CHAPTER 62

NUMERICAL MODELS OF HUGE TSUNAMIS OFF THE SANRIKU COAST

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Abstract

Although numerical computations of the generation and propagation of tsunamis are successfully achieved in recent years, modeling of their wave sources is still a big problem. Three kinds of wave source model, that is statistical, oceanographic and fault model, are studied in this paper.

It is found that the first model gives reasonable wave heights as shown in the previous paper, the second one presents roughly one half of those for the first model and the last one produces too small wave heights.

Based on the analysis of computed results, nature of undulations off from the shore boundary, directivity of wave propagation and the spindle-shaped leading part are discussed.

Comparing magnitude of various wave parameters for the leading wave along the minor axis of the wave source, it is shown that the long wave approximation modified by the slope effect illustractes the tsunamis in deep region of the sea and the slope effect is most dominant in shallow region.

INTRODUCTION

For the numerical computation of the generation and propagation of tsunamis, one of the most important problem is the knowledge of the wave sources, such as their location, their dimensions and the motion of the sea beds. A set of empirical equations were proposed by the author in 1974 which gave dimensions of source ellipse assuming the earthquake magnitude M or the tsunami magnitude m[1] (Iwasaki, 1974, hereafter refered to as the previous paper). A wave source derived from these equations is called a statistical model in this paper since these equations are obtained from data collected statistically.

However concerning on the motion of the sea bed, uniform distribution with a rumped function is assumed in this model. This seems to much simple to be realistic.

Dr. I. Aida tried to extract information on the crustal deformation from available data for the Tokachi-oki Tsunami in 1968[2]. Performing numerical experiments for several models with various distributions of sea bed deformation in the wave source area and comparing amplitudes of water surface oscillations thus calculated with actual tide gauge records, he selected a model. Since bay oscillations were not taken into account in his analysis which should be included in actual records, his method of selection was somewhat immature. Neverthless this is one of more developed model. Since it is derived by comparison with actual marigrams, this model is called an oceanographic model.

The Alaskan 1964 Earthquake was an epoch-making which investigation started the study of huge earthquakes along submarine ditches since survey on the bed deformation due to the earthquake was successfully achieved which made possible the interpretation of source mechanism[3]. Until today there were three other examples for which survey on the bed deformation were accomplished, that is the Kanto 1923 Earthquake, the Nankaido 1946 Earthquake and the Chilean 1960 Earthquake. Owing to these experiences such huge earthquakes are explained as accompanied by the rupture of lithosheres. Thus although huge earthquakes off the Sanriku Coast have their sources in quite deep areas which make bottom survey impossible, estimation of source mechanism was presented by Dr. Kanamori[4][5][6]. He urged that the most reliable and quantitative data concerning the deformation of the lithosphere could be obtained by his method. Distributions of sea bed deformations thus derived are called fault models.

It is intended in this paper to obtain the most creditable information of tsunami waves by comparing results of numerical computations for three kinds of source models mentioned above with each other and with observed values. Then a number of special characteristics inherent to tsunami waves are discussed.

SOURCE MODELS FOR THREE TSUNAMIS

The Sanriku Coast is the name of the coast which situates in northern part of the main island of Japan facing the Pacific Ocean stretching from 38° N to 41.5° N. Although a lot of tsunamis have attacked there in the past, there were only three cases for which scientific records were obtained among those which originated off this coast and caused severe damage. They are the Sanriku Tsunami in 1896, the Sanriku Tsunami in 1933 and the Tokachi-Oki Tsunami in 1968.

Fig. 1 shows the region of computation where the Japan Sea Trench lies almost parallel with the Japan Island arcs.

Numerical Computation scheme and the boundary conditions are mostly similar with those in the previous paper in which inertia terms and friction terms are included. Alteration is that Corioli's terms are added and the time and space steps are changed from 10 seconds to 6 seconds and from 20 km to 10 km respectively on account of resolution of networks. Depth at the shore boundary is assumed as 20 meters so that the sea bed is not exposed by negative waves.

Fig 2 shows the wave sources of the statistical models for three tsunamis mentioned above which locations are estimated from inverse-refraction diagrams. Dimensions are computed by the equations proposed by the previous paper such that,

Fig. 1 Japan Islands and the Japan Sea Trench.

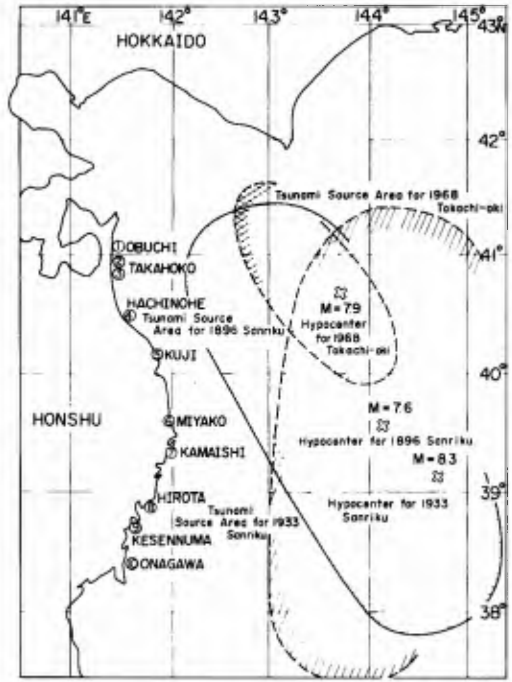


Fig. 2 Wave Sources of the Statistical Models for Three Tsunamis.

$$\ell = 10 \exp \left[(M - 6.27) / 0.76 \right] \tag{1}$$

$$\varepsilon = \tanh \left[1.5 \tanh \left(\left(\frac{\pi}{2012} \right)^{1/2} \cdot \ell^{2/3} \right) \right] \tag{2}$$

$$2a = \ell / \varepsilon \tag{3}$$

$$2b = \ell (1 - \varepsilon^2)^{1/2} / \varepsilon \tag{4}$$

$$S = \pi ab \tag{5}$$

$$m = 2.61 M - 18.44 \tag{6}$$

$$E_t = 10 \exp \left[0.6 m + 11.4 - \log_{10} 9.8 \right] \tag{7}$$

$$\eta_{max} = (2E_t / w_o S)^{1/2} \tag{8}$$

, in which m and M are the magnitudes of tsunami and earthquake respectively. ε, ℓ, a, b, S are the eccentricity, the distance between foci in km, the half length of the major axis in km, that of the minor axis in km and the area of the source in square km respectively. And E_t is the tsunami energy in ton-meter units. Table 1 shows the dimensions of wave sources for these three tsunamis.

Table 1. Dimensions of the Wave Sources for Statistical Models

Tsunami		M	2a(km)	2b(km)	η_{max}(m)	T(sec)
Sanriku	1896	7.6	307	272	8.15	96
Sanriku	1933	8.3	420	190	7.54	60
Tokachi-oki	1968	7.9	168	94	9.18	60

Although the earthquake magnitude of Sanriku 1896 was very small if compared with that of Sanriku 1933, the source area derived from the inverse refraction method was comparable with the latter. Dr. Kanamori illustrated this was due to the abnormal slow deformation with the time constant of about 100 secs of the former which was out of the instrument response at that time[6]. So, from eqs. (1) to (7), M is estimated so as to produce the dimensions of such source areas, which is M= 8.12. η_{max} in table 1. is derived by eq. (8) using this value. T is the duration time of earth movement and is selected to be approximately equal to the time constant proposed by Dr. Kanamori.

For another two tsunamis, source dimensions are derived using values of M in table 1 and T are given as illustrated in the previous paper.

Fig. 3 shows the oceanographic model for the Tokachi-oki 1968 presented by Dr. Aida[2]. It is noted that there is a region of subsidence in northwest part which is seperated by the longitude of 143°E from a region of upheaval where the maximum dislocation is roughly 5 m.

For another two tsunamis, such model is not obtained.

Fig. 4 shows fault parameters defining a faulting plane and its motions, which are a breadth W, a length L, a dislocation D, a dip direction ϕ, a dip angle δ and a slip angle λ with a duration time T and a rupture velocity ν. Displacements of the sea bottom can be computed from such fault parameters. Dr. Maruyama[7] and also Dr. Mansinha and Smylie[8] presented theories for them. Fig. 5 shows an example in which abscissa is in unit of breadth of the fault plane. In lower figure, the adverse dip slip planes with the dip angles are shown. In the upper figure, the vertical displacements of the surface of the earth are shown in unit of dip slip component. The length of horizontal bars show the horizontal displacements.

Fig. 6 shows the sea bed displacements for the Sanriku 1933 drawn from fig. 5 taking rapture velocity ν 3.5 km/sec, dip angle δ 45°, dip direction ϕ 90° and the time duration T 10 sec which were presented by Kanamori[4]. As shown in this figure, a zone of upheaval is recognized in the east half and a zone of subsidence in the west half. The motion is the normal faulting without the strike component. There is some ambiguity on the wave source motion. Thus we assume the displacement started at the fault line and propagated to east- and westward.

Fig. 7 shows for the 1969 Tsunami presented by Dr. K. Abe[9]. It was said this earthquake was caused by the low angle thrust faulting with a considerable strike-slip component, in which the ocean side underthrust beneath the continent.

Wave sources thus obtained are summerized in table 2.

Table 2 Wave sources

Tsunamis	Statistical models	Oceanographic models	Fault models
1896	x	–	–
1933	x	–	x
1968	x	x	x

MAXIMUM WAVE ELEVATIONS ABOVE STILL WATER LEVEL

A lot of data on the inundation levels were reported by many sources. Fig. 8, 9 and 10 show such data collected. Northern to Hachinohe, the coastline is monotonous with sloping beach. Since these area was still undeveloped, reliable informations were not available except few places. Remaining area is composed of a large number of bays with various size, shape and topography. Five bays which are Kuji, Miyako, Kamaishi, Hirota and Kesennuma and Onagawa are shown in more detail with dotted contour lines at bay entrances. It is noted that remarkable discrepancy is

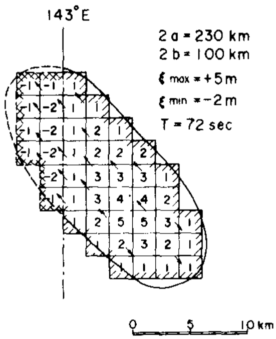


Fig. 3 The Oceanographic Model for the Tokachi-oki Tsunami in 1968 by Aida.

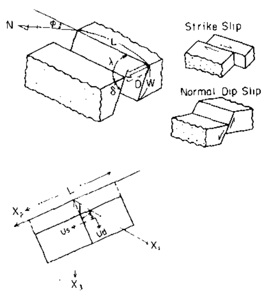


Fig. 4 Fault Parameters which define a Faulting Plane and its Motion.

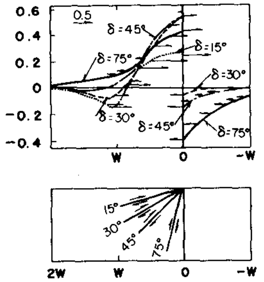


Fig. 5 Vertical and Horizontal Displacements of the Sea Bed due to the Adverse Dip Slip Faulting.
(The Vertical Scale is in Unit of Dip Slip Component.)

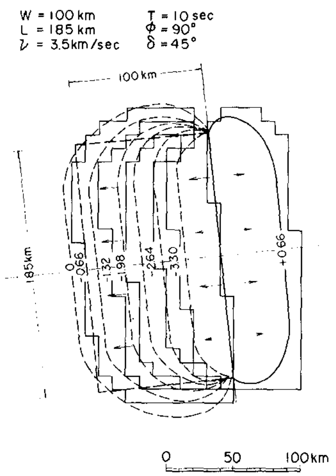


Fig. 6 The Fault Model for the Sanriku Tsunami in 1933.

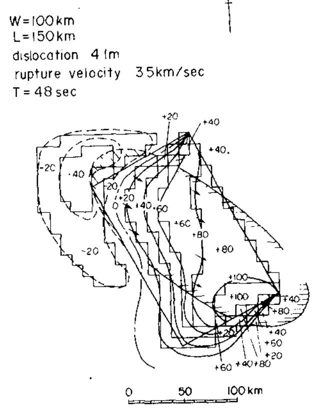


Fig. 7 The Fault Model for the Tokachi-oki Tsunami in 1968 by Abe.

recognized between maximum wave heights even in a same bay. So it is extremely difficult to examine justifiability of source models by comparison of numerical results with observed values.

One procedure tried in this paper is to calculate wave heights at 5m deep for sloping beach areas or those at bay mouths for remaining areas from those at output points which are 20 kms off the shore boundaries by the Green's law. These are shown in frames also in figs 8, 9 and 10.

It is noticed that computed wave heights by the statistical model give reasonable values at Kuji, Miyako, Kamaishi, Hirota and Kesennuma for the Sanriku 1896 Tsunami, but too large at Hachinohe and Onagawa. Also those derived for the Sanriku 1933 by the statistical model which are denoted by S in frames give acceptable values except Onagawa. However the statistical model for the Tokachi-oki 1968 produces too large wave heights.

The computed wave heights by the oceanographic model are probably too small and it seems that they should be doubled.

Results by the fault models are mostly very much small and those for the 1933 tsunami should be multiplied by 18 times in order to give comparable values with results by the statistical model. Similarly those for the 1968 tsunami are to be multiplied by 8 times.

However it should be noted that such estimation is reduced simply by comparison of numerals shown in figures without detailed inspection on hydrodynamic effects due to locality.

SURFACE OSCILLATIONS AT OUTPUT POINTS IN NEAR-COAST OFFSHORE REGION

In spite of the difference of the wave heights with each other, it is possible to examine oscillation pattern at output points in nearcoast offshore region since water depths of these points are deep enough such that effects of non-linear terms in equations of motion can be neglected and surface oscillations thus computed are linearly proportional to the amount of sea bed deformation which make possible relative comparison of wave pattern with each other by adjusting vertical scales.

Fig. 11 shows surface oecillations of the Sanriku 1933 Tsunami, in which full lines are results by the statistical model and dotted lines are plotted of values multiplied by 20 times of results by the fault model It is noticed that oscillation patterns are not so different except Obuchi and Hachinohe where free oscillations may be invoked in the water body bounded by the Hokkaido and the Honshu which are superposed on the oscillations over the continental shelf. Since the wave source is roughly parallel with the meridian line, the tsunami waves proceed to the coast as if one dimensional waves. So such indifferent pattern of undulations with the distributions of the sea bed movement lead an expectation such that one depressed and one elevated wave may be appeared forcedly at first which are not be effected remarkably by higher order components of the bed movement and following oscillations may be free oscillations over the

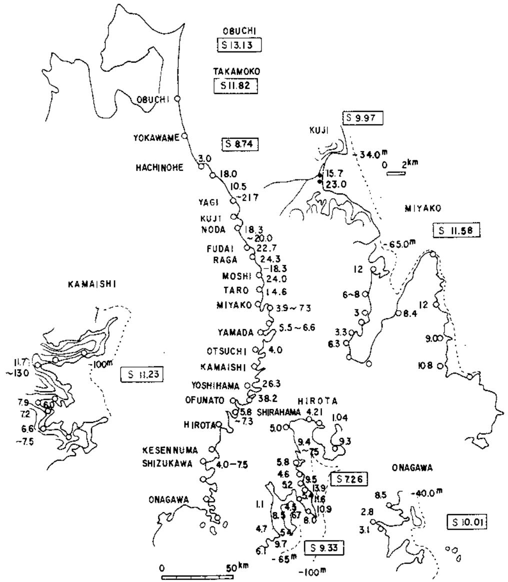


Fig. 8 Maximum Water Levels above MSL in meters for the Sanriku Tsunami in 1896. (Figures in Frames are computed Values. F for the Fault Model.)

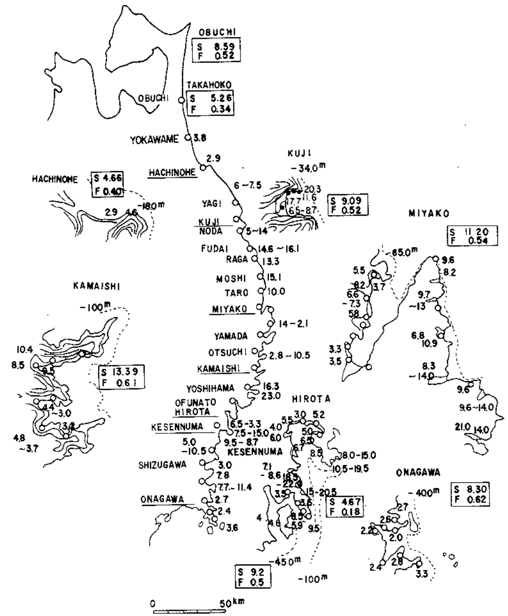


Fig. 9 Maximum Water Levels above MSL in meters for the Sanriku Tsunami in 1933. (Figures in Frames are computed Values. S for the Statistical Model F for the Fault Model.)

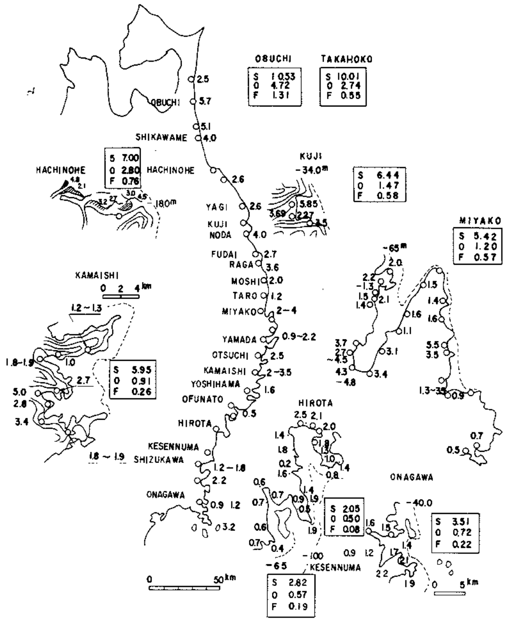


Fig. 10 Maximum Water Levels above T.P. in meters for the Tokachi-oki Tsunami in 1968. (Figures in Frames are computed Values.
S for the Statistical Model
O for the Oceanographic Model
F for the Fault Model.)

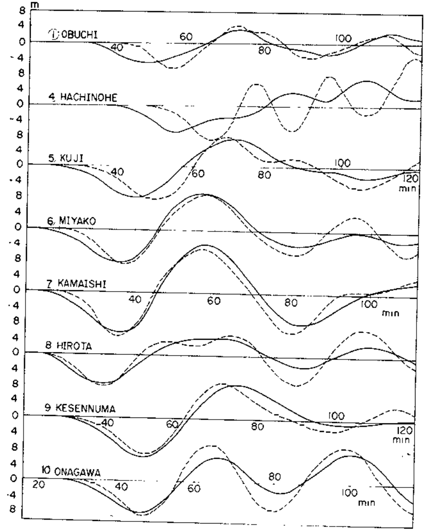


Fig. 11 Surface Oscillations of the Sanriku Tsunami in 1933.

continental shelf.

Oscillations of the 1968 tsunami are somewhat more complicated as shown in fig. 12. Full lines show waves by the statistical model, broken lines are obtained by the oceanographic one which are multiplied twice and dotted lines are those by the fault model multiplied by eight times. It is noticed that the first oscillations reflect the different pattern of the source movement. For the oceanographic and the fault models, the negative waves appear at first at Takahoko and Hachinohe and the positive waves appear at Miyako and Kamaishi which are also reported in field survey. At Kuji the sign of the first waves is opposite with each other. The marigram of this tsunami supports the oceanographic model. Free oscillations of the water body on the continental shelf and those of the water body bounded by the Hokkaido and the Honshu are also noticed.

By the way, readings of actual marigrams are plotted in figures for Hachinohe and Miyako by lines connecting white circles. However they seem to be incorrect informations, since time scale of original recording is too much reduced to make possible one week long recording in a sheet which is attached around a drum rotating once during a week.

SPATIAL WAVE FORMS

Fig. 13 and fig. 14 show spatial wave forms of the 1968 tsunamis by the statistical and the fault models respectively, in which horizontal axis are taken along the 41° N latitude between the most eastward end of the wave source and the coastal boundary at Takahoko. Figures a) are on the progressive stage, figures b) are when the leading fronts have arrived just at the coast and figures c) are when the maximum waves appear there.

The non uniform distributions of the sea bed displacement are reflected on the wave forms in figures a) and b) for progressive stages. The steepness of the leading waves is the order of 10^{-5}, which is very small when compared with those of surface waves.

At the coast, there occured standing waves and the maximum deviations from the still water are nearly twice of those of the progressive waves.

The spatial wave forms along the major and the minor axis of the source ellipse computed by the fault model for the 1933 tsunami are shown in fig. 15, in which lowest figures show the bottom topography. Along the major axis a depression wave is generated at first, which is disappeared in relatively short period. However along the minor axis, waves are propagated as if one dimensional waves, in which the leading part is composed of one negative and one positive wave. The dispersive nature is recognized and there seems to be so called ripples behind the leading part.

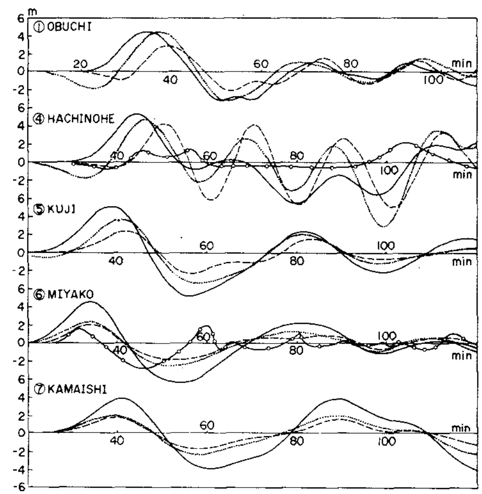


Fig. 12 Surface Oscillations of the Tokachi-oki Tsunami in 1968.

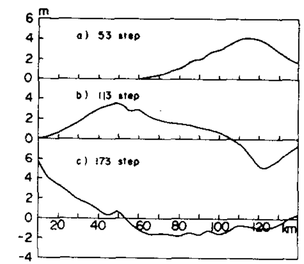


Fig. 13 Spatial Wave Forms of the Tokachi-oki Tsunami in 1968 by the Statistical Model.

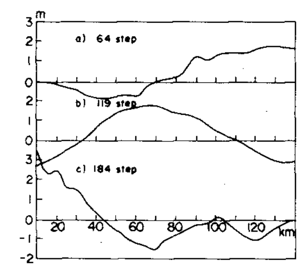


Fig. 14 Spatial Wave Forms of the Tokachi-oki Tsunami in 1968 by the Fault Model.

WAVE PARAMETERS ALONG THE MINOR AXIS

Various wave parameters are computed for the leading wave along the minor axis, which are the non-linear parameter a/h, the long wave parameter $(h/l)^2$, the slope parameter $h_x l/ h$ and the Ursell's parameter $U= al^2/h^3$, in which a, h and h_x are amplitude, depth and slope respectively and Ursell's parameters are computed by Dr. Raichlen and Dr. Hammack's method for which the inflection point of the leading waves is selected to define the slope and points of intersection of the tangent with the undisturbed and the highest wave elevations define the length l by their horizontal distance.

Fig. 16 shows these no-dimensional parameters with depth as abscissa. Where it is deeper than 3000 meters, the long wave parameter is nearly equal to the slope parameter and is much larger than the non-linear parameter. Shallower than 3000 meters, the slope parameters are dominant to the other two. So in a deep sea tsunami can be approximated by the long wave theory with modification by the slope effect. However in a shallower region theories which take into account the slope effect such as Green's law are most powerful for explaining tsunami waves.

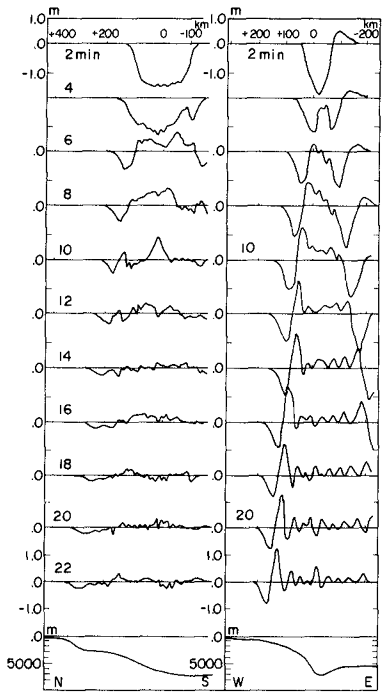
TWO DIMENSIONAL PROPAGATION

Thus far various aspects of tsunami waves are discussed mostly in one dimensional. However computations are made in two dimensional horizontal scheme, so features on two dimensional propagation are studied here.

Fig. 17, 18 and 19 show wave contours at 10, 20 and 30 minutes after the outbreak of the Sanriku earthquake in 1933 in which contours are drawn in every 10 cms by thin full lines and those in every 50 cms by heavy lines from results by the fault model. In fig. 15 it is recognized that the first dropdown proceeds westwards followed by ascension as reaction and the first rising up proceeds eastward with the following falling limb with large size. In fig. 17 these are found in relatively limited area which are spindle-shaped. Behind them, corrugated patches parallel with the major axis are noticed. As shown in fig. 18 and 19, wave fronts are becoming destorted by effects of bottom topography tending to be parallel with bottom contours. However the frontal spindles seem their size and the maximum wave height unchanged remarkably during procession.

The theory of the long wave approximation in shallow water with constant depth derived by Dr. Kajiura gives the following equation for the surface elevations.[10]

$$\eta (x, y, t) = \iint_S H_s \cdot P \, ds \tag{9}$$



a) along the Major Axis.

b) along the Minor Axis.

Fig. 15 Spatial Wave Forms of the Sanriku Tsunami in 1933 by the Fault Model.

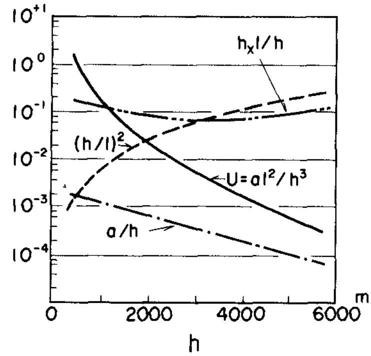


Fig. 16 Wave Parameters of the Leading Wave along the Minor Axis.

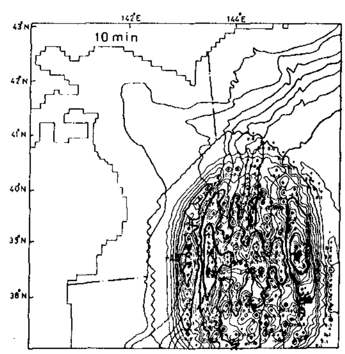


Fig. 17 Wave Contours at 10 min. after the Outbreak of the Sanriku Earthquake in 1933.

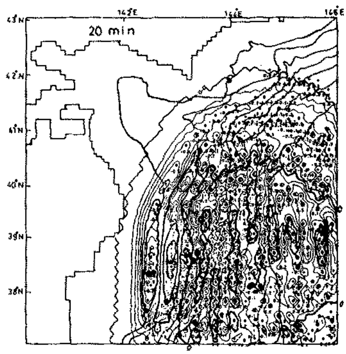


Fig. 18 Wave Contours at 20 min. after the Outbreak of the Sanriku Earthquake in 1933.

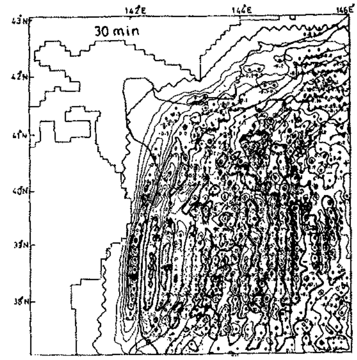


Fig. 19 Wave Contours at 30 min. after the Outbreak of the Sanriku Earthquake in 1933.

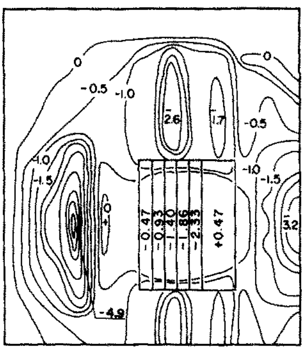


Fig. 20 Wave Contours at 20 min. after the Outbreak of an Earthquake computed by Kajiura's Theory setting the Distributions of the Sea Bed Displacements at the Sanriku Earthquake in 1933. Depth is assumed uniformly 3000 m.

, in which $P = -\frac{1}{2\pi} ct((ct)^2 - \bar{r}^2)^{-\frac{3}{2}}$, $c = (gh)^{1/2}$ and $\bar{r}^2 = (x-x_o)^2 + (y-y_o)^2$. Hs is the water surface elevation above still water level at the wave source (x_o , y_o), h is the water depth and ds is a surface element of the source area S respectively.

Assuming the same distributions of the surface elevation with those of the bottom displacements of the fault model for the 1933 tsunami and taking the water depth as 3000 m uniformly, wave height distributions 20 minuits after the outbreake of the earthquake are roughly estimated by eq. (9) which are shown in fig. 20. The spindle-shaped hump and depression around both ends of the minor axis and corrugated patches around both ends of the major axis are also noticed, which can be illustrated that contributions from source subelements are inaugurated along the minor axis and are cancelled along the major axis with each other since in the wave source the positive half and the negative half lie almost parallel with the major axis.

CONCLUSIONS

Although slight modifications are still necessary on the method of assuming locations and dimensions of wave sources, it is concluded that statistical models give reasonable wave heights at near-coast offshore points. Oceanographic model is not recommended since trial and error procedure to obtain it is troublesome and method of evaluation for selection of the most suitable model is doubtful.

Perhaps fault models are the most reliable ones as quoted by Dr. Kanamori on relative distibutions of sea bed deformations. However the absolute values proposed until now are too much small.

The leading part of tsunami is generated so as to reflect the pattern of the sea bed dislocation, propagates under the effect of dispersion and exites the forced oscillation in near-coast region where free oscillations follow it which periods are governed by oscillation systems there.

For the Sanriku 1933 tsunami, it is found a depression wave generated along the major axis disappeared in relatively short period and one drop down and one ascension form the leading part along the minor axis which proceed to the shallower region without change the wave height remarkably. This leading part forms spindle -shaped two dimensionally. Such directivity of tsunami is illustrated by the bottom movement with opposite sign against the major axis.

ACKNOWLEDGEMENT

The author wishes his acknowledgement to Master of Eng. Akira Mano, a graduate student of Tohoku University for his excellent achievement of

numerical calculations employing NEAC 2230-700 and CDC 6600.

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