvement. The potential

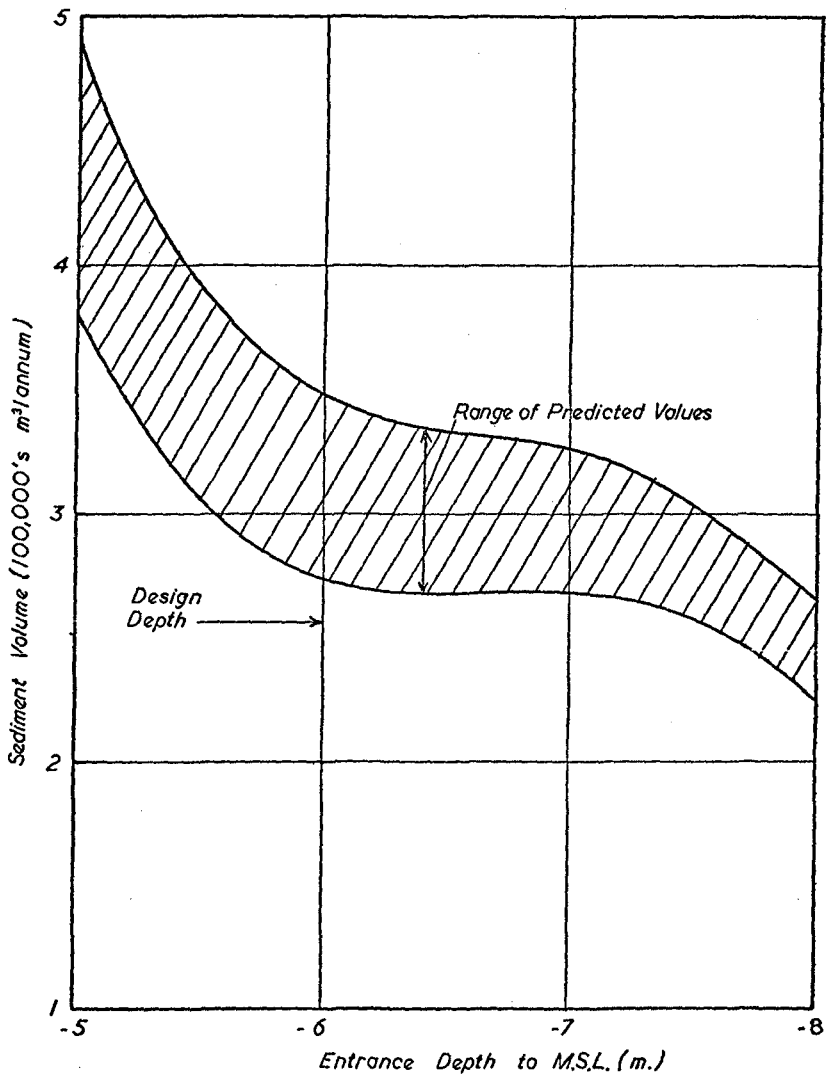


Figure 9. Predicted Annual Dredging Volumes against Entrance Depth

flow model does not account for frictional effects in the vicinity of the basin and the diffraction model should only be used for simple breakwater configurations. However, development work concerning all these aspects will be continued and it is hoped that it will be possible to include such phenomena as the formation of rip currents and mass transport and hence additional forces causing onshore-offshore sediment movement. Nevertheless results from the model are realistic and comparisons to the physical model are reasonable.

An important factor when considering the use of higher order models is the practical limits of computer time. However, there are few detailed field measurements available which do suggest that extra computing effort can be justified. A considerable part of the effort expended in the formulation of the present model was devoted to keeping the execution time to within reasonable bounds and still allow several wave conditions to be processed.

The sediment model is not universal in as far as there are certain parameters that must be calibrated from field data. The consistency of the in-situ sediment concentration profiles collected at Duynfontein and fitted to the model is very good. However, there is a general scarcity of detailed field or experimental data for either concentration profiles or sediment transport rates under waves and it was therefore not possible to generalise the model. Consequently the results from further development of the model to include such non-linear effects as mass transport will be difficult to confirm.

The overall model can be used to examine several different harbour or basin configurations very rapidly and is therefore a very useful engineering tool for the planning of new works or for investigation into modification to existing works.

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