CHAPTER 67

LONGSHORE TRANSPORT PREDICTION - SPM 1973 EQUATION

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ABSTRACT

The 1973 Shore Protection Manual (SPM) predicts longshore transport rates that are 83% higher than its 1966 predecessor, for the same wave conditions. This upward revision is the result of concurrent increases due to (1) deletion of all laboratory data used to establish the 1966 prediction, (2) addition of Komar's (1969) field observations, and (3) limiting energy flux values computed from previously unused data obtained at Santa Barbara (Johnson, 1952). A derivation based on conservation of energy shows that P_q is the longshore component of

the energy flux confined between two wave orthogonals spaced a unit distance apart in the longshore direction, and that a term previously identified as the onshore component of energy flux is identical with the total energy flux in the direction of wave travel between these orthogonals. Use of submerged weight transport rates has no engineering benefit at the present time because: (1) all available data are in terms of volume rates, (2) conversion to submerged weight requires estimates of the void ratio and sand grain density which have been assumed constant in practice, and (3) the engineering problem needs volume rates which would require reconversion back to volume rates if an immersed weight prediction were established.

INTRODUCTION

Energy flux method. Experience indicates that the best way to predict longshore transport at a given site is to adopt proven values from nearby sites, or to estimate values from surveyed changes in sand volume at suitable places along the shore. However, such data are often not available. In the absence of actual field-based estimates, the recommended procedure is to use the energy flux method.

The energy flux method empirically relates longshore transport rate, Q, to a computed variable called the energy flux factor, $P_{y_{c}}$, by an equation of the form:

$$Q = K P_{ls}$$
(1)

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The energy flux factor depends on some combination of wave direction, height, and period. The coefficient of proportionality, K, is empirically determined.

The relation between Q and P_{ls} has been published in many forms, with wide variations in the suggested value of K. This paper compares two published versions of equation (1): the SPM and TR4 versions. (SPM is the "Shore Protection Manual" of the Coastal Engineering Research Center [CERC], U.S. Army Corps of Engineers. SPM replaces the CERC Technical Report Number 4 [TR4], "Shore Protection, Planning and Design". SPM has been issued under two dates [1973, 1975], but both editions are identical for the material referred to in this paper. The third, and final, edition of TR4 [1966] is used for comparison with SPM.)

Units. The units in this paper are those used in SPM. The longshore transport rate, Q, is a volume per unit time, reported as cubic yards per year (yd^3/yr) . This rate measures volume of beach sand in place, including voids. This is the volume rate of importance in beach erosion and shoaling studies.

The other side of equation (1) is the energy flux factor, P_{ls} , a power per unit length of shoreline, reported here as foot - pounds per second per foot of shoreline (ft - lbs/sec/ft). The proportionality constant, K, has units to balance the equation (yd³ - sec/lb - yr).

It is believed that these units give the most effective engineering formulation at the present time. A later section of this paper discusses the applicability of immersed weight rates of transport instead of volume rates, and the physical meaning of the longshore energy flux factor.

To convert the volume rate and the energy flux factor units of this paper to their metric equivalents, the following conversions are required:

Q:
$$1 \text{ yd}^3/\text{yr} = 0.76 \text{ m}^3/\text{yr}$$
 (2)

 $P_{e_{a}}$: 1 ft - lb/sec/ft = 4.45 newton-m/sec/m (3)

<u>Purpose</u>. The energy flux prediction in SPM (Figure 4-37 and equation 4-40 on pages 4-100 and 4-101) replaces the energy flux prediction in TR4 (Figure 2-22, page 175). A comparison (Figure 1) between the two versions shows that the SPM prediction yields transport rates that are 83% higher than those in TR4, for the same wave conditions.

The purpose of this paper is to document the reasons for this significant upward revision of the predicted longshore transport rate.

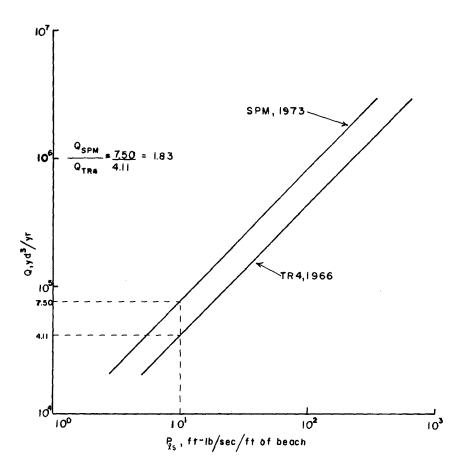


FIGURE 1. UPWARD REVISION OF LONGSHORE TRANSPORT PREDICTION

THEORY

For a field engineer applying a design curve to predict longshore transport rate, the background behind the curve is usually not of immediate interest. However, sooner or later questions arise about the method, or about the relation between the energy flux method and the apparently different methods available from other sources. For this reason, the following sections describe the basic theory behind the energy flux method and how it relates to the immersed weight rate of transport.

Derivation of P_{ls} . The energy flux factor, P_{ls} , is a quantity related to the power, P*, supplied by the waves to the coast. The equation for P*, from small amplitude wave theory, is

$$P^* = C_g E \tag{4}$$

where C_g is the group velocity of the wave and E is the energy density. Group velocity is related to wave speed, C, by

$$C_{g} = nC$$
 (5)

where n is a factor that has a value of 1/2 in deepwater and 1 in shallow water. The energy density depends only on wave height, H

$$E = \gamma H^2 / 8 \tag{6}$$

 γ is the weight density of water.

The power, P^* , is in units of energy per second per unit length along the wave crest (ft - lb/sec/ft). In general, the value of P^* will change as the wave travels to shore. A point on a wave crest moving from deepwater to the shore describes a curved path that gradually approaches perpendicular to the bottom contours and the shoreline (Figure 2). This path is a wave orthogonal. Conservation of energy, according to the small amplitude wave theory used to obtain equation (4), requires that no energy can cross through a wave orthogonal, so that between any two orthogonals, the total wave power is constant in the direction of wave travel. Total wave power is the product of P^* and the length, w, along the wave crest between the orthogonals. Thus, from conservation of energy, with symbols from Figure 2,

$$W_0 P_0^* = W_1 P_1^*$$
 (7)

For the remaining steps of the derivation, it is necessary to assume that the bottom contours are straight and parallel, although not necessarily evenly spaced. If this is the case, then for a given wave condition, any wave orthogonal has exactly

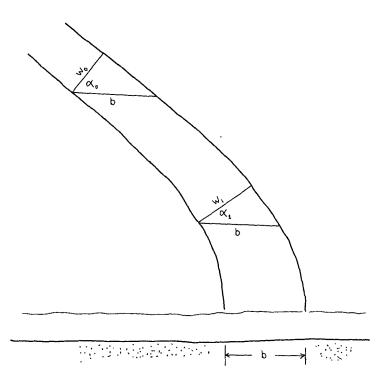


FIGURE 2. WAVE ORTHOGONALS EVENLY SPACED IN LONGSHORE DIRECTION OVER STRAIGHT, PARALLEL, BOTTOM CONTOURS

the same shape as any other. Thus, if the longshore distance between two orthogonals is b at the shoreline, then the longshore distance between those orthogonals remains b at any distance from the shoreline (Figure 2).

This constancy in longshore spacing, b, between orthogonals allows getting rid of the variable distance, w, in equation (7), since

$$w = b \cos \alpha \tag{8}$$

where α is the angle between the wave crest and the shoreline. Thus, the total wave power between wave orthogonals becomes

Total Wave Power = b P*
$$\cos \alpha$$
 (9)

Divide both sides of equation (9) by b and set b equal to 1 foot. Equation (9) then becomes total wave power per unit length or shoreline, given as P

$$\mathbf{P} = \mathbf{P}^* \cos \alpha \tag{10}$$

P and P* are both power per unit distance, but the unit distance is the longshore spacing between orthogonals in the case of P and a distance along the wave crest in the case of P*. From energy conservation (equation [7]), the value of P defined by equation (10) is a constant that does not change along the wave path. Equation (10) for P is formally identical with the equation (7.2.5) of Longuet-Higgins (1972) for his F_x which is

called the "onshore component of the energy flux" (Longuet-Higgins, 1972, p. 210). However, the derivation just shown indicates that P (or F_x) is constant in the direction of wave travel, rather than in the onshore direction.

At any point along the wave path, the wave power per unit shoreline has a magnitude (P) and a direction (α), and therefore P can be broken into components. The longshore component is

 $P_{\rho} = P \sin \alpha \tag{11}$

from Figure 2. Since α will change as the wave refracts while P remains constant, it is evident that P_{ℓ} varies along the wave path. Equation (11) can be rewritten using equations (10), (4), and (5) and a trig identity to get:

 $P_{\ell} = \frac{l_2}{c_q} C_q E \sin 2 \alpha \qquad (12)$

Longuet-Higgins (1970, p. 210) has stated that P_{ℓ} has no obvious physical meaning, and that it should be banished from the literature. However, according to the derivation of this paper, P_{ℓ} has a physical meaning: it is the longshore component

of the energy flux between two wave orthogonals spaced a unit distance apart in the longshore direction.

Since P_{l} has units of energy per second per foot, it is called a longshore energy flux. The basic assumption behind the energy flux method is that the longshore transport rate, Q, depends on P_{l} evaluated at the outer edge of the surf zone where the waves break. This evaluation requires approximation, since wave breaking is outside the linear wave theory used to develop the equation for P_{l} . To indicate the approximation involved, the subscript s (surf zone) is added to P_{l} , and equation (12) is written

$$P_{ls} = \frac{1}{2} C_{b} E_{b} \sin 2\alpha_{b}$$
(13)

The subscript, b, indicates evaluation of the group velocity (equation [5]), energy density (equation [6]), and wave direction (α) , all at the breaker point.

The appropriate height to use for H in the energy density equation (6) is the root-mean-square height. However, by custom, most coastal engineers use the significant height which is proportional to the rms height. Because of this and other approximations, the term P_{ks} is described as the "energy flux factor", and in effect, it is calibrated for significant wave height.

Equation (13) has been formulated in explicit terms using wave direction, height, and period (SPM, Table 4-8, p. 4-97). These and other relations are the subject of a separate report in preparation by the authors which has as its aim a complete documentation of the energy flux method.

The derivation presented above benefited from the work by Walton (1972) and Longuet-Higgins (1970).

<u>Immersed Weight</u>. From a scientific viewpoint, a number of investigators (Bagnold, 1963; Komar and Inman, 1970; Longuet-Higgins, 1972) recommend using the immersed weight rate of transport, I_{g} , rather than Q. The immersed weight rate leads

to a dimensionally homogeneous equation with a dimensionless coefficient, instead of the peculiar units that K has in equation (1). The immersed weight is related to the volume rate by

 $I_{q} = a^{t} \Delta \gamma Q \qquad (14)$

where a' = volume solids/volume sand in place and $\Delta \gamma$ is the difference in specific weight between sand grain and water.

In practical application, the immersed weight formulation does not now improve the engineering prediction. The required engineering quantity is a volume rate of sand in place (Q), and all the existing data were originally measured in terms of Q, or in Q equivalents. Therefore, in order to develop the immersed weight formulation from existing data, it is necessary to estimate values of a' and $\Delta \gamma$ and convert Q values to I $_{\ell}$ by equation (14). Then, to use the immersed weight formulation to solve a problem, one must reverse the procedure and convert back to the required Q.

Available data have led those investigators who have worked with I_{ℓ} to assume that both a' and $\Delta \gamma$ are constants. To the extent that this is a fact, I_{ℓ} is directly proportional to Q, independent of any other variables. If this is the case, nothing is gained toward an engineering solution by using I_{ℓ} , and something may be lost since the procedure would add two unnecessary calculations.

It appears fairly certain that most sand grains on beaches with longshore transport problems are quartz, so that specific gravity in equation (14) is not expected to vary very much. It is less certain that the porosity of the littoral sands is effectively constant. The I_{ℓ} formulation will be necessary if future work on the soil mechanics of beaches shows that average a' varies significantly from one locality to another.

EVOLUTION OF ENERGY FLUX PREDICTIONS

The initial application of what has become the energy flux method to predict longshore transport rate appears to have been made by the Los Angeles District, U.S. Army Corps of Engineers in the 1940s (U.S. Army Corps of Engineers, Los Angeles District, 1948; Eaton, 1951).

Since then there have been a number of empirical equations that relate volume longshore transport rate to longshore wave energy flux. They range chronologically from Watts (1953) to the present form, SPM (1973), including the six which are listed in Table 1. Column A of Table 1 lists the published reference, column B gives the equation as presented in the original paper, with the original units described in column C. In column D, the equation is presented with the units used in the SPM (1973). Figure 3 is a graphical comparison of these column D equations.

Watts' (1953) equation (row 1, Table 1) was based on four monthly field data points collected at South Lake Worth Inlet in Florida. The longshore transport rate was measured by surveying the amount of sand pumped into a detention basin at the inlet. $P_{\ell S}$ (E_T in Watts' paper) was computed using significant wave height and period taken from the analysis of the wave

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TABLE 1. SIX ENERGY FLUX PREDICTIONS FOR LONGSHORE TRANSPORT RATE

	A	В	С	D
	REFERENCE	EQUATION AS GIVEN IN REFERENCE, Q equals:	UNITS USED IN COLUMN B ≁Transport/ Wave Power→	EQUATION IN SPM UNITS* Q equals:
1.	Watts (1953)	0.011 $(E_{T})^{0.9}$	<u>yd³</u> <u>ft·lb</u> day day ft	11,130 (P _{ls}) ^{0.9}
2.	Caldwell (1956)	210(E _i) ^{0.8}	<u>yd³</u> <u>10⁶ ft·lb</u> day day ft	10,810 (P _{ls}) ^{0.8}
3.	Savage (1962)	1.30 E a	$\frac{yd^3}{day} \frac{10^4 \text{ ft} \cdot 1b}{day \text{ ft}}$	4110 P _{ls}
4.	TR4 (1966)	130 E a	<u>yd³</u> <u>10⁶ ft·lb</u> day day ft	4110 P _{ls}
5.	Das (1972)	$0.000193 \times 10^{-4} E_{a}$	$\frac{yd^3}{day} \frac{ft \cdot 1b}{day \ ft}$	6090 P ls
6.	SPM (1973)		$\frac{yd^3}{yr}$ $\frac{ft\cdot lb}{sec ft}$	7500 P _{ls}

*SPM units given in Row 6, Column C.

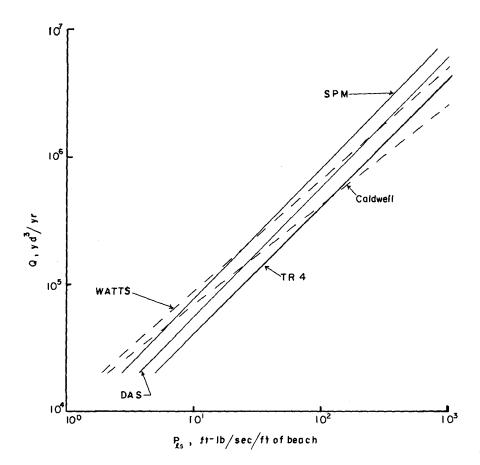


FIGURE 3. FIVE PREDICTIONS OF Q FROM ENERGY FLUX (1953 - 1973) records of a pressure gage installed at the seaward end of a pier located eleven miles north of South Lake Worth Inlet. Wave direction was obtained from visual observations.

Caldwell (1956) calculated five more field data points. The longshore transport was measured by comparing successive sets of surveys of the beach along the 11,000 foot study area immediately south of the jetties at Anaheim Bay, California. P_{ls} (E_i in Caldwell's paper) was calculated from wave records of a step resistance wave gage installed on the seaward end of the Huntington Beach Pier, located about six miles south of Anaheim Bay. Wave direction was obtained from wave hind-casting and wave refraction analysis using synoptic weather charts. The equation in row 2 of Table 1 was determined by Caldwell using his five and Watts' four data points.

Savage (1962) added numerous laboratory data points to those of Watts (1953) and Caldwell (1956) to produce a curve described by the equation in row 3 of Table 1. This relation was eventually presented as Figure 2-22, page 175 in TR4 (1966), the only change being in P_{lS} units (E_a in Savage [1962] and TR4 [1966]) as is shown in column C of Table 1.

Das (1972) added field data points from Komar (1969) and Moore and Cole (1960) to those used by Savage (1962) to obtain the equation in row 5 of Table 1. However, in determining this equation, Das deleted those laboratory data based on experiments with lightweight sediment.

As can be seen in Table 1, Watts and Caldwell used P_{ls} raised to a power less than 1 (0.9 and 0.8 respectively). The other references showed linear equations in P_{ls} . Das, in unpublished work leading to his 1972 paper, found no statistical advantage to using a power equation if the linear coefficient is chosen correctly.

Figure 3 shows that, in the range of the most commonly occurring values of longshore transport rate $(10^5 \text{ to } 10^6 \text{ yd}^3/\text{ year})$, Watts' (1953) prediction comes closest to the curve now given in SPM (1973). In this range of transport rates, both Watts and SPM predict the highest values of Q for a given P_{ls} , of the six equations in Table 1. It is interesting that Watts' (1953) was the earliest equation; SPM (1973) the most recent.

CHANGE FROM TR4 TO SPM

Although the difference between the SPM curve and the early curve of Watts is small, the difference between SPM and its immediate predecessor, the TR4 curve, is large (Figure 3). As shown earlier on Figure 1, the SPM curve gives a transport rate that is 83% higher than the TR4 curve for the same wave conditions. There are three principal reasons leading to the higher placement of the SPM curve, as illustrated on Figure 4 and described below.

Deletion of Lab Data. The TR4 curve is based on nine field observations and 150 laboratory data points (Table 2). The SPM curve is based on 23 plotted field observations, but no laboratory data. The laboratory data were deleted in SPM because additional field information had become available and because the numerous laboratory data overwhelmed the few field data in locating the curve. The effect of the laboratory points is indicated by the strippled field on Figure 4.

TABLE 2. NUMBER OF DATA POINTS - TR4 AND SPM PREDICTIONS

Lab	TR4	SPM
Krumbein, 1944	15	0
Saville, 1950	9	0
Shay and Johnson, 1951	99	0
Sauvage and Vincent, 1954	17	0
Savage, 1962	10	0
	100	U
Field		
Watts, 1953	4	4
Caldwell, 1956	5	5
Komar, 1969	_0*	14
	9	23

*Komar's data became available after TR4 curve was published.

Komar's Field Data. The field measurements of Komar (1969) added fourteen data points to the curve (Figure 4). In general, these field data plotted above the TR4 trend, which was heavily weighted by the lower laboratory results and the lower field data of Caldwell (1956). Caldwell's data fall in the lower part of the region identified as "Field Data, SPM and TR4" on Figure 4.

Santa Barbara Estimate. Johnson (1952) published values of Q, P_{Q}^{*} , and wave period for intervals of high longshore transport rates at Santa Barbara. These Santa Barbara data have been identified as exceptional (M. P. O'Brien, personal communication, 1969), but they had not previously been used with longshore transport predictions because the data lack wave direction.

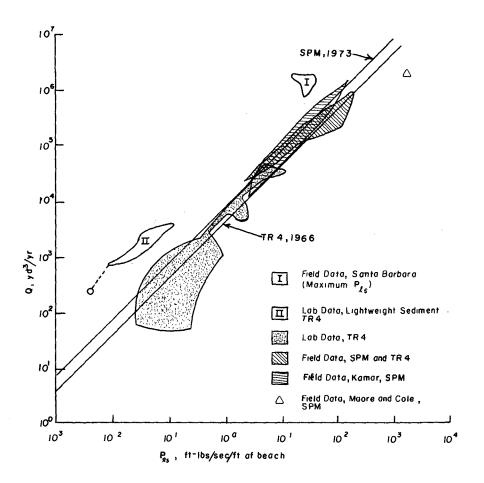


FIGURE 4. DATA USED TO ESTABLISH SPM AND TR4 LONGSHORE TRANSPORT PREDICTIONS In order to benefit from the Santa Barbara data, the equation for P_{ℓ} , equation (12), was written as the product of two terms, one being P^{\star}_{O} and the other dependent only on wave direction.

$$P_{\ell} = M P_{O}^{\star}$$
(15)

where

$$M = \cos \alpha_0 \sin \alpha_b \tag{16}$$

By using Snell's law, an estimated breaker depth, and trial and error, it is possible to get a maximum value of M (Galvin, 1969). Using this maximum value of M in (15) with Johnson's data for P_{O}^{\star} yields equivalent values of P_{ls} that are well above the TR4 curve and above even the SPM curve (Region I on Figure 4).

These Santa Barbara data were not plotted on the SPM curve (Figure 4-37 of SPM), but their existence added confidence to the placement of the SPM curve. Since 1972 when the SPM curve was developed, new data have come available from CERC studies at Channel Island Harbor, California, which also support the higher curve.

CONCLUSIONS

1. The SPM prediction for the volume rate of sediment transport, Q, as a function of the longshore energy flux factor, $P_{y_{e}}$, is, in the recommended units,

$$Q = 7500 P_{ls}$$
 (17)

This equation gives volume transport rates 83% higher than the rates from the predecessor curve in TR4, for the same wave conditions (Figure 1).

2. The 83% increase from TR4 to SPM is due to deleting the TR4 laboratory data (Table 2), adding Komar's (1969) field data, and determining feasible upper limits for Johnson's (1952) previously unused Santa Barbara data (Figure 4).

3. The recommended units for Q and $P_{g,g}$ are yd^3/yr and

ft - lb/sec/ft, respectively, or their metric equivalents in equations (2) and (3). The volume rate, Q, is preferred over the immersed weight rate, I_{g} , at least until an I_{g} curve can be constructed without assuming values for specific gravity and void ratio of the beach sands. Even if I_{g} can be shown necessary because of significant variability in void ratio, Q is still the variable of engineering interest. 4. P_{ℓ} has a readily understood physical meaning. It is the longshore component of the energy flux conserved between two orthogonals spaced a unit distance apart in the longshore direction (Figure 2). $P_{\ell S}$ is P_{ℓ} evaluated at the seaward edge of the surf zone.

5. A review (Table 1) of six predictions of Q developed over a twenty-year interval indicates that the first prediction (Watts, 1953) is the one closest to the last (SPM, 1973).

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