CHAPTER 45

STABILITY OF A SAND BED UNDER BREAKING WAVES

by

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ABSTRACT

The possible effect on the stability of a porous sand bed of the flow induced within the bed during the passage of near-breaking or breaking waves is considered. It is found that the horizontal flow rather than the vertical flow within the bed may affect its stability. An approximate analysis, used in geotechnical computations of slope stability, indicates that a momentary bed failure is likely to occur during the passage of the steep front slope of a near-breaking wave. Experimental results for the pressure gradient along the bottom under near-breaking waves are presented. These results indicate that the pressure gradient is indeed of sufficient magnitude to cause the momentary failure suggested by the theoretical analysis. The loss of stability of the bed material due to the flow induced within the bed itself may affect the amount of material set in motion during the passage of a near-breaking or breaking wave, in particular, in model tests employing light weight bed material. The failure mechanism considered here is also used as the basis for a hypothesis for the depth of disturbance of the bed in the surf zone. The flow induced in a porous bed is concluded to be an important mechanism which should be considered when dealing with the wave-sediment interaction in the surf zone.

INTRODUCTION

One of the most challenging and important problems facing the coastal engineer is that of sediment transport caused by wave action. The difficulties involved in this problem, in particular if one considers the wave-sediment interaction in the surf zone, are quite obvious when one realizes our extremely limited understanding of the fluid motion associated with near-breaking or breaking waves.

In view of the enormous difficulties associated with an accurate description of the processes involved in the wave-sediment interaction in the surf zone, it is quite surprising that simple empirical relationships for the rate of alongshore sediment transport have been obtained. Thus, the empirical relationship obtained by Komar and Inman (1970) suggests the immersed weight transport rate to be proportional to the alongshore component of wave energy flux. Longuet-Higgins (1972) interpreted the transporting force in terms of the radiation stress concept and presented a semi-theoretical argument for the applicability of a relationship of the type suggested by Komar and Inman (1970). Although existing empirical relationships for the alongshore rate at sand transport are extremely useful for engineering purposes they do represent a considerable simplification of the actual processes involved in the wave-sediment interaction. If we are ever to rise above the level of pure empiricism when dealing with the wave-sediment interaction taking place in the surf zone it is obvious that we need a better understanding of the fundamental mechanisms involved in the process.

The purpose of this paper is to present evidence of the possibly very important role played in the wave-sediment interaction in the surf zone by the flow induced in the porous bed. The recognition of the possible importance of this mechanism was arrived at through discussions with Dr. William R. James at the U.S. Army, Coastal Engineering Research Center. While diving in the surf zone his observation that "just prior to the passage of the crest of a near-breaking wave the bottom seemed to explode" gave the impetus to consider the flow induced in the porous bed as a possible explanation of this "explosion". The initial idea was that the pressure distribution on the bottom associated with a near-breaking wave would induce a vertical flow out of the porous bed of sufficient magnitude to produce bed failure, piping. The effects of a vertical flow into and out of a porous bed on the initiation of sediment movement have previously been considered by Martin (1970) in the case of steady flow. Considering wave motion as a special application, Martin (1970) concluded that this would not induce a vertical flow of sufficient magnitude to affect the bed stability significantly. When dealing with waves approaching breaking this conclusion still holds, however, when recognizing the large horizontal pressure gradient associated with the steep forward slope of a near-breaking wave, it is intuitively clear that a significant horizontal, rather than vertical, flow may be induced in the porous bed. This horizontal flow within the bed will exert a force on the bed material and may produce a horizontal shear failure of the bed.

An analysis of stability of a plane horizontal bed, consisting of a cohesionless sediment, subject ot the flow induced within the bed by a pressure distribution on the bed surface is outlined. For a detailed presentation of this development the reader is refered to Madsen (1974). The result of this analysis shows that a momentary failure of the bed may occur due to the flow induced within the porous bed provided the magnitude of the horizontal pressure gradient exceeds a critical value. The order of magnitude of this critical horizontal pressure gradient does not seem to be beyond the range one would expect to be associated with near-breaking waves. However, no information on the actual value of this quantity is available, so to assess the order of magnitude of the pressure gradient along the bottom associated with near-breaking waves an experimental investigation was undertaken in a laboratory wave flume.

The results of the experiments clearly indicate that the critical pressure gradient is exceeded for the wave conditions tested, thus rendering active the mechanism of momentary failure of the bed caused solely by the flow induced within the porous bed. From a discussion of the implications of this mechanism, it appears that it may affect the rate of sediment transport in the surf zone, in particular it may play an important role in movable bed model tests using light weight granular material. The momentary bed failure is also used to hypothesize a possible mechanism responsible for burial of tracer particles in the surf zone. Although this discussion is of a rather qualitative nature it does indicate that the effect of the flow induced in the porous bed may be an important factor, which should be considered when investigating the wave-sediment interaction in the surf zone.

THEORETICAL MODEL OF BED INSTABILITY

When a surface wave travels over a impermeable bottom it exerts a pressure on the bottom. For waves of permanent form this pressure may be predicted from existing wave theories which indicate that the wave associated bottom pressure under the crest exceeds that under the trough. Considering now the bed to consist of a slightly porous material, such as sand, the pressure variation along the bed surface would induce a flow in the porous bed. Reid and Kajiura (1957) considered the flow induced in a porous bed in the context of evaluating the wave attenuation due to percolation. From their results it may be inferred that it is reasonable for an only slightly permeable bed material to neglect the unsteadiness when evaluating the pore pressure distribution within the bed. The accuracy of this approach was recently substantiated experimentally by Sleath (1970).

In contrast to these previous investigations the present study focusses its attention on waves approaching breaking. It is a well-known fact that the surface profile of near-breaking waves becomes increasingly asymmetric with a forward leaning crest having a steep front slope and a flat back. In a qualitative sense this asymmetry of the surface profile will also be present in the pressure distribution on the bottom. In the following analysis we retain formally this asymmetry by assuming an instantaneous pressure distribution on the horizontal bed surface given by

$$p_o = \bar{p}_o + p_o^+(x) \qquad (1)$$

as illustrated in Fig. 1. The average pressure, $\bar{p}_o$, in Eq. 1 corresponds to the average weight of the water above the bed, i.e., $\bar{p}_o = \rho g h$ where h is the depth of undisturbed water of density ρ. Unsteadiness has been neglected in accordance with Reid and Kajiura (1957) and $p_o^+(x)$ is the instantaneous bottom pressure associated with the wave motion. As a first approximation the pressure distribution may be assumed periodic with a wave length L as indicated in Fig. 1.

With the coordinate system shown in Fig. 1 and introducing the pore

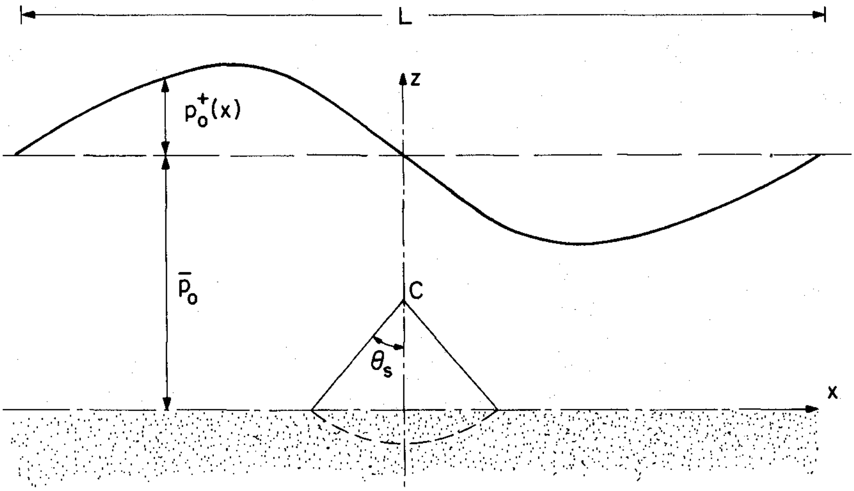

Figure 1: Definition of Coordinate System. Bottom pressure distribution and limiting slip circle indicated.

pressure in excess of hydrostatic pressure

$$p^+ = p - \bar{p}_o + \rho g z \tag{2}$$

where p is the actual pore pressure, the governing equation for p^+ becomes, Darcian flow in the bed being assumed

$$\nabla^2 p^+ = \frac{\partial^2 p^+}{\partial x^2} + \frac{\partial^2 p^+}{\partial z^2} = 0 \tag{3}$$

with the boundary condition

$$p^+ = p_o^+(x) \quad ; \quad z = 0 \tag{4}$$

$$p^+ \to 0 \quad ; \quad z \to -\infty \tag{5}$$

With the assumed periodicity of the pressure distribution, $p_o^+(x)$, it is an easy task to find the solution for $p^+(x,z)$ by using the Fourier series representation of $p_o^+(x)$.

Corresponding to the predicted pore pressure distribution within the bed the flow induced in the bed may be found from Darcy's law. Through the action of viscous forces this flow of pore fluid will act on the grains of the bed material giving rise to a force on the solids in the direction of flow. This force, the seepage force F_s, is per unit volume equal to the pressure gradient (Terzaghi and Peck, 1967).

$$F_s = (F_{s,x}, F_{s,z}) = -\nabla p^+ = -(\frac{\partial p^+}{\partial x}, \frac{\partial p^+}{\partial z}) \quad (6)$$

Disregarding, for the moment, the unsteadiness of the problem, failure of the bed due to a purely vertical flow out of the bed is generally believed to take place when the vertical seepage force, $F_{s,z}$, exceeds the effective gravity force acting on the solids (Terzaghi and Peck, 1967). If the density of the saturated bed material is denoted by ρ_t, the effective density of the bed material will be $\rho_t' = \rho_t - \rho$, and the critical vertical gradient can be expressed as

$$(F_{s,z})_{crit} = -(\frac{\partial p^+}{\partial z})_{crit} = (\rho_t - \rho)g \quad (7)$$

The action of this force is often considered in geotechnical problems where it may produce failure of the soil, piping. The interaction of the vertical flow into and out of the bed with the turbulent flow above the bed was considered by Martin (1970) and Martin and Aral (1971) who studied its influence on incipient motion of particles on the bed water interface.

To evaluate the effect of a horizontal flow induced in a porous bed, consider a thin horizontal slice of thickness δ at the bedwater interface. The force resisting the horizontal seepage force would be the mobilized internal friction between the grains at a depth δ. For a cohesionless material this intergranular friction may be expressed in terms of the effective (intergranular) normal stress and the angle of internal friction, ϕ, of the material. The balance of these forces leads to an expression for the critical horizontal pressure gradient

$$(F_{s,x})_{crit} = -(\frac{\partial p^+}{\partial x})_{crit} = -(\frac{\partial p_o^+}{\partial x})_{crit} = (\rho_t - \rho)g \tan\phi \quad (8)$$

where the horizontal gradient of the pressure associated with the wave motion is introduced since a thin slice close to the surface of the bed is being considered.

For steady open channel flow the magnitude of the horizontal pressure gradient is insignificant and it is not surprising that its effect was neglected by Martin (1970). However, for a wave motion, in particular as the waves approach breaking, the horizontal pressure gradient associated with the steep forward slope of the crest may be considerable. In fact, it may be shown by solving Eqs. 3, 4, and 5 for an assumed asymmetric pressure distribution such as the one indicated in

Fig. 1, that the maximum horizontal pressure gradient exceeds the maximum vertical gradient by a considerable amount. This observation, in conjunction with the indication of Eqs. 7 and 8 that the critical pressure gradient in the vertical direction exceeds that in the horizontal direction, since ϕ generally is less than 45°, suggests that the horizontal rather than the vertical flow induced in the porous bed is likely to affect the stability of the bed.

To investigate further the likelyhood of the horizontal pressure gradient exceeding the critical value given by Eq. 8 let us for simplicity assume that the vertical pressure distribution associated with the wave motion is hydrostatic. Then,

$$p_o^+ = \rho g \eta \tag{9}$$

where η is the free surface elevation relative to the stillwater level and Eq. 8 becomes

$$-\frac{1}{\rho g}\left(\frac{\partial p_o^+}{\partial x}\right)_{crit} = -\left(\frac{\partial \eta}{\partial x}\right)_{crit} = \frac{\rho_t - \rho}{\rho} \tan \phi \tag{10}$$

Choosing values representative of a relatively loose sand in seawater (Terzaghi and Peck, 1967, Tables 6.3 and 17.1), the value of the right hand side of Eq. 10 is found to be of the order 0.5. One can readily imagine that the steep forward slope of a near-breaking wave exceeds this value of 0.5, thus indicating that the horizontal flow induced in the bed indeed is likely to be of sufficient magnitude to affect the stability of the bed.

At this point it should be emphasized that, whereas the neglect of unsteadiness was justified when obtaining the pore pressure distribution within the bed, the neglect of unsteadiness in the analysis of the bed stability makes the prediction of bed failure a formal one. The time interval during which an elemantary bed volume is affected by a sufficiently large horizontal pressure gradient to induce failure, i.e., the time it takes the steep forward slope of a near-breaking wave to pass overhead, is relatively short and inertia terms should be considered where analyzing the details of the bed stability. Thus, the predicted failure is not to be interpreted in the geotechnical sense of the word, i.e., the failure will not cause a soil packet to be forced out of the bed. Rather, the predicted failure should be interpreted as a "momentary failure" taking place for the short period of time during which the horizontal pressure gradient exceeds the critical gradient given by Eq. 10.

Keeping this interpretation of the predicted failure in mind a more realistic, yet simple, analysis of the stability of the bed may be performed once the pore pressure distribution within the bed is determined from Eqs. 3, 4, and 5. A comparatively simple analysis due to Bishop (Terzaghi and Peck, 1967), which is used in geotechnical evaluations of slope stability, is used in the present context, and

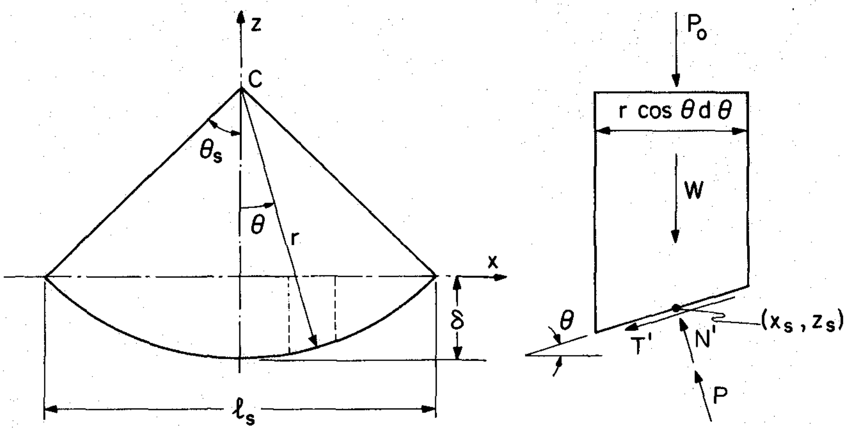

Figure 2: Limiting slip circle, Notation and the forces acting on an elementary segment of the material above the slip circle

leads to further insight into the extent of the bed failure than did the preceding simple analysis. The formal application of slope stability analyses to unsteady problems involving water waves has previously been performed by Henkel (1970) and as presented at this conference by Dr. Mitchell.

For this analysis the bed failure is assumed to take place along a circular arc, the slip circle indicated in Fig. 1. For the purpose of briefly outlining the analysis, the details may be found in Madsen (1974), the limiting slip circle is shown in Fig. 2 along with the forces acting on a small bed element above the slip circle. Bishop's simplified method assumes that the forces acting on the sides of the small element have no vertical component. Thus, with the notation indicated in Fig. 2, vertical equilibrium of the forces acting on a small element gives

$$P_o + W + T' \sin\theta - (N' + P)\cos\theta = 0 \tag{11}$$

in which

$$\begin{aligned}
P_o &= \text{external pressure force; Eq. 1} = (\bar{p}_o + p_o^+(x_s))r\cos\theta\, d\theta \\
W &= \text{total weight of slice} = \rho_t g\, z_s r\cos\theta\, d\theta \\
T' &= \text{frictional force} = \tau_f\, r\, d\theta = \sigma_n'\tan\phi\, r\, d\theta \\
N' &= \text{intergranular, effective normal force} = \sigma_n'\, r\, d\theta
\end{aligned} \tag{12}$$

$$P = \text{pore pressure force; Eq. 2} = (\bar{p}_o + p^+ + \rho g z_s)\ r\ d\theta \qquad (12)$$

where the shear stress, τ_f, mobilized at failure is related to the effective normal stress, σ_n', through

$$\tau_f = \sigma_n' \tan \phi \qquad (13)$$

Introducing Eq. 12 in Eq. 11 leads to an equation for the effective normal stress along the slip circle which may be used in conjunction with Eq. 13 to evaluate the mobilized shear stress along the slip circle. This mobilized shear stress will resist the driving forces associated with the external pressure force. A measure of this resistance is the moment of the shear forces around the center of the slip circle and when this moment is equated to the moment of the driving forces, the external pressure, an equation for the angle θ_s (see Fig. 2) defining the limiting slip circle is obtained. It is emphasized that this determination of the angle θ_s may be carried out numerically in the general case, provided the pore pressure distribution within the bed is known. In view of the formal application of this type of analysis to the present problem some simplifying assumptions are introduced which allow an analytical expression for θ_s to be obtained. The assumptions made are that the external pressure distribution, $p_o^+(x)$ shown in Fig. 1, is antisymmetric around $x = 0$, and that the pressure varies linearly over the horizontal extent of the slip circle, l_s in Fig. 2, and has a magnitude of

$$-\frac{1}{\rho g}\left\langle\frac{\partial p_o^+}{\partial x}\right\rangle_{l_s} = \alpha \qquad (14)$$

The implications of these assumptions are discussed by Madsen (1974); here we just state the resulting approximate equation which, in spite of the many assumptions, captures the essense of the analysis,

$$\frac{1}{3}\,\frac{\sin^2\theta_s}{1 - \theta_s \cot\theta_s} = \frac{\rho_t - \rho}{\rho}\,\frac{\tan\phi}{\alpha} \qquad (15)$$

from which the angle θ_s, defining the limiting slip circle, may be obtained if the value of the right hand side is known.

Whether or not the material above the limiting slip circle is in a state of momentary failure depends on whether or not Eq. 15 has a solution for θ_s. Defining the right hand side of Eq. 15 as a stability number

$$C_s = \frac{\rho_t - \rho}{\rho}\,\frac{\tan\phi}{\alpha} \qquad (16)$$

with the dimensionless pressure gradient, α, given by Eq. 14, it is found that for $C_s > 1$ no solution for θ_s can be found from Eq. 15. For a given material and fluid a large value of C_s is associated with a small value of the pressure gradient, α. That no solution for θ_s can be found in this case is simply expressing the fact that the pressure gradient is of insufficient magnitude to cause a bed failure. The left hand side of Eq. 15 approaches the limit of unity as $\theta_s \to 0$, which is indicative of a very shallow failure. Thus, $\theta_s \to 0$ expresses the limiting condition for which failure of the bed occurs and corresponds to a value of the stability number $C_s = 1$. In terms of the corresponding pressure gradient, the critical gradient, this corresponds to

$$\alpha_{crit} = \frac{\rho_t - \rho}{\rho} \tan \phi \tag{17}$$

which is seen to be the same result as previously obtained, Eq. 8. Finally, for large values of the pressure gradient, $C_s < 1$, Eq. 15 has solutions for θ_s, thus indicating that failure does occur and that the bed material above the limiting slip circle is in a state of momentary failure during the short time of passage of the large pressure gradient. The extent of this failure below the bed surface, δ in Fig. 2, may be obtained if for example the horizontal extent of the slip circle, l_s, is known. From geometrical considerations one then obtains

$$\frac{\delta}{l_s} = \frac{1 - \cos\theta_s}{2 \sin\theta_s} \tag{18}$$

Fig. 3 presents a graphical solution of Eqs. 15 and 18 as a function of the stability number C_s, and is, Madsen (1974), valid only so long as $\theta_s < \frac{\pi}{2} - \phi$.

The somewhat more involved analysis of the bed stability has provided some insight into the extent of the bed failure, although the predicted value of the critical horizontal pressure gradient was the same as that obtained from the simplified analysis. It is again emphasized that the neglect of unsteadiness in the stability analysis makes the predicted failure a formal one, which is to be interpreted as a "momentary failure". The theoretical results have, however, indicated that such a momentary failure is highly likely to occur since the critical horizontal pressure gradient α_{crit} is of the order 0.5, which is a value certainly expected to be exceeded under the steep forward slope of a near-breaking wave.

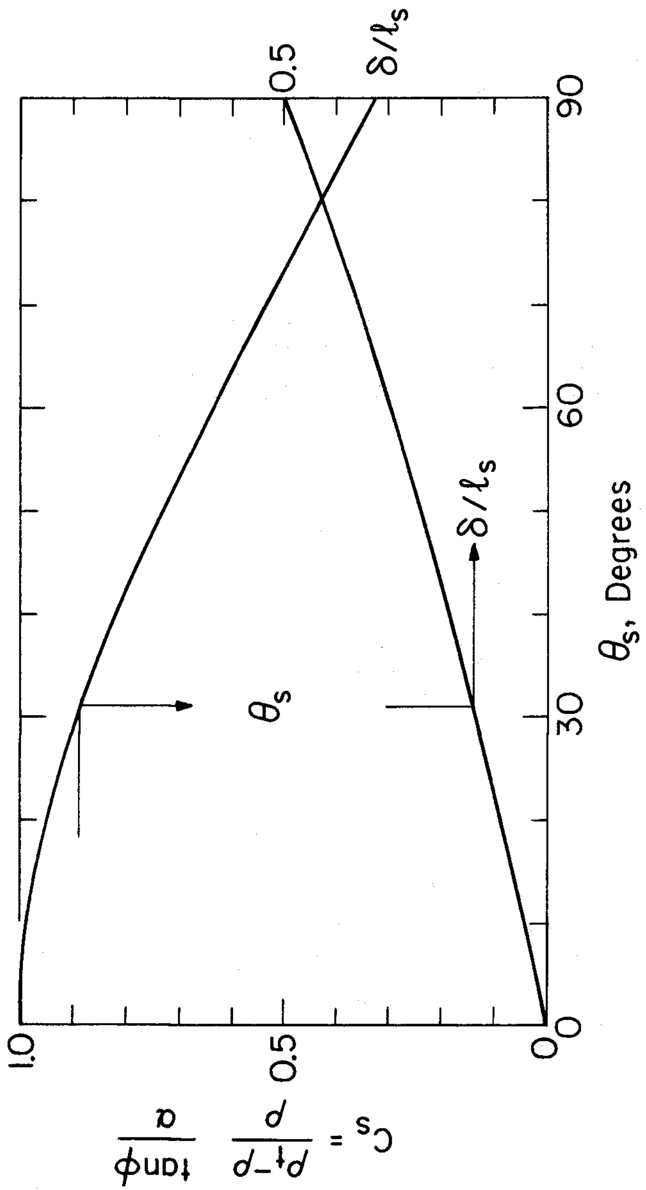

Figure 3: Graphical solution of Eqs. 15 and 18 as a function of the stability number C_s. $C_s > 1$ no solution. Valid only for $\theta_s < \pi/2 - \phi$.

EXPERIMENTAL DETERMINATION OF THE PRESSURE GRADIENT

The theoretical considerations presented in the preceding Section showed the all important role played by the horizontal gradient of the pressure on the bottom associated with waves in the surf zone in determining the possible influence of the flow induced in the porous bed on the bed stability. In order to establish the magnitude of this pressure gradient some laboratory experiments were performed in a 30 m long, 75 cm wide and 90 cm deep wave flume in the Ralph M. Parsons Laboratory at M.I.T. This flume is equipped with a variable speed drive flap-type generator capable of producing sinusoidal waves. Approximately 18 m from the generator the flume terminates in a 1 on 10 sloping impermeable beach consisting of a steel frame with plywood boards.

The wave motion taking place in the flume is measured by parallel wire resistance-type wave gages, which measure the surface fluctuations at one point as a function of time. The wave measurements are recorded on Permapaper using one of the two channels of a Sandborn, Model 296, recorder.

To measure the pressure variation on the bottom under breaking waves, a 30 by 30 cm square of the plywood beach was cut out a distance of 2.5 m up the slope. This square was replaced by two aluminum plates mounted flush with the plywood surface. One of these plates contained the pressure taps which were placed approximately along the centerline of the flume a distance of 1.7 cm apart. This distance was dictated by the space requirements necessary to connect the pressure taps to the system of pipes and copper tubing, which in turn connected the pressure taps to the pressure chambers on the two sides of the diaphragm of a differential pressure transducer, Pace Engineering Co., Model KP15, series 20442. The other aluminum plate provided the necessary access to the region below the beach. The measured pressures were recorded on the second channel of the Sandborn recorder. Thus, when the two pressure taps in the beach were connected to the pressure transducer the pressure difference, Δp, between these two positions, 1.7 cm apart, was recorded. If only one tap was hooked up the other side of the diaphragm being connected to the constant hydrostatic pressure in the fluid under the beach, the recorded pressure would correspond to the pressure at one point on the bottom associated with the wave motion.

In principle the analysis is simple keeping our main goal in mind. According to the theory presented in the previous Section, we would expect the mechanism discussed there to be active when the bottom pressure gradient exceeds the value of 0.5 ρg. Thus, if the pressure difference between the two pressure taps in the bottom of the wave tank exceeds $\Delta p/\rho g = 0.5 \cdot 1.7$ cm $= 0.85$ cm, the intended mission is completed. However, this pressure difference is indeed small and since it is associated with the passage of the steeply sloping front of a near-breaking wave, it may have only a very short duration which raises questions about the response characteristics of the pressure measurement system.

Characteristics of the Pressure Measuring System

Expecting to measure pressure differences of the order 1 cm of water, i.e. 1 gm/cm^2, the diaphragm to be used in the pressure transducer should be very sensitive. Since the natural frequency decreases with increasing sensitivity of the diaphragm the choice of diaphragm involves a compromise. A reasonable compromise turned out to be a diaphragm having a maximum pressure range of 350 gm/cm^2 for which it was still possible to electronically amplify the signal such that pressure differences of the order 1 cm of water could be recorded on the Sandborn recorder. The drawback of using this heavy diaphragm is its susceptibility to mechanical vibrations. This difficulty was partially overcome by performing the experiments during periods of low traffic intensity on the adjacent street.

The response characteristics of the system were obtained from "balloon tests" the details of which are presented by Madsen (1974). These tests showed the pressure measuring system to have a natural frequency of the order $f_o = 27\ sec^{-1}$ and a damping coefficient $\mu = 0.7$, i.e. close to critical damping. This damping was achieved by filling one of the copper tubes connecting the pressure taps to the transducer with steel wool shavings.

Based on these response characteristics it is concluded that signals of periods longer than 0.2 sec would be accurately recorded by the system with recorded pressure differences corresponding to those obtained from a static calibration of the system. Static calibration of the pressure transducer set-up showed this to have a linear response and the electronic amplification of the signal was chosen such that a 10mm deflection of the pen on the Sandborn recorder corresponded to a pressure difference of 10mm of water.

Characteristics of the Generated Waves

Due to the shortage of funds for experimental equipment only one pressure transducer was available and this quite clearly necessitates that the position of the pressure taps be close to the point of breaking of the incident waves. Invoking the approximate analogy between the pressure gradient along the bottom and the slope of the free surface it is evident that the breaking wave to be measured must be significantly larger than 1.7 cm, the distance between the pressure taps, in order for the measured pressure difference to express accurately the pressure gradient along the bottom.

Galvin (1968) obtained criteria for the occurrence of various breaker types, spilling or plunging, on laboratory beaches. From his Fig. 1 it is clear that the more gently breaking wave, the spilling breaker, is associated with the smaller slope of the front of the wave crest as breaking is approached. This indicates that the pressure gradient along the bottom would be smaller for a spilling breaker than for a plunging breaker, hence breaking waves of the spilling type seem crucial in the present context. Guided by the result of Galvin (1968) that spilling breakers occur when

$$\frac{H_o}{L_o} \geq 4.8 \tan^2\beta \tag{19}$$

where H_o and L_o are the deep water wave height and wave length respectively, and tan β is the beach slope, one obtains for the present experimental set-up (tan β = 0.1) that the incident wave steepness should exceed the value 0.05 to ensure breakers of the spilling type. To generate a wave of this considerable steepness it was necessary to choose a relatively short wave period, T = 0.95 sec, for a water depth of h_o = 38 cm in the constant depth portion of the flume.

Immediately following the start of the wave generator a uniform wave train of height H = 9 cm existed in the flume. This corresponds to a deep water wave steepness of the order 0.07 which according to Galvin's criterion should be sufficient to produce spilling breakers. Initially the breakers were indeed of the spilling type, although showing some plunging characteristics, and they all broke in approximately 10 cm water depth. After some time, however, the initially uniform wave train developed some strong beat features with some waves breaking before some after reaching the initial break-point. Eventually, after 15 to 30 minutes, a quasi-steady condition was reached where some waves broke even before reaching the slope, some broke approximately 2 m up the slope, others 3 m up the slope. Although the beat effect exhibited a strong periodicity, with approximately every fifth wave breaking at a certain location, this variation in breaker position seemed to be a rather undesirable situation.

The details of the observed wave anomaly are presented by Madsen (1974), where a possible explanation of its appearence may also be found. Here it suffices to say that the wave instability seems to be associated with the development of a long period standing wave in the flume. Rather than avoiding the problems associated with the observed variability in breaker position by for example running the experiments in bursts, i.e., starting the wave maker from rest and only use the first waves for the measurements, it was decided to live with this anomaly since it to some extent could be used to our advantage.

The reason for this choice of wave conditions, H $\simeq$ 9.0 cm, T = 0.95 sec in 38 cm of water, was the observation stated previously that approximately one out of five waves broke 2 m up the slope whereas one of five waves broke approximately 3 m up the slope. Thus, positioning the pressure taps, recording the pressure difference between two points on the bottom, 2.5 m up the slope, i.e., in 13 cm of water, would serve the purpose of measuring the pressure difference associated with a wave of broken form as well as that associated with a wave approaching breaking. This was particularly found desirable since the wave breaking 2 m up the slope was a definite spilling breaker which, as previously mentioned, is the crucial breaker-type to investigate in the present context.

Experimental Results

With the account given in the preceding sections of the characteristics of the pressure transducer set-up and the incident waves the experimental results obtained may be presented.

As previously mentioned the basic purpose of the experiments was to demonstrate that the pressure gradient along the bottom associated with near-breaking or breaking waves was of sufficient magnitude to cause a momemtary failure of a porous bed as predicted by theory. With the pressure transducer set-up as described we may measure and record the pressure difference, Δp, between the two pressure taps, $\Delta x = 1.7$ cm apart. This pressure difference may in turn be used to evaluate the approximate magnitude of the pressure gradient along the bottom

$$\frac{\partial p_o^+}{\partial x} \simeq \frac{\Delta p}{\Delta x} \qquad (20)$$

The choice of wave conditions and position of the pressure taps, in 13 cm of water, allow the measurement of the pressure difference associated with spilling breakers, breaking just prior to passing the pressure taps, as well as the pressure difference associated with waves approaching plunging. The surface profile, η, obtained at the location of the pressure taps is presented in Fig. 4 and demonstrates clearly the arrival of an already broken wave,indicated by S, followed by a wave approaching breaking, indicated by P. In addition to the surface profile, Fig. 4 also shows the measured pressure difference between the two pressure taps. The somewhat ragged appearance of the recorded pressure difference is due to mechanical vibrations as previously mentioned. With the calibration of the pressure transducer set-up being 10 mm pen deflection corresponding to 10 mm water column it is readily seen from Eq. 20 that the critical pressure difference, Δp_{crit}, indicating that the momentary failure will occur, is given by $\Delta p_{crit}/\rho g = 0.5 \cdot \Delta x = 8.5$ mm. As is evident from the results presented in Fig. 4 this critical pressure difference is indeed exceeded in the experiments by the spilling breaker as well as by the wave approaching plunging. In view of the different appearance of the surface profiles of the two different breaker types it is interesting to note that their associated pressure differences are highly similar.

By analyzing several records similar to that presented in Fig. 4 it was found that the maximum pressure difference for the already broken spilling breaker was between 10 and 15 mm water column with a pronounced mean value of 12 mm. The corresponding values for the wave approaching plunging were 10 to 13 mm with a mean value of 11 mm. This corresponds to a mean value of the maximum pressure gradient along the bottom

$$\frac{1}{\rho g}\left|\frac{\partial p_o^+}{\partial x}\right|_{max} \simeq \begin{array}{ll} 0.71 & \text{(spilling breaker)} \\ 0.65 & \text{(plunging breaker)} \end{array} \qquad (21)$$

which clearly demonstrates that the pressure gradient found from these experiments is of sufficient magnitude to render active the mechanism of momentary failure of the bed discussed in the preceeding Section.

The above information is in essense all we set out to obtain. However,

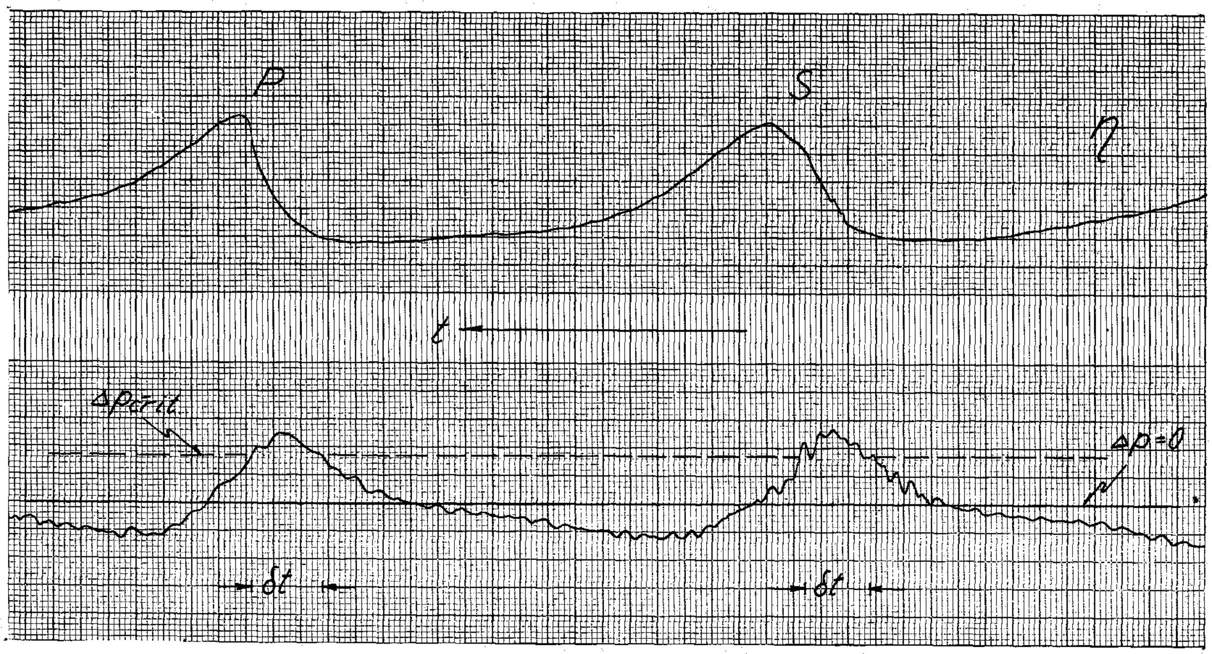

Figure 4: Typical surface profile, η, and pressure difference, Δp, corresponding to an already broken spilling breaker, S, and a wave approaching plunging, P. Critical value Δp_{crit} = 8.5 mm water column indicated along with time interval δt during which $\Delta p > \Delta p_{crit}$.

in order to estimate the order of magnitude of the spatial extent of the momentary failure additional information on for example the horizontal extent of the failure, l_s in Fig. 2, is necessary. This extent is clearly related to the distance, δx, over which the average pressure gradient, defined by Eq. 14, exceeds its critical value. Considering the parameters involved, it seems obvious that the distance $\delta x \sim l_s$ is proportional to the height of the wave breaking overhead, H_B, with the constant of proportionality being primarily a function of breaker type. From the present experiments it is possible to estimate the average value of the pressure gradient during the time interval, δt, see Fig. 4, when the critical pressure gradient is exceeded. For the spilling breakers it is found that

$$\frac{1}{\rho g}\left\langle\frac{\partial p_o^+}{\partial x}\right\rangle_{\delta t} = 0.64 \tag{22}$$

which may be taken as a representative value of α in Eq. 14. The time interval δt is found from the experiments to be $\delta t = 0.11$ sec (± 0.01 sec) and the problem is that of transforming this time interval, δt, into an equivalent horizontal distance $\delta x \sim l_s$ over which the average horizontal pressure gradient is given by Eq. 22.

The simplest possible relationship among space and time would be of the form $\delta x = c\,\delta t$. An approximate value of c may be obtained by measuring the pressure variation with time at a point and compare the magnitudes of the quantities $\partial p/\partial t$ and $\partial p/\partial x$. From such an analysis (see Madsen, 1974) it is found that a value of $c \simeq 44$ cm/sec seems appropriate during the time interval δt and leads to an estimate of $l_s/H_B = O(0.5)$ for the present experiments. It should be noted that the experimentally determined value of c is quite different from the value obtained from say linear wave theory ($c \simeq 100$ cm/sec). In view of the strong nonlinearities and rapid changes in wave form as breaking is approached this can hardly be considered surprising.

DISCUSSION OF THE RESULTS

An analysis considering only the forces associated with the flow induced within the porous bed itself by the pressure exerted on the bed surface by a passing near-breaking or broken wave showed that a failure of the bed would take place if the horizontal gradient of the wave induced bottom pressure exceeded the value

$$\frac{1}{\rho g}\left|\frac{\partial p_o^+}{\partial x}\right|_{max} = \frac{\rho_t - \rho}{\rho}\tan\phi \tag{23}$$

Corresponding to a relatively loose sand the value of the right hand side of Eq. 23 is approximately 0.5 and a laboratory investigation showed that this critical value of the pressure gradient along the bottom was indeed exceeded by breaking waves of the spilling as well as of the plunging type, thus implying that a failure of the type considered here is likely to occur under natural conditions.

The analysis of the bed instability due to the flow within the bed did not account for the inertia of the material in failure. Thus, the prediction of failure during the short time interval as the steep forward slope of a near-breaking wave passes, cannot be interpreted in the geotechnical sense, i.e., the failure does not result in the removal of the material above the limiting slip circle. Rather, the predicted failure should be interpreted as a "momentary failure" causing the bed material above the slip circle to lose its stability.

The implication of this momentary failure caused solely by the flow within the bed itself is that the bed material in failure is unable to resist any additional force. In particular, sand grains on or near the bed-water interface would be unable to resist any force due to the turbulent flow of water above the bed. The instability of the bed due to the flow induced in the bed may consequently significantly influence the amount of bed material set in motion during the passage of a near-breaking wave and hence play an important role in the wave-sediment interaction taking place in the surf zone. Qualitatively the above discussion agrees with the bottom "explosion" described in the Introduction.

When realizing that movable bed model tests often are performed employing very light weight bed material, such as plastic, it is quite clear from Eq. 23 that, due to the low value of $(\rho_t - \rho)/\rho$, the effects of the flow induced in the bed may be pronounced and possibly even extend beyond the surf zone. Some evidence of this was brought out in the discussions following the presentations by Dr. Kamphuis and Dr. Ince at the Fourteenth Coastal Engineering Conference. Although they reported to have had considerable success in using light weight materials in reproducing topographical changes outside the region of breaking waves they had experienced considerable difficulties in reproducing features influenced by breaking waves. The effect is readily observed in a demonstration wave flume using a beach consisting of light weight granular material. Subject to incident waves one can observe the beach material oscillate with the incident waves to a considerable depth below the bed surface.

Although not causing material to be removed from the bed, the momentary failure due to the flow induced in the bed may cause a slight motion of the material above the limiting slip circle. This slight motion may cause individual grains to slide past each other and exchange position. This "mixing" process may be demonstrated by the simple experiments just outlined, and may cause a particle initially on the surface of the bed to slowly work its way into the bed to a depth given by the maximum depth of the failure, δ in Fig. 2.

The above hypothesis would lead to an estimate of the maximum burial depth of a tracer particle or as it is sometimes referred to: the depth of distrubance, in the surf zone. A rough estimate of the order of magnitude of this quantity may be obtained from the present investigation if we take the value of $[(\rho_t - \rho)/\rho] \tan\phi = 0.5$ and the value of α corresponding to Eq. 22. In this case the stability number, Eq. 16, has a value of $C_s = 0.5/0.64 = 0.78$ and Fig. 3 yields

$$\delta = 0.22\ l_s \tag{24}$$

From the estimate of $l_s \simeq 0(\frac{1}{2} H_B)$ obtained in this investigation, Eq. 24 indicates a burial depth, or depth of disturbance, of the order $\delta \simeq (0.11\ H_B)$ where H_B is the wave height of the breaking wave. Although the above result is limited to the particular experimental conditions for which the estimates of C_s and l_s were obtained it does agree in form as well as in the order of magnitude with the results for the depth of disturbance reported by King (1951) who found depths of disturbances of the order $0.04\ H_B$. Recent results by Williams (1971) indicate disturbance depths far greater than those reported by King (by a factor of up to 10), in addition Williams found grain size to have a negligible effect contrary to the results of King (1951) who found the depth of disturbance to increase with increasing grain size. The present hypothesis suggests that the depth of disturbance may vary from one location to another being affected primarily by local wave conditions (breaker type) and to a lesser degree by sediment characteristics. Thus, in a qualitative sense the predicted depth of disturbance based on the hypothesis presented here seem quite reasonable.

Some evidence to the effect that mechanisms other than bed form migration are responsible for the burial of tracer particles in the surf zone may be found in Clifton et al. (1971) who found bed forms to be absent in the high energy breaker zone. Also, James (1974) mention results of field investigations in the nearshore zone which show tracer dispersion depths within the bed exceeding 14 cm whereas a nearby bottom elevation monitor showed maximum changes in bottom elevation to be less than 3 cm. These observations along with the hypothesis for the depth of disturbance presented here, which clearly needs further confirmation to be considered more than a hypothesis, suggest that one should be cautious in interpreting measured burial depths of tracer particles in the surf zone as being indicative of the thickness of the layer of bed material being transported along the shore by the combined action of waves and wave induced currents.

It is hoped that the present investigation, although in many aspects of a preliminary nature, has served its purpose of pointing out that the effect of the flow induced in the porous bed is a mechanism which should be considered in many problems, in particular where dealing with the wave-sediment interaction in the surf zone. To quantitatively take the effect into consideration when evaluating say the longshore rate of sediment transport awaits further progress in our understanding of the mechanisms involved in this process.

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