CHAPTER 127

WAVE EFFECTS ON BUOYANT PLUMES

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Abstract

When a buoyant plume is discharged into water where
wave motions exist, the axis of plume is bent over and
horizontally fluctuated.   The dilution rate of the plume
is affected by this motion.   Experiments are carried out
to investigate the dilution rate of plumes   when they
arrive at the free surface.   An analogy to the case of
buoyant plumes discharged into uniform steady cross streams
is applied to analyse the experimental data.   Within the
experimental range, it is found that the dilution rate is
inversely proportional to the square of the ratio of the
water depth to the diameter of outlet and is proportional
to the ratio of the discharge velocity to a characteristic
horizontal velocity of the ambient fluid.   The entrainment
coefficient is given as a function of a densimetric Froude
number for small Froude number and is a constant, 0.28, for
bigger Froude number than 4.3.

Introduction

It is well known that the buoyant plume discharged from

a point source into a cross stream shows a different dilution
rate compared with that into still water.   The difference
is considered due mainly to two causes.   The first cause
is that the total length of the plume is longer in a cross
stream than in still water.   Consequently, the plume in a
cross stream has a bigger chance to be diluted by the clean
ambient fluid.   The second reason is that the section of
the plume in a cross stream can not be approximated by a
circle, but by a "horse shoe" shape which is a result of the
bigger upward motion near the central part of the section.
In the plume of this kind, the entrainment of ambient fluid
is caused not only by the turbulence due to plume motion
but also by this upwards motion, while in case of a plume
in still water the turbulence is only one major cause of
entrainment.

     Several researchers have treated buoyant plumes in
uniform steady cross streams.   The present paper reports
experimental results of buoyant plumes discharged into water
waves.

Theoretical consideration

     It is assumed that the axis of the plume is nearly
horizontal, and that the horizontal movement of the plume
is almost equal to that of the ambient fluid.   The vertical
movement of the plume is, however, governed by the buoyancy
due to the density difference and the initial momentum, and
is quite different from that of the ambient fluid.   The
difference in the vertical velocities is the main cause to
yield the turbulence and entrain the ambient fluid.   The
entrainment is expressed by a coefficient α which is the
ratio of the entrainment velocity to the velocity difference.

     The present paper does not treat a detailed structure
of the plume, but only discusses values on the axis of the
plume, which are considered important from the practical
point of view.

Sections of the plume are assumed circle. The velocity
and density distributions are assumed to take the "top-hat"
shape and follow a similarity law. Caution is needed to
the difference bet ween these assumptions and the reality.
Although we have just mentioned that the upward motion of
water near the center of the plume was one of the main
causes of bigger entrainment, no consideration of the phe-
nomenon is taken into the above assumptions. All the effects
are included in values of α which is to be experimentally
determined.

The following four equations, integrated over a section
of the plume, are basic equations which still contain the
unsteady terms.

Equation of continuity

$$\frac{\partial D^2}{\partial t} + U\frac{\partial D^2}{\partial x} = 4\alpha Dw \tag{1}$$

Equation of motion in vertical direction

$$\frac{\partial(D^2 w)}{\partial t} + U\frac{\partial(D^2 w)}{\partial x} = \frac{\Delta\rho}{\rho}gD^2 \tag{2}$$

Equation of conservation of density defficiency

$$\frac{\partial(D^2\Delta\rho)}{\partial t} + U\frac{\partial(D^2\Delta\rho)}{\partial x} = 0 \tag{3}$$

Path of plume

$$\frac{\partial y}{\partial x} = \frac{w}{U} \tag{4}$$

in which α denotes the entrainment coefficient, D the
diameter of the section of plume, w the vertical velocity
in the plume, U the horizontal velocity of the ambient
fluid, ρ the density and $\Delta\rho$ the density difference.

Since it is not easy to solve these equations analyti-
cally, the following consideration is introduced to obtain
approximate expressions.

The plume discharged while the ambient fluid is flowing
rightwards will rise diagonally rightwards. Then, the
direction of motion of the ambient fluid changes leftwards.
The plume, too, changes the direction of its ascent. New
part of the plume which now begins to be discharged locates

below the old part of the plume, and these two parts of the
plume do not interact each other.   If the old part of the
plume is reflected with respect to a vertical line at which
the plume arrives at the end of the rightward movement, the
whole plume is regarded similar to the plume in a unidirec-
tional stream.   Accordingly, if a characteristic velocity
of the ambient fluid is well defined, the theoretical
considerations used in the analysis of plumes in a cross
stream can be applied with a sufficient accuracy to the
present case in order to estimate the dilution rate of the
plume at the free surface.

We select the mean velocity U averaged over a half
wave period as a representative velocity.   Within the range
that the wave motion is approximated by long waves, the
horizontal velocity is vertically uniform.

These assumptions are used in averaging the above set
of equations.   Such terms as D and w which vary periodical-
ly with respect to time may have some contributions through
the interaction terms such as $D^2 w$ to the analysis.   For
example, the initial velocity, w_0 , of the plume can vary
periodically.   The discharge of the plume is determined by
the piezometric head between the constant head tank and
the outlet of the plume at the bottom of channel.   Water
pressure in the channel fluctuates to some extent due to
wave motion, and the term $D^2 w$ does not vanish after the
averaging.   The fluctuation is, however, considered small
enough to be neglected in the analysis.   Thus, contribu-
tions of such terms are included in the value α , which can
be a function of F_{rU} or F_{rw_0} due to this approximation.

Averaging the equations (1) to (4), we have the same
set of equations as in the case of the plume in a uniform
cross stream, and the solutions are given in the reference
(4).

Dilution rate at the free surface is given by

$$\frac{\Delta\rho}{\Delta\rho_0} = \frac{1}{4\alpha^2} k \left(\frac{h}{D_0}\right)^{-2}$$

(5)

where h denotes the water depth and D_0 the diameter of the outlet.

Experimental equipment and procedure

A channel, 7.4 m long, 75 cm high and 50 cm wide, at the Asian Institute of Technology was used in the experiment. Two piston-type wave generators placed at each end of the channel can be operated synchronously to generate standing waves.   The channel was filled with salt water.

Fresh water is storaged in the two tanks, each with a capacity of 180 litres and pumped up to the constant head tank, from which fresh water is discharged through a 5/8 in. pipe.   Water overflowed the constant head tank returns to the storage tank and is repumped up.

The outlet is placed at the point where we have the maximum horizontal velocity under standing wave motion. Diameters of the outlet used in the experiments are 0.5 cm, 0.75 cm and 1.0 cm.

The discharge is determined by measuring the decrease of water quantity in the storage tank.

In order to know the dilution rate, samples were taken near the water surface, and their salinity and concentration of chromium were measured.   Initial   concentration   of chromium of the fresh water was 5000 $\mu g/l$.

A spectrophotometer was used to determine the concentration of chromium of the samples, and as the accuracy of this method was better than that of the measurement of salinity, only the results obtained by the spectrophoto-analysis are used in the following analysis.

The samples are taken from three points; one point just above the outlet, two points 50 cm apart from this point up- and downstream along wave direction.   After having examined that the experimental condition arrived at steady condition and having stopped the wave generator, water samples were taken.   Thickness of the surface layer which

was composed of the diluted plume were 3 to 5 cm, and the
inlets of the samplers were inserted at 1-2 cm below the
free surface.   Intake velocity was controlled not to
entrain the water from the lower layer.   No sampling was
carried out at deeper points because the sampling only from
the plume itself is impossible due to the horizontal fluc-
tuation of the path of the plume caused by wave motion.

    Wave periods were determined from the rotation of the
wave generator, and wave height was measured from its
spacial distribution recorded along the side wall.

    A mixture whose density is the same as that of water
was made from benzene and carbon tetrachloride, and small
particles of the mixture were introduced in the tank to
measure the maximum excursion distance.   Theoretical
estimate of the excursion distance given by the theory of
long waves of small amplitude agreed very well with the
experimental results.

    Although standing waves were used in the experiment,
it is possible to transfer the present results to that
under progressive waves, because the excursion length is
so small compared with the wave length that the spacial
variations of horizontal velocity do not affect so much
the dilution of the plumes.

    Experimental conditions were as follows.

$h = 12.5, \quad 25, \quad 35 \, \text{cm}$

$h/D_0 = 12.5 \sim 70$

$F_{rU} = 1 \sim 7$

$k = \dfrac{w_0}{U} = 3 \sim 30$

$w_0 = 0.32 \sim 3.00 \, \text{m/sec}$

$\Delta \rho_0 = 0.01 \sim 0.05 \, \text{g/cc.}$

$H = 1.4 \sim 5 \, \text{cm}$

$T = 2.9 \sim 6.0 \, \text{sec}$

where F_{rU} is a densimetric Froude number defined with the
horizontal velocity U , H the wave height of progressive
waves and T the wave period.

Experimental results and discussions

(1). Classification of plumes.

Plumes are classified according to how many times the wave motion is repeated until they arrive at the free surface. A plume is called "nearly vertical" if the plume reaches the free surface within one half wave period. An "intermediate" plume is one that reaches the free surface within one wave period. Others are called "nearly horizontal". Thickness of the surface layer produced by the "nearly vertical" plume was the smallest, and the "nearly horizontal" plume gave the thickest surface layer. Figure 1 shows the classification of the plume in terms of the initial velocity of the plume, the horizontal velocity of the ambient fluid, the diameter of the outlet and the water depth. Except for the cases of smaller relative depth than 30, major factor which determines the type of plume is the relative velocity.

(2). Dilution rate of the plume at the free surface.

Abraham gave a formula of dilution for plume in still water as follows;

$$\frac{\Delta\rho}{\Delta\rho_0} = 9.7 F_{rw_0}^{2/3} \left(\frac{h}{D_0}+2\right)^{-5/3} \tag{6}$$

which might be a good approximation for the case that the plume rises nearly vertically under small or negligible effects of the horizontal velocity of the ambient fluid. If the phenomenon is well approximated by the plume in a cross stream, the ratio $\Delta\rho/\Delta\rho_0$ may be proportional to $(h/D_0)^{-2}$.

The least scatter of experimental results obtained is shown in Fig.2, where the empirical relationship is

$$\frac{\Delta\rho}{\Delta\rho_0} = 0.17 F_{rw_0}^2 \, k^{-1}\left(\frac{h}{D_0}\right)^{-2} \tag{7}$$

or

$$\frac{\Delta\rho}{\Delta\rho_0} = 0.17 F_{r_U}^2 k\left(\frac{h}{D_0}\right)^{-2} \tag{8}$$

Comparison with Eq.(5) gives

$$\alpha = 1.21 F_{rU}^{-1} \tag{9}$$

In the above equations, F_{rU} and F_{rw_0} are the densimetric Froude number, with respect to the horizontal velocity of the ambient fluid U, and the initial velocity of the plume w_0, respectively.

(3). Entrainment coefficient.

Figure 3 shows α as a function of F_{rU}, after evaluating it by Eq.(5). For large values of F_{rU}, α is better approximated by a constant, 0.28, which is a little smaller than 0.33 for cases of the plume in a cross stream.

The major reason of this difference seems to depend on the difference of the definition of U.

For smaller values of F_{rU}, α is expressed as a function of F_{rU}. Tan gave

$$\alpha = f(k F_{rw_0}^{5/8}) \tag{10}$$

but in our cases, it is discovered more appropriate to express α as function of F_{rU}.

Cares are needed when we use the results given in Fig. 3. For α is bigger than unity, for very small values of F_{rU}. The coefficient α is the ratio of the entrainment velocity to the velocity which excites the entrainment. It is, therefore, physically impossible that α can exceed unity without any amplification mechanism. This fact tells that our model fails.

In addition, caution should be paid on the fact that values of α scatter very much for $h/D_0 < 16$. In the vicinity of the outlet, no similarity law is applicable due to the effect of high initial velocity. This affects the values of α, too.

Even with these faults, results shown in Fig.3 are practically applicable to compute the dilution rate of the plume.

(4). Empirical formula of the dilution of the plume.

Within the present experimental range where wave motion
is sufficiently approximated by long waves of small amplitude,
U in Eq.(8) is rewritten in terms of h and H and we have

$$\frac{\Delta\rho}{\Delta\rho_0}=\frac{1}{1.27}\frac{1}{\alpha^2}\frac{w_0}{\sqrt{gh}}\frac{h}{H}\left(\frac{h}{D_0}\right)^{-2}$$
$$\alpha=1.21F_{rU}^{-1}\quad F_{rU}\leqq 4.3$$
$$=0.28\qquad F_{rU}\geqq 4.3$$

(11)

Conclusions

An empirical formula, Eq.(11), is established to
compute the dilution rate of the plume discharged into
wave motion which can be approximated by long waves.

Entrainment coefficient, α , is also given by Eq.(11),
but caution is needed because the physical basis of α for
small Froude number is ambiguous.

Nevertheless Eq.(11) is sufficient enough for practical
purposes.

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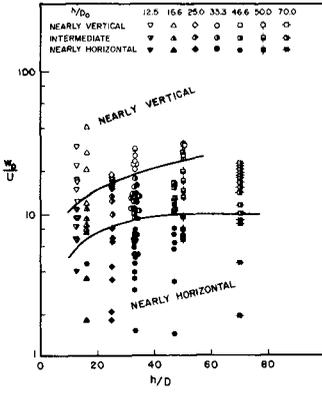
Fig.1 Classification
of plumes
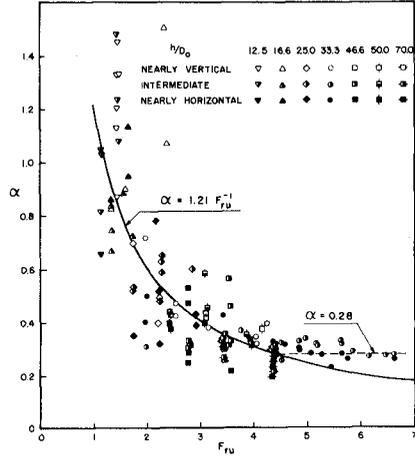
Fig.3 Entrainment coefficient
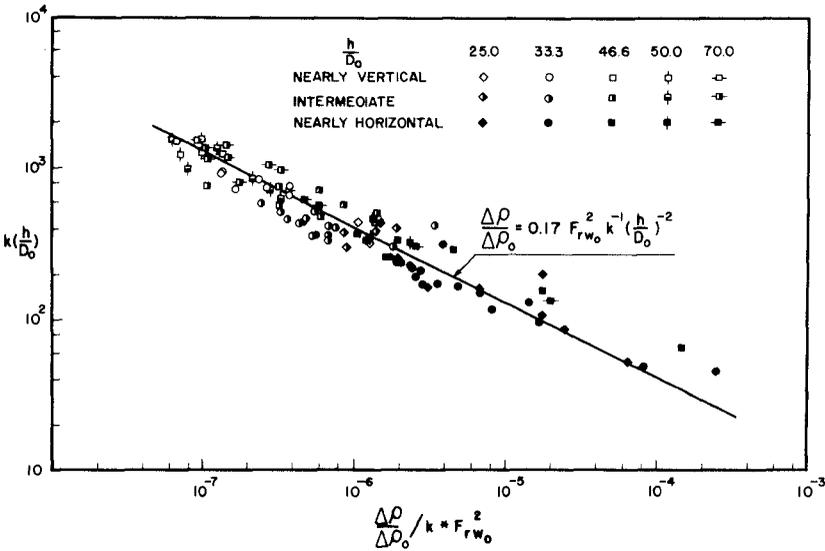
Fig.2 Dilution rate of plumes at free surface