CHAPTER 8

APPLICATIONS OF EMPIRICAL FETCH-LIMITED SPECTRAL FORMULAS TO GREAT LAKES WAVES[1]

Paul C. Liu, M. ASCE[2]

ABSTRACT

Two episodes of Great Lakes waves for which both wind and wave data are simultaneously available are used to examine the applicability of the empirical fetch-limited spectral wave formulas developed by JONSWAP, Mitsuyasu, Liu, and Sverdrup-Munk-Bretschneider. Comparing the results hindcast from the formulas with those recorded shows that, for hindcasting significant wave heights, Liu's formula gives better results for less than fully developed waves, while formulas by JONSWAP, Mitsuyasu, and Sverdrup-Munk-Bretschneider give better results for fully developed waves. In hindcasting average wave periods and peak-energy frequencies, all the formulas result in a deviation of up to 2 s and 0.5 rad s^{-1}, respectively. These results can be used as a reference in evaluating and interpreting wave predictions made by these formulas as applied to the Greak Lakes.

1. INTRODUCTION

Various surface wave studies in recent years have results in the development of several sets of empirical fetch-limited spectral formulas of practical interest in wave predictions. These formulas are all characterized by the dimensionless fetch parameter $X_o = gX/U_*^2$, where g is the acceleration of gravity, X the fetch distance, and U_* the frictional wind velocity. In terms of X_o, the significant wave height $H_{1/3}$, peak-energy radian frequency ω_p, and spectral energy density $S(\omega)$ as a function of frequency ω can be expressed by

$$H_{1/3} = A \frac{U_*^2}{g} X_o^{a} \tag{1}$$

$$\omega_p = B \frac{g}{U_*} X_o^{-b} \tag{2}$$

$$S(\omega) = C\, g^2 \omega^{-5} X_o^{-C} \exp\left[-D X_o^{-d} \left(\frac{U_* \omega}{g}\right)^{-4}\right] F \tag{3}$$

in which A, B, C, and D = the empirical coefficients; a, b, c, and d = the empirical exponentials; and F = an empirical spectral shape function. Several authors have developed various sets of numbers for these empirical constants. Table 1 represents a summary of the results obtained from these studies. Data

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[1] GLERL Contribution No. 93.

[2] Physical Scientist, Great Lakes Environmental Research Laboratory, National Oceanic and Atmospheric Administration, Ann Arbor, Michigan 48104.

TABLE 1.--Empirical Constants Used by Various Authors in Eqs. (1), (2), and (3)

Author	$A \times 10^2$	B	C	D	a	b	c	d	F
Garratt (1973)	1.264	4.10			0.600	0.300			
Hasselmann et al. (JONSWAP, 1973)	5.060	6.80	0.348	2,665	0.500	0.330	0.220	1.320	$\gamma^{\exp\frac{-(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}}$ *
Kononkova et al. (1970)		14.80				0.360			
Liu (1971)	1.740	8.57	0.400	5,500	0.542	0.333	0.250	1.333	1
Mitsuyasu (1971)	4.520	6.28	1.337	1,948	0.504	0.330	0.312	1.320	1

* γ = ratio of maximum spectral energy to maximum of fully developed Pierson-Moskowitz spectrum $\sim$ 3.3.

σ = spectral width factor, $\sigma = 0.07$, $\omega < \omega_p$; $\sigma = 0.09$, $\omega > \omega_p$.

collected from many oceans and lakes, as well as laboratory wave tanks, were used. Table 1 reveals that some of the constants derived by these authors are quite close; some of the constants, however, differ significantly. It is of interest to note that, although the authors used their own wave measurements in developing their empirical formulas, they all made an effort to incorporate the results of well-known wave studies published during the past 20 years. Obviously, some of the differences in the coefficients can be attributed to the authors' subjectivity in adjusting the regression line to fit their own data. Because of these differences and because of the simple nature of the formulas, case studies have been made to examine the applicability of the formulas by using available wave data from the Great Lakes. This paper presents two episodes of wave conditions for which recorded data are compared with those hindcast by the formulas of JONSWAP, Mitsuyasu, and Liu. In addition, conventional hindcasts from the Sverdrup-Munk-Bretschneider formulas (Shore Protection Manual, Vol I, 1973) are also included for further comparisons.

2. CHARACTERTISTICS OF THE FORMULAS

Before proceeding to the case studies, it is of interest to examine some of the basic characteristics of the formulas that use hypothetical wind conditions. Because frictional wind velocity was not available for this study, logarithmic wind profile and a drag coefficient, C_{10} of 10^{-3} have been used in this paper for converting wind speed at different levels to U_*. Fig. 1 presents a set of examples of wave spectra calculated by empirical formulas (3) for 10-m level wind speeds of 5 and 40 m s^{-1} and fetch distances of 10 and 100 km. As the four cases shown in the figure combine long and short fetches with high and low wind speeds, respectively, the magnitude of spectral energy and peak-energy frequency varies significantly from case to case. However, the relative differences among the three empirical spectra are similar for all cases. In general, JONSWAP and Mitsuyasu spectra are fairly close, especially at high wind speeds, while Liu spectra yield much lower energy content. The energy at the spectral peaks for JONSWAP and Mitsuyasu are approximately an order of magnitude larger than those of Liu. An explanation for this great difference lies in the fact that the wave data used in developing the JONSWAP and Mitsuyasu formulas are from fully developed seas, whereas the wave data from Lake Michigan used in Liu's formula are not.

With energy spectra calculated from the empirical formulas, the spectral moments can be readily obtained. The basic parameters, such as significant wave heights and average wave periods, can then be derived from the moments, based on the theoretical results of Longuet-Higgins (1952) and Rice (1945), as:

$$H_{m_0} = 4{m_0}^{1/2} \tag{4}$$

and

$$T_{m_{0,2}} = 2\pi(m_0/m_2)^{1/2} \tag{5}$$

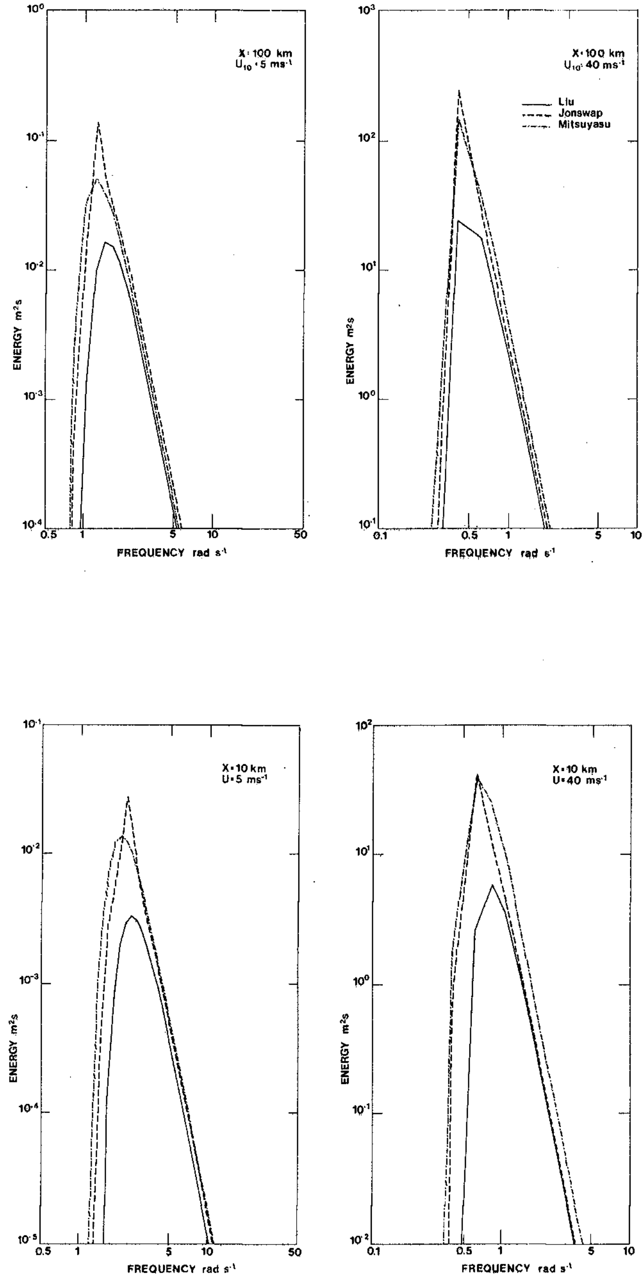

Fig. 1 Examples of wave spectra calculated by empirical formulas.

in which the nth moment of a wave spectrum is given by

$$m_n = \int_0^{\infty} \omega^n S(\omega)\ d\omega. \tag{6}$$

Here the average wave period $T_{m0,2}$ is presumably equivalent to the wave period given by the Sverdrup-Munk-Bretschneider method. The significant wave height H_{m0} calculated by Eq. (4) should approximate $H_{1/3}$, calculated by (Eq. 1). A correlation of calculated H_{m0} and $H_{1/3}$ is given in Fig. 2. In the case of the JONSWAP method, H_{m0} tends to be larger than $H_{1/3}$. A slightly greater trend is also shown in Mitsuyasu's case. Since Liu's empirical equiation for $H_{1/3}$ was a direct derivation from the spectral moments, the close correlation shown is not surprising. In the following discussions, $H_{1/3}$, ω_p, and $T_{m0,2}$ will be calculated from the various empirical formulas and compared with actual data.

3. APPLICATIONS OF THE FORMULAS

Two episodes of Great Lakes waves, with both wind and wave data available, are used to examine the applicability of the various empirical formulas. The first episode occurred in Lake Michigan. Wave data were recorded by a staff gage installed on a research tower located 2 km offshore from Muskegon, Michigan, in 16 m of water (Fig. 3). The wind anemometer was located at 10 m above the lake surface on the same tower. The episode covered approximately 4 days during October 25th through 28th of 1971. The wind conditions, which are typical of the Great Lakes, are shown in Fig. 4. The wind speeds during this episode were quite unsteady with the predominant direction from the south and south-southeast providing moderate fetch distances of the order of 50 km.

Fig. 5 represents the significant wave heights $H_{1/3}$ recorded during this episode as compared with those hindcast by empirical formulas. Here and in the following figures in this paper, the heavy solid lines indicate recorded data; the dotted lines, the long-short dashed lines, the dashed lines, and the light solid lines indicate the hindcast data by Sverdrup-Munk-Bretschneider, Mitsuyasu, JONSWAP, and Liu, respectively. The results of Sverdrup-Munk-Bretschneider, Mitsuyasu, and JONSWAP, shown in Fig. 5, are strikingly close. In this episode, they all overestimated the recorded data by a factor of 2. Liu's results are relatively lower than the others since the energy contents given by Liu's formula are generally less than the others. In this case, Liu's results are closer to the recorded results than the others.

Fig. 6 shows the comparisons for the average wave period $T_{m0,2}$. The relative differences between the results of different authors seem quite similar and distinctive. Sverdrup-Munk-Bretschneider, for the most part, furnished the highest estimate, followed in order by Mitsuyasu and JONSWAP. Liu hindcast relatively lower average wave periods for the same wind condition.

The results for the peak-energy frequency ω_p are shown in Fig. 7. The Sverdrup-Munk-Bretschneider formula does not give estimations for ω_p; hence,

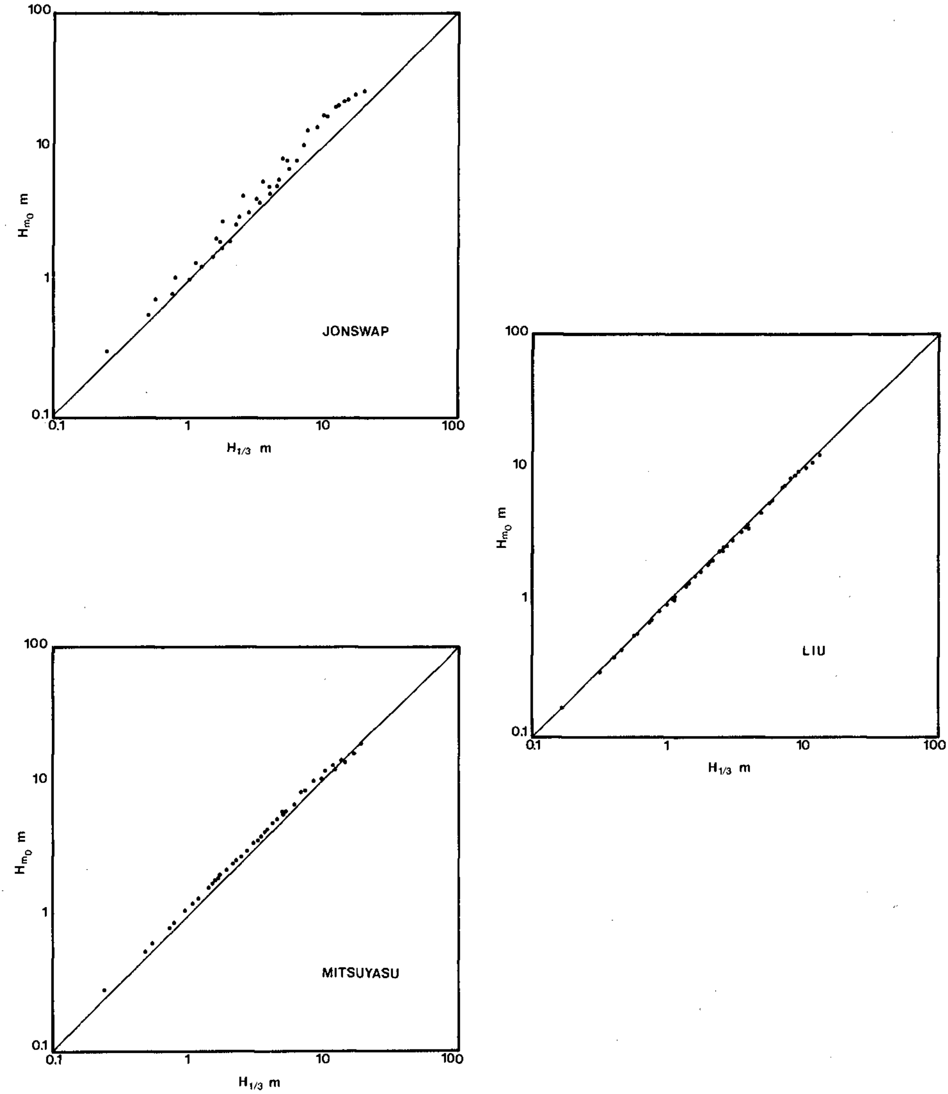

Fig. 2 Correlations of H_{m_0} and $H_{1/3}$ calculcated by empirical formulas.

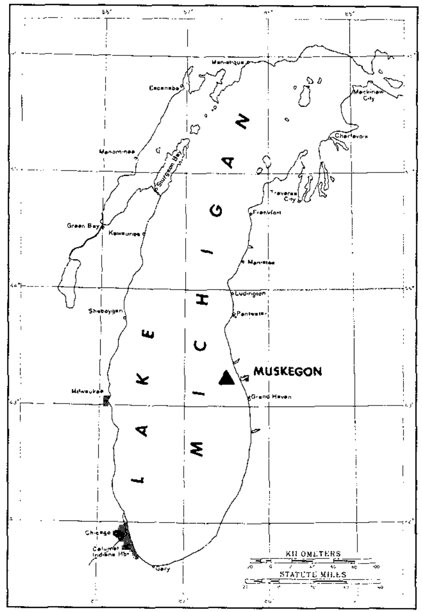

Fig. 3 Location of wave gage in Lake Michigan.

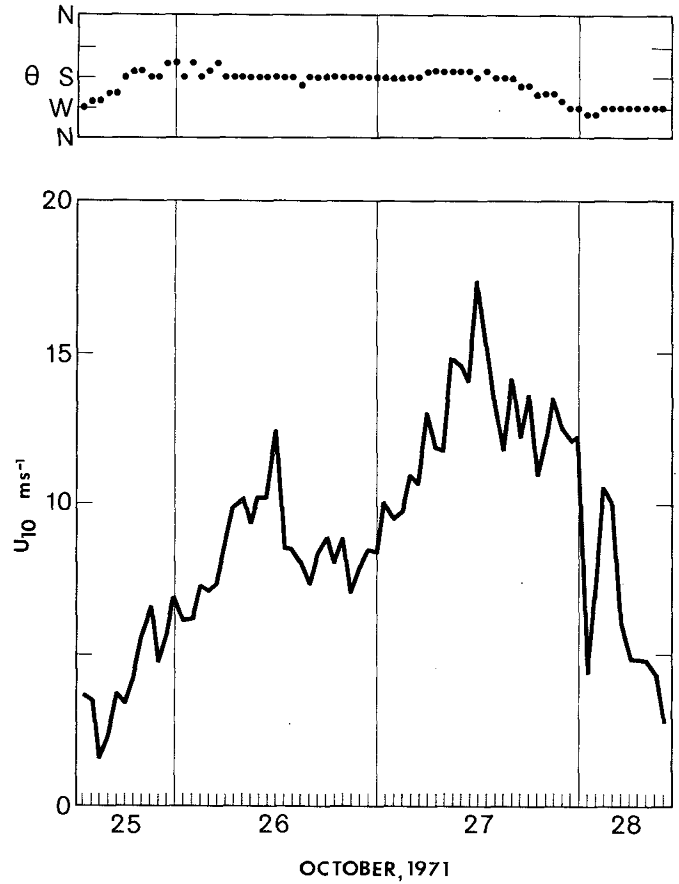

Fig. 4 Wind conditions during Lake Michigan episode.

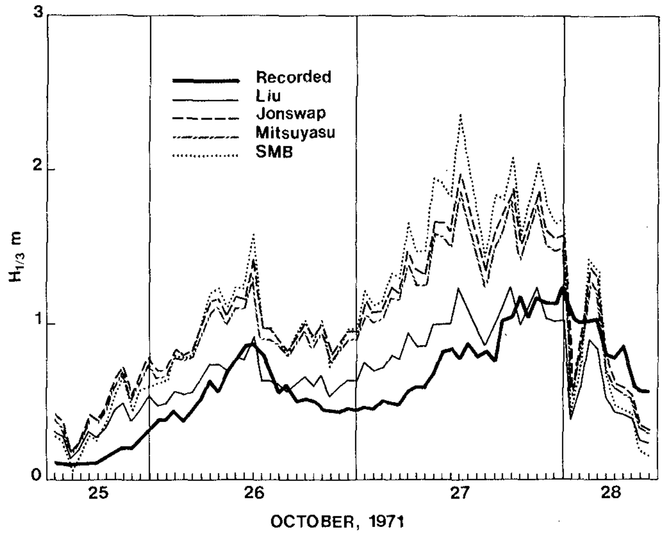

Fig. 5 Comparison of hindcast and recorded $H_{1/3}$ during Lake Michigan episode.

SMB overpredicts.

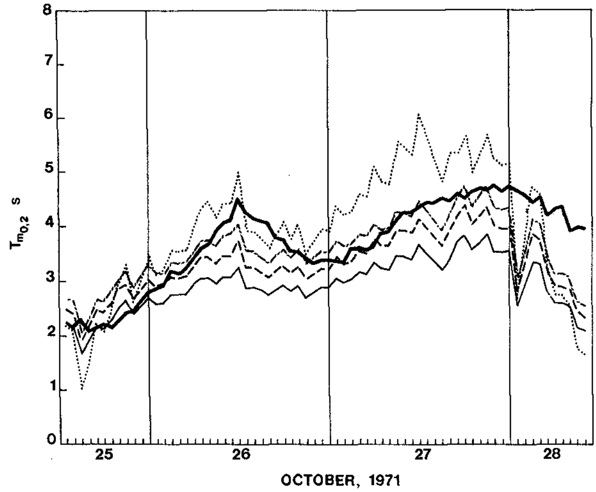

Fig. 6 Comparison of hindcast and recorded $T_{m_{0,2}}$ during Lake Michigan episode.

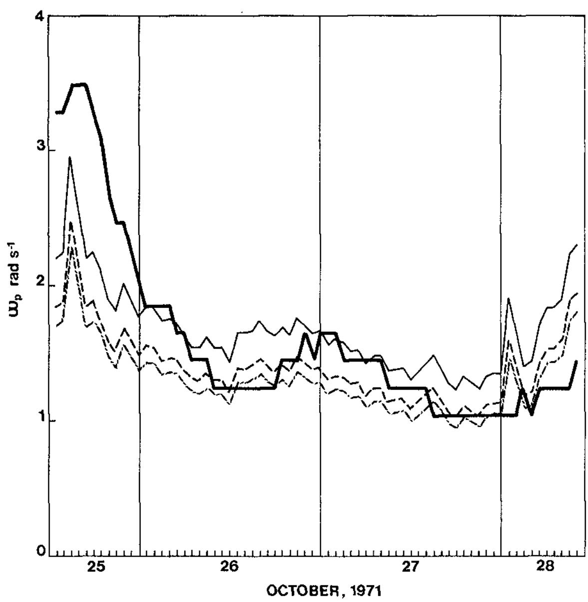

Fig. 7 Comparison of hindcast and recorded ω_p during Lake Michigan episode.

only JONSWAP, Mitsuyasu, and Liu are compared. The relative differences are again quite similar and distinctive. Liu hindcast a higher ω_p, followed by JONSWAP and then Mitsuyasu.

The second episode is from Lake Ontario. The wave gage, a waverider buoy manufactured by Datawell of Holland, was located 30 km northeast of Oswego, New York, in 150 m of water. The solid circle labeled Oswego-2 on the map, Fig. 8, indicates the waverider location. The open circle on the map shows the location of an instrumented buoy where the wind data used in this analysis were recorded. The wind anemometer on the buoy was 4 m above the lake surface.

Fig. 9 gives the wind conditions during this 4-day episode of October 7-11, 1972. The wind speeds were typically unsteady, with directions starting from the north, with a moderate fetch of 40 km, changing to west with a long fetch of 200 km or more when the storm intensified, and gradually switching back to north after the storm subsided.

The comparisons for the hindcast and recorded significant wave heights for this episode are shown in Fig. 10. The relative differences between the empirical results remain the same. The results of this episode are different from the first episode, however, in the sense that Liu's hindcasts, which were closer to the recorded results than the others in Fig. 5, are clearly underestimations in Fig. 10. The hindcasts of Sverdrup-Munk-Bretschneider, Mitsuyasu, and JONSWAP are again close to each other in this episode and also close to the recorded data during the growth part of the storm. All the empirical formulas significantly underestimated $H_{1/3}$ when the wind field decayed and directions switched from long fetches to short fetches.

Figs. 11 and 12 show the comparisons for $T_{m_{0,2}}$ and ω_p, respectively, for this episode. The results are generally similar to those discussed in Figs. 6 and 7. It seems that, at any given time, one of the formulas tends give a better hindcast than the others, but none of the formulas is able to maintain the close hindcast throughout the episode. The inconsistency, therefore, casts doubt over the applicability of all the formulas.

Two examples of comparing computed spectra with the recorded spectra are shown in Fig. 13. The accuracy of the spectrum hindcast by the formulas depends mainly on the accuracy of corresponding formulas in hindcasting significant wave height and peak-energy frequency. If the hindcast significant wave height and peak-energy frequency values are close to those recorded, then the hindcast spectrum will certainly be close to that recorded and vice versa. The comparisons of spectra shown in the figure represent the typical results that can be expected from these empirical formulas.

4. SUMMARY AND CONCLUDING REMARKS

It has been the intent of this paper to present an assessment of the usefulness and applicability of the various empirical formulas in connection with surface waves in the Great Lakes. The assessment has been performed by comparing waves that were hindcast by the formulas with those actually recorded during two storm episodes in Lakes Michigan and Ontario. Based on the detailed

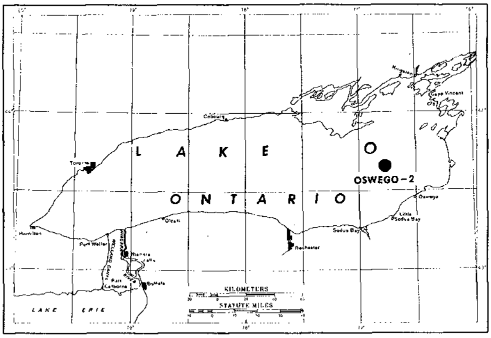

Fig. 8 Location of wave gage in Lake Ontario.

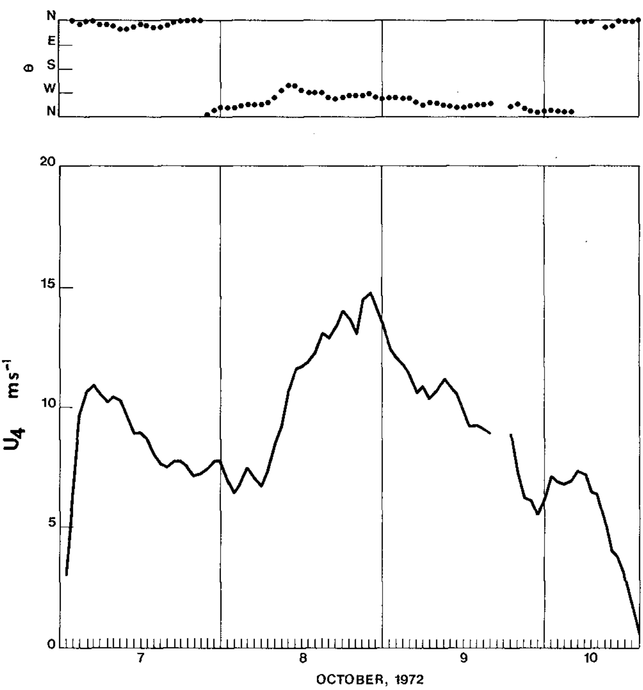

Fig. 9 Wind conditions during Lake Ontario episode.

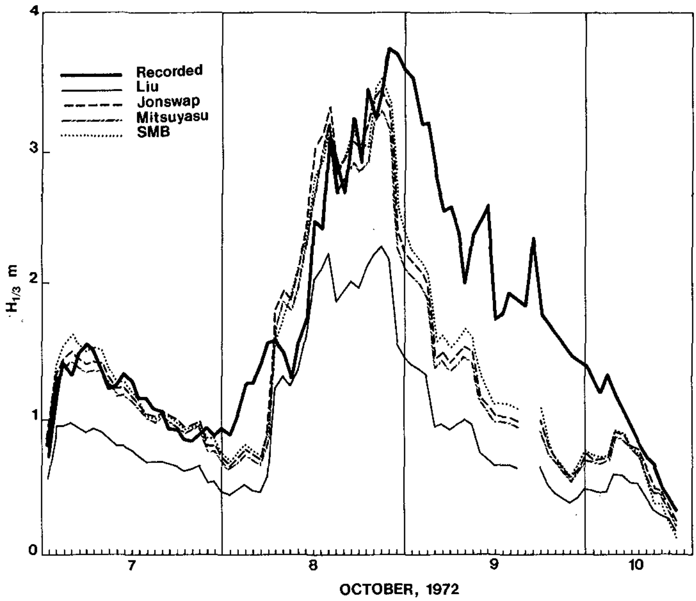

Fig. 10 Comparison of hindcast and recorded $H_{1/3}$ during Lake Ontario episode.

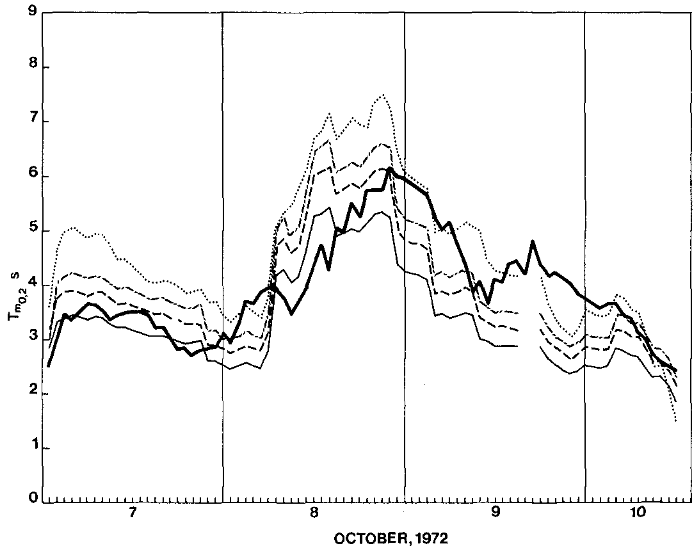

Fig. 11 Comparison of hindcast and recorded $T_{m_{0,2}}$ during Lake Ontario episode.

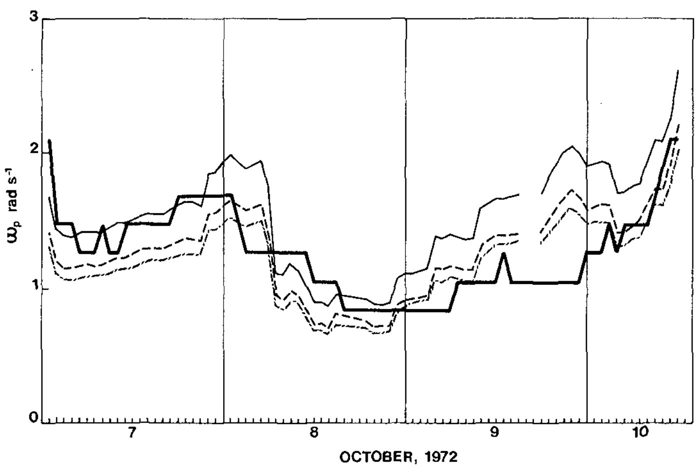

Fig. 12 Comparison of hindcast and recorded ω_p during Lake Ontario episode.

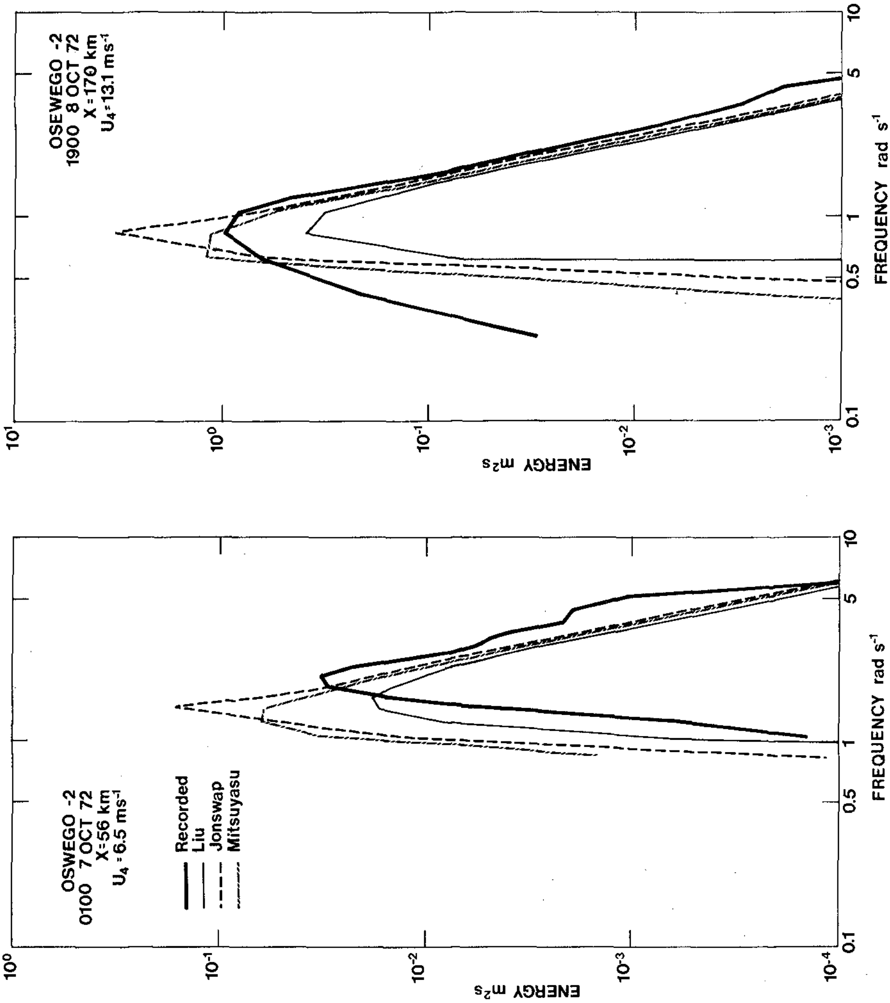

Fig. 13 Examples of hindcast and recorded wave spectra.

comparisons for the three parameters $H_{1/3}$, $T_{m0,2}$, and ω_p, results can be summarized as follows:

a. Significant Wave Height, $H_{1/3}$. The hindcasts by the JONSWAP, Mitsuyasu, and Sverdrup-Munk-Bretschneider methods are generally close to each other and greater than Liu's hindcast by about 50 percent. During the Lake Michigan episode, Liu's results were closer to the recorded results than the others; thus, the other formulas yielded overestimations. During the Lake Ontario episode, JONSWAP, Mitsuyasu, and Sverdrup-Munk-Bretschneider's results were closer to the recorded results than Liu's during the growth part of the storm; hence Liu's results appear to be underestimations.

At first glance, the results rendered by the two episodes seem to be inconsistent. The inconsistency, however, can be clarified by further examination of the two respective wind fields. During the Lake Michigan episode, the 10-m wind speeds started at 2 m s^{-1} and gradually increased to 10 m s^{-1} and higher; thus, the wave spectra were mostly under development. During the Lake Ontario episode, on the other hand, the wind speeds at the 4-m level had reached over 10 m s^{-1} at a very early stage of the storm. The wave spectra under this wind field tended to be fully developed. From these facts, it appears that Liu's formula is applicable for the less than fully developed waves, whereas JONSWAP, Mitsuyasu, and Sverdrup-Munk-Bretschneider formulas are applicable for the fully developed waves. This is consistent with the characteristics of the data from which the corresponding formulas were derived.

The above discussions apply only to the growth part of the storms. All the formulas underestimated the significant wave height when the wind field decayed, especially when the decay involved changes in wind direction as well as reductions in fetch distances. As the formulas were neither derived from, nor intended for, decaying processes, the results are by no means unexpected.

b. Average Wave Period, $T_{m0,2}$. The difference between high and low estimates as hindcast by the formulas for a given time is about 2 s. The recorded data generally lie within this range. Therefore, the error in hindcasting average wave periods by any formula is about 2 s or less. None of the empirical results can be considered substantially close to the recorded results throughout either of the two episodes.

c. Peak-Energy Frequency, ω_p. The difference between high and low ω_p hindcast for a given time is about 0.5 rad s^{-1}. With the exception of the low wind portion of the Lake Michigan episode and the decaying portion of the Lake Ontario episode, the recorded ω_p lies within the range of the empirical hindcasts. An accurate hindcast of ω_p will lead to accurate hindcasting of wave spectra if the hindcast of significant wave height is also accurate. None of the formulas seems to be able to provide an accurate ω_p. During both episodes, Liu's hindcasts were relatively close to the recorded results through a major protion of the storm. JONSWAP and Mitsuyasu's hindcasts came closer to the recorded results at the peak of the storm.

These results, while generally fulfilling most of the objectives of this paper, are by and large both disappointing and encouraging. These results are disappointing because they demonstrate that simple fetch-limited spectral formulas have not provided substantial applicability to the prediction of Great

Lakes waves. Undoubtedly, parameters neglected by the formulas, such as duration and atmospheric stability, play important roles not known at the present time and need to be further explored. On the other hand, these results are encouraging because they provide information on the limitations and error ranges that can be expected in the application of these formulas to predicting Great Lakes waves. Until further developments and additional studies are made, the results presented in this paper will continue to be useful.

5. REFERENCES

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