CHAPTER 36

WATER WAVES ON A BILINEAR SHEAR CURRENT

by

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Abstract

A water wave theory is presented to describe waves propagating on a bilinear shear current flowing in the direction of the waves. The theory is derived assuming an ideal fluid in which a current exists, having a vertical velocity profile which varies linearly from a mean water level velocity of U_S, an interfacial velocity U_I at depth, d, and a bottom velocity U_B. The theory is developed first for small amplitude waves and then extended to any arbitrary order by a numerical perturbation technique for symmetric waves. For measured waves, an irregular form of the theory is presented to provide a representation of these waves for analysis.

Introduction

In the design of offshore structures, it is necessary to use water wave theories that incorporate into their formulation a mean current, as currents are always present under design conditions. In the past, several techniques have been tried to incorporate the current. First, a constant current, having the same velocity over the depth and flowing in the direction of the wave, was assumed as, for instance, in the Stream Function wave theory (Dean, 1965). Recently, Dalrymple (1974) developed the linear shear current theory, which extended the Stream Function wave theory by allowing the inclusion of a current which varied linearly over the depth. This model could be carried to any order, thus extending the analyses of Biesel (1950) and Tsao (1959). (See also Dalrymple, 1973.)

In this paper, a better model is proposed, which fits an ambient current with a velocity profile which varies linearly over the depth from a mean water level velocity of U_S to an interfacial velocity, U_I at some depth,d. From U_I, the velocity again varies linearly to the bottom velocity, U_B. By including the interfacial velocity, the designer is allowed more flexibility in modeling the design current. This model, the bilinear shear current theory, is developed first for small amplitude symmetric waves and then for finite

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amplitude waves; in either case, the waves are characterized by the wave height, H, the wave period, T, the water depth, h, and the current parameters (U_S, U_I, d, U_B). A final form of the theory is presented for the representation of measured wave data, which is characterized by the measured, digitized, free surface elevations and the current parameters.

Mathematical Formulation of the Boundary Value Problem

Several assumptions must be made a priori to enable the formulation of a boundary value problem. First, the waves are assumed to be long crested, which makes the problem two-dimensional and, secondly, the waves propagate without change in form. With this last assumption, the coordinate system may be translated with the wave celerity, C, thus rendering the wave motion steady in time. Next the fluid is assumed incompressible, or mathematically,

$$\frac{\partial(U + u - C)}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

where U is the ambient current and (u,v) are the horizontal and vertical wave-induced water particle motions in the (x,y) directions. A stream function, $\psi(x,y)$ may now be defined by

$$\left.\begin{aligned} -\frac{\partial\psi}{\partial y} &= U + u - C \\ \frac{\partial\psi}{\partial x} &= v \end{aligned}\right\} \tag{2}$$

Lastly, the current is assumed to be well-established and the effect of viscosity is neglected. The applicable equations of motion then are the Euler equations, Lamb (1945).

$$(U + u - C)\,\frac{\partial(U + u - C)}{\partial x} + v\,\frac{\partial(U + u - C)}{\partial y} = -\frac{1}{\rho}\,\frac{\partial p}{\partial x} \tag{3}$$

$$(U + u - C)\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = -\frac{1}{\rho}\,\frac{\partial p}{\partial y} - g \tag{4}$$

Cross-differentiating to eliminate the pressure, p, and the acceleration due to gravity, g, and substituting the stream function yields

$$-\frac{\partial\psi}{\partial y}\,(\nabla^2\psi) + \frac{\partial\psi}{\partial x}\,(\nabla^2\psi) = 0 \tag{5}$$

where ∇^2 is the two-dimensional Laplacian operator. This equation requires that the fluid vorticity, $\nabla^2\psi$, be constant along a streamline, therefore, the equation may be integrated to yield

$$\nabla^2\psi = f(\psi) \tag{6}$$

where $f(\psi)$ is the vorticity distribution function. The classical theories of Airy (1845) and Stokes (1847) correspond to the irrotational case, $f(\psi) = 0$ everywhere. In this paper, $f(\psi)$ is assumed to be a constant in each of the two fluid regions depicted in Figure 1.

The theoretical form of the ambient shear current is expressed as

$$U(y) = \begin{cases} U_B + (U_I - U_B)\left(\frac{h+y}{h-d}\right), & \text{for } -h \le y < -d + \zeta(x) \\ U_S + (U_S - U_I)\left(\frac{y}{d}\right), & \text{for } -d + \zeta(x) \le y \le \eta(x) \end{cases} \tag{7}$$

Here, $\zeta(x)$ and $\eta(x)$ are the displacements of the interface and the free surface due to the passage of the wave.

Substituting $U(y)$ into Equation 6 results in two differential equations:

$$\begin{aligned} \nabla^2\psi_1 &= -\left(\frac{U_I - U_B}{h-d}\right) \quad \text{for} \begin{cases} -h \le y < -d + \zeta(x) \\ 0 \le x < L \end{cases} \\ \nabla^2\psi_2 &= -\left(\frac{U_S - U_I}{d}\right) \quad \text{for} \begin{cases} -d + \zeta(x) \le y \le \eta(x) \\ 0 \le x < L \end{cases} \end{aligned} \tag{8}$$

where L is the wave length. The subscripts on the stream functions denote the fluid regions for which they are applicable; ψ_1, the lower region and ψ_2, the surface region.

To fully prescribe the boundary value problems for ψ_1, ψ_2, boundary conditions must be specified. At the horizontal bottom, no vertical flow is allowed,

$$\frac{\partial\psi}{\partial x} = 0 \text{ on } y = -h \tag{9}$$

For periodic waves, the stream function must be periodic over a wave length.

$$\psi_i(x,y) = \psi_i(x+L,y), \quad i = 1, 2. \tag{10}$$

At the free surface, the pressure must be a constant. To mathematically express this condition, the Bernoulli equation, which is valid along a streamline, is used:

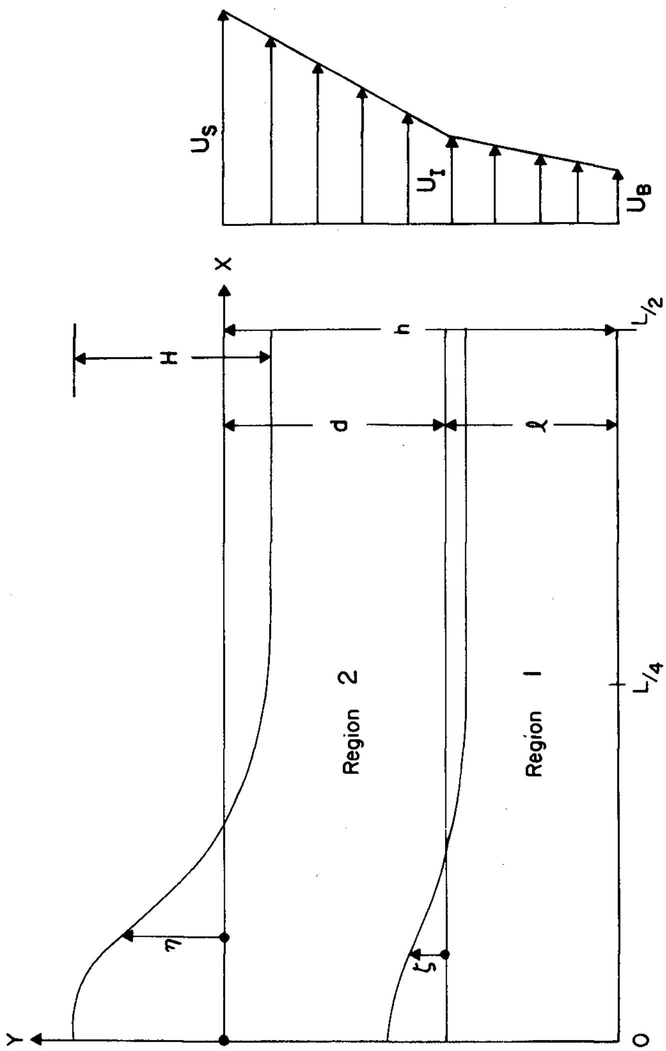

FIGURE 1 DEFINITION SKETCH FOR A WAVE ON A BILINEAR CURRENT, AS SEEN BY OBSERVER MOVING WITH THE WAVE

$$y + \frac{(U + u - C)^2 + v^2}{2g} + \frac{p}{\rho g} = Q(\psi), \text{ a constant.} \tag{11}$$

The boundary condition, called the Dynamic Free Surface Boundary Condition (DFSBC), then is expressed on the free surface streamline as

$$\eta + \frac{\left[\left(\frac{\partial\psi_2}{\partial x}\right)^2 + \left(\frac{\partial\psi_2}{\partial y}\right)^2\right]}{2g} = \overline{Q}, \text{ a constant on } y = \eta(x) \tag{12}$$

Also on the free surface, it is specified that the presence of the wave does not change the mean water level; that is, $\eta(x)$ must have a zero mean.

$$\frac{1}{L}\int_0^L \eta(x)\, dx = 0 \tag{13}$$

For small amplitude waves, it is convenient to use an alternative form of the condition, called the Kinematic Free Surface Boundary Condition (KFSBC), which requires the free surface to be a streamline,

$$-\frac{\partial\psi}{\partial y}\frac{\partial\eta}{\partial x} = \frac{\partial\psi}{\partial x} \text{ on } y = \eta(x) \tag{14}$$

Note that this requirement is true by definition when using a stream function representation of the fluid flow.

Across the interface between the two fluid regions, the velocities and pressures must be continuous. From the Bernoulli equation applied to the uppermost streamline in Region 1 and the lowermost streamline in Region 2, the pressures across the interface will be continuous if the horizontal and vertical velocities are continuous. Therefore, the interfacial boundary conditions may be specified as

$$\left.\begin{aligned} \frac{-\partial\psi_1}{\partial y} &= \frac{-\partial\psi_2}{\partial y} \\ \frac{\partial\psi_1}{\partial x} &= \frac{\partial\psi_2}{\partial x} \end{aligned}\right\} \text{ on } y = -d + \zeta(x) \text{ for all } x \tag{15}$$

Finally, it is required that the interfacial displacement have a zero mean

$$\frac{1}{L}\int_0^L \zeta(x)\, dx = 0 \tag{16}$$

For a small amplitude wave, it is again convenient to use the alternative condition

$$-\frac{\partial\psi}{\partial y}\frac{\partial\zeta}{\partial x}=\frac{\partial\psi}{\partial x} \quad \text{on } y=-d+\zeta(x) \tag{17}$$

Small Amplitude Bilinear Shear Current Theory

For small waves, all nonlinear terms in the boundary conditions are linearized using the rationale that terms of O(H/L) are small, therefore, terms of $O((H/L)^2)$ are negligible.

The free surface and interfacial displacements are assumed to be sinusoidal in x and given by

$$\eta(x) = \frac{H}{2}\sin kx$$
$$\zeta(x) = \frac{b}{2}\sin kx \tag{18}$$

where k is the wave number ($k = 2\pi/L$).

The stream functions are assumed to be of the following form

$$\psi_1(x,y) = -(U_B-C)y - (U_I-U_B)\frac{\left(hy+\frac{y^2}{2}\right)}{(h-d)} + D\sinh k(h+y)\sin kx \tag{19}$$

$$\psi_2(x,y) = -(U_S-C)y - (U_S-U_I)\frac{y^2}{2d} + (A\sinh ky + B\cosh ky)\sin kx \tag{20}$$

These stream functions satisfy the periodicity requirements and the bottom boundary condition. The coefficients, A, B, and D must now be found to satisfy the remaining boundary conditions.

Using the KFSBC, Equation 14, and retaining only the linear terms.

$$B = (U_S-C)\frac{H}{2} \tag{21}$$

The DFSBC must be satisfied at the free surface. In a linear analysis, the DFSBC is expanded in a Taylor Series about y = 0 and analytically continued up to $y = \eta$. Keeping only the first term,

$$\overline{Q} = \frac{\left[\left(\frac{\partial\psi_2}{\partial y}\right)^2 + \left(\frac{\partial\psi_2}{\partial x}\right)^2\right]}{2g} + \eta\frac{\partial}{\partial y}\left\{y + \frac{\left[\left(\frac{\partial\psi_2}{\partial x}\right)^2 + \left(\frac{\partial\psi_2}{\partial y}\right)^2\right]}{2g}\right\} \text{on } y = 0 \qquad (2\ )$$

Substituting for ψ_2 and retaining only the linear terms,

$$A = \frac{H}{2}\,\frac{(g + (U_S-C)(U_S-U_I)/d)}{(U_S-C)k} \qquad (23)$$

in order that $\overline{Q}$ be a constant.

At the interface, the linear kinematic condition can be written as

$$\frac{\partial\psi_1}{\partial x} = (U_I-C)\,\frac{\partial\zeta}{\partial x} \quad \text{on } y = -d \qquad (24)$$

or

$$D = \frac{(U_I-C)b}{2\sinh k\ell} \qquad (25)$$

The requirement that the vertical velocities be continuous across the interface yields

$$D = \frac{-A \sinh kd + B \cosh kd}{\sinh k\ell} \qquad (26)$$

The last boundary condition to be satisfied is the equality of the horizontal velocities across the interface,

$$\frac{(U_I-U_B)}{\ell}\,\frac{b}{2} - Dk \cosh k\ell = \frac{(U_S-U_I)}{d}\,\frac{b}{2} - (Ak \cosh kd - Bk \sinh kd) \qquad (27)$$

Substituting for $\frac{b}{2}$, D, and B in terms of A yields, after some algebra,

$$\left[\left[\frac{(U_S-U_I)}{d} - \frac{(U_I-U_B)}{\ell}\right]\frac{1}{(U_I-C)} + k \coth k\ell\right]\left[(U_S-C)^2 - \left(g + (U_S-C)\frac{(U_S-U_I)}{d}\right)\frac{\tanh kd)}{k}\right]$$

$$= g + (U_S-C)(U_S-U_I)/d - (U_S-C)^2 k \tanh kd. \tag{28}$$

This is the dispersion relationship which relates the wave number, k, to the given characteristics of the wave and current.

The final forms of the stream functions are

$$\psi_1(x,y) = -(U_B-C)y - (U_I-U_B)(hy + \frac{y^2}{2})/\ell + \frac{H}{2}\left[(U_S-C)\cosh kd - \frac{(g + (U_S-C)(U_S-U_I)/d)}{(U_S-C)k}\sinh kd\right]\frac{\sinh k(h+y)}{\sinh k\ell}\sin kx \tag{29}$$

$$\psi_2(x,y) = -(U_S-C)y - \frac{(U_S-U_I)}{2d}y^2 + \frac{H}{2}\left[(U_S-C)\cosh ky + \frac{(g + (U_S-C)(U_S-U_I)/d)}{(U_S-C)k}\sinh ky\right] \cdot \sin kx \tag{30}$$

The small amplitude form of the bilinear shear current theory, Equations (27), (28), and (29), generalized the work of a number of previous investigators. For example, Sir G. I. Taylor (1955), investigating wave breaking by bubble breakwaters in infinitely deep water, treated the case for $U_B = U_I = 0$ and $h \to -\infty$. Thompson (1949), earlier treated the case of a shear current in the lower layer. Binnie and Cloughley (1971) investigated the problem of stationary waves (C=0) on the same current profile, i.e., $U_S = U_I$. All of the results of these earlier investigators are a special case of the bilinear shear current theory, as, in fact, is the wave propagating in still water.

Finite Amplitude Bilinear Shear Current Theory

To extend the bilinear shear current theory to large waves, that is, when (H/L) is not necessarily small, all the nonlinear terms in the boundary conditions must be retained and, further, the free surface conditions must be applied directly on the free surface. To do this efficiently, some of the boundary conditions are specified in a least squares form. The boundary conditions are thus rewritten as:

$$E_1 = \frac{2}{L}\int_0^{L/2} (Q(x) - \overline{Q})^2\, dx \quad \text{where } \overline{Q} = \frac{2}{L}\int_0^{L/2} Q(x)\, dx \text{ on } y = \eta(x) \tag{31}$$

$$E_2 = \frac{2}{L}\int_0^{L/2} \eta(x)\, dx \tag{32}$$

$$E_3 = \eta(0) - \eta(\tfrac{L}{2}) - H \tag{33}$$

$$E_4 = \frac{2}{L}\int_0^{L/2} \left(-\frac{\partial\psi_2}{\partial y} + \frac{\partial\psi_1}{\partial y}\right)^2 dx \tag{34}$$

$$E_5 = \frac{2}{L}\int_0^{L/2} \left(\frac{\partial\psi_2}{\partial x} - \frac{\partial\psi_2}{\partial x}\right)^2 dx \tag{35}$$

and

$$E_6 = \frac{2}{L}\int_0^{L/2} \zeta(x)\, dx \tag{36}$$

The error terms E_1, E_4, and E_5 are the mean square error to the DFSBC and the interfacial conditions. The E_2, E_3 and E_6 terms are the constraint terms on the solution, stating that the mean water levelremain unchanged, the wave height be specified as H, and the interface not be displaced by the wave. For an exact solution to the boundary value problem then, all the E_i (i=1,6) would be zero. Note also that the symmetry of the wave has been used to reduce the range of the integration to only L/2.

It is convenient to define an objective function, OF, which must be minimized towards zero, using a Lagrange multiplier approach. (See, for example, Hildebrand, 1965).

$$OF = E_1 + \lambda_1 E_2 + \lambda_2 E_3 + E_4 + E_5 + \lambda_3 E_6 \tag{37}$$

where the λ_i (i=1,3) are the Lagrange multipliers. Again, if OF is zero, or very small, the problem is solved.

The stream function for each region is assumed to be given by a series expansion of the following form

$$\psi_1\,(x,y) = -\,(U_B-C)y - (U_I-U_B)(hy + y^2/2)/(h-d) + \sum_{n=NN+2}^{NN+\left(\frac{NN-2}{2}\right)+1} X(n) \cdot \sinh k_n\,(h+y)\,\cos k_n x \tag{38}$$

where $k_n = 2(n - (NN+1))\,\pi/L$

$$\psi_2\,(x,y) = -(U_S-C)y - \frac{(U_S-U_I)y^2}{2d} + \sum_{n=3,5}^{NN-1} \left(X(n)\,\sinh\frac{(n-1)\pi y}{L} + X(n+1)\,\cosh\frac{(n-1)\pi y}{L}\right) \cdot \cos\frac{(n-1)\pi x}{L} \tag{39}$$

The parameter, NN, is related to the order of the wave theory; NN=2· order +2. These stream functions satisfy the governing differential equations, (8), and the periodicity and bottom boundary conditions exactly. The X(n) are unknown constant coefficients, which then must be chosen to reduce OF towards zero. The other unknowns are the wave length, L, and the value of the free surface and interface streamlines, $\psi_2(x,\eta)$ and $\psi_2(x, -d+\zeta)$.

The free surface and interface displacements are obtained by solving (39) by substituting the appropriate value of the streamline.

To solve for the unknowns, a trial set of X(n) are necessary. These can be obtained by neglecting the shear current in the upper layer, keeping only the U_S term, assuming a first order wave, setting $\psi_2(x,\eta) = 0$ and using the wave length obtained from (28). The trial values of X(3) and X(4) are then obtained from (39), by examining the wave crest, ($y = H/2$, $X = 0$) and the wave trough, ($y = -H/2$, $x = L/2$). The remaining X(n) are set to zero.

With these trial X(n), the value of OF is quite large. An iterative numerical perturbation technique is thus used to minimize OF with respect to the X(n). To facilitate this, the objective function is quasi-linearized by expanding all terms of OF at iteration (j + 1) in a first order Taylor series in $X'(n)$ at iteration (j) where $X'(n)$ are small changes in the X(n).

$$OF^{j+1} = OF^{j} + \sum_{n=1}^{NN + \left(\frac{NN-2}{2}\right) + 1} \frac{\partial(OF^{j})}{\partial X(n)} X'(n) \tag{40}$$

Next, OF^{j+1} is minimized with respect to the X(n) and the Lagrange multipliers. This results in (3NN+6)/2 equations for the same number of unknowns. By solving these equations by matrix techniques for the $X'(n)$, new values of the X(n) are obtained.

$$X(n)^{j+1} = X(n)^{j} + \alpha\, X'(n)^{j} \tag{41}$$

Substituting these new values into OF results in a smaller error. This procedure is then iterated until OF is acceptably small. Typically this requires about twenty iterations. Note that α in (41) is, in general, near one; however, for near breaking waves, an instability results if α is not less than one-half.

Analysis of a Measured Wave Propagating on a Bilinear Shear Current

The bilinear current wave theory may also be modified for use in the analysis of measured wave data where the free surface displacement and the current is known and can be approximated by a bilinear shear current. The governing equations and the boundary conditions at the interface are the same as before. The kinematic free surface condition is modified so as to ensure the predicted free surface displacement, η_{p_i}, at each digitized time point, i, corresponds to the measured free surface displacement, η_{m_i}. There are assumed to be I data points corresponding to the total free surface profile.

The stream functions for the two regions are

$$\psi_1(x,y) = -(U_B-C)y - (U_B-U_I)\frac{(hy+y^2/2)}{(h-d)} + \sum_{n=3NN+5}^{4NN+5} X(n)\sinh\frac{2\pi m_n(h+Y)}{L} \cdot \cos\left(\frac{2\pi m_n t}{T} + X(3m_n+3)\right) \tag{42}$$

$$\psi_2(x,y) = -(U_S-C)y - (U_S-U_I)y^2/2d + \sum_{n=1,2..}^{NN}\left[X(3n+1)\sinh\frac{2n\pi y}{L} + X(3n+2)\cosh\frac{2n\pi y}{L}\right] \cdot \cos\left[\frac{2n\pi t}{T} + X(3n+3)\right] \tag{43}$$

where m_n=n - (3NN+4) and NN is now the order of the wave theory. The unknowns again are the X(n), and L, T, $\psi_2(x,\eta)$, $\psi_2(x, -d+\zeta)$, which are defined as X(1), X(2), X(3) and X(3NN+4) for convenience.

The stream functions are periodic in L, ψ_1 satisfies the bottom boundary condition, and a phase angle, X(3n+3), necessary to fit an irregular water surface, is introduced.

To determine the X(n) which best satisfies the remaining boundary conditions, an objective function is defined:

$$OF^j = \frac{1}{I}\sum_{i=1}^{I}(Q_i-\bar{Q})^2 + \frac{1}{I}\sum_{i=1}^{I}\left(-\frac{\partial\psi_2}{\partial y} + \frac{\partial\psi_1}{\partial y}\right)_i^2 + \frac{1}{I}\sum_{i=1}^{I}\left(\frac{\partial\psi_2}{\partial x} - \frac{\partial\psi_1}{\partial x}\right)_i^2 + \frac{1}{I}\sum_{i=1}^{I}(\eta_m-\eta_p)_i^2 + \frac{1}{L}\int_0^L \zeta(x)\,dx \tag{44}$$

Here the integral form for least squares is replaced by a summation over the I data points. Again, if OF is equal to zero, the boundary value problem would be solved exactly. The numerical perturbation procedure is exactly the same as in the previous case, given a trial set of X(n) , obtained as before, a better set of coefficients is found by minimizing OF^{j+1} in its quasi-linearized form with respect to all the X(n) , and solving for the $X'(n)$ which are added to the $X(n)^j$ to obtain the $X(n)^{j+1}$. This procedure is repeated until O^j is acceptably small.

In this application it is assumed that only wave length and the coefficients are affected by the region below the interface; therefore, in the least squares procedure, the wave period, T, and the NN phase angles are determined solely by the fit to the free surface conditions.

Results and Comparisons with Previous Theories

Due to the lack of laboratory or field data for the empirical verification of either form of the bilinear shear current theory, analytic validity must be used to verify the theories. As an example, a shallow water wave was generated propagating on a bilinear shear current flowing against the wave direction. In Table 1, the errors to the boundary condition at the free surface and the interface are given as well as the characteristic values of the wave and current. The maximum error occurs in the mean displacement of the interface, with an error, in this case, of 0.0689 ft. (0.021m), which is acceptably small. More iterations or terms in the series solution reduces the error further, thus substantiating the validity of the solution.

Table 1. Dimensionless Errors to Boundary Conditions of a Shallow Water Wave Propagating Against a Bilinear Shear Current

Given Data: H = 6.29 ft. (1.92m), T = 10 sec., h = 10.0 ft. (3.05m), U_S = -1.0fps (-0.305m/sec.), U_I = -0.2fps (-0.06m/sec.) at d = 5.0 ft. (1.52m), U_B = 0.0fps.

Twelfth order wave theory with 12 iterations, with resulting wave length of 201.59 ft. (61.44m).

E_1/h (Eq.31)	E_2/h (Eq.32)	E_3/h (Eq.33)	E_4/C^2 (Eq.34)	E_5/C^2 (Eq.35)	E_6/h (Eq.36)
2.44×10^{-6}	2.37×10^{-3}	1.62×10^{-3}	5.30×10^{-6}	1.57×10^{-6}	6.89×10^{-3}

To illustrate the necessity of an adequate representation of both the wave and the current, a comparison was made between the bilinear shear current, the linear shear current and the Stream Function wave theories. For each theory, a 50-foot (15.2m) high wave was generated in 100 feet (30.48m) of water with a 3.0fps (1.9m/sec.) mean water level current. The difference between a constant current (Stream Function) velocity profile and the bilinear velocity profile (U_I=U_B=0 and d=25 feet (7.62m)) under the wave crest is quite large and would obviously result in a vast disparity in wave forces. The linear shear current theory is shown in both its linear and finite amplitude form. The finite amplitude theory compares more favorably with the bilinear theory than does the Stream Function as it is a better model to the bilinear current. The linear theory results in quite large errors as it is unable to represent the wave profile correctly. These data are shown in Figure 2.

Finally, the irregular form of the bilinear theory was compared with the irregular forms of the linear shear current theory (Dalrymple, 1974) and the Stream Function theory (Dean, 1965). The measured wave was taken from

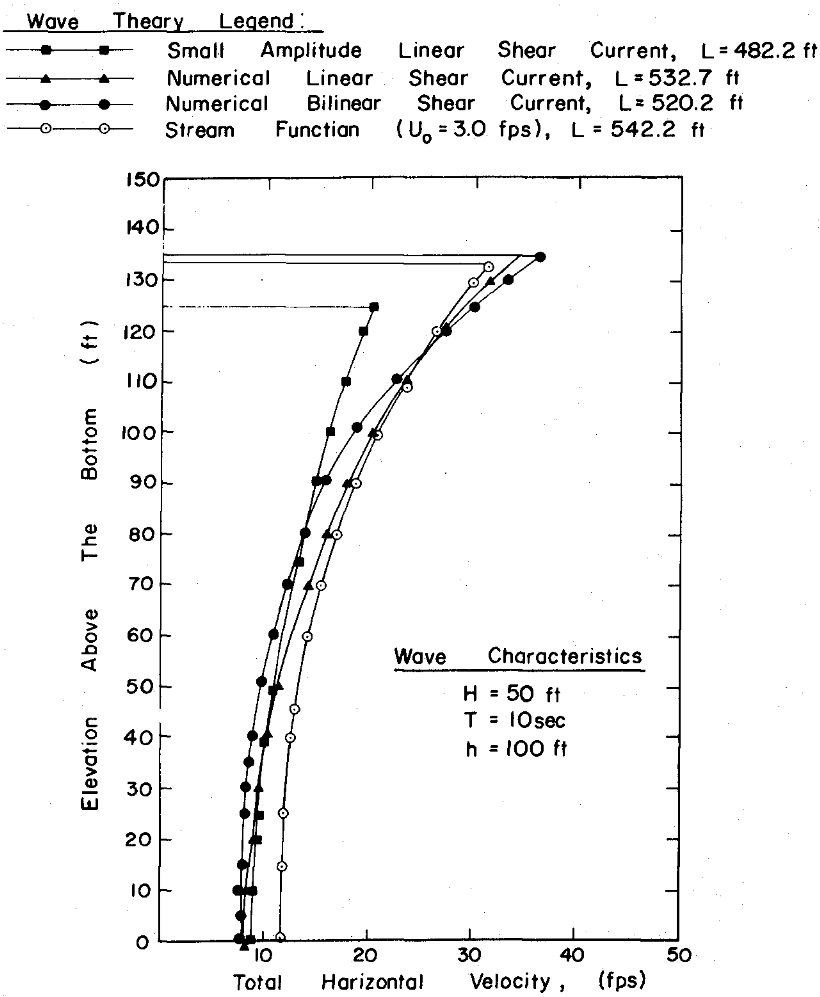

FIGURE 2 COMPARISON OF LINEAR AND BILINEAR HORIZONTAL VELOCITY PROFILES UNDER THE CREST OF THE CASE A WAVE FOR 3 FPS CURRENT AT STILL WATER LEVEL (y = 0)

Dean (1965), his Figure 5, and represents a wave 39 feet (11.89m) in height with a 14-second period propagating in 98 feet (29.9m) of water. For the Stream Function theory, no current was assumed, for the linear shear current theory, U_S=2.0fps (.61m/sec.), U_I=U_B=0, and d=25 feet (7.62m). The representation of the free surface and the fits to the boundary conditions were approximately the same in magnitude for all of the wave models; however, the differences in the predicted currents under the wave crest are quite significant. Also, the wave lengths of the predicted waves varies with the velocity profile as is shown at the top of Figure 3. With the exception of the small amplitude wave theory, the bilinear shear current has the shortest wave length; it also has the less current over the water depth. The more current over the water depth, the longer the wave (or the more the influence of the current on the wave).

Conclusions

Three representations of water waves propagating on a bilinear shear current flowing in the direction of the wave have been presented. As has been shown the models are analytically valid. The empirical validity awaits adequate field or laboratory studies.

The importance of including the correct form of the ambient current is to be emphasized. There are large disparities in maximum velocities and wave length for the same waves propagating on different currents, which would be greatly enhanced in wave force calculations (as the drag force is proportional to (U + u)/U + u/).

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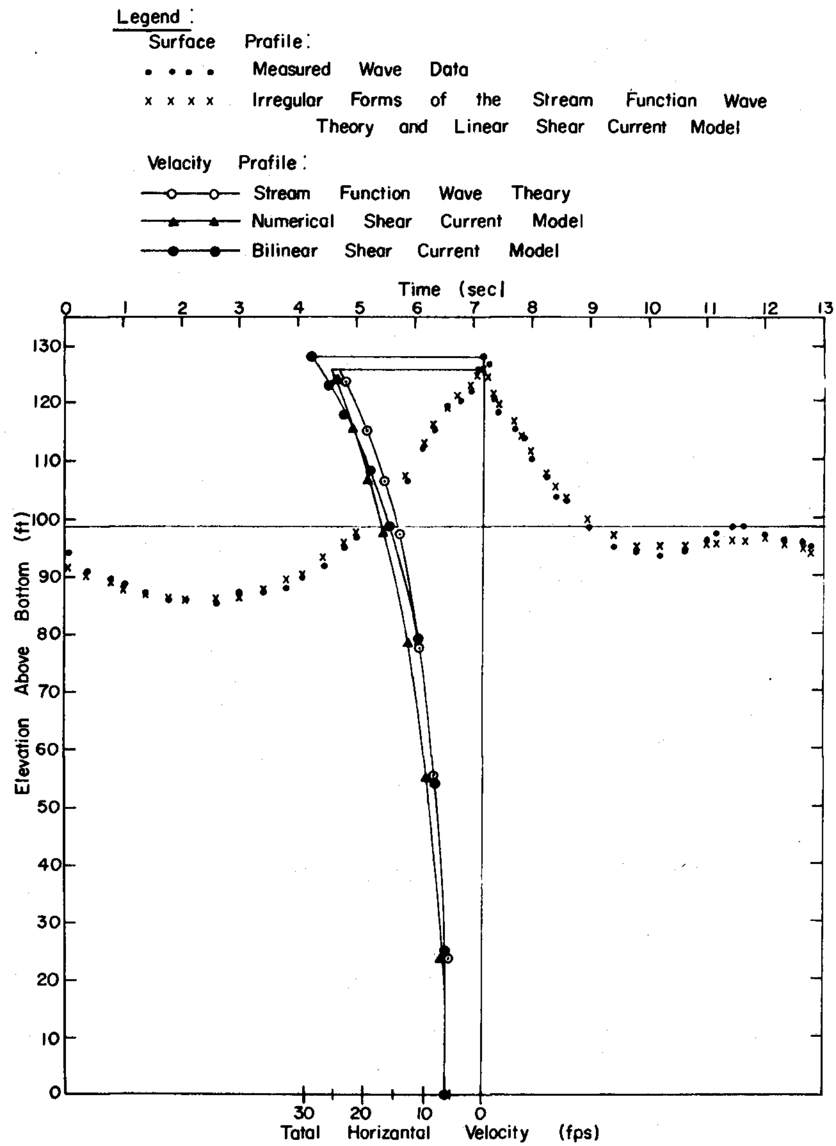

FIGURE 3 MEASURED WAVE FIT BY STREAM FUNCTION WAVE THEORY AND LINEAR AND BILINEAR SHEAR CURRENT MODELS, WITH TOTAL HORIZONTAL VELOCITY PROFILES PREDICTED UNDER WAVE CREST.

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