CHAPTER 40

COMPUTATION OF LONGSHORE CURRENTS

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ABSTRACT

The steady state profile of the longshore current induced by regular, obliquely incident, breaking waves, over a bottom with arbitrary parallel bottom contours, is predicted. A momentum approach is adopted. The wave parameters must be given at a depth outside the surf zone, where the current velocity is very small. The variation of the bottom roughness along the given bottom profile must be prescribed in advance. Depth refraction is included also in the calculation of wave set-down and set-up. Current refraction and rip-currents are excluded. The model includes two new expressions, one for the calculation of the turbulent lateral mixing, and one for the turbulent bottom friction. The term for the bottom friction is non-linear. Rapid convergent numerical algorithms are described for the solution of the governing equations. The predicted current profiles are compared with laboratory experiments and field measurements. For a plane sloping bottom, the influence of different eddy viscosities and constant values of bottom roughness is examined.

1. INTRODUCTION

When breaking waves approach a straight coastline at an oblique angle, a mean current tends to be set up parallel to the coastline. The prediction of such longshore currents and the associated sediment transport is of prime importance for the coastal engineer.

It has been general practice to predict the alongshore sediment transport based on simple refraction calculations. When this method is used, all wave orthogonals stop at the breaker line, and the bathymetry in the surf zone is excluded in the prediction. To calculate the profile of the longshore current would be a better starting point, and then calculate the sediment transport. Such a prediction is not yet possible, but the present paper is a step in the above-mentioned direction. A recent survey of the subject has been given by Longuet-Higgins (1972).

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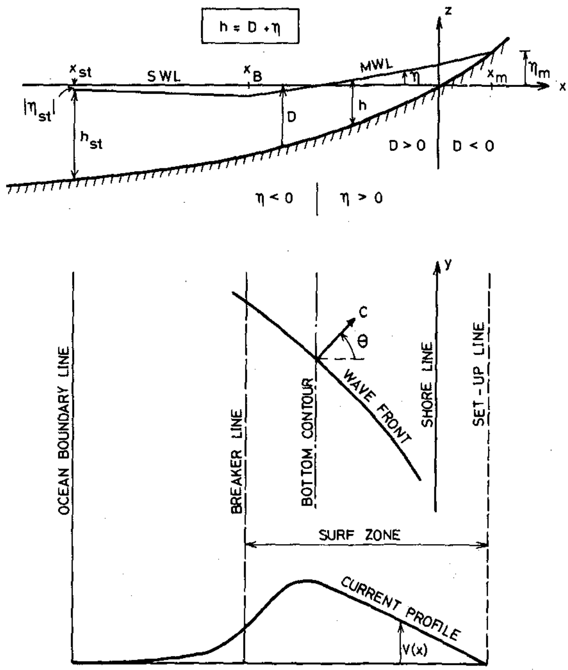

Fig. 1-A Cross section and plan

A steady state theoretical model is presented here, which for a given arbitrary variation of parallel depth contours and bottom roughness predicts the profile (mean over depth) of the longshore current (see Fig. 1-A) from given values of regular wave parameters. Continuous breaking in the surf zone is assumed. Depth refraction is included, also in the calculation of wave set-down and set-up. Current refraction and rip-currents are excluded. The model does not assume infinite water depth, where the ocean boundary condition (see Fig. 1-A) is formulated.

In contrast to most other models, the present study uses a non-linear bottom shear stress term. The model has the same two independent parameters (breaker height to depth ratio and bottom roughness) as Thornton (1971). The approach is different in this respect from Longuet-Higgins (1970), which has three parameters (breaker height to depth ratio, a constant C (in the friction term), and a constant N (in the mixing term)).

The applied numerical method is easy to use for arbitrary variations of bottom profile and roughness. The examples presented in the paper have a constant bottom slope and roughness, however.

2. ASSUMPTIONS AND SHORTCOMINGS

As regards the sea bed, we assume straight and parallel bottom contours (but allow for an arbitrary bottom profile), so Snell's law can be applied. Furthermore the depth in the surf zone must be monotonously decreasing. The waves are regular Stokes waves of the lowest order, i.e. the phase velocity is $0(H^0)$, particle velocities are $0(H^1)$, momentum, pressure and energy are $0(H^2)$, et cetera, where H is the wave height, and '0' denotes 'order of magnitude'. In the surf zone, furthermore, the usual shallow water approximations are made. The breaking criterion used is

$$H_B = \beta\, h_B \tag{2.1}$$

in which h_B is the water depth at the breaker line (Fig. 1-A). β is a dimensionless figure (of the order 1), which is assumed constant through the surf zone, thus implying spilling breakers. So in the surf zone, the wave height is determined by Eq. 2.1, without indices.

Outside the surf zone, energy losses are neglected, although a bed shear stress is included also in this zone, see Eq. 3.4 a. The effect of this inconsistency is considered small. The ocean boundary line (see Fig. 1-A) is so chosen, that the actual current velocity is very small here. Along this line we also assume parallel wave orthogonals and constant wave height. So we end up with a one-dimensional model, since rip-currents are neglected. Thus the variations in the mean water level (η) and in the current velocity (V) are dependent only on the distance from the shore line. Current refraction (see Jonsson et al. (1971), and Jonsson (1971 b)) is neglected. This can to some extent be justified, using the argument that the angles of incidence are normally small where the current velocities are high (i.e. in the surf zone) and vice versa. For the detailed assumptions leading to expressions for the turbulent bed shear and the lateral turbulent mixing, reference is made to Chapter 4.

The current velocity is assumed parallel to the coast and constant over depth in any vertical. Wind effects are excluded.

It would be fair to mention also, what are considered to be the major shortcomings of our model: These are the use of linear wave theory and of regular waves, and the neglect of current refraction and rip-currents. (In a recent study, James (1972) introduced finite amplitude waves. Battjes (1974) used irregular waves, but on the other hand neglected lateral mixing.) Also the mixing length hypothesis introduced in Chapter 4 may be doubtful, especially outside the breaker line.

3. THE MOMENTUM EQUATIONS

In this study a momentum approach is adopted for a steady state situation. So the equilibrium equations perpendicular to and parallel to the shore line must be formulated. It should be noticed that all forces and stresses in these equations are mean values over the wave period T. All forces in vertical sections are first integrated over depth, and are given per unit horizontal length.

EQUILIBRIUM PERPENDICULAR TO THE SHORE LINE

Assuming no net shear stress at the bottom, only pressure and (normal) momentum forces have resulting components at right angles to the coast. (Shear forces at the ends of the element in Fig. 3-A cancel out.)

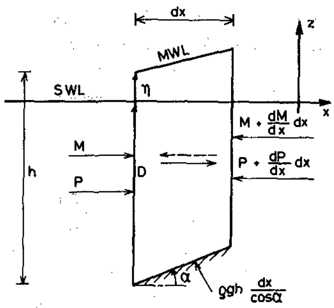

Fig. 3-A
Forces having components perpendicular to the shore line

M and P being momentum and total pressure forces over depth h ($= D + \eta$, see Fig. 1-A) per unit length in the y-direction, horizontal equilibrium yields

$$d(M+P)/dx + \rho\ g\ h \tan\alpha = 0 \tag{3.1}$$

ρ and g are density and acceleration due to gravity. Note the difference between D and h, being undisturbed and actual depth, respectively. The 'excess normal stress' σ_{xx} is defined by

$$\sigma_{xx} \equiv M + P - \frac{1}{2}\rho\ g\ h^2 \tag{3.2}$$

For normal incidence, σ_{xx} is simply the radiation stress. If Eq. 3.2 is inserted into Eq. 3.1, we find, using also $h = D + \eta$ and $\tan\alpha = -dD/dx$

$$d\sigma_{xx}/dx = -\ \rho\ g\ h\ d\eta/dx \tag{3.3}$$

Eq. 3.3 constitutes in fact two ordinary differential equations, one inside and one outside the breaker line. If σ_{xx} is known (Chapter 4), Eq. 3.3 can be integrated to yield the mean water level η (wave set-down and set-up, Chapter 5).

EQUILIBRIUM PARALLEL TO THE SHORE LINE

Neglecting the wind, we have three types of forces having resultant components parallel to the shore line. Firstly there is the 'driving force' T_w, which is the flux of y-momentum across a plane parallel to the shore line, created by the oblique water particle velocities. The gradient of T_w is balanced by the bed shear stress τ_b and by the gradient of the horizontal shear force T_v due to turbulent mixing.

As we neglect dissipation outside the surf zone, the gradient of T_w vanishes here. So, again, we deal with a system of two equations. With the sign conventions in Fig. 3-B we get:

Outside the surf zone $\quad -\tau_b + dT_v/dx = 0 \quad$ (3.4 a)

In the surf zone $\quad dT_w/dx - \tau_b + dT_v/dx = 0 \quad$ (3.4 b)

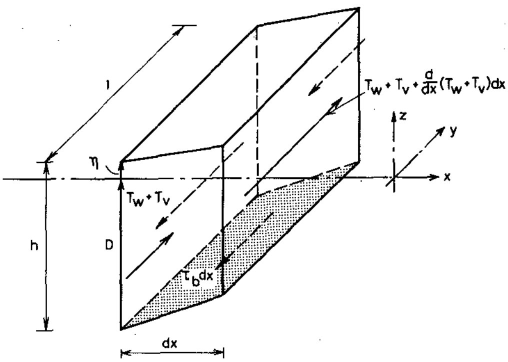

Fig. 3-B
Forces parallel to the shore line

The equilibrium conditions, Eqs. 3.3 and 3.4, are in fact formally correct for a one-dimensional situation, as considered here. The problem is that neither of the four quantities σ_{xx}, T_w, τ_b, and T_v can be determined with any great accuracy.

4. CALCULATION OF FORCES AND STRESSES

In this chapter we shall derive expressions for σ_{xx} (to be used in Eq. 3.3), as well as for T_w, τ_b, and T_v (to be used in Eq. 3.4).

PRESSURE AND MOMENTUM FORCES

The 'wave normal stress' σ_{xx} can be calculated from the momentum flux tensor (see for instance Longuet-Higgins (1970) p. 6780, Eq. 12), or taken directly from Jonsson (1971a) Eq. 2.9, by putting $\varepsilon = 0$, $\delta = \pi/2$, and substituting α by θ, or from Jonsson and Jacobsen (1973) Eq. 2

$$\sigma_{xx} = \frac{1}{16}\rho\, g\, H^2\, G + \frac{1}{16}\rho\, g\, H^2(1+G)\cos^2\theta \qquad (4.1)$$

in which

$$G \equiv 2kh/\sinh 2kh \qquad (4.2)$$

k is the wave number $\equiv 2\pi/L$, where L is the wave length. θ is the angle of incidence, Fig. 1-A.

The wave shear force T_w is also easily calculated from the momentum flux tensor, or taken directly from Jonsson (1971a) Eq. 2.10, by putting $\varepsilon = 0$, $\delta = \pi/2$, and substituting α by θ. With the sign convention in Fig. 3-B we find

$$T_w = -\frac{1}{16}\rho\, g\, H^2(1+G)\sin\theta\cos\theta \qquad (4.3)$$

BOTTOM FRICTION

The essence of the calculation of the bottom shear stress presented below is the introduction of a simple interpolation formula, based on expressions for a pure current and a pure wave motion, assuming rough turbulent flow. (A similar consideration, for waves and currents going in the same direction, was proposed with some success by Jonsson (1966).)

Consider first waves at right angles to the current direction. The particle velocities just above the wave boundary layer are shown in Fig. 4-A. It is now assumed that the instantaneous bottom shear stress $\vec{\tau}_b'$ is

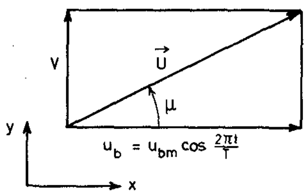

Fig. 4-A
Instantaneous particle velocity $\vec{U}$ near the sea bed. u_{bm} is the maximum wave particle velocity

in the direction of the vector sum $\vec{U}$ of the longshore current velocity V and the instantaneous wave particle velocity u_b

$$\vec{\tau}_b' = f' \frac{1}{2} \rho \, |\vec{U}|\vec{U} \tag{4.4}$$

The problem is hereafter to determine a reasonable expression for the friction factor f'. For $\mu = 0^o$ (no current, see Fig. 4-A), f' equals the wave friction factor f_w, as defined by Jonsson (1967). Similarly, for $\mu = 90^o$ (no waves), f' equals the current friction factor f_c. So, in the absence of measurements and theory, the following simple interpolation formula is suggested

$$f' = f_w + (f_c - f_w) \sin \mu \tag{4.5}$$

Through μ, f' becomes a function of time t. Introducing Eq. 4.5 in Eq. 4.4 we find the following expression for the mean value of the bottom shear stress parallel to the coast

$$\tau_b = f_s \frac{1}{2} \rho \, V^2 \tag{4.6}$$

with

$$f_s = f_c + \left(\frac{2}{\pi} \sqrt{1+(u_{bm}/V)^2} \; E(m) - 1\right) f_w \tag{4.7}$$

In Eq. 4.7, u_{bm} appears from Fig. 4-A, and E is a complete elliptic integral of the second kind, with parameter m given by

$$m = \frac{u_{bm}^2}{u_{bm}^2 + V^2} \tag{4.8}$$

(In the actual calculations it proved necessary to deal with the complementary parameter.) For a weak current, Eqs. 4.6 and 4.7 reduce to $\tau_b = (1/\pi) \; f_w \; \rho \; u_{bm} \; V$, which equals the expression used by Thornton (1971), see also Lundgren and Jonsson (1961).

The wave friction factor f_w is a function of the ratio between wave particle amplitude at the bottom, a_{bm}, and the Nikuradse sand roughness, k_N. In this study the expression proposed by Jonsson (1963,1967) is used, as modified by Skovgaard et al. (1975). The modification is simply that a constant value, $f_w = 0.24$, is used for $a_{bm}/k_N < 2$. (Recently Riedel et al. (1973) have made comprehensive measurements of f_w in an oscillating water tunnel. For $a_{bm}/k_N > 1$, their values are apparently only about 60% of those used here, and so will result in a stronger longshore current, all other things being equal.)

The current friction factor f_c is a function of the ratio between the water depth and the roughness. The usual expression for f_c: $f_c = 0.32/\log_e^2(11h/k_N)$ was used in a slightly modified form to avoid the singularity at $h = k_N/11$. For details, see Olsen and Vium (1974).

Up to this point, bottom friction has been calculated for wave incidence perpendicular to the current. Considering the uncertainty in the determination of τ_b, however, Eqs. 4.6 and 4.7 will be used without correction for θ being different from zero. An example of the calculated variation of f_s is shown in Fig. 4-B. The bottom is a plane slope. The amplification of the current friction factor due to the wave is quite clear, as expected most noticeable where the current is weak.

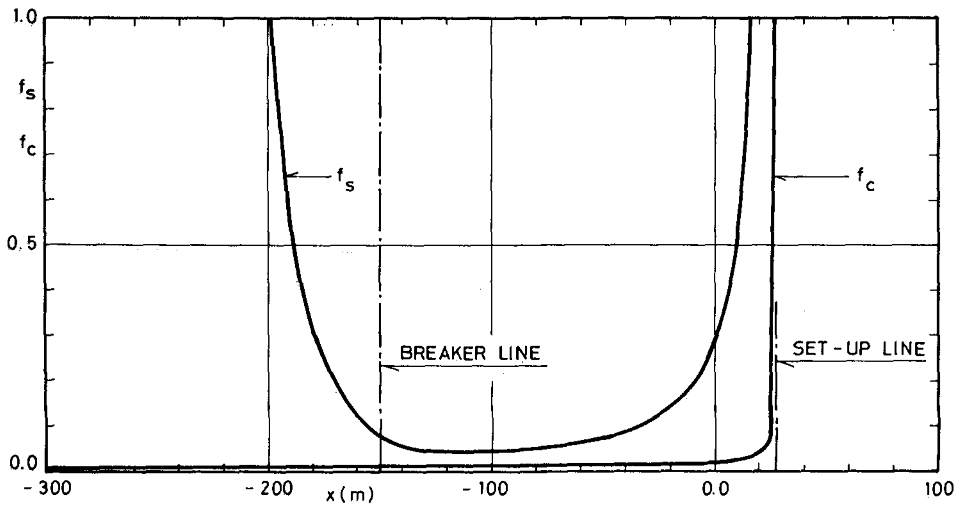

Fig. 4-B Friction factors f_s (current + wave) and f_c (pure current) for a plane slope 1:50. $T = 8$ s, $H_{st} = 2$ m, $\theta_{st} = 30°$ ($\Rightarrow \theta_o \approx 45°$), $h_{st} = 10$ m ($\Rightarrow x_{st} \approx 500$ m), $\beta = 0.8$, $k_N = 0.10$ m

TURBULENT MIXING

Finally we shall evaluate the lateral shear force T_v over total depth, due to turbulent mixing. In our model we have adopted a mixing length consideration, relating the mixing length to the wave particle amplitude. Using the classical Prandtl approach, we write the correlation expression for the shear stress as the product of the mean values of the absolute values of the turbulent velocity fluctuations u' and v', i.e.

$$\tau/\rho = -\overline{u'v'} \approx \overline{|u'|}\ \overline{|v'|} \tag{4.9}$$

In the x-direction we then assume

$$\overline{|u'|} \approx \frac{2}{\pi} u_m \cos\theta = \frac{4}{T} a_m \cos\theta \tag{4.10}$$

$a_m = u_m T/(2\pi)$ being the amplitude of the wave particle motion. In the y-direction we find, with $a_m \cos\theta$ as a mixing length

$$\overline{|v'|} \approx \ell \, dV/dx \approx a_m \cos\theta \, dV/dx \tag{4.11}$$

Since the eddy viscosity ν_T is defined from $\tau/\rho \equiv \nu_T \, dV/dx$, we find from Eqs. 4.9, 4.10, and 4.11

$$\nu_T = \frac{4}{T} a_m^2 \cos^2\theta \tag{4.12}$$

$$T_v = \rho \, h \, \nu_T \frac{dV}{dx} \tag{4.13}$$

In the surf zone, shallow water approximations are introduced in Eq. 4.12. Outside the breaker line, Eq. 4.12 is also used (with a mean-over-depth value of a_m^2), although a quite different mixing mechanism exists here. It is expected that Eq. 4.12 overestimates the eddy viscosity in this region.

An analogous approach was originally proposed by Thornton (1971). He uses shallow water approximations also outside the surf zone, and his value of ν_T is half that given by Eq. 4.12. (It is difficult to have any strong opinion of which of the two expressions for ν_T is the best. Nor is the effect on the velocity profile large, see Fig. 7-B. It should be mentioned, however, that an analytical error has crept into Thornton's expression for dT_v/dx, which is the first term on the right hand side of his Eq. 37. The water depth ought to be moved in between the brackets, since it is a function of x and thus also should be differentiated. The error stems from his Eqs. 8 and 36, where differentiation and integration seem to be performed in the wrong order.)

5. NUMERICAL SOLUTION

Having found expressions for σ_{xx}, T_w, τ_b, and T_v in Chapter 4, it is now possible to solve the momentum equations 3.3 and 3.4. Since we neglect the 'feed-back' of the current on the wave motion, it is possible to solve the momentum equation perpendicular to the shore independently of the one parallel to the shore. (So V is in fact neglected in the calculation of η.) Using the water depths thus obtained (i.e. including set-down and set-up), the longshore current emerges as a solution to the momentum equation parallel to the shore.

WAVE HEIGHT AND BREAKING DEPTH

Outside the breaker line, the <u>wave height</u> is determined from the assumption of constant transmission of wave power between orthogonals. From Eqs. 23 - 25 and 39 in Skovgaard et al. (1975) we get with $K_f' \equiv 1$

$$\left[\frac{H}{L_o}\right]^2 = \left[\frac{H_{st}}{L_o}\right]^2 \frac{1+G_{st}}{1+G} \frac{c_{st}}{c} \frac{\cos\theta_{st}}{\cos\theta} \tag{5.1}$$

where G is defined by Eq. 4.2. Here the phase velocity is $c = c_o \tanh kh$, with $c_o = L_o/T = gT/(2\pi)$; the suffix o refers to the reference value for deep water. In the surf zone, the wave height is determined by $H = \beta \, h$.

The angle θ in Eq. 5.1 is determined from Snell's law $c/\sin\theta = c_{st}/\sin\theta_{st}$, i.e. (suffix st referring to the ocean boundary line)

$$\cos\theta = \sqrt{1-(c/c_{st})^2 \sin^2\theta_{st}} \tag{5.2}$$

Remark that only for $(c_o/c_{st})\sin\theta_{st} < 1$, we can define a reference value θ_o for deep water. This is the reason, why our model is formulated with an arbitrary bottom depth at the ocean boundary line. In a laboratory experiment, one can choose wave period and wave maker angle independently of each other. So one can easily find a situation, where

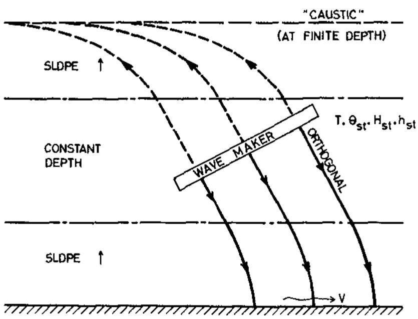

Fig. 5-A
Experiment where θ_o is not defined

θ_o simply is not defined. Such a case corresponds to an imaginary caustic behind the wave maker as sketched in Fig. 5-A. If the present model had demanded deep water data as input, we could not compare calculations with an experiment like that shown in Fig. 5-A.

The water depth h_B at the breaker line is determined by the condition, that the wave height must be continuous at this line, using Eqs. 2.1 and 5.1. This results in a transcendental equation with one root h_B, which is determined by a Newton-Raphson iteration.

WAVE SET-DOWN AND SET-UP

The variation in the MWL outside the breaker line is determined by Eq. 3.3, with H determined by Eq. 5.1. The solution to this first-order initial value problem is (Jonsson (1973)) $\eta = -H^2G/(16\,h)$ for $x_{st} \le x < x_B$, see Fig. 1-A. This corresponds in fact to $\eta = 0$ at infinite water depth. From the above solution we find at the ocean boundary line: $\eta_{st} = -H_{st}^2\,G_{st}/(16\,h_{st})$. For a given x, $D = h - \eta$ is given, and not h. Therefore the equation for η is transcendental.

The variation in the MWL in the surf zone is determined by Eq. 3.3, with $H = \beta\,h$. The initial condition for this first-order problem is $\eta = -\beta^2\,h_B/16$ for $x = x_B$. This corresponds to $G = 1$ (i.e. shallow water approximation) in the above expression for η at x_B, and so gives a slight discontinuity here. The solution is, using Jonsson and Jacobsen (1973)

$$[1 + 8/(3\,\beta^2) - 2\,D\,c^*]\,\eta = -D + 5\,h_B/6 + c^*(D^2 + \eta^2 - h_B^2) \tag{5.3}$$

for $x_B \le x \le x_m$. In Eq. 5.3, $c^* \equiv g\sin^2\theta_{st}/(2\,c_{st}^2)$. For the determination of x_m, see below. For a given x, i.e. D known, Eq. 5.3 is a quadratic equation in η.

The maximum set-up, η_m, is determined from Eq. 5.3 with $D = -\eta_m$

$$\eta_m = \frac{5}{16}\beta^2 h_B(1-\frac{6}{5}c^* h_B) \qquad (5.4)$$

It is perhaps surprising to observe that in this model, the maximum set-up is independent of the bottom profile in the surf zone, for the same value of the water depth at the breaker line. η_m then determines x_m for a given beach profile.

Thornton (1971) p. 299, neglects the effect of waves approaching at an angle $\neq 0$ in his calculations of η. An evaluation of the effect of refraction on η_m is given by Jonsson and Jacobsen, Fig. 2.

THE LONGSHORE CURRENT

The current velocity distribution V(x) is determined by Eq. 3.4. The differential equations turn out to be of the form

$$g_1(x)\, d^2V/dx^2 + g_2(x)\, dV/dx + f(x,V) = g_3(x) \qquad (5.5)$$

where $g_3(x)$ vanishes outside the surf zone (Eq. 3.4 a). $g_1(x)$, $g_2(x)$, $g_3(x)$, and $f(x,V)$ are known functions of x and V. So V is the solution to two second-order, non-linear, ordinary differential equations. The boundary conditions are: Vanishing current for $h = h_{st}$ and $h = 0$, and matching of the current and its gradient at the breaker line. Note that x_{st} must be so chosen that the actual velocity (in an experiment or in a field case) is very small here.

The matched (at $x = x_B$) boundary value problem is solved as a sequence of pairs of linear boundary value problems, using Newton's method (i.e. Taylor expansion of the non-linear term), see e.g. Bailey et al. (1968) pp. 153-156. As initial value in the Newton iteration was used $V(x) \equiv 0$ for $x_{st} \leq x \leq x_m$. The linear boundary value problems are solved by piecewise interior orthogonal collocation (or global spline-collocation), see Skovgaard (1973) Secs. 4 and 5. A basic idea of orthogonal collocation is that the solution of the differential equation is represented by a finite series of orthogonal polynomials. The unknown coefficients in this representation are found by satisfying the associated conditions and the differential equations at an appropriate number of selected points. In this project shifted Jacobi polynomials were used. Most of the calculations were performed with the simplest form of Jacobi polynomials, namely Legendre polynomials, and with about 20 terms in the series for each differential equation. The described numerical model was programmed in PL/I, using version 5.4 of the IBM OS PL/I (F) compiler. All floating point calculations were done with approximately 16 decimal digits.

6. COMPARISON WITH MEASUREMENTS

In this chapter some comparisons will be made between calculated longshore currents and observations. These are few, however, if only for the reason that many problems arise in the selection of suitable data. For laboratory measurements a closer inspection of the data will sometimes reveal that a fully developed rough turbulent flow was not present - as assumed in the mathematical model. Field measurements, on the other hand, are difficult to perform, so necessary information as input to our model is often lacking. A special problem is a reasonable choice of bottom roughness. In both cases the prediction of β (= H/h in the surf zone) is also difficult.

LABORATORY EXPERIMENTS

Fig. 6-A shows the calculated current profile, corresponding to data from laboratory Exp. No. 29 by Putnam et al. (1949).

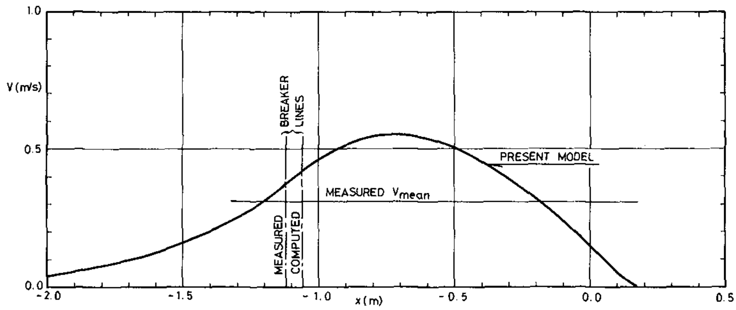

Fig. 6-A Putnam et al. (1949). Plane slope 0.098.
$T = 0.95$ s, $H_{st} = 0.093$ m, $\theta_{st} = 49.°8$, $h_{st} = 0.50$ m,
$\beta = 0.8$ (estimated), $k_N = 0.006$ m

The present test has a well defined bottom (1/4" pea gravel, bonded with a thin grout) so the applied value of the Nikuradse roughness parameter of 0.006 m is rather reliable. With a chosen value of β = 0.8 (measured value of H_B/h_B is 0.72) a reasonable agreement with the rather vaguely defined mean longshore current is observed. It is noted that the values H_{st} and θ_{st} were calculated from h_{st} (arbitrarily chosen) and from measured quantities at the breaker line, using linear theory.

Galvin and Eagleson (1965) gave more detail, presenting current velocity profiles at several stations along a plane, smooth, concrete laboratory beach. In Fig. 6-B, calculated velocity profiles corresponding to β-values of 0.8 and 1.42 are shown together with measurements from Test No. 3 in Ser. III. The reason for trying also a value as high as 1.42 are the findings of Komar and Gaughan (1973). This value turned out to give a very good prediction of breaker heights, when one uses linear wave theory. Although this is no real justification for using the value uncritically in this study, the prediction of the maximum current velocity and of the breaker point is seen to be good. Another reason for using a higher value of β here than for Putnam's experiment, is the difference in wave steepness in the two cases (bed slope were almost equal): It seems that breaking was of the plunging type in the latter case, while it in Putnam's Exp. No. 29 probably was a transition phenomenon between spilling and plunging. (At the breaker line, Galvin and Eagleson measured β = 1.13.) The high calculated velocities near and outside the breaker line could be caused by the choice of a too high eddy viscosity in this region, as mentioned in Chapter 4, see also Fig. 7-B.

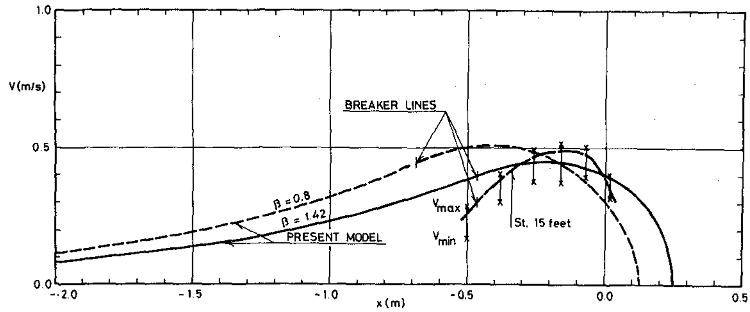

Fig. 6-B Galvin and Eagleson (1965) Ser. III, Test No. 3.
Plane slope 0.109 (nearshore average).
$T = 1.125$ s, $H_{st} = 0.051$ m, $\theta_{st} = 27^{\circ}$, $h_{st} = 0.351$ m,
$k_N = 0.0003$ m (estimated)

FIELD MEASUREMENTS

An attempt has been made also to predict the longshore current for an actual beach. To this end an observation from Trancas Beach (Jan. 19, 1962), as reported by Ingle (1966), has been used. The result is shown in Fig. 6-C. A rather good agreement between theory and measurement is

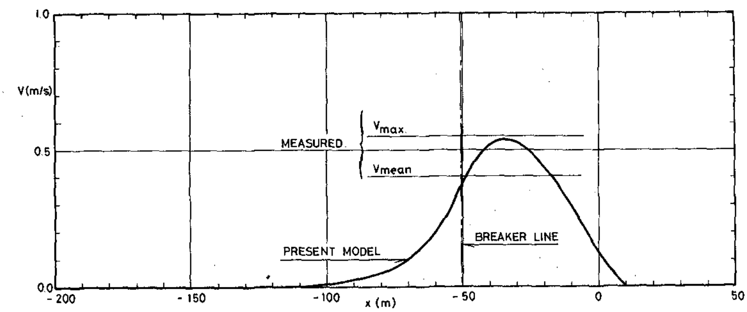

Fig. 6-C Ingle (1966). Trancas Beach, Jan. 19, 1962.
Slope 0.0244 (estimated).
$T = 8.5$ s, $H_{st} = 0.74$ m, $\theta_{st} = 18\overset{\circ}{.}1$, $h_{st} = 4.00$ m,
$\beta = 0.8$ (estimated), $k_N = 0.02$ m (estimated)

observed for $\beta = 0.8$. This is perhaps a little surprising, since breakers were probably plunging. On the other hand, Fig. 6-B already demonstrated that V_{max} is not very sensitive to which value is chosen for β. Furthermore, Ingle's data were incomplete, so the water depth at the breaker line had to be calculated as H_B/β, and β was chosen equal to 0.8. This value was then used to calculate H_{st} and θ_{st} (from H_B and θ_B) at the arbitrary start depth 4.00 m. Due to incomplete information for the bottom profile, a plane slope was assumed.

7. SOME EFFECTS OF β, k_N AND ν_T

It has already been inferred from Fig. 6-B that the calculated maximum current velocity is not too much affected by the chosen ratio between breaker height and water depth, β, within certain reasonable limits, naturally. Nor is the calculated width of the surf zone very sensitive to reasonable changes in β. On the other hand the position of the surf zone changes very much, as can be seen: Both the breaker line and the set-up line move drastically.

As regards the bottom roughness, k_N, we have tried in the examples of Chapter 6 not to make manipulations with the numerical input values chosen. In each case, a bottom roughness is estimated after careful inspection. (For a natural sand beach, see for instance Skovgaard et al. (1975).) This is important to observe, since it turns out that such an estimate is indeed crucial in our model. This can be seen from Fig. 7-A, showing

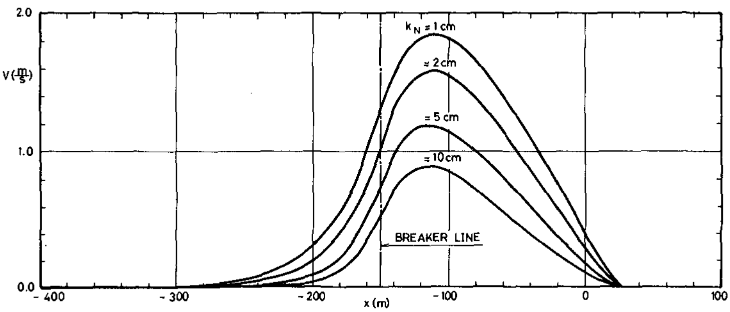

Fig. 7-A Influence of bottom roughness on calculated velocity profiles (Otherwise data as in Fig. 4-B)

calculated velocity profiles for four different roughnesses. It shows that, in this region of roughness parameters, a reduction of the roughness by a factor of ten will increase the maximum velocity in the present model by a factor of two. (It can be inferred from the figure that had we chosen a value of k_N = 5 cm in Fig. 6-C, the agreement with Ingle's measurement would become poorer.) Generally, it should be emphasized that very little is known about the bottom roughness of natural beaches under the combined effects of (breaking) waves and currents. The k_N-values chosen in Fig. 7-A do not necessarily represent what can be expected to be found in nature.

Finally, it has been investigated how sensitive the results are to a change in the eddy viscosity ν_T, Eq. 4.12, since this expression is based on some rather daring assumptions. The results are presented in Fig. 7-B. It appears that reducing ν_T by a factor of two (which is about the same as using Thornton's expression, see Chapter 4) does not have any large effect on the longshore current. The calculated maximum velocity is only increased by some 10%. Velocities outside the breaker line are as a whole reduced, naturally. Since it is felt that the present model probably exaggerates mixing outside the surf zone, the effect of a drastic reduction of ν_T in this region was investigated also. Reducing it by a factor of 100 changes the total mass flux significantly, as would be expected. The reduction in V_{max} is only about 10%, however.

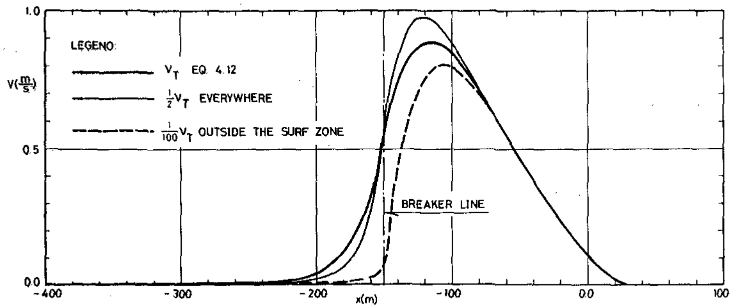

Fig. 7-B Influence of eddy viscosity on calculated velocity profiles (Data as in Fig. 4-B)

8. CONCLUSIONS

The proposed method makes it possible to predict (in a steady state) the profile of the longshore current, induced by regular waves, over a bottom with arbitrary parallel bottom contours, when the wave parameters are known at some arbitrary depth, where the actual velocity is very small. Also the breaker height to depth ratio (assumed constant through the surf zone), and the bottom roughness variation must be prescribed. The variations in the mean water level (set-down and set-up) are included in the mathematical model, with proper regard taken to the effect of refraction.

The method includes two new expressions, one for the calculation of the turbulent lateral mixing, and one for the bottom friction. In contrast to most other works, the friction term is non-linear in this study.

The governing differential equations for the current are of the second order, ordinary and non-linear. The matched, two-point boundary value problem is solved as a sequence of pairs of linear, matched boundary value problems, using Newton's method. This linear differential equations are solved by piecewise orthogonal collocation.

The maximum current velocity depends heavily on the bottom roughness, but rather little on the chosen breaker height to depth ratio. On the other hand, the predicted position of the surf zone is very much affected by the latter quantity. Therefore both the roughness and the breaker height to depth ratio should be chosen with great care in each case.

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