CHAPTER 12

PARAMETRIC REPRESENTATION OF A WIND-WAVE FIELD

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SUMMARY

The applicability of various spectral shape parameters is discussed. The wave height distribution from 60 actual wave recordings is computed and compared to the Rayleigh distribution. The behaviour of various wave period parameters is discussed. Based on results from field data as well as numerical computations, it is concluded that some of the spectral wave parameters frequently used today may not be suitable for characterizing the wave field.

INTRODUCTION

Offshore activity in the North Sea has increased the need for wave data for the design of offshore structures. To overcome the increasing amount of wave data collected in the field, a parametric representation of the data is highly preferable.

This paper considers the usefulness of certain currently used wave parameters and distributions such as wave spectra, spectral width parameters, wave period parameters, and wave height distribution. It should be noted that the International PIANC Commission in their recent wave study /1/ recommended and stressed the need for such studies wherever possible.

The litterature on this topic is relatively extensive and the results do not appear to be consistent. Some of the discrepancies from the past may be explained from the fact that reliable wave data from the field may be difficult to collect, especially during severe weather conditions, and that the wave recording equipment applied may not have been reliable. A recent test of the shipborne Tucker wave recorder /2/ concludes that "the pressure signal contains a relatively high level of noise of any kind, in particular in cases of

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high wave conditions" /3/.

However, the accelerometer buoy produced by Datawell, Netherlands (the "Waverider" buoy), appears to be reliable /4/.

THE WAVE ENERGY SPECTRUM

It is generally believed today that the wave energy spectrum is able to express all the linear statistical properties of a wind-wave surface. However, the shape of the wave energy spectrum appears to be controversial. Based on the wave data available in 1964, Pierson and Moskowitz /5/ proposed a spectrum of the type

$$S_{PM}(f) = \alpha g^2 2 \pi^{-4} f^{-5} \exp \left[-\frac{5}{4} (\frac{f}{f_p})^{-4} \right]$$
(1)

where $S_{pM}(f)$ is the spectral density, f is the frequency (inverse of the wave period), f is the peak frequency (the frequency at which the spectral density reaches^D its maximum value) and α is Phillips' constant. This spectrum is widely used today for engineering applications /6, 7, 8, 9/.

More recently, K. Hasselmann et.al. /10/ arrived at another spectral formulation for fetch-limited, wave-growth conditions during the first Joint North Sea Wave Project (JONSWAP). This spectrum was shown to be much more sharply peaked than the corresponding Pierson Moskowitz type of spectrum. K. Hasselmann et. al. were able to fit the spectral shape of their observations to the following analytical expression:

(2)

$$S(f) = \alpha g^{2} (2\pi)^{-4} f^{-5} \exp\left[\frac{-5}{4} \left(\frac{f}{f_{p}}\right)^{-4}\right]$$
$$\gamma \exp\left[\frac{-(f-f_{p})^{2}}{2\sigma^{2} f_{p}^{2}}\right]$$
$$\sigma = \begin{cases} \sigma_{a} \text{ for } f < f_{p} \\ \sigma_{b} \text{ for } f > f_{p} \end{cases}$$

where

where S(f) is the spectral density of the spectrum (denoted the JONSWAP spectrum), α is the Phillips'/ll/ constant (which was shown to vary considerably with the fetch and the wind speed), g is the acceleration of gravity, γ is the ratio of the maximal spectral energy to the maximum of the corresponding Pierson-Moskowitz spectrum (l) with the same values of α and f; σ_{a} and σ_{b} define the left and right side widths, respectively, of the spectral peak. Fig. l illustrates the physical interpretation of the parameters α , f, γ , σ_{a} and σ_{b} . It is noted that for $\gamma = 1$, the JONSWAP spectrum reduces to a Pierson-Moskowitz type spectrum.





The shape factor γ was found to have values up to 7 with an average value of 3.3. Therefore the wave energy spectrum appears to be much more peaked during wave growth conditions than in the PM-spectrum. Fig. 2 illustrates the JONSWAP spectrum with a γ factor equal to 7 and the corresponding Pierson-Moskowitz spectrum. For a brief review of the JONSWAP results see /12/.

The JONSWAP results were obtained during short-fetched, light wind conditions. However, recently obtained storm wave energy spectra indicate that the spectral shape compares reasonably well to the JONSWAP spectrum during more severe wave conditions also /13, 14, 15, 16/.

The spectral shape of the wave energy field is important to consider for a number of engineering applications. When transfer functions for linear systems are applied, a too low spectral peak in the wave-spectral input may underestimate the forces on an oscillating structure tuned to the peak frequency. D. Hoffmann /9/ stresses the importance of varying the spectral shape when wave loads on ship structures are considered. Wave group formation (i.e. that large waves tend to succeed each other in one single run) tends to be more pronounced for a narrow (or sharply peaked) spectrum /17, 18, 19, 20/. P. Bruun /21/ stresses the importance of wave-group occurrencies in the wave field for the breakdown of breakwaters exposed to extreme sea states. Also, for the slow-drift oscillations of moored structures (i.e. oscillations with periods much larger than the periods present in the wave spectrum) the shape of the wave spectrum appears to be an important factor /16, 22/. These oscillations tend to respond to the wave height envelope period /23, 24, 25/ rather than to the individual wave period.



Fig. 2. Illustration of the sharply peakedness of the JONSWAP spectrum. α = arbitrary. The frequency scale is normalized to the peak frequency f. Two values of γ are shown, γ = 7 (full line) and $\gamma \stackrel{P}{=} 1$ (broken line). σ_a = 0.07 and σ_b = 0.09 which correspond to the average values found during the JONSWAP experiment /10/.

SPECTRAL SHAPE PARAMETERS

The large variations in the spectral shape found for field data /8, 9, 10, 26/ indicate the need of spectral shape parameters. One parameter which has been applied extensively is the spectral width parameter ϵ defined by ${m_2}^2$

$$\varepsilon^2 = 1 - \frac{m_2}{m_0 m_4} \tag{3}$$

where the moment m of the wave-energy spectrum is defined by

$$m_n = \int_0 S(f) f^n df$$
(4)

The spectral width parameter was introduced by Cartwright and Longuet-Higgins /27/ in order to describe whether the wave energy was concentrated within a narrow frequency band (ε +0) or not (ε +1).

Another parameter ν , applied by Longuet-Higgins /17/ to describe the narrowness of the spectrum, is defined

$$v^2 = \frac{m_2 m_0}{m_1^2} - 1 \tag{5}$$

A similar parameter v_1 , also suggested by Longuet-Higgins /29/, is defined

$$v_1^2 = 1 - \frac{m_1}{m_2 m_0} \tag{6}$$

A parameter $\rm Q_p$ to describe the peakedness of a wave spectrum, was introduced by Goda /18/ $\,$ and is defined by

$$Q_{\rm p} = \frac{2}{m_0^2} \int_{0}^{1} f[S(f)]^2 \, df \tag{7}$$

All these parameters are supposed to characterize the distribution of the wave energy on the frequency scale. However, it appears that some of these parameters depend on the choice of the high-frequency cut-off when the moments (4) are computed. Figs. 3-6 show the dependency of these parameters on the choice of the high-frequency cut-off frequency. The JONSWAP spectrum (2) was applied for S(f), and the integrations were carried out numerically.

It turns out that the parameter Q_p is the only one which is not dependent on the high-frequency cut-off choice. In addition, the parameter seems to distinguish very well between a very sharply peaked JONSWAP spectrum (γ =7) and a Pierson-Moskowitz type spectrum (γ =1). It is therefore recommended that this parameter be applied rather than the parameters $\epsilon,$ v or v₁.

From Fig. 6, it may even be considered to establish a relationship between γ and $Q_{\rm p}.$ This relationship is shown graphically in Fig. 7 for a specific choice of $\sigma_{\rm a}$ and $\sigma_{\rm b}.$

The reason why ε varies with the choice of the high-frequency cut-off in the numerical integrations, is that the wave energy spectra are found to have a high-frequency tail proportional to f⁵. When the fourth moment of such a



Fig. 3. The spectral width parameter ε as a function of the high-frequency cut-off choice. ε is computed from (3). α = arbitrary. The frequency scale is normalized to the peak frequency f. Two values of γ are shown. $\sigma_a = 0.07$. $\sigma_b \stackrel{P}{=} 0.09$.



Fig. 4. The parameter V as a function of the high-frequency cut-off choice. V is computed from (5). α = arbitrary. The frequency scale is normalized to the peak frequency f_p. Two values of γ are shown. σ_{a} = 0.07. σ_{b} = 0.09.



Fig. 5. The parameter v_1 as a function of the high-frequency cutoff choice. v_1 is computed from (6). α = arbitrary. The frequency scale is normalized to the peak frequency f. Two values of γ are shown. σ_a = 0.07. σ_b = 0.09.



Fig. 6. The peakedness parameter Q_p as a function of the highfrequency cut-off choice. Q_p is computed from (7). ' α = arbitrary. The frequency scale is normalized to the peak frequency f. Three values of γ are shown. σ_a = 0.07. σ_b = 0.09.





spectrum is computed, the result will be proportional to the natural logaritm of the high-frequency cut-off. In fig. 3, ε is shown to vary between 0.4 and 0.8 for a Pierson-Moskowitz type spectrum, dependent on the cut-off frequency. In addition, ε appears to distinguish poorly between a very sharply peaked (γ =7) wave energy spectrum and a Pierson-Moskowitz type spectrum (γ =1). These facts may explain some of the conflicting results reported in the litterature /14, 30, 31/.

It is possible to obtain a rough estimate of ϵ directly from a wave record /18/ by applying the formula

$$\varepsilon^2 = 1 - \left(\frac{N_0}{N_1}\right) \tag{8}$$

where N_1 is the number of maxima and N_0 is the number of zero-up-crossings in the wave record. However, the number N_1 will depend on the resolution of the wave recorder; a recorder with a large resolution will record more maxima than a recorder with a small resolution and thus enlarge the value of ε . This effect will be similar to the effect of choosing a larger highfrequency cut-off when ε is computed from the moments of the spectrum.

THE WAVE HEIGHT DISTRIBUTION

When the spectrum is narrow, it can be shown that the wave height distribution is approximately a Rayleigh distribution /32/. This distribution has been compared to the actual distribution of the wave heights recorded, and the agreement appears to be good /18, 28, 30, 32, 33/.

In order to compare wave height distribution from actual wave recordings to the Rayleigh distribution, 60 recordings of wind-generated waves were examined. The data were collected with a "Waverider" accelerometer buoy located outside Utsira, Norway. The location is shown in Fig. 8. The water depth was about 100 m. The duration of each recording was between 8 and 20 minutes.



Fig. 8. Location of the "Waverider" wave recorder. From /16/.

The data applied for the analysis were collected from three storms, occurring in October, November and December 1970. All of the recordings were selected so that some waves larger than 4 m were present in each recording.

Wave heights lower than 0.5 m were excluded because these waves were expected to be affected by the resolution of the recording insturment. By excluding these waves, the results were expected to improve /33/.

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Fig. 9 shows that the wave heights follow the Rayleigh distribution comparatively well. In order to include all of the 60 recordings on the same diagram, the average wave height \overline{H} was used as unity for each of the recordings.



Fig. 9. The distribution of the wave heights from 60 wave recordings. The wave heights are determined from the zero-up-cross method. The heights are normalized to the average wave height of each recording.

THE WAVE PERIOD PARAMETERS

The distribution of the wave periods appears to be more controversial than the distribution of the wave heights. Bretschneider /34/, using semi-empirical relationships, proposed a Rayleigh distribution of the zero-up-crossing wave period squared (T_2^2) which has been applied with variable success /18, 33/. Recently, however, Longuet-Higgins /28/ has arrived at a theoretical wave



Fig. 10. Illustration of the average periods T and T $m_{0,2}$ for a very sharply peaked JONSWAP spectrum. α = arbitrary. The frequency scale is normalized to the peak frequency f. γ = 7. σ = 0.07. σ = 0.09. T is determined according to (11). a T is determined from p(9). T is determined from (10).



Fig. 11. Illustration of the average periods T_{mol} and T_{mol} for a Pierson-Moskowitz type spectrum. $\alpha = \frac{m_{Dl}}{ar_{Dl}}$ The frequency scale is normalized to the peak frequency f. $\gamma = 1$. $\sigma = 0.07$. $\sigma_{b} = 0.09$. T_p is determined according^Pto (11). T_{mol} is determined from (9). T_{mo2} is determined from (10).

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period distribution which seems to compare well with observations.

A number of wave period parameters have been proposed for practical applications. Based on the moments of the spectrum, an average wave period T_{mol} may be defined /9, 28/.

$$T_{mOl} = \frac{m_0}{m_1}$$
(9)

Sometimes the average periode T_{m00} is also defined /9, 30, 35, 36/.

$$\Gamma_{m02} = \left(\frac{m_0}{m_2}\right)^{\frac{1}{2}}$$
(10)

In /9/, T_____ is denoted "the average apparent period".

The peak period T (inverse of the peak frequency f) is obtained by derivating the wave spectrum equation:

 $\frac{d}{df} [S(f)] = 0 \tag{11}$

This period corresponds to the frequency of maximum energy density in the wave spectrum /9, 10/.

The period parameters T_{m01} and T_{m02} were computed by numerically integrating the JONSWAP spectrum. Figs. 10 and 11 show the results from the computation. For a very sharply peaked JONSWAP spectrum (γ =7), T_{m01}/T_p was found to be ~ 0.885 while T_{m02}/T_p was found to be ~ 0.880. For a Pierson-Moskowitz type spectrum (γ =1), T_{m01}/T_p ~ 0.785 while T_{m02}/T_p ~ 0.735.

The results were also found to depend on the high-frequency cut-off choice as shown in Fig. 12. The numbers above are given for a high frequency cut-off choice approximately equal to 4 f_{p} .

For the JONSWAP spectrum, the main part of the energy will be concentrated within a narrow frequency band about the peak frequency. The largest waves (the most important for engineering purposes) will have their periods very close to the peak period because these waves are expected to contribute most to the wave spectrum. However, the computations in this paper show that for a relatively narrow, singel-peaked wave energy spectrum with a peak period of, say, 10 seconds, the "average period" (T $_{\rm MO1}$ or T $_{\rm MO2}$) is expected to fall within the range 7.1 - 8.9 seconds, dependent of the high-frequency cut-off choice and the peakedness of the spectrum. This computation therefore suggests that the average wave periods computed by means of the moments of the spectra will conclusion is consistent with Goda /30/ which states that T $_{\rm meq}^{02}$.

Another wave_period parameter frequently applied is the average "zero-up-cross" wave period ${\rm T_z}$. This parameter is defined as the ratio between the number of zero-up-crossing of the wave trace and the total duration of the wave record. This parameter depends on the resolution of the recorder. A larger number of zero-up-crossing will be recorded if the resolution is improved. By means of numerical simuation, it has also been shown by Goda /18/ that ${\rm T_z}$ varies with the spectral shape.



Fig. 12. The "average" wave periods T_{m01} and T_{m02} as a function of the spectral shape parameter Y and the high-frequency cutoff choice. T_{m01} is normalized to the peak period T and the high-frequency cut-off choice is normalized to the peak frequency f. The following parameters in the JONSWAP spectrum were applied: α = arbitrary, f = arbitrary, $\sigma_a = 0.07, \sigma_b = 0.09$.

Based on field data, a low correlation has been found /37, 38, 39/ between \overline{T}_{a} and the period of the maximum energy density. It is therefore concluded that the period \overline{T}_{a} does not appear to be a convenient parameter to characterize the periods of the sea waves.

However, recent findings indicate that the periods of the highest waves in a wave record might be applicable. Thompson /37/ suggests that the average period in trains of large waves in the wave record might be a useful period parameter to apply, and he shows that this period will be very close to the spectral peak period T. Goda /30/ concludes that the significant wave period is statistically stable. This parameter appears to be only about 5% smaller than the spectral peak period.

The number 5% is in good agreement with the relation between the peak period $\rm T_{p}$ and the significant wave period given by Bretschneider /40/.

$$r_{\rm p} = \frac{\sqrt[4]{5/4}}{1} r_{\rm S}$$
(12)
$$\approx 1.06 r_{\rm c}$$

Also, Earle/41/ considered the relationship between the period of the max. wave height T_{HMAX}, T_S and \overline{T}_2 . He also finds that \overline{T} needs a correlation of about + 20%; $HMAX, T_S \approx 0.8 T_S$. For T_{HMAX} , he finds it approximately equal to T_S , but T_{HMAX} is less statistically HMAX, stable than T_S .

A small check on these results was applied on wave data from the North Sea. A quarter of the 60 recordings previously mentioned were selected for spectral analysis. Wave recordings from a growing wind-wave field were digitized and the spectrum computed by means of an FFT algoritm. The peak period T was determined from the spectrum and the period of the maximum wave height was found directly from the wave record. Fig. 13 shows the results. Also shown, is the significant wave height determined from $4\sqrt{m_0}$. Fig. 13 indicates that the period of the maximum wave tends to slightly underestimate the period of the spectral peak. The finding is consistent with the results from Goda's investigation /30/, where the period of the maximum wave was found, on the average, to be only about 5% lower than the period of the spectral peak.



Fig. 13. Comparison of the spectral peak period T_p and the period of the maximum wave height for 15 actual wave recordings from Utsira. The recordings are selected from a growing windwave field.

It is concluded that the peak period T and the significant wave period T_S characterize the periods of the largest waves in the recording better than the various average wave periods \overline{T}_z , T_{m01} or T_{m02} do. It is therefore recommended that T_p or T_S are applied rather than the various "average" periods.

CONCLUSIONS

The results derived in this paper may be summarized as follows:

- 1. The spectral width parameter is shown to vary considerably with the high-frequency cut-off choice. The spectral peakedness parameter Q_p , introduced by Goda /18/, appears to be a more convenient parameter with which to characterize the spectral distribution.
- 2. The wave height distribution from a wind-wave field compares very well to a Rayleigh distribution.
- The period of the spectral peak or the significant wave period will correspond relatively close to each other and will characterize the period of the largest waves present in the recording better than the "average" wave periods do.

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PARAMETRIC REPRESENTATION

<u>APPENDIX - NOTATION</u>		
f	=	frequency or the inverse of the wave period
fp	Ξ	frequency of the spectral peak
g	=	acceleration of gravity
Ĥ	=	average wave height
m	=	n'th moment of the spectrum
N	=	number of zero-up-crossings in a wave record
N	.=	number of maxima in a wave record
Qp	=	spectral peakedness parameter
S	=	spectral energy density distribution
T _{m01}	}	"average" wave periods
T p) =	period corresponding to the spectral peak frequency f $${\rm p}$$
T	Ξ	significant wave period
Tz	=	zero-up-crossing wave period
\bar{T}_z	=	average zero-up-crossing wave period
α	=	Phillips' constant in the wave energy spectrum
γ	=	spectral shape factor in the JONSWAP spectrum
$\left. \begin{array}{c} \varepsilon \\ v \\ v_1 \end{array} \right\}$	=	spectral width parameters
σ_{a}^{σ}	=	parameters to describe the broadness of the spectral peak in the JONSWAP spectrum

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