

Wainae Harbor, Island of Oahu

PART II

COASTAL SEDIMENT PROBLEMS

Lumahai Beach, Island of Kauai



CHAPTER 65

QUANTITATIVE DESCRIPTION OF SEDIMENT TRANSPORT BY WAVES

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1. Introduction

With the apparent desire of man to move some of his activities into the offshore region the problems associated with the assessment of the impact of large structures in this environment will be of increasing importance. One of the impacts of a large offshore structure, such as for example the Atlantic Generating Station proposed by Public Service Electric and Gas Company of New Jersey, would be its effect on the wave and current pattern in the vicinity of the structure. These changes in wave and current conditions will induce changes in the sediment transport pattern and may disturb an existing equilibrium thus causing large changes in bottom topography in the vicinity of the structure. These topographical changes may extend all the way to the adjacent shoreline and thus cause deposition in some and erosion in other areas. To assess the severity and extent of topographical changes induced by an offshore structure an ability to quantify not only the effects of the structure on the wave and current pattern but also the mechanics of the interaction of the resulting fluid motion with the bottom sediment is clearly needed. The purpose of this paper is to establish quantitative relationships for the fluid-sediment interaction in this environment.

To avoid misunderstandings it should be pointed out from the outset that the results obtained in the following are limited to non-cohesive sediments and to reasonably well behaved wave conditions. The former of these limitations does not seem severe in view of the fact that the bottom sediments for the major part of the continental shelf may be characterized as cohesionless; the latter limitation excludes the direct application of the results in the immediate vicinity of the structure and in the surf zone where the conditions are complicated by the occurrence of wave breaking. It is, however, hoped that the results of the present research are of a sufficiently general nature to provide at least some insight also into these complicated processes.

With these limitations in mind the first question to ask if concerned with the sediment transport caused by a moving fluid would be: when does the sediment start to move? At the outset of this study a review of the literature on the initiation of sediment movement in oscillatory unsteady flow revealed that there were as many answers as there were publications pertaining to this question. By reanalyzing some of these previously published results

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(2) Research Assistant, Ralph M. Parsons Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139 and making use of Jonsson's (1966) wave friction factor concept, which enables one to determine the magnitude of the maximum bottom shear stress associated with an oscillatory fluid motion, the differences between various investigations are essentially reconciled. In Section 2 it is shown that the Shields Criterion obtained for unidirectional steady flow is applicable as a general criterion for the onset of sediment movement also in oscillatory unsteady flow.

With a general criterion for the initiation of sediment movement established the next logical question to ask is: at which rate is sediment being transported once the critical bottom shear stress is exceeded? This question, in the context of purely oscillatory flow, has previously been addressed in a search for quantitative relationship by Einstein and co-workers at the University of California at Berkeley in the early sixties (Einstein, 1972), i.e., prior to the availability of the work of Jonsson (1966) on the wave friction factors. In Section 3 the Berkeley data is reanalyzed, using the wave friction factor concept, in terms of more fundamental parameters than those employed by Einstein and co-workers. It is found that their experimental data are well represented by a quasi-steady application of the empirical Einstein-Brown relationship (Brown, 1950) for sediment transport in unidirectional steady flow. Provided the equivalent sand roughness of the bottom is taken as the sediment grain size when evaluating the bottom shear stress for the purpose of predicting the rate of sediment transport it is found that the quasisteady application of the Einstein-Brown relationship represents the Berkeley data obtained for a plane bed as well as for a bed exhibiting bed forms (ripples).

Some aspects of the application of the general sediment transport relationship, which was established in Section 3 based on experiments performed in purely sinusoidal flow, to predict net sediment transport rates in the coastal environment are discussed in Section 4. The net sediment transport in this wave dominated environment arises from differences in the rate at which sediment is transported back and forth with the wave motion. These differences arise from second order effects such as wave asymmetry, wave induced mass transport and superposed tidal or wind-induced currents. Due to our limited knowledge of the near-bottom turbulent flow conditions associated with unsteady boundary layer type flows the discussion of the factors producing a net sedIment transport is rather qualitative, and serves primarily to identify topics in desperate need of further research. It is, however, felt that the quantitative relationships for the sediment transport by waves may serve as the framework for further quantitative studies of sediment transport in the coastal environment. The presentation is rather brief and a general reference is given to Madsen and Grant (1976) for a more detailed discussion.

2. Initiation of Sediment Movement in Oscillatory Flow

For unidirectional steady flow a widely accepted criterion for the initiation of sediment motion on a plane bed is given by Shields Criterion (Shields, 1936). This criterion essentially expresses the critical value of the ratio of entraining to stabilizing forces acting on a sediment grain on the sedimentfluid interface. The entraining force is related to the shear stress exerted on the bottom by the moving fluid, the stabilizing force is related to the submerged weight of a sediment grain and when the ratio of the two forces, referred to as Shields Parameter, exceeds a critical value sediment movement is initiated. The Shields Criterion is an empirical relationship which is quite general in that it applies for any fluid, flow and sediment characteristics so long as the sediment is cohesionless.

For oscillatory unsteady flows, such as the to and fro motion of the near bottom fluid particles under waves, several empirical criteria for the onset of sediment movement have been advanced. Bagnold (1946) and Vincent (1958) thus relate the amplitude of the near-bed fluid particle motion relative to the bed and the period of oscillation corresponding to the critical condition of initiation of sediment motion. A set of curves, each corresponding to particular sediment characteristics results. Relationships of this kind are usually limited by the range of experimental conditions from which they were derived and are not of the general nature of the Shields Criterion for unidirectional steady flow. The considerable differences between the critical conditions for initiation of sediment movement under waves exhibited by the comparison of some 13 such relationships (Silvester and Mogridge, 1971) clearly demonstrate this.

More general cirteria for the initiation of sediment movement under waves have been proposed by Horikawa and Watanabe (1967) and Kajiura (1968). Both of these investigations evaluate the stability of a single grain on the sedimentfluid interface based on the concept of the maximum bottom shear stress associated with the oscillatory flow. Madsen and Grant (1975) presented Bagnold's data on initiation of motion in oscillatory flow in the form of a Shields Diagram in a discussion of a paper by Komar and Miller (1973). Madsen and Grant (1975) utilized the results of the comprehensive study of Jonsson (1966) to evaluate the maximum bottom shear stress associated with an oscillatory flow and this procedure was adopted by Komar and Miller (1975) in their reply. Despite the demonstration of the general validity of the Shields Criterion provided by Komar's and Miller's (1975) analysis their final recommendation for the quantitative description of the initiation of sediment movement under waves is essentially equivalent to their earlier paper.

In an unsteady flow one might expect that an inertia force in addition to a drag force, expressed by the bottom shear stress, contributes to the entraining force acting on a sediment grain on the sediment-fluid interface. An approximate analysis (Madsen and Grant, 1976) as well as a comparison with experimental data, however, shows that the entraining force is adequately represented by the bottom shear stress. Thus, adopting Jonsson's results, it is possible to evaluate the shear stress exerted on the bottom by the oscillatory fluid motion above the bed and present various experimental data for the onset of sediment motion in oscillatory flow in a Shields Diagram based on a Shields Parameter

$$\Psi_{\rm m} = \frac{\tau_{\rm om}}{\rho_{\rm g}(\rm s-1)d} \tag{1}$$

in which ρ is the fluid density, g is the acceleration of gravity, s is the specific gravity of the sediment material, d is the grain diameter and τ_{om} is the maximum bottom shear stress defined by

$$\tau_{\rm om} = \frac{1}{2} f_{\rm w} \rho u_{\rm b}^2$$
(2)

in which u_b is the maximum velocity of the fluid relative to the bed and f_w is the wave friction factor as given by Jonsson (1966).

From knowledge of the oscillatory motion, the fluid and sediment characteristics Jonsson's results may be used to evaluate the value of τ_{om} and hence the Shields Parameter given by Eq. (1). Rather than presenting the results in a conventional Shields Diagram the more practical presentation using the parameter

$$S_* = \frac{d}{4v} \sqrt{(s-1)gd}$$

in place of the boundary Reynolds number is used. The experimental results analyzed are summarized in Table 1 and presented graphically in Fig. 1.

All experiments on the initiation of sediment movement were performed using an initially flat bed and the equivalent boundary roughness was taken as the sediment diameter. From the sediment and fluid characteristics S_{\star} is obtained from Eq. (3). When no information on the fluid temperature was available the kinematic viscosity of the fluid (in all cases water) was taken as 10^{-5} ft²/sec (9.3 10^{-3} cm²/sec). The various sets of experimental data are presented by the heavy vertical lines in Fig. 1 and are identified by a letter in the left hand side of the diagram indicating the investigator (e.g., M for Manohar, 1955) and by the symbol identifying the particular sediment (e.g., S1) whose characteristics may be found in Table 1.

The range of the critical values of the Shields Parameter obtained by a particular investigator for a given sediment indicates a variation of at most some 30% around the mean value. In the context of sediment transport and, in particular, when realizing the subjectiveness involved in determining the point of incipient sediment motion (defined as the condition when one or two grains are dislodged and move a few places) this scatter must be considered reasonable. The possibility, however, remains that the scatter exhibits a systematic variation with period of oscillation as suggested by Komar and Miller (1973). By examining the data it was, however, found that no general trend of the variation of the critical value of the Shields Parameter with period of oscillation was exhibited by the data. In some cases $\Psi_{\rm m}$ increased with decreasing period (e.g., VPU, Vincent, 1958) in others the reverse was true (e.g., MS5, Manohar, 1955) and in some $\Psi_{\rm m}$ varied randomly with period (e.g., Horikawa and Watanabe, 1967).

From the preceding discussing it is concluded that the variation exhibited by individual experiments is due primarily to experimental scatter. When comparing the sets of experimental data by Bagnold (1946) and Manohar (1955) who both used an oscillating plate in their studies, it is, however, seen that the results of Bagnold consistently plot below those of Manohar. This difference, which was also noted by Komar and Miller (1973), could be attributed to imperfect motion of the tray in the case of Bagnold's experiments. The close agreement between Bagnold's results and those of Rance and Warren (1968), who used an oscillating water tunnel, suggests that this reason must be discarded and the differences must be attributed to individual differences in determining just when initial sediment movement occurs.

(3)

| Type of | Inves- | | | Spec. | Diameter | Range of Periods | Number of |
|------------------|---|-------------|--------|---------|----------|---------------------|--------------|
| Exp. | tigator | Material | Symbol | Gravity | (mm) | (sec) | Exp. |
| ng Plate | Bagnold (1946) | Sand | BS1 | 2.65 | 3.30 | 1.0- 4.8 | 7 |
| | | Sand | BS2 | 2.65 | 0.80 | 1.4- 7.8 | 6 |
| | | Sand | BS3 | 2.65 | 0.36 | 2.2- 7.0 | 6 |
| | | Sand | BS4 | 2.65 | 0.16 | 0.8-10.5 , | 8 |
| | | Sand | BS5 | 2.65 | 0.09 | 3.1-15.7 | 5 |
| | | Coal | BC1 | 1.30 | 8.00 | 2.4- 7.0 | 6 |
| | | Coal | BC2 | 1.30 | 2.50 | 2.1-12.5 | 6 |
| | | Coal | BC3 | 1.30 | 0.36 | 2.1-14.3 | 8 |
| | | Stee1 | BST | 7.90 | 0.60 | 1.1-2.7 | 4 |
| Oscillati | Manohar (1955) | Sand | MS1 | 2.63 | 1.98 | 2.4- 8.5 | 18 |
| | | Sand | MS2 | 2.60 | 1.83 | 2.7- 8.2 | 19 |
| | | Sand | MS3 | 2.60 | 1.01 | 3.1-10.2 | 15 |
| | | Sand | MS 4 | 2.63 | 0.79 | 3.2-10.5 | 18 |
| | | Sand | MS5 | 2.65 | 0.28 | 3.1-11 | 21 |
| | | Glass | MG1 | 2.54 | 0.61 | 3.9-13 | 19 |
| | | Glass | MG2 | 2.49 | 0.24 | 3.7-14.5 | 21 |
| | | Polystyrene | MP | 1.052 | 3.17 | 9.2-27 | 13 |
| | | Polyviny1 | MPC | 1.28 | 3.17 | 6.5-14 | 13 |
| Laboratory Waves | Vincent (1958) | Sand | VS1 | 2.65 | 0.63 | 1.5- 2.1 | 4 |
| | | Sand | VS2 | 2,65 | 0.46 | 1.5- 1.9 | 3 |
| | | Sand | VS3 | 2.65 | 0.24 | 1.0- 2.7 | 10 |
| | | Pumice | VPU | 1.38 | 1.20 | 0.9- 1.7 | 5 |
| | | Plastic | УP | 1.46 | 0.39 | 0.8- 1.5 | 8 |
| | Horika- wa and Watana- be (1967) | Sand | HWS | 2.65 | 0.20 | 0.8- 2.2 | 17 |
| Water Tunnel | | Sand | RWS1 | 2.65 | 0.82 | 5.2-16.1 | 4 |
| | Rance & Warren (1968) | Sand | RWS2 | 2.65 | 0.39 | 5.0-15.0 | 4 |
| | | Sand | RWS 3 | 2.65 | 0.24 | 6.0-13.8 | 4 |
| | | Coal | RWC | 1.30* | 7.00 | 3.6-15.7 | 6 |
| | | Limestone | RWL | 2.72* | 4.10 | 5.9-13.9 | 4 |

* Estimated values.

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Table 1: Experiments on Initiation of Motion Presented in Figure 1.



Figure 1: Experimental Observations of the Initiation of Sediment Movement in Oscillatory Flow.

When considering all the experimental data presented in Fig. 1 which were obtained from quite different experimental set-ups and for a wide range of periods (0.8-16 sec) and materials (s = 1.052 - 7.90) and keeping in mind the subjectiveness involved in obtaining the experimental results the overall scatter is not alarming. In this respect it should be pointed out that experimental results for unidirectional flow also exhibit some scatter. The general trend of the data indicate a critical value of the Shields Parameter slightly above that associated with the Shields Curve determined from unidirectional steady flow experiments. Despite this minor descrepancy, the conclusion of the results presented in Fig. 1 is that: Shields Criterion for the initiation of sediment movement as derived from steady unidirectional flow conditions serves as a quite accurate and general criterion for the initiation of sediment movement as derived the boundary shear stress is properly evaluated.

Since it is to be expected that a bed generally would exhibit bed forms rather than being flat the Shields criterion may seem to be of limited usefulness. The general applicability of Shields Criterion, as demonstrated in Fig. 1 for the purpose of defining the threshold conditions for a flat bed does, however, establish the importance of the Shields Parameter in quantifying the fluid-sediment interaction for unsteady flow conditions. Thus, when the critical value of the Shields Parameter is exceeded, sediment transport is initiated and the rate of sediment transport may be expected to be related to the value of the Shields Parameter.

3. Rate of Sediment Transport in Oscillatory Flow

For a unidirectional steady flow the sediment will, once it is set in motion, be transported in the direction of flow. Hence, in a steady current, the answer to the question of sediment transport rate once the critical shear stress is exceeded would be equivalent to establishing a relationship between fluid and sediment properties as well as flow characteristics and the rate at which sediment is transported. In the oscillatory unsteady flow associated with a wave motion the answer is somewhat more involved.

In an oscillatory flow, the flow above the bed and hence the sediment transport is constantly varying in magnitude as well as direction. To the first approximation the near-bottom fluid velocity associated with a wave motion may be described by linear wave theory as a purely oscillatory motion. Consequently, if the threshold value of the bed shear stress is exceeded during the wave period the amount of sediment transported forward (in the direction of wave propagation) during half of the cycle will equal the amount being transported backwards during the other half of the cycle by virtue of the symmetry of the motion. This means that, to the first approximation, no net sediment transport is associated with a wave motion. Now, water waves do not induce a purely sinusoidal flow near the bed. Nonlinear effects such as wave asymmetry and wave induced mass transport currents are likely to disturb the equilibrium between the amounts of sediment transported forward and backward during a wave period, thus producing a net sediment transport. It is, however, important to realize that such a net sediment transport is brought about by the, possibly small, difference between the, possibly large, quantities of sediment moving forward and backward with the waves.

The preceding qualitative discussion of the mechanics of sediment transport by waves points out the undesirable, but unavoidable, problem of determining a small difference between two large quantities, if one attempts to derive a relationship for the net sediment transport due to wave action. It does, however, pose the fundamental question: What is the rate at which sediment moves forward and backward in a purely oscillatory flow? The answer to this fundamental question is tantamount to the successful solution of the problem of net sediment transport caused by wave action.

As suggested by the analysis of the initiation of sediment motion data presented in the previous section one would expect the Shields Parameter defined by Eq. (1) to be a physically significant parameter in quantifying fluid-sediment interaction. For this reason the experimental data on the average rate of sediment transport in a purely oscillatory flow obtained at the University of California at Berkeley by Einstein and co-workers are reanalyzed in an attempt to establish an empirical relationship

 $\overline{\phi} = \overline{\phi}(\Psi_{m}) \tag{4}$

(5)

$$\frac{1}{\phi} = \frac{q_s}{wd}$$

is the average sediment transport rate, q_s , nondimensionalized by the fall velocity w of an equivalent spherical sediment grain of diameter d.

For given sediment and fluid properties the fall velocity, w, may be obtained (Madsen and Grant, 1976, Fig. 6) and from the measured sediment transport rate the value of the dimensionless sediment transport function, Eq. (5) is readily evaluated. Since the experiments analyzed here were performed for an initially flat bed it seems reasonable to take the equivalent sand roughness of the boundary as the sediment grain size. Whether bed forms developed during the experiments is not quite clear; however, a particular set of experiments where bed forms definitely were present (Manohar, 1955) will be analyzed later. At this point we may regard the experiments performed by Kalkanis (1964) and Abou-Seida (1965) to correspond to a plane bed. From knowledge of the oscillatory motion of the plate, and taking the equivalent sand roughness of the boundary to be the sediment grain diameter Jonsson's (1966) results may therefore be used to obtain the value of the maximum boundary shear stress, and the value of $\Psi_{\rm m}$ may be determined. The results obtained in this manner are plotted in Fig. 2 of ϕ versus $\Psi_{\rm m}$.

From Fig. 2 it is noted that the sediment transport rate drops off for values of the Shields Parameter of the order 0.035. This is not surprising since the threshold of sediment movement in terms of the Shields Parameter as determined in Section 2 corresponds to values of Ψ_m of the order 0.04 to 0.05 for the sediments used by Kalkanis (1964). It is, however, worthwhile to note the fact that sediment transport does occur for these rather low values of the Shields Parameter, whereas Manohar (1955) as mentioned in Section 2 from direct observations of the movement of sediment grains consistently found critical values of the Shields Parameter higher than those indicated by

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Figure 2: Empirical Relationship for the Average Rate of Sediment Transport in Oscillatory Flow (Plane Bed).

Shields Criterion. The difference between these results, both obtained from oscillating plate experiments, must therefore be attributed to the difference between the methods used to define the condition of incipient sediment movement.

For values of the Shields Parameter somewhat greater than the critical value the results presented in Fig. 2 indicate a rather well defined functional relationship among the two parameters of the type

(6)

(7)

$$\overline{\phi} \propto \Psi_{\rm m}^3$$

as evidenced by the dashed straight line drawn onto the diagram.

The experimental data from which this empirical relationship is derived exhibit a scatter similar to the scatter exhibited by the same data when plotted in terms of the parameters used by Einstein and co-workers (Einstein, 1972). It should, however, be noted that the trend of the data in Fig. 2, for large values of the Shields Parameter, is in reasonable agreement with the empirical relationship suggested by the straight line, whereas the Einstein relationship (Einstein, 1972, Figure 16) in this region deviates from the experimental data. Furthermore, it should be noted that, in the present analysis of the Berkeley data, it was not found necessary to introduce any correction factor for the experimental results obtained by Abou-Seida (1965) with fine sediments. This very convenient finding may partially be attributed to the particular choice of the dimensionless sediment transport function, Eq. (5), made in the present analysis.

The rather well defined empirical relationship, Fig. 2, between the average rate of sediment transport in oscillatory flow and the Shields Parameter which was obtained in the preceding section bears a strong resemblance to the Einstein-Brown formula for the sediment transport in unidirectional steady flow. This empirical relationship, suggested by Brown (1950), reads

 $\phi = 40 \Psi^3$

where the bar over ϕ and the subscript m have been omitted to indicate that this formula applies in steady flow.

In the context of fluid-sediment interaction a similarity between steady unidirectional and unsteady oscillatory flow was previously noted in Section 2 when establishing a criterion for the initiation of sediment movement. In the present context of rate of sediment transport in an oscillatory flow, the similarity between Eqs. (6) and (7) suggests that a quasi-steady application of the Einstein-Brown relationship may represent the Berkeley data. Hence, one is led to adopt a sediment transport relationship of the form

$$\phi(\mathbf{t}) = 40 \Psi^3(\mathbf{t}) \tag{8}$$

in which $\phi(t)$ is the instantaneous value of the sediment transport function

$$\phi(t) = \frac{q_{\rm g}(t)}{wd} \tag{9}$$

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q (t) being the instantaneous rate of sediment transport and

$$\Psi(\mathbf{t}) = \frac{\tau_o(\mathbf{t})}{(\mathbf{s}-\mathbf{1})\rho \mathbf{gd}}$$
(10)

is the instantaneous value of the Shields Parameter.

The basic assumption inherent in the application of the Einstein-Brown formula in this quasi-steady manner is that the response time of the rate of sediment transport, $q_{\rm S}(t)$, is short relative to the time it takes the Shields Parameter, $\Psi(t)$, to change appreciably. Limited evidence for the very short response time of the sediment to the time varying forces associated with an oscillatory flow was found in Section 2 where the maximum value of the Shields Parameter was found to govern the initiation of sediment movement. Although this finding by no means constitutes a proof of the applicability of Eq. (8), it is taken to support the adoption of Eq. (8) as the basis for analyzing sediment transport rates in unsteady oscillatory flow.

The problem in applying Eq. (8) in oscillatory unsteady flow becomes that of determining the instantaneous value of the Shields Parameter or, as seen from Eq. (10), the instantaneous value of the boundary shear stress, $\tau_0(t)$, associated with an oscillatory flow. To tackle this problem, the expression for the maximum boundary shear stress given by Jonsson (1966) may be generalized to reflect the temporal variation associated with an oscillatory flow above the bed

$$\tau_{o}(t) = \frac{1}{2} f_{w} \rho \left| u(t) \right| u(t)$$
(11)

in which

is the near-bottom velocity relative to the bed, having a maximum value u_{b} and a radian frequency $\omega = 2\pi/T$ with T being the period of oscillation.

This simple form, Eq. (11) for the time-varying boundary shear stress preserves the varying direction as well as varying magnitude of the shear stress. It neglects a possible phase difference between the velocity and the shear stress, which according to Jonsson (1966) is small for fully turbulent flow in the boundary layer and immaterial in the present context. Furthermore, the assumption of $f_{\rm W}$, the wave friction factor, being constant throughout the period of oscillation will be invoked in the following. Both of these assumptions indicate the applicability of Eq. (11) to be limited to conditions when the boundary layer flow is highly turbulent, which incidentally is the case for the Berkeley experiments presented in Fig. 2.

Introducing Eq. (12) in Eq. (11) and making use of Eqs. (1) and (2) lead to an instantaneous value of the Shields Parameter, Eq. (10), given by

$$\Psi(t) = \frac{com}{(s-1)\rho g d} |cos\omega t| cos\omega t = \Psi_m |cos\omega t| cos\omega t$$
(13)

(12)

The sediment transport formula, Eq. (8), may therefore be written $\phi(t) = 40\Psi^{3}(|\cos\omega t| \cos\omega t)^{3}$

(14)

corresponding to purely oscillatory flow with a turbulent boundary layer.

This formula clearly has a zero mean value when averaged over a full period of oscillation. Thus indicating, as it should, that the amount of sediment transported in the positive direction (when $\cos \omega t > 0$) equals the amount transported in the negative direction (when $\cos \omega t > 0$) when the flow is purely sinusoidal. From the assumption, inherent in the adoption of Eq. (8), of instantaneous response of the sediment transport rate to changes in the transport function, $\Psi(t)$, it follows that sediment will be transported only when the value of the Shields Parameter exceeds the critical value for initiation of movement, Ψ_{mc} , whose value is readily determined from Fig. 1. Hence, time-averaging Eq. (14) over the time interval during which $\cos \omega > 0$ and $\Psi(t) > \Psi_{mc}$ leads to a prediction of the time average transport rate, ϕ . The result of this analysis is the full line shown in Fig. 2! For values of Ψ_m/Ψ_{mc} greater than two the average sediment transport rate is closely described by the equation

$$\overline{\phi} = 12.5 \Psi_{\rm m}^{3} \tag{15}$$

This equation is the equation for the straight line drawn in Fig. 2, which represents the experimental data reasonably well. The curved part of the line shown in Fig. 2 reflects the small value of the Shields Parameter relative to the critical value corresponding to initiation of sediment movement and has been drawn corresponding to $\Psi_{mc} = 0.04$. Since the Barkeley experiments used in Fig. 2 were regarded as corresponding to conditions of a flat bed the boundary roughness was taken as the sediment grain diameter and the influence of turbulence is limited to a thin boundary layer. In fact, the boundary layer thickness can be estimated from Jonsson (1966) corresponding to the experimental conditions and it can be shown that the boundary layer thickness is of the order 5 times the boundary roughness. Hence, sediment movement is for the experiments analyzed so far restricted to a very thin layer near the bottom. This ensures a rapid response of the sediment transport rate to changes in the transporting forces as was assumed in the quasi-steady application of the Einstein-Brown formula. Furthermore, the limited vertical extent of the region in which sediment is being transported would suggest that if one were to characterize the transport as suspended or bed load, the present experiments would be categorized as bed load. That a bed load formula, here chosen as the Einstein-Brown formula, is successful in representing the experimental data can therefore hardly come as a surprise.

Experimental confirmation backed up by physical reasoning supported the quasi-steady application of the Einstein-Brown formula to describe sediment transport on a plane bed. Experimental evidence, Manohar (1955) and Carstens et al. (1969), however, shows that for flow conditions exceeding only slightly those corresponding to initiation of sediment movement the resulting sediment transport leads to the formation of bed forms, ripples. This points out that a sediment transport relationship restricted to plane bed conditions is of limited practical importance.

For a bed exhibiting bed forms, the equivalent sand roughness of the boundary will be related to the scale of the bed forms. The boundary layer thickness and hence the region of turbulent fluid motion may consequently be of considerably larger extent than corresponding to a plane bed. Kennedy and Locher (1972) thus report measurements of the sediment concentration in a layer of considerably thickness above a rippled bed. These observations may cast some doubt on the notion of the sediment transport being characterized as bed load as well as possibly invalidating the assumption of instantaneous response of the sediment transport rate to changes in the transporting forces. Even if these questions are disregarded there still remains the problem of determining the transporting forces, the bottom shear stress, which now may be a function of the scale of the bed forms.

The approach to the similar problems encountered in the context of unidirectional steady flow, Einstein (1950), is to separate the boundary resistance into two components: one, a skin friction component based on the sediment grain size the other being a form drag component which is associated with the bed forms. The former of these components is regarded as expressing the transporting force. In view of the similarities between steady unidirectional and unsteady oscillatory flow already uncovered in this investigation of fluidsediment interaction it seems natural to pursue an analogous line of approach in the present context of unsteady flow. Hence, one may adopt Eq. (8) as the basic sediment transport relationship with an evaluation of the bottom shear stress, Eq. (11), appearing in the Shields Parameter based on the boundary roughness being taken as the sediment grain size, i.e., essentially disregarding the presence of the bed forms.

The approach outlined in the foregoing may be tested against some experiments performed by Manohar (1955) who undertook a study of bed form geometry and migration using the oscillating plate set-up. In contrast to the Berkeley experiments which were presented in Fig. 2, the oscillations of the sediment carrying plate were in this set of experiments not purely sinusoidal. An asymmetric motion was achieved by changing the radian frequency of the flywheel driving the plate when this was at its extreme positions while the excursion amplitude was held constant. The resulting forward and backward motions of the plate were essentially sinusoidal and choosing the subset of Manohar's (1955) experiments in which sediment transport occurred only during the forward motion a test of the procedure suggested above for sediment transport rates in the presence of bed forms is available. Thus, taking

$$\Psi_{\mathbf{m}}' = \frac{\tau'}{(\mathbf{s}-\mathbf{1})\rho g \mathbf{d}}$$
(16)

where the prime indicates that the boundary roughness is taken as the grain diameter, i.e., corresponding to skin friction, we may present Manohar's (1955) experiments whether bed forms were present or not as done in Fig. 3.

It is seen from Fig. 3 that the bed remained flat in some of the experiments. As shown by Manohar (1955) the reason for this is that ripples form only for a certain range of values of the transporting force. For low values of the transporting force, Ψ_m^* , the sediment transport is insufficient to cause the development of bed forms whereas the sediment transport is so intense for high values of Ψ_m^* that bed forms are washed away.



Figure 3: Empirical Relationship for the Average Rate of Sediment Transport in Oscillatory Flow (Plane and Rippled Bed).

The excellent agreement between the experimental data and the quasisteady application of the Einstein-Brown relationship, the full line in Figs. 2 and 3, suggests that the procedure of disregarding the presence of the bed forms and taking the boundary roughness as the sediment grain size for the purpose of evaluating the transporting forces may be applied with some confidence.

4. Some Aspects of the Prediction of Net Sediment Transport and Resulting Topographical Changes in the Coastal Environment

The sediment transport relationship, Eq. (8), established in the previous section may be used as the basis for a discussion of the problems involved in quantifying the factors which produce a net sediment transport in the coastal environment. It is emphasized that the following discussion, despite the experimental support of Eq. (8) presented in Section 3, must be considered qualitative since it involves considerable generalizations which await experimental confirmation before they may be accepted with confidence. With this in mind the sediment transport relationship, Eq. (8), may be generalized to read

$$\vec{\phi}(t) = 40 [\vec{\Psi}'(t)]^3$$
(17)

in which $\vec{\phi}(t)$ is the instantaneous dimensionless sediment transport vector given by Eq. (5) with the sediment transport rate, $\vec{q}_s(t) = (q_{s,x}(t), q_{s,y}(t))$.

The instantaneous value of the bottom shear stress based on the grain roughness is given by

$$\vec{\tau}'(t) = \frac{1}{2} \rho f_{g}[\vec{u}(t)]^{2} \frac{\vec{u}(t)}{|\vec{u}(t)|}$$
(18)

in which f_g is a generalized friction factor and $\dot{u}(t) = (u(t), v(t))$, is the instantaneous velocity vector.

Introducing Eq. (18) into (17) the following expressions in terms of the components of the sediment transport vector result

$$q_{s,x}(t) = 40wd \left[\frac{\frac{1}{2}\rho f_{s}(u^{2}(t) + v^{2}(t))}{(s-1)\rho gd}\right]^{3} \frac{u(t)}{\sqrt{u^{2}(t) + v^{2}(t)}}$$
(19)

$$q_{s,y}(t) = 40wd[\frac{\frac{1}{2}\rho f_{s}(u^{2}(t) + v^{2}(t))}{(s-1)\rho gd}] \frac{v(t)}{\sqrt{u^{2}(t) + v^{2}(t)}}$$
(20)

Eqs. (19) and (20) express the instantaneous rate of sediment transport in the x and y direction, respectively. In principle it is possible to evaluate these equations if the instantaneous velocity and the appropriate value of the generalized friction factor f_{a} are known. In general there is little interest in knowing the instantaneous value of the sediment transport rate. The quantity of importance is the time averaged value of the transport rate, i.e., the net sediment transport rate, since this quantity through the sediment continuity equation determines the rate of topographical changes. For the purpose of predicting the net sediment transport rate one would therefore time average Eqs. (19) and (20)

$$\vec{q}_{s,net} = \frac{1}{T} \int_0^T (q_{s,x}(t), q_{s,y}(t)) dt$$
(21)

where the integration is carried out over the time interval, formally identified as 0 to T', during which the sediment is in motion, i.e., when $|\overline{\Psi}'(t)| > \Psi_{mc}$. For a periodic motion of period T the average is, of course, taken over the period.

Once the net sediment transport rates have been determined the sediment continuity equation may be used to evaluate the rate of change in bottom elevation, $\partial n/\partial t$,

$$\frac{\partial q}{\partial \mathbf{x}}_{s,net,\mathbf{x}} + \frac{\partial q}{\partial \mathbf{x}}_{s,net,\mathbf{y}} = -(1-\varepsilon) \frac{\partial n}{\partial t}$$
(22)

in which the factor $1-\varepsilon$, where ε is the porosity of the sediment, is introduced to account for the fact that the net sediment transport rates are obtained in terms of the actual volume of sediment transported.

The preceding discussion explains the general use of the sediment transport relationship for the prediction of topographical changes. If the water motion is a sinusoidal oscillation, as discussed in Section 3, the net sediment transport is, of course, zero. The second order effects which, when added to a basic sinusoidal wave motion, will produce a net sediment transport are: (1) effect of a sloping bottom; (2) wave asymmetry; (3) wave induced mass transport currents; (4) currents other than mass transport currents.

4.1 The Effect of a Gently Sloping Bottom. The influence of a gently sloping bottom on the rate of sediment transport may qualitatively be examined by considering the entraining forces acting on a sediment grain on the sedimentfluid interface. Even under the influence of a purely sinusoidal wave propagating towards shallower water (up-slope) the gently sloping bottom will give rise to an asymmetry in the forces acting on a sediment grain and hence induce a net sediment transport. Thus, under the wave crest the bottom shear stress acting up the slope will be counteracted by the component of the submerged weight of the particle acting down the slope whereas the two forces will both act in the down-slope direction under a trough. This asymmetry in entraining forces will result in a net sediment transport towards deeper water. No experimental data are available on the influence of bottom slope on the rate of sediment transport in oscillatory flow. Until such data are available this effect cannot be quantified with confidence.

4.2 The Effect of Wave Asymmetry. For a small amplitude wave progressing over a horizontal bed, linear wave theory predicts a purely sinusoidal orbital

velocity above the bed. However, if finite amplitude, i.e., nonlinear, effects are considered the wave profile is no longer symmetric about the mean water level. For nonlinear waves the wave crests become more peaked (higher and steeper) than the wave troughs (shallower and flatter). This lack of symmetry of the surface profile is also reflected in the near-bottom velocity which shows a larger forward velocity of shorter duration under the wave crests and a smaller backward velocity of longer duration under the troughs than predicted by small amplitude wave theory. In principle one might therefore take the friction factor, $f_{\rm s}$, in Eq. (19) as Jonsson's wave friction factor $f_{\rm w}$ and evaluate the net sediment transport from Eq. (21). This procedure is applied to the experiments performed by Manohar (1955), of which a subset was presented in Fig. 3, with reasonably good agreement between predicted and observed net transport rates (Madsen and Grant, 1976, Fig. 9).

4.3 The Effect of Wave Induced Mass Transport Currents. When the analysis of the viscous flow in the bottom boundary layer associated with progressive waves is advanced to include second order effects a steady streaming in the direction of wave propagation is predicted based on the assumption of laminar flow in the boundary layer. This effect, which is referred to as mass transport, has been investigated theoretically by Longuet-Higgins (1953) and by Unluata and Mei (1970) under the assumption of laminar flow. For laminar flow in the bottom boundary layer this steady streaming, which immediately above the bottom is in the direction of wave propagation, will produce a non-zero time average shear stress in the direction of wave propagation acting on the bottom. This, in turn, would indicate that wave induced mass transport currents will result in a net sediment transport in the direction of wave propagation when the asymmetrical shear stress variation is introduced in the sediment transport relationship, Eq. (17).

Recent results reported by Bijker et al. (1974) clearly demonstrate that there are serious reasons to doubt the validity of Longuet-Higgins' solution in the case of a strongly turbulent boundary layer flow. As the roughness of the slope over which the waves propagate is increased from smooth to sand roughness to rippled bed the observed value of the near-bottom mass transport velocity is found to be increasingly smaller than the theoretical results based on a laminar flow assumption. In fact, the experiments with the slope roughness consisting of artificial ripples show a complete reversal of the direction of the near-bottom mass transport velocity. Bijker et al. (1974) conclude that much more data are needed to clarify the nature of mass transport in water waves when the boundary layer flow is turbulent.

In the present context of sediment transport by waves mass transport in addition to wave asymmetry discussed in Section 4.2 is undoubtedly an important factor in producing a net sediment transport. This makes it doubly unfortunate that the state of our knowledge is such that we cannot predict even the direction of this velocity with confidence not to mention its magnitude. For this reason further progress must be awaited before the effects of mass transport can be incorporated in the sediment transport relationship for the purpose of predicting net sediment transport rates associated with progressive waves. 4.4 <u>The Effect of Currents</u>. As a final second order effect which would produce a net sediment transport even if the wave motion were purely sinusoidal, the effects of a weak steady current superposed on the wave motion are considered. The action of these currents, for example, tidal or wind-induced, when combined with a wave motion will produce a net rate of sediment transport. If the current is weak, i.e., essentially a second order effect, the wave motion may be considered a stirring agent which by itself produces no net sediment transport. It does, however, make sediment available for transport by a current, although this current by itself would be incapable of even initiating sediment movement.

Assuming that the instantaneous velocity vector is known, this may be introduced in the general sediment transport relationship and one may numerically integrate Eq. (21) to obtain the time average value of the net sediment transport rates in the x and y direction, respectively. Thus, it is in principle quite simple to apply the general sediment transport relationship to determine the net sediment transport associated with the combined action of waves and currents. The major obstacle in performing this analysis is, however, the determination of the appropriate value of the generalized friction factor $f_s=f_{out}$ for the combined action of waves and currents. Presently it is possible to estimate with some confidence the bottom shear stress only for a pure wave motion or for a pure current from knowledge of the bottom roughness. As shown by Madsen (1976) the friction factor, f_w , for a pure wave motion may be an order of magnitude larger than the friction factor, fc, for a pure current of comparable magnitude, thus leaving the value of the friction factor for the combined action of waves and currents rather uncertain. In addition to this problem there is also some indication of a rather peculiar behavior of the sediment transport resulting from a weak current superimposed on a wave motion when the bottom exhibits bed forms. Thus, Inman and Bowen (1963) reported a net sediment transport in the direction opposite of the weak current superimposed on a wave motion. Although Inman's and Bowen's experiments and data analysis have certain deficiencies, as discussed by Madsen and Grant (1976), their observation and the experimental investigation of Natarajan (1969) show that the quantitative description of sediment transport over a rippled bed resulting from the combined action of waves and currents may present problems in addition to the problem of the determination of the wave-current friction factor f

Disregarding the problems mentioned above Grant and Madsen (1976) adopted a formula suggested by Jonsson (1966a) for the wave-current friction factor to investigate the topographical changes in the vicinity of the tip of a semiinfinite breakwater. The waves were assumed to be normally incident on the breakwater and a uniform current parallel to the breakwater produced with the diffracted wave pattern a spatially varying net sediment transport. The details of this investigation may be found in the above paper or in Madsen and Grant (1976). Here, the resulting prediction of erosion and deposition rates a distance of two wave lengths behind the breakwater are presented in Fig. 4. From Fig. 4 it is seen that no change, i.e., no sediment transport, occurs a distance of 3-4 wavelengths into the shadow region behind the breakwater. This is due to the decrease of wave activity in this region and emphasizes the importance of wave motion as being the stirring agent making sediment available for transport by currents which by themselves would be incapable of initiating sediment movement.





5. Concluding Remarks

It is not excluded that further analyses of the experimental data obtained by the Berkeley group may produce different sediment transport relationships for unsteady flow, just as the Einstein-Brown formula is far from being the only relationship for sediment transport in steady flow. Further documentation of the quasi-steady application of the Einstein-Brown sediment transport relation, suggested by the present investigation, is called for before this approach may be accepted with confidence.

The discussion given in Section 4 demonstrates an urgent need for futher research and an improved understanding of the nature of the flow in the oscillatory turbulent bottom boundary layer associated with waves and currents in order to treat the problem of sediment transport in the coastal environment in a general manner. The problems of the quantitative description of the fluid flow and its interaction with a solid bottom in the coastal environment must be overcome before one can hope to establish an accurate sediment transport model for this environment. It is hoped, however, that the present study has produced a framework within which sediment transport in the coastal zone may be approached in a rational manner. At the very least the present research serves to point out that it is absurd to attack the problem of sediment transport in the coastal environment without considering the influence of waves.

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References:

- Abou-Seida, M.M. (1965), U. Cal., Berkeley, Hydr. Engrng. Lab., Rep. 1. HEL-2-11, 78pp.
- 2.
- Bagnold, R.A. (1946), Proc. Roy. Soc. London, A, 187, p. 1-15. Bijker, E.W. et al. (1974), Proc. 14th Conf. Coastal Engrng., ASCE, 3. рр. 447-465.
- Brown, C.B. (1950), In: Rouse H., Ed., Engineering Hydraulics, John Wiley 4. and Sons, Inc., N.Y., 1039pp.
- Carstens, M.R. et al. (1969), U.S. Army, CERC, Tech. Memo No. 28, 39pp. 5.
- 6. Einstein, H.A. (1950), U.S. Dept. Agric. S.C.S., Tech. Bull. No. 1026.
- 7. Einstein, H.A. (1972), In: Meyer, R.E., Ed., Waves on Beaches and Resulting Sediment Transport, Academic Press, New York, 462pp.
- 8. Grant, W.D. and O.S. Madsen (1976), Proc. 3rd Inter-Agency Conf. Sed., W. Res. Council, p. 6.28-6.38.
- Horikawa, K. and A. Watanabe (1967), Coastal Engrng. Japan, 10, p. 39-57. 9.
- 10. Inman, D.L. and A.J. Bowen (1963), Proc. 8th Conf. Coastal Engrng., ASCE, p. 137-150.
- 11. Jonsson, I.G. (1966), Proc. 10th Conf. Coastal Engrng., ASCE, 1, p. 127-148.
- 12. Jonsson, I.G. (1966a), Basic Res. Pro. Rep. No. 11, Tech. U. Denmark, p. 1-12.
- 13. Kajiura, K. (1968), Bull. Earthquake Res. Inst., U. Tokyo, 46, p. 75-123.
- Kalkanis, G. (1964), U.S. Army, CERC, Tech. Memo No. 2, 38pp. Kennedy, J.F. and F.A. Locher (1972), In: Meyer, R.E., Ed., <u>Waves on</u> 14.
- 15. Beaches and Resulting Sediment Transport, Academic Press, N.Y., 462pp.
- Komar, P.D. and M.C. Miller (1973), J. Sed. Petrology, 43, 4, p. 1101-1110. 16.
- Komar, P.D. and M.C. Miller (1975), J. Sed. Petrology, 45, 2, pp. 362-367. Longuet-Higgins, M.S. (1953), Phil. Trans. Roy. Soc. London, A, 903, 17. 18.
- pp. 535-581.
- 19. Madsen, O.S. (1976), In: Stanley, D.J. and D.J.P. Swift, Marine Sediment Transport and Environmental Management, J. Wiley and Sons, N.Y., 600pp. Madsen, O.S. and W.D. Grant (1975), J. Sed. Petrology, 45, 2, p. 360-361. Madsen, O.S. and W.D. Grant (1976), R.M. Parsons Lab., MIT, Tech. Rep.
- 20.
- 21. No. 209, 105pp.
- 22.
- Manohar, M. (1955), U.S. Army, BEB, Tech. Memo, No. 75, 121pp. Natarajan, P. (1969), Ph.D. Thesis, U. of London, Imperial College. Rance, P.J. and N.F. Warren (1968), Proc. 11th Conf. Coastal Engrng., ASCE, 23. 24.
- p. 487-491.
- Shields, A. (1936), Mitteil. Preuss. Versuchsanst. Fur Wasserbau and 25. Schiffbau, Berlin, No. 26.
- Silvester, R. and G.R. Mogridge (1971), Proc. 12th Conf. Coastal Engrng., 26. ASCE, 2, p. 651-667.
- 27. Unluata, U. and C.C. Mei (1970), J. Geophys. Res., 75, 36, p. 7611-7617.
- 28. Vincent, G.E. (1958), Proc. 6th Conf. Coastal Engrng., ASCE, p. 326-354.