

CHAPTER 152

SURGE IN THE SOUTHEAST BASIN, LONG BEACH HARBOR, CALIF.

By Basil W. Wilson¹, F. ASCE
James A. Hendrickson² and Juan Jen³, A.M. ASCE

INTRODUCTION

Recent studies of the hydrodynamic responses of Los Angeles-Long Beach Harbors to the effects of long period waves [conducted in 1967-68 for the U.S. Army Corps of Engineers, Los Angeles District (35)], showed that the basins had characteristic modal periods of oscillation, which could be excited, apparently, on rather rare and largely unpredictable occasions. Available field data for confirmation of the mathematical results were largely non-existent at the time and satisfactory correlations were greatly hampered. In the interim the development of the Southeast Basin, Long Beach Harbor, for the reception of fast container ships has led to more detailed study and measurement of the responses of this particular basin to long period waves. The purpose of this paper is to present some of the original theoretical results of the 1968 study and examine their correlation with data of recent acquisition. Their bearing upon the motions in surge and sway of ships moored within the Southeast Basin will also be examined briefly in the light of some simple measurements of ship motions.

FREE UNDAMPED OSCILLATIONS OF HARBOR BASINS

Many approaches have been taken to the solution of the problem of the free oscillations of a basin of irregular planform and variable (or uniform) depth (4, 5, 11-14, 20, 25, 34, 35). Some general review of these has been given elsewhere (32,33) and accordingly it will not be repeated here. Interest at the moment,

¹ Consulting Oceanographic Engineer, Pasadena, California

^{2,3} Senior Engineers, Science Engineering Associates, Newport Beach, California, (Division of Dames and Moore)

specifically, is in results of the method followed by Wilson, et al (35).

The procedure follows from the equations of motion and continuity of a water particle in long waves of low amplitude, free from forced excitation and damping. These equations, referred to an x - y co-ordinate system in the horizontal plane, can be combined to yield (cf. 9, §193) a single equation governing water surface elevation η , namely

$$\frac{\partial^2 \eta}{\partial t^2} = g \left[\frac{\partial}{\partial x} \left(h \frac{\partial \eta}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \eta}{\partial y} \right) \right] \quad (1)$$

in which $h(x, y)$ is variable water depth according to location (x, y) and g the acceleration from gravity.

If the solution to Eq. (1) is taken of the form:

$$\eta = \zeta(x, y) \cos \omega t \quad (2)$$

where ζ is an amplitude function in (x, y) independent of time t and ω is an eigenfrequency (angular) of a mode of free motion, then substitution of Eq. (2) in Eq. (1) transforms the latter to

$$\frac{\partial}{\partial x} \left(h \frac{\partial \zeta}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \zeta}{\partial y} \right) + (\omega^2 / g) \zeta = 0 \quad (3)$$

On expansion Eq. (3) becomes

$$h \left(\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right) + \frac{\partial h}{\partial x} \frac{\partial \zeta}{\partial x} + \frac{\partial h}{\partial y} \frac{\partial \zeta}{\partial y} + (\omega^2 / g) \zeta = 0 \quad (4)$$

This equation can be integrated in closed form for only a limited number of relatively simple geometrical shapes of basins. However it may be solved numerically by a finite difference technique, which follows essentially the principles discussed by Stoker (24, §10.13).

In order to develop a proper degree of flexibility in the analysis, the equation is considered in relation to a rectangular co-ordinate grid system (Fig. 1a) whose elemental distances δ in the directions of x and y are not necessarily equal. Thus the regime at any single point 0 can be specified in relation to that at each

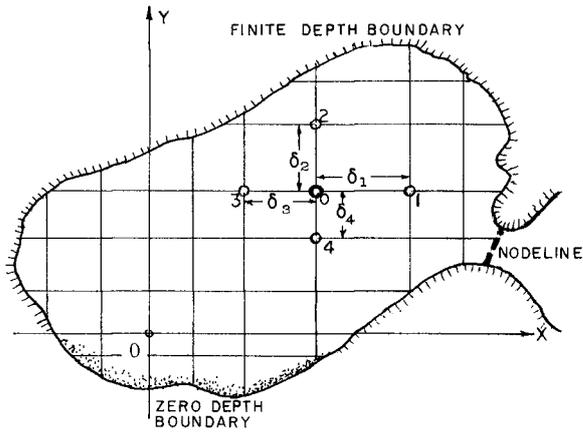


FIG. 1(a) - SCHEMATIC DIAGRAM OF RECTILINEAR COORDINATE GRID SYSTEM OVERLAID ON PLAN OF BAY OR HARBOR

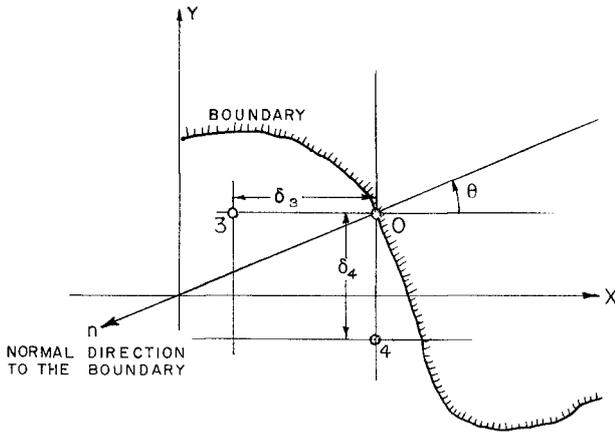


FIG. 1(b) - SCHEMATIC DETAIL OF GRID POINT DEFINING A BOUNDARY

of the four adjacent grid points, 1 to 4, in terms of the incremental distances δ_1 , δ_2 , δ_3 , and δ separating the points. Thus the differential terms of Eq. (4) can be expressed as:

$$(i) \quad \frac{\partial \zeta}{\partial x} = \frac{\zeta_1 \delta_3^2 + \zeta_0 (\delta_1^2 - \delta_3^2) - \zeta_3 \delta_1^2}{\delta_1 \delta_3 (\delta_1 + \delta_3)} \quad 5(a)$$

$$(ii) \quad \frac{\partial^2 \zeta}{\partial x^2} = 2 \left\{ \frac{\zeta_1 \delta_3 - \zeta_0 (\delta_1 + \delta_3) + \zeta_3 \delta_1}{\delta_1 \delta_3 (\delta_1 + \delta_3)} \right\} \quad 5(b)$$

and

$$(i) \quad \frac{\partial \zeta}{\partial y} = \frac{\zeta_2 \delta_4^2 + \zeta_0 (\delta_2^2 - \delta_4^2) - \zeta_4 \delta_2^2}{\delta_2 \delta_4 (\delta_2 + \delta_4)} \quad 6(a)$$

$$(ii) \quad \frac{\partial^2 \zeta}{\partial y^2} = 2 \left\{ \frac{\zeta_2 \delta_4 - \zeta_0 (\delta_2 + \delta_4) + \zeta_4 \delta_2}{\delta_2 \delta_4 (\delta_2 + \delta_4)} \right\} \quad 6(b)$$

with

$$(i) \quad \frac{\partial h}{\partial x} = \frac{h_1 \delta_3^2 + h_0 (\delta_1^2 - \delta_3^2) - h_3 \delta_1^2}{\delta_1 \delta_3 (\delta_1 + \delta_3)} \quad 7(a)$$

$$(ii) \quad \frac{\partial h}{\partial y} = \frac{h_2 \delta_4^2 + h_0 (\delta_2^2 - \delta_4^2) - h_4 \delta_2^2}{\delta_2 \delta_4 (\delta_2 + \delta_4)} \quad 7(b)$$

In Eqs (5) through (7), subscripts 0 to 4 identify the values of the function ζ at the points 0 to 4.

By substitution of Eqs (5) through (7) in Eq. (4) and by suitable algebraic manipulation, Eq. (4) can be converted to the form:

$$(\nu_k - a_0) \zeta_0 + a_1 \zeta_1 + a_2 \zeta_2 + a_3 \zeta_3 + a_4 \zeta_4 = 0 \quad (8)$$

in which the eigenvalue ν_k (an unknown) is related to any k th eigenfrequency ω_k ($k = 1, 2, 3, \dots, N$) by

$$\nu_k = \omega_k^2 / g \quad (9)$$

and the coefficients a_i ($i = 0, 1, 2, 3, 4$) are constants calculable in terms of

the depths h_1 and the distance increments δ_1 (35).

If there are N reticulation points of the type 0 in the network, N equations of the type of Eq. (8) can be formed for the N unknowns of the function ζ_n ($n = 1, 2, 3, \dots, N$) in association with N possible eigenvalues $\nu_k = 1, 2, 3, \dots, N$. Consequently an $N \times N$ matrix can be formed for determination of the eigenfrequencies ω_k and the surface elevations ζ_n from the corresponding eigenvectors.

Where the grid network intersects the boundaries of the basin, special conditions apply and some of the N equations of type Eq. (8) must express these conditions. In general the boundary can be of three forms; a zero-depth boundary such as a beach or sloping wall; a finite-depth boundary such as a vertical wall or cliff; and an open-mouth boundary such as a harbor entrance or bay-mouth. As shown in Fig. 1(b), through the versatility of a variable grid spacing, it is possible to ensure that a typical boundary condition can be expressed in terms of a grid point 0 located exactly on the boundary, at suitably small distances from adjacent grid-points 3 and 4 (for the configuration shown).

Zero-Depth Boundary:

The condition of zero depth ($h_0 = 0$) causes the first terms of Eq. (4) to vanish. Eqs (5a) and (6a) correspondingly reduce to

$$\frac{\partial \zeta}{\partial x} = (\zeta_0 - \zeta_3) \delta_3 \quad (10a)$$

$$\frac{\partial \zeta}{\partial y} = (\zeta_0 - \zeta_4) / \delta_4 \quad (10b)$$

while Eqs (7a) and (7b) become:

$$\frac{\partial h}{\partial x} = -h_3 / \delta_3 \quad (11a)$$

$$\frac{\partial h}{\partial y} = -h_4 / \delta_4 \quad (11b)$$

Applications of Eqs (10) and (11) to Eq. (4), then yield the special zero-depth boundary condition:

$$\{(h_3/\delta_3^2) + (h_4/\delta_4^2) - \nu_k\} \zeta_0 - (h_3/\delta_3^2) \zeta_3 - (h_4/\delta_4^2) \zeta_4 = 0 \quad (12)$$

Finite-Depth Boundary:

If the boundary at point 0 in Fig. 1(b) is one of finite depth, then there can be no horizontal flow of water normal to the boundary at that point. This requires that $\frac{\partial \zeta}{\partial n} = 0$ where n expresses the direction normal to the boundary. If θ be the angle which the normal direction makes with the x -axis, then

$$\frac{\partial \zeta}{\partial n} = \frac{\partial \zeta}{\partial x} \cos \theta + \frac{\partial \zeta}{\partial y} \sin \theta = 0 \quad (13)$$

By use of Eqs (10), Eq. (13) then becomes the special finite depth boundary condition.

$$(\delta_3 \tan \theta + \delta_4) \zeta_0 - \delta_4 \zeta_3 - (\delta_3 \tan \theta) \zeta_4 = 0 \quad (14)$$

Open-Mouth Boundary:

At an open-mouth "boundary" (fictitious, of course), by which a basin or bay is considered connected to another body of water, there will be, in general, both horizontal flow and vertical surface elevation of the water. The condition of existence of an eigenfrequency oscillation in the basin, however, usually requires that a nodal condition ($\zeta = 0$) shall prevail along some nodal line across the mouth. If the basin entrance is a comparatively narrow one, without much neck or channel, it is usually safe to assume that the direct line of shortest distance connecting the promontories of the mouth will mark the position of a node, across which the flow is normal. If the mouth is a broad entrance to a bay the position of a node line will be less accurately definable, though its position may always be assumed, and, in this, sound judgement and experience can be helpful. Strictly the condition across the mouth needs to be defined by matching it with the motion behavior of the external body of water, but in this study this refinement and complication was considered unnecessary.

The boundary equation for a grid-point on the node line, is then simply

$$\zeta_0 = 0 \quad (15)$$

VELOCITIES AND DIRECTIONS OF OSCILLATORY FLOW

The horizontal flow velocity, V , of the water at any point (x, y) in the free mode of oscillation (Fig. 2a) may be defined as to direction s by the angle ψ which the s -direction makes with the axis of x . For the long waves and small amplitudes which basin oscillations normally invoke, V may be considered uniform with depth. In terms of the velocity potential ϕ prevailing at the point (x, y) and the component velocities u and v , its value is

$$V = (u^2 + v^2)^{\frac{1}{2}} \approx \frac{\partial \phi}{\partial s} \tag{16}$$

while the value of the potential ϕ itself is given by the generalized Bernoulli equation for the free surface, namely

$$\frac{\partial \phi}{\partial t} = g\eta \tag{17}$$

From Eqs (2), (16) and (17), then, the velocity becomes

$$V = (g/\omega) \frac{\partial \zeta}{\partial s} \sin \omega t \tag{18}$$

Usually, because the velocity fluctuation is sinusoidal, the major interest is in knowing its maximum value, V_{\max} , (at $\sin \omega t = 1$) and its direction of flow, s . In relation to values ζ_i ($i = 0, 1, 2, 3, 4$) for grid points surrounding a central point $i = 0$ (Fig. 2b), the vector magnitude and direction of velocity can be determined from:

$$V_{\max} = (g/2\delta_1 \delta_2 \delta_3 \delta_4 \omega) [(\delta_2 \delta_4)^2 \{ \delta_3(\zeta_1 - \zeta_0) + \delta_2(\zeta_0 - \zeta_3) \}^2 + (\delta_1 \delta_3)^2 \{ \delta_4(\zeta_2 - \zeta_0) + \delta_2(\zeta_0 - \zeta_4) \}]^{\frac{1}{2}} \tag{19}$$

and

$$\tan \psi = \frac{\delta_1 \delta_3}{\delta_2 \delta_4} \cdot \frac{\delta_4(\zeta_2 - \zeta_0) + \delta_2(\zeta_0 - \zeta_4)}{\delta_3(\zeta_1 - \zeta_0) + \delta_1(\zeta_0 - \zeta_3)} \tag{20}$$

Thus velocities and directions of oscillatory flow are readily determined once the matrix solution of ζ has been achieved.

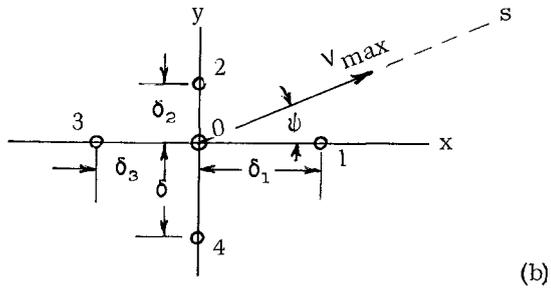
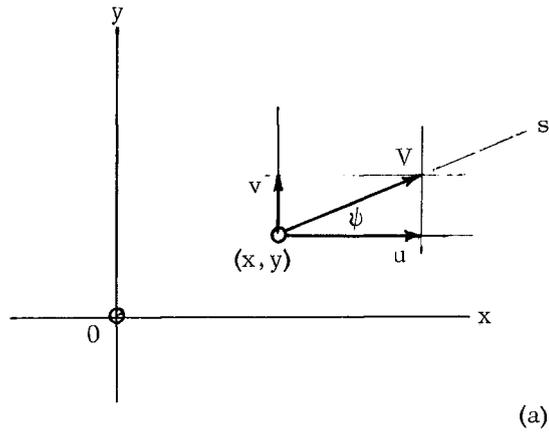


FIG. 2 - SCHEMATIC REPRESENTATION OF VELOCITY AT A POINT; (a) IN PLAN VIEW; (b) IN THE NUMERICAL GRID

NUMERICAL SOLUTION FOR SOUTHEAST BASIN SHAPE OF 1968

The Southeast Basin of Long Beach Harbor as it existed in 1967-1968 is shown in Fig. 3. Depths within the basin varied from about 35 ft to 55 ft in the deepest spot within Basin Six. The modelling of this basin for the numerical computation is shown by the grid of 88 points in Fig. 4 which cover the area and define the boundary. The sequence of points from 1 to 38 all lie upon the boundaries while the remainder from 39 to 88 encompass the interior area.

Results of the numerical calculations are given most readily in the form of contoured diagrams of normalized surface elevations expressing the generalization of Eq. (2), namely:

$$(\eta_k)_n = (\zeta_k)_n \cos \omega_k t \quad (21)$$

where $k(= 1, 2, 3, \dots)$ expresses the mode number and n the grid number ($n = 1$ to 88), for the condition that amplitudes are a maximum ($\cos \omega_k t = 1$). Thus Figs 5(a) and (b) give the mode shapes for the two lowest modes ($k = 1$ and $k = 2$), with contour lines in increments of 5, normalized to maximum positive surface elevation of 100. Arrows and small figures alongside grid points define directions and velocities in units per sec, normalized to maximum surface height variation (double amplitude) of one unit. Because values of velocity are of secondary interest in the present paper no attempt has been made to preserve the legibility of the velocity figures, as computer-plotted in these diagrams.

The fundamental mode oscillation ($k = 1$), with period $T_1 = 14.2$ mins, has just one node at the mouth of the basin (Fig. 5a); this is the so-called "pumping" mode for the Southeast Basin. In the second mode oscillation ($T_2 = 7.0$ mins), there are two nodes (Fig. 5b) and the entire basin oscillates in what is effectively a uninodeal seiche between Basin Six and the Southeast Basin. Basin Six, however, at the same time displays its pumping mode response to the node at the entrance of the coupled basins.

In Fig. 6(a) the third mode oscillation ($T_3 = 3.7$ mins) turns out to be predominantly a diagonal (cross) seiche between extremities of the Southeast Basin.

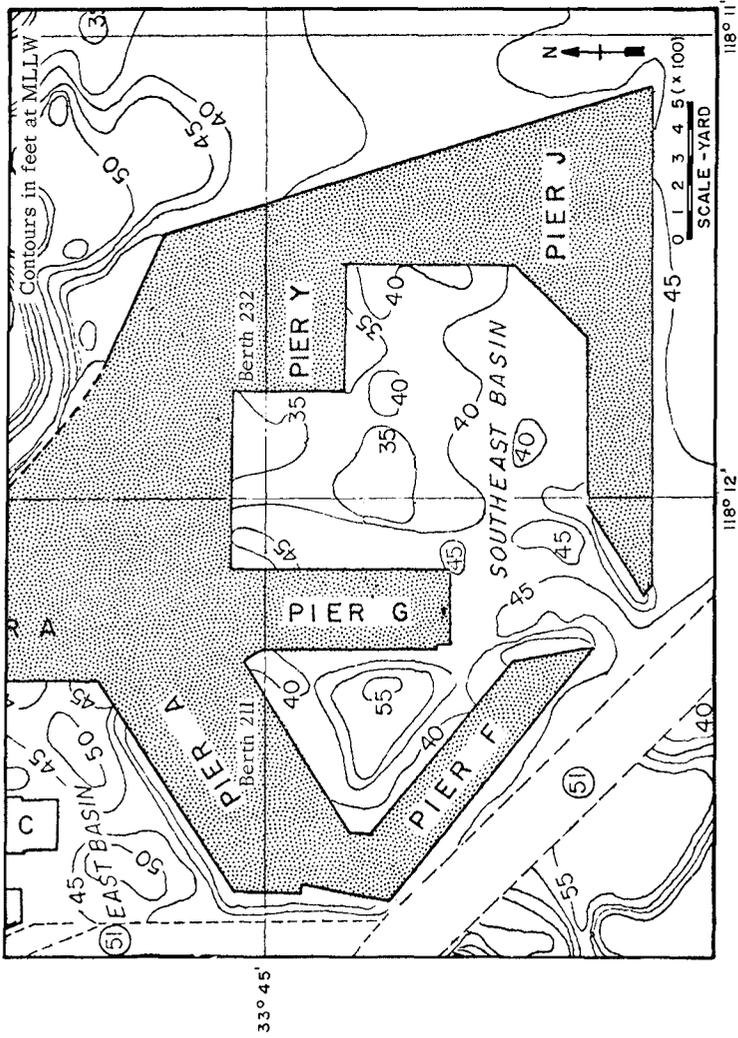


FIG. 3 - BATHYMETRY OF SOUTHEAST BASIN, LONG BEACH HARBOR (1967)

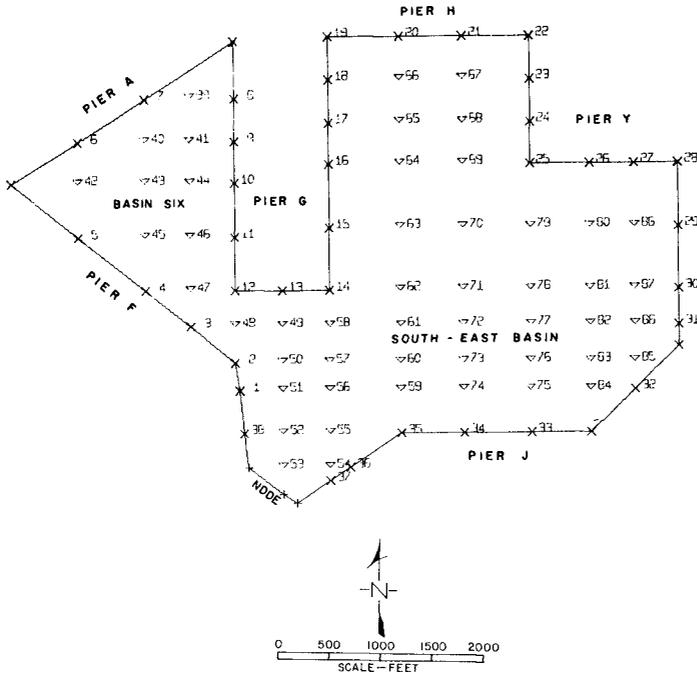


FIG. 4 - NUMERICAL GRID FOR MODELLING SOUTHEAST BASIN SHAPE OF 1967 - 1968

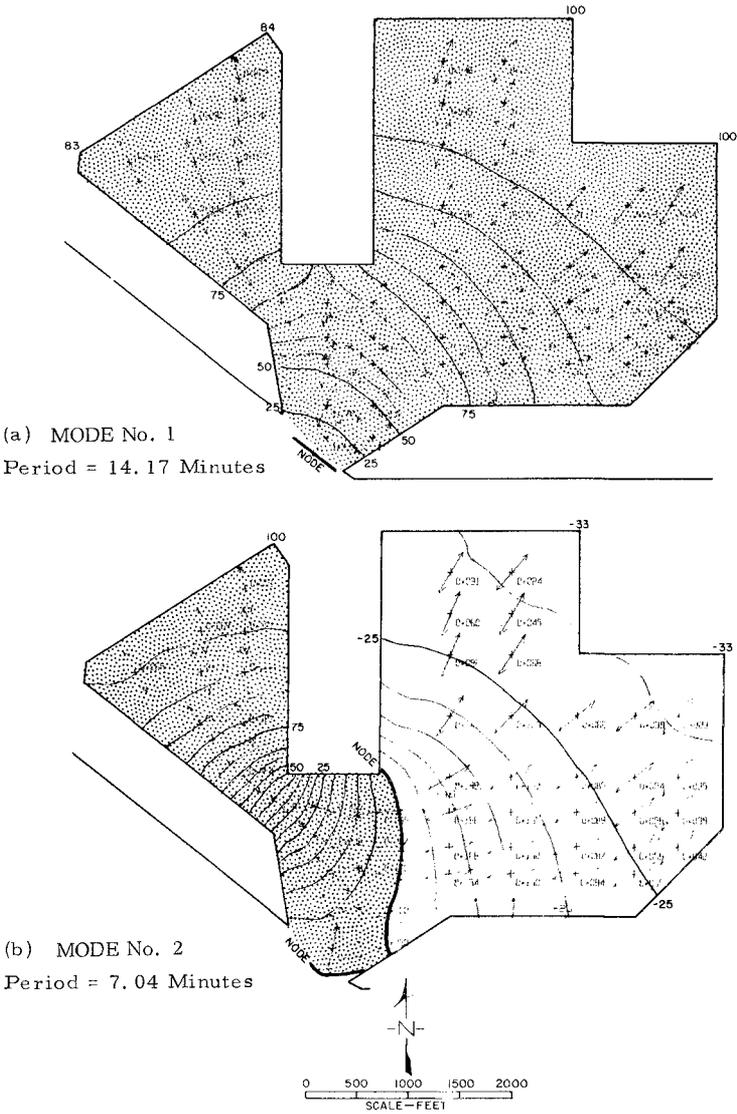


FIG. 5 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1968)

(a) 1st MODE; (b) 2nd MODE

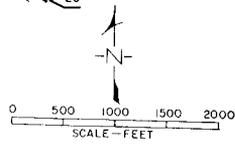
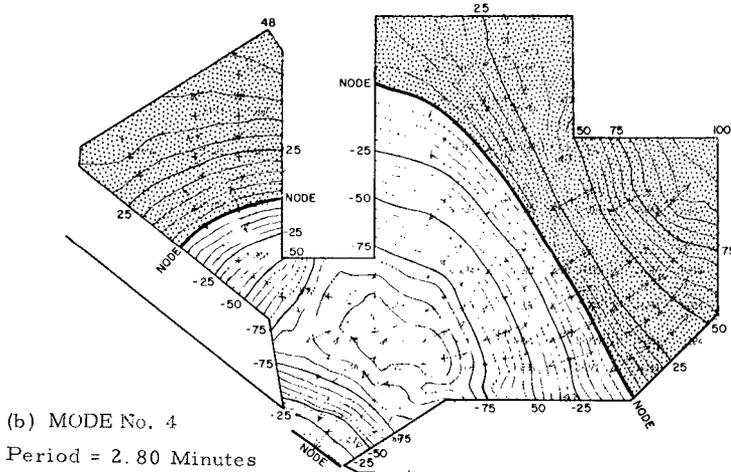
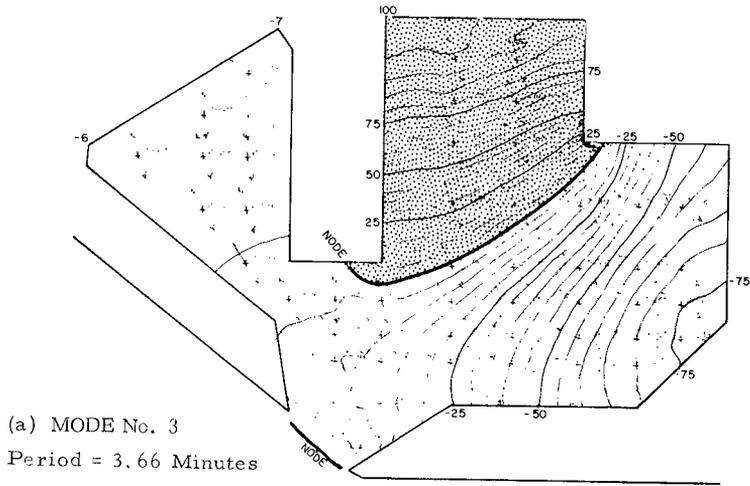


FIG. 6 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1968)

(a) 3rd MODE; (b) 4th MODE

Basin Six hardly responds at all.

The fourth mode oscillation ($T_4 = 2.8$ mins, Fig. 6b) is essentially the pure bi-nodal oscillation of the open-mouth connected basins. The Southeast Basin, with a normalized amplitude of 100 at the northeast corner, as compared with 48 in the north corner of Basin Six, obviously dominates the oscillation.

Configurations of higher modal oscillations are not given here for the basin shape of 1967-68, though they were recorded to the 10th mode (35). Numerical calculations were not pursued beyond the 10th mode because of the inherent decline in accuracy of the numerical method for large values of k .

CONFIRMATION OF THE NUMERICAL MATRIX SOLUTIONS FOR FREE OSCILLATIONS

By way of verifying the approach taken and the results thus far given, two different methods were used to evolve corroborative information. The first of these employed the time-marching technique of Leendertse (14) in propagating a long wave of given period into the Southeast Basin to establish its own oscillation. The second procedure used the impedance principle first developed by Rayleigh in 1877 (21) in acoustical problems and adapted by Neumann in 1944 to the study of seiches in connected bodies of water (17,18).

Numerical Time-Marching Technique

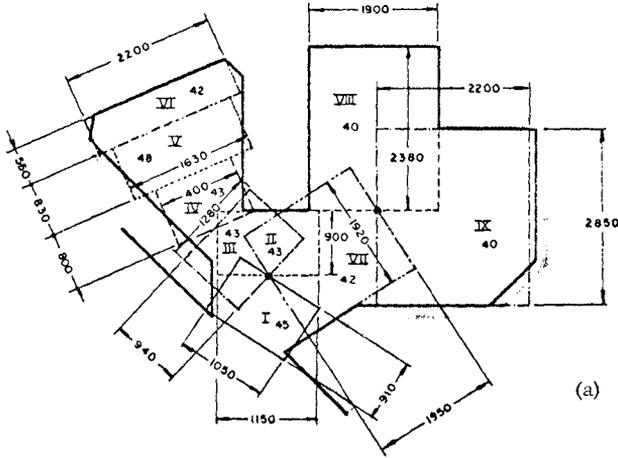
This method uses the integrated equations of motion and continuity, governing the propagation of long period waves, in two sets of difference equations for a step-by-step solution in increments of time of the water surface elevation and water particle velocities (14). The program of computation can be applied to a much finer grid than that used in the matrix solution and can thus define the shape and depths of a basin relatively more accurately. However, the method cannot establish an eigen period (free oscillation) except by trial and error solutions of effects from a large number of input periods, taken very close together.

As a check on the matrix solution results given in Fig. 6(b) a 2.8 mins period long wave was impressed on the entrance to the Southeast Basin with an arbitrary 15 cms amplitude at the star-positions just outside the basin mouth (Fig.7). The propagation of the wave was followed at time-steps of 6 secs over a 260-point spatial grid covering the Southeast Basin areas. The program, developed (and applied) by Leendertse (14), included the effects of bed friction and Coriolis force (though the latter effects would be almost negligible in so small a basin). After 85 time-steps the motion inside the basin reached the stable oscillation represented in Fig. 7. The oscillation established a node in a straight line across the entrance and a second node within the basins, characterizing it as the binodal resonant oscillation with respect the entrance, typical of an open-mouth basin. Fig. 7 is seen to be in good general agreement with Fig. 6(b), both as regards nodal and antinodal positions and also relative amplitudes. The mode shape contours of Fig. 7, however, may be expected to be more accurate than those of Fig. 6(b), because of the much larger number of grid points (260), [as compared with 88 for Fig. 6(b)], used in definition of spatial area and depth within the basins.

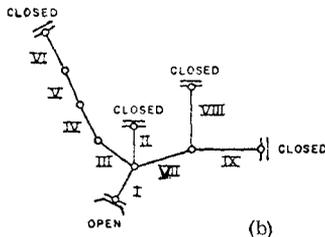
Neumann's Impedance Method of Seiche Analysis

The second method used in general confirmation of the matrix solutions required that the variable shape and depth of the Southeast Basin and Basin Six be approximated as a series of connected basins of uniform depth, width and length which are open-ended at connecting points and closed at terminal points and, of course, open-ended at the mouth. The adopted manner of dividing up the Southeast Basin into such a system is illustrated in Fig. 8(a) which gives also the "individual basin" dimensions of length, width and depth. The accompanying schematic circuit diagram for this array is shown in Fig. 8(b), in which the individual basins, in series or parallel, are designated from I to IX.

The details of the method for solving the eigen-frequencies of oscillation of the combined basin system will not be given here. The reader is referred to Defant (4), O'Brien (19) or Wilson (33) for particulars. The eigen-frequencies



(a)



(b)

FIG. 8 - "IMPEDANCE" MODEL FOR SOUTHEAST BASIN, LONG BEACH HARBOR AS A CONNECTED SYSTEM OF RECTANGULAR BASINS (a) BASIN DIMENSIONS (b) EQUIVALENT CIRCUIT DIAGRAM

ω_k are obtained as the roots of two parametric equations, one of the variables of which is ω_k . Details of these equations may be found in Wilson (33) or Wilson, et al (35).

The sequence of modal periods evolving from the solutions to these equations are given in Table 1 in comparison with the modal periods derived from the matrix solutions.

TABLE 1 - COMPARISON BETWEEN MODAL PERIODS OF FREE OSCILLATION OF SOUTHEAST BASIN, LONG BEACH HARBOR, CALCULATED BY TWO METHODS

Method	Modal Period, T_m - (mins)									
	m=1	2	3	4	5	6	7	8	9	10
Matrix	14.2	7.0	3.7	2.80	2.02	1.80	1.76	1.58	1.54	1.44
Impedance	12.7	5.2	4.3	2.83	2.20	1.60	1.48	1.42	1.41	1.07

The comparison, although not too good as to accuracy, is quite acceptable as to order of magnitude. The test is a particularly severe one for the impedance method; its accuracy is only as good as the approximations made in simulating the Southeast Basin and Basin Six as an array of interconnected rectangular basins. It serves nevertheless to confirm the validity of the matrix solutions, whose accuracy also is necessarily limited by the matrix number (88 x 88) or the memory capacity of the computer.

NUMERICAL SOLUTION FOR SOUTHEAST BASIN SHAPE OF 1972

Developments in the Southeast Basin in the last few years have been towards reclaiming more warehouse and loading space and reducing the amount of water space in the basin. The new shape of the basin as of 1972 is shown in Fig. 9 which gives also the numerical grid adopted, in this case, with 83 points to identify the boundary and bathymetry. Identified also in Fig. 9 are two corner positions,

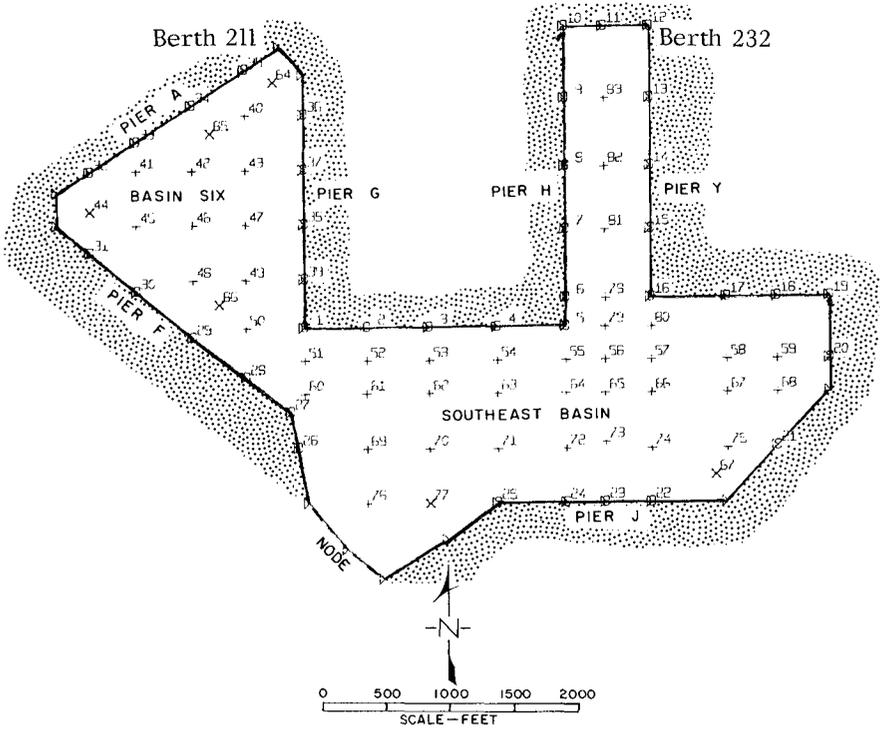


FIG. 9 - NUMERICAL MODEL OF SOUTHEAST BASIN (1971)

Berth 211 in Basin Six and Berth 232 in the Southeast Basin slip, at which special tide gages were established for recording long wave activity in the basins. These will be referred to later.

For the matrix solution of the new basin shape the whole basin system was assumed to have a uniform depth of 45 ft. In practice this depth has been increased to 55 ft throughout most of the Southeast Basin, but as the calculations were performed in 1967-68 for the lesser depth of 45 ft this must be borne in mind when comparisons are made (later) between calculated results and measurements.

The sequence of Figs 10 to 14 give the matrix solutions of the lowest modes of oscillation of the new basin shape, determined again on the assumption that a node develops across the mouth of the basin.

The lowest four modes, as might be expected, are quite similar in character to the corresponding modes of free oscillation for the 1968 basin. This is readily seen by comparison of Figs 5 and 10 and Figs 6 and 11. For modes higher than the fourth, the similarity ends and the behavior of the coupled basins becomes one of mutual interaction. It is noteworthy in Fig. 11(a) that in the third mode ($k=3$), the slip in the Southeast Basin is responding strongly to an oscillation which, though bi-nodal for the entire basin system, is uninodal for the slip itself, as judged by the fact that the internal node is approximately at the mouth of the slip.

Fig. 12(a) can be recognized as a mode ($T = 1.80$ mins) which develops a tri-nodal seiche between the northern extremity of Basin Six and the eastern extremity of the Southeast Basin. The slip has a rather weak response.

At $T = 1.65$ mins, (Fig. 12 b), Basin Six develops a strong uninodal oscillation between the north and west corners, while the Southeast Basin and slip oscillate rather moderately in what may be described as a clover-leaf pattern, the stem of which is the core of the white area enveloping the mouth of the slip and the petals the three shaded areas outside.

At $k=7$, $T = 1.58$ mins, the next mode (Fig. 13 a), Basin Six continues to oscillate strongly between the north and west corners, while a moderately strong

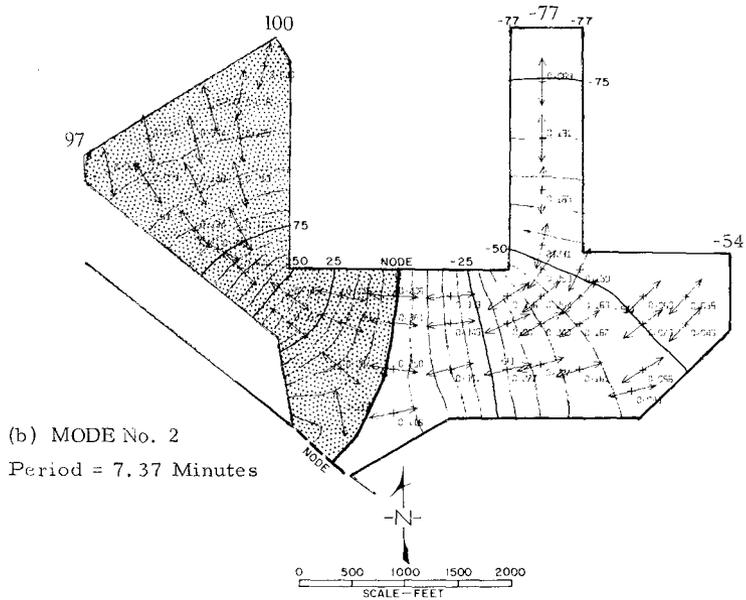
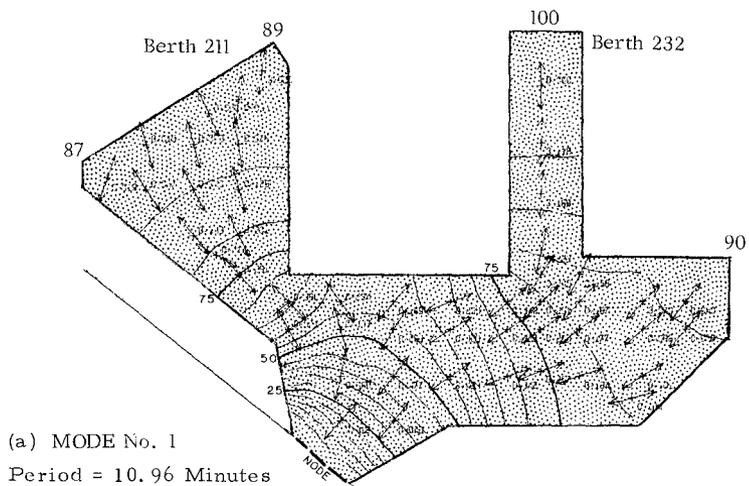


FIG. 10 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1971)
(a) 1st MODE; (b) 2nd MODE

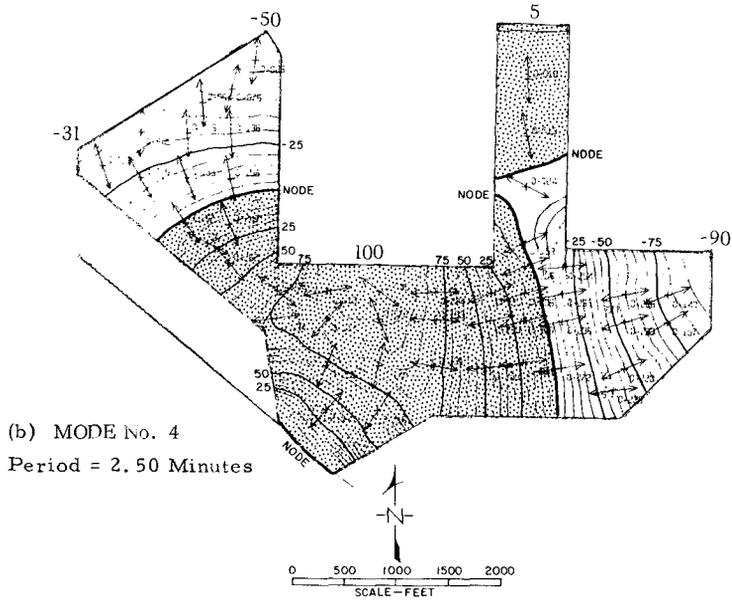
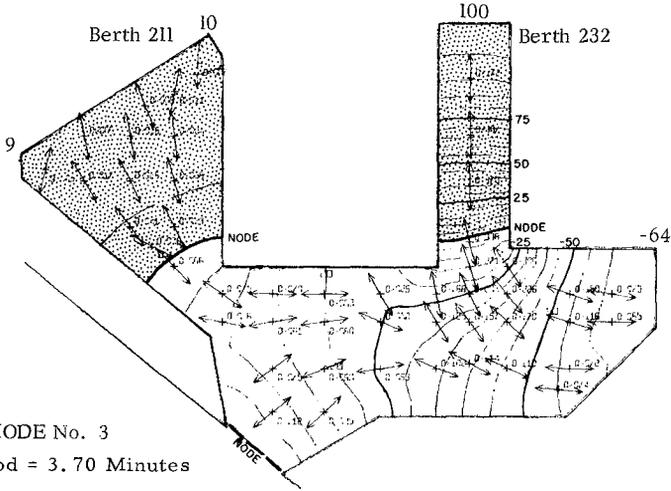


FIG. 11 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1971)
(a) 3rd MODE; (b) 4th MODE

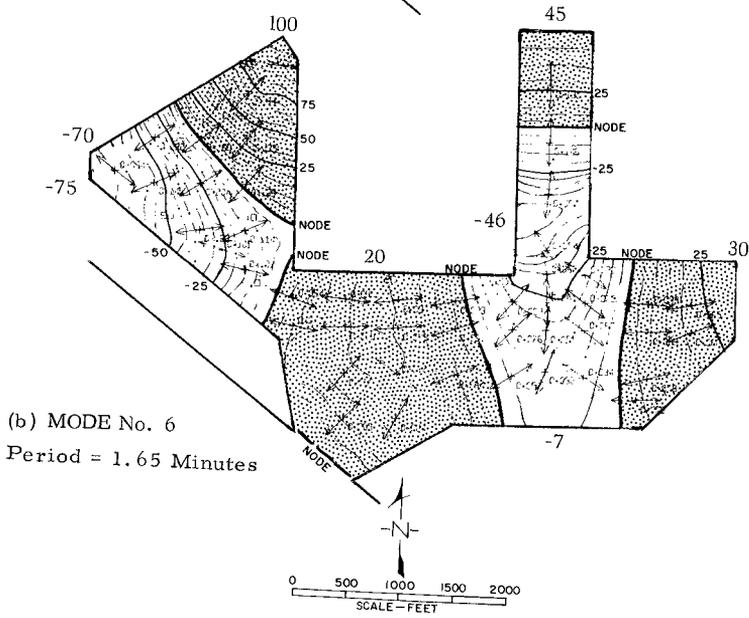
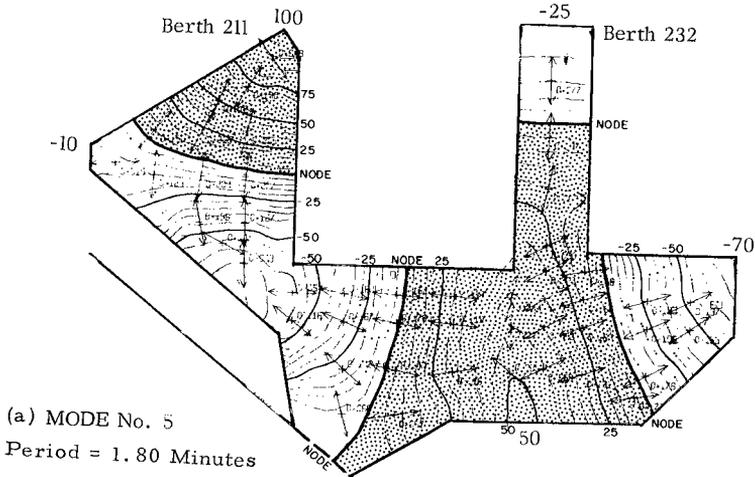


FIG. 12 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1971)
(a) 5th MODE; (b) 6th MODE

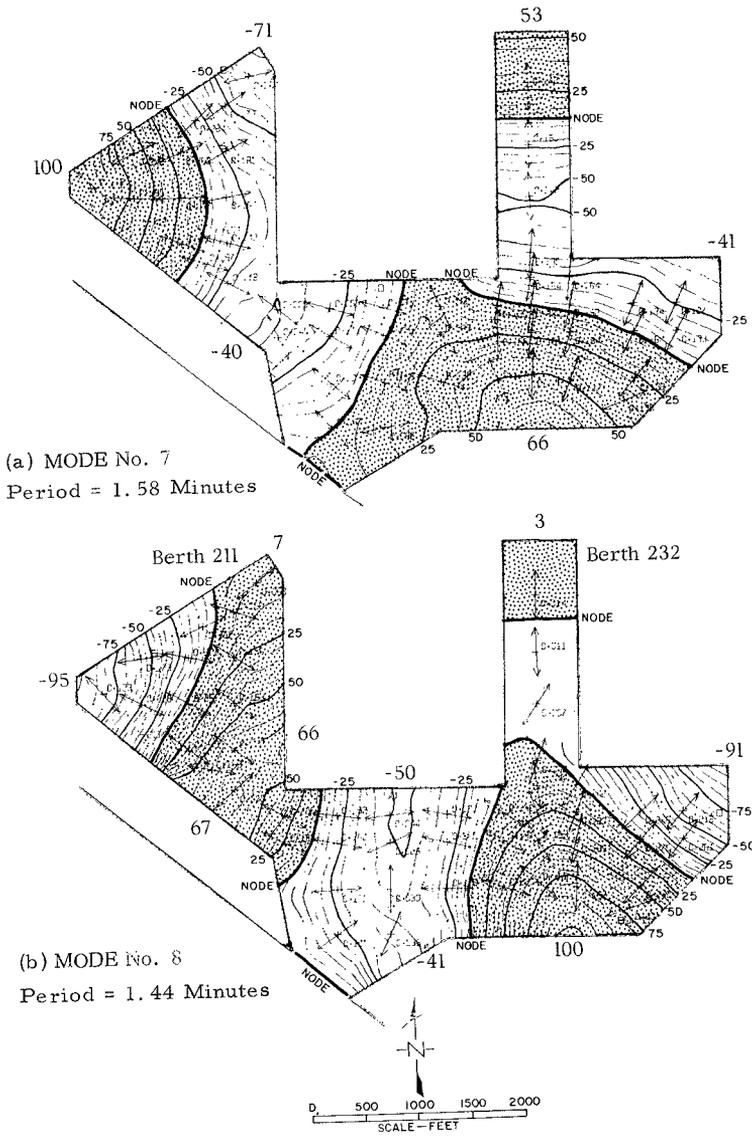


FIG. 13 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1971)

(a) 7th MODE; (b) 8th MODE

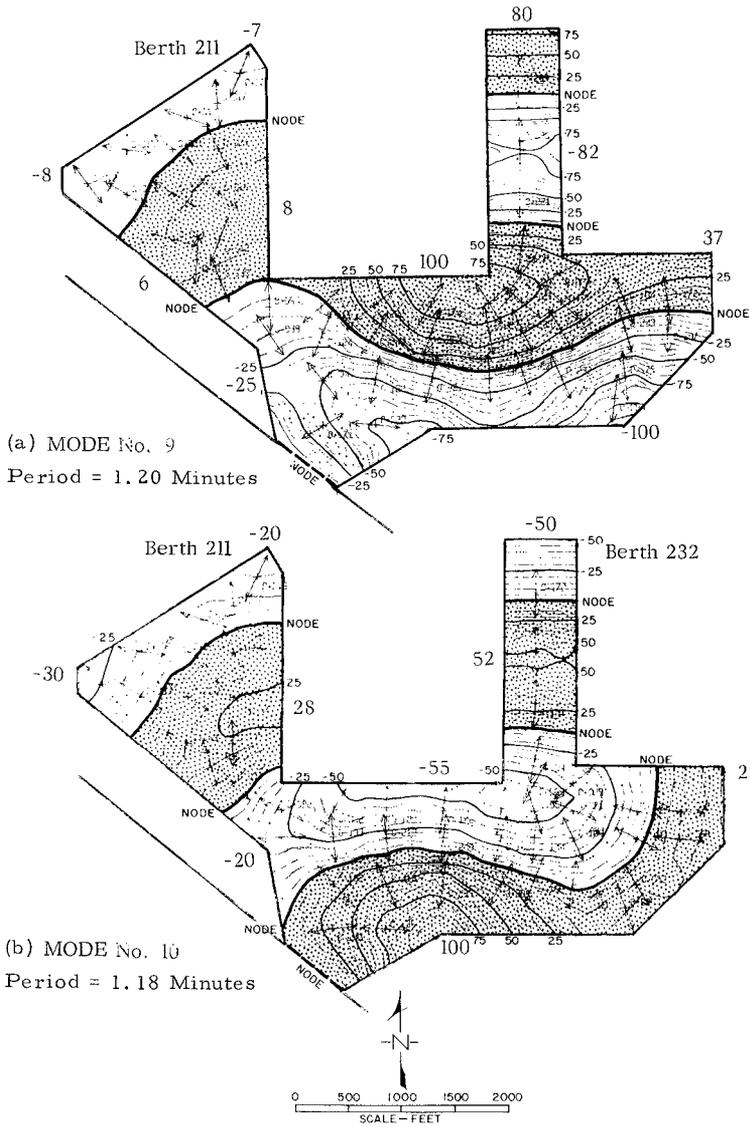


FIG. 14 - MODES OF FREE OSCILLATION OF SOUTHEAST BASIN (1971)
(a) 9th MODE; (b) 10th MODE

binodal oscillation develops between the head of the slip and south arm of the Southeast Basin (Pier J).

In the eighth mode ($k = 8$; $T = 1.44$ mins, Fig. 13 b), the oscillation is predominantly a quadrinodal seiche between the western and eastern extremities of the Southeast Basin, inclusive of Basin Six. In the slip at Berth 232 there is almost no response.

Fig. 14 (a) for the next higher mode ($k = 9$; $T = 1.20$ mins) suggests that the Southeast Basin develops a strong tri-nodal seiche between the northern extremity of the slip and the southern boundary at Pier J. The oscillation in the slip itself virtually conforms to a binodal seiche as if the slip was a closed-end basin. At the same time the oscillation in the area between Pier G and Pier J is virtually a uninodal transverse seiche with a node approximately axial to this part of the basin in an east-west direction.

Finally in the tenth mode (as far as calculations were pursued for $k = 10$; $T = 1.18$ mins) Fig. 14 (b) shows that the oscillation is a slight variation of the ninth mode, but has larger participation in Basin Six and weaker response in the remaining Southeast Basin.

In summary, the modal periods of free oscillation for the Southeast Basin in its 1971 configuration form the sequence

$$T \approx 11.0; 7.4; 3.7; 2.5; 1.80; 1.65; 1.58; 1.44; 1.20; 1.18; \dots \text{mins} \quad (22)$$

It is perhaps unfortunate that the numerical calculations could not have been pursued to higher modes than the tenth ($k = 10$), to reveal periods of response down to values as low as $T = 30$ secs. However it was judged that the close proximity of modal periods for $k > 8$ (as for example, T_8 and T_9 in Figs 14) and general declining accuracy would preclude reliability of results. It would have required a very much larger computer memory than was available in 1967-68 for coping with an enlarged $N \times N$ matrix to have increased the accuracy of results for $k > 10$, via a finer network of points covering the basin area and boundaries. In this connection it should be noted that pre-existing studies have concluded that

the critical periods for surge motion of ships in harbors cover mainly the period range from about 20 secs to 2 mins with periods in the neighborhood of 1 min predominantly important (2, 6, 7, 8, 22, 26, 30, 35). At least part of this period range is covered by the calculations of this paper.

It is rather easy to infer from Figs 14(a) and (b), that, had the calculations been pursued to the next few higher modes, the node near the mouth of Basin Six would advance farther into the basin, while that deeper in the basin would separate into two parts, each enclosing an approximately circular sector of unshaded area at the north and west corners of the basin. Such an oscillation in Basin Six, at expected periods of about 1 min, would then assume a 'cloverleaf' pattern with the three corners (petals) of Basin Six responding with in-phase elevation in opposition to the motion of the central part (stem) of the basin. The significance of this will be discussed later in relation to observational data.

OBSERVATIONAL EVIDENCE OF LONG WAVE ACTIVITY IN THE SOUTHEAST BASIN

Incidence of Surge in the Southeast Basin

It is no secret that Los Angeles-Long Beach harbors have been afflicted by troublesome surge problems on rather infrequent occasions over 60 years, ever since the construction of these ports. Long before the development of the Long Beach outer harbor, comprising the East and West Basins and the Southeast Basin, ship surging disturbances were being experienced on occasion in the outer harbor of Los Angeles, particularly in the East Channel (35). Fig. 15, which is adapted from Leybold (15), gives measurements of 1935 of typical effects in the East Channel. These disturbances would penetrate part way up the Main Channel (Fig. 15, inset) but die out within about a mile of its entrance. At the Berth 174 tide gage deep in the inner harbor, on the other hand, it was a common, almost daily, occurrence to find fairly prominent 60 min oscillations superimposed on the high tide. The latter phenomenon is believed to be a resonant response of the inner Los Angeles harbor and Cerritos Channel (Fig. 15, inset) to a continental shelf

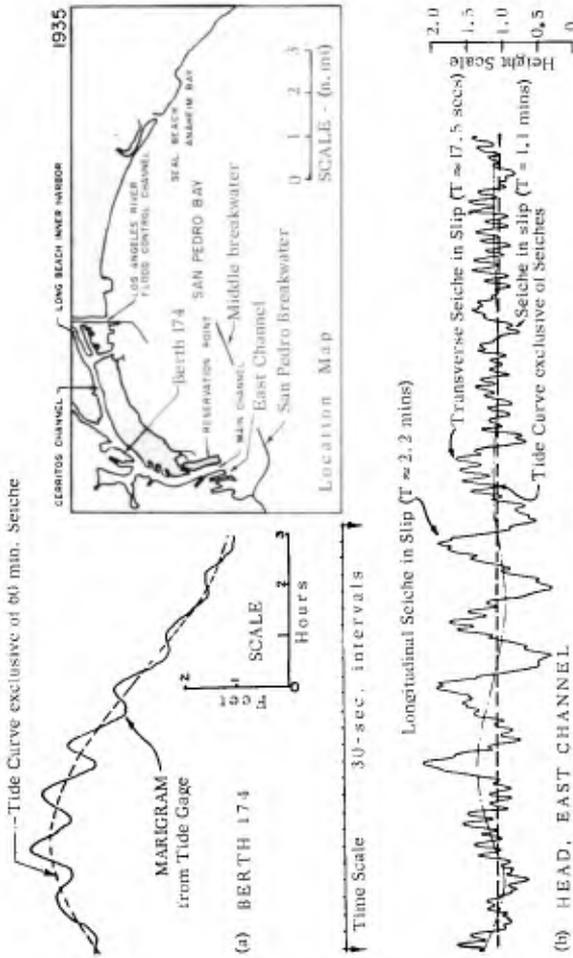


FIG. 15 - DATA INDICATIVE OF LONG PERIOD WAVE ACTIVITY IN LOS ANGELES HARBOR IN 1935-36: (a) TYPICAL TIDE CURVE, BERTH 174, L. A. INNER HARBOR (circa 1935-36); (b) MARIGRAM FOR HEAD OF EAST CHANNEL, L. A. OUTER HARBOR, DECEMBER 19, 1935 (adapted from Leybold, 1937)

oscillation off San Pedro Bay (31,35) and, so far from being troublesome, is beneficial in flushing the inner harbor with mild currents.

The incidence of surge action is believed to be secular in its frequency and severity, waxing and waning in apparent out of phase relationship with the sun-spot cycle (29,35). Fig. 16, which shows the height of long period wave oscillations (solid black graph) as a function of time in relation to local sea level and weather parameters, for a tide gage station at Pierpoint (end of Pier A, East Basin, Fig. 3), suggests that from 1964 to 1967 activity was on the increase. The measure of "height of surge" in Fig. 16 was taken as the band width of the trace on the tide gage, which at a chart speed of 1 inch per hour, tended to fuse oscillations of less than 3 mins period. As expected from previous studies of surge action in harbors (27,29), there is no correlation between local weather effects and magnitude of surge, the latter being related rather to large cyclonic storms in the adjacent or distant oceans.

Fig. 17 presents in greater detail a portion of Fig. 16 covering the period between August 26 and September 14, 1967. Here, in the interval September 1 to 6, the surge index (solid black diagram) reached a highest peak, and various moored ships were reported in trouble at their berths. It may be noted, in this case, that peaks of the silhouette diagram correlate well with times of high tide. a peculiarity of surge action that had been noted in 1913 by Muñoz, a consultant to Los Angeles Harbor (16), and has been noted also in Table Bay Harbor, Cape Town, South Africa (29). It is probable that normal tidal influx into the harbor induces a degree of free oscillation at high tides which accounts for the peaks at these times and that it increases the magnitude of any long wave activity present from other sources.

In 1967-68 there existed no factual information on the surge responses of the Southeast Basin other than the calculations reported in this paper. However, some statistical information of wharfinger's estimates of surge conditions, made on occasions of berth occupancies, was used to develop criteria of relative susceptibilities of berths in Basin Six to "medium" surge conditions (35). These criteria

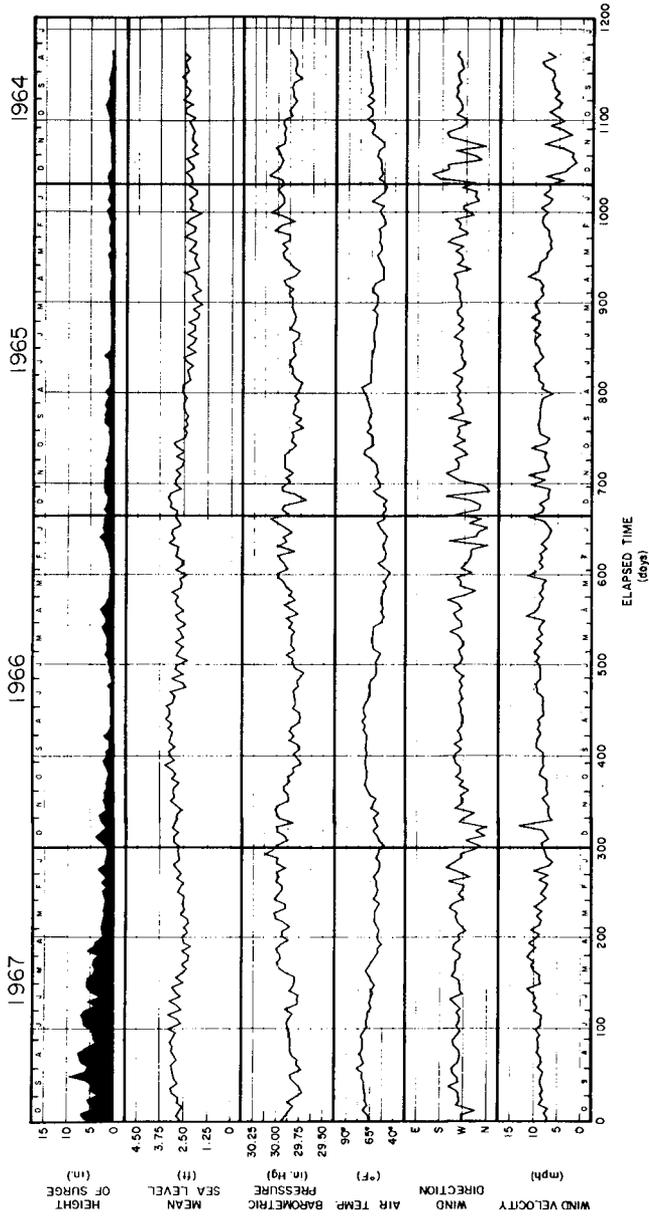


FIG. 16 - CONDENSED HISTORIES OF COMPARATIVE SURGE, TIDE AND LOCAL WEATHER CONDITIONS AT LONG BEACH HARBOR, 1964-67

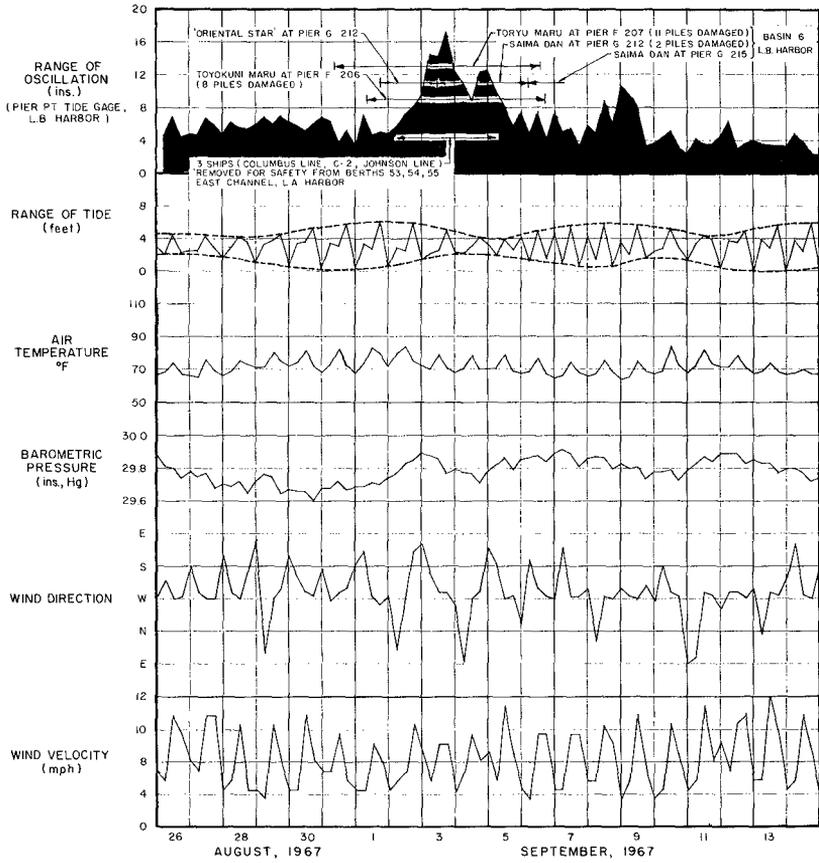


FIG. 17 - DETAILED HISTORIES OF COMPARATIVE SURGE, TIDE AND LOCAL WEATHER CONDITIONS AT LONG BEACH HARBOR, AUGUST 26 - SEPTEMBER 14, 1967

are shown in Fig. 18 as percentage figures flanking the berth numbers. The percentages express the mean annual probability (as of 1965-66) that medium surge would prevail during occupancy of a berth by a ship. The definition of "medium surge" was quite nebulous, but expressed a wharfinger's opinion that the surge was not "serious", but also not "light", and therefore presumably was able to cause motion in a moored ship, though not to the extent of being really troublesome.

Fig. 18 shows that Berth 210 on Pier A in Basin Six had the highest relative susceptibility to surge (21%) of any of the berths occupied in the period August 1965 to June 1966. Berths 207 and 208 at the southwest corner of the basin showed the highest susceptibility of 17 percent. Berth 213, was apparently not occupied, as also berths 204, 205, and 215, near the entrance of Basin Six. From the susceptibility distribution covering berths 206 to 212, it may be inferred that the surge oscillation most likely to give such a distribution would be one having the nodal lines shown in Fig. 18 and would probably be in the nature of a "cloverleaf" seiche, which by inference from the matrix calculations of the last section would be expected to have a period approximating 1 min.

It is possible to infer what the period of such an oscillation would be by another approach. Thus by regarding the access to one of the corners of the triangular Basin Six as a triangular canal of uniform depth, it is known from Lamb (9, §186) that the surface oscillation along the axis of such a canal would take the form

$$\eta \propto J_0(kx) \quad (23)$$

in which J_0 is a Bessel function of zero order, x the distance from the vertex of the canal along the axis or bisector of the angle and k a wave number defined by

$$k^2 = \sigma^2/gh \quad (24)$$

σ being the angular frequency of the oscillation and h the water depth. The Bessel function $J_0(kx)$ has unit value at $x=0$ (the vertex), zero value at $kx=2.41$ and maximum negative value (-0.428) at $kx=3.83$.

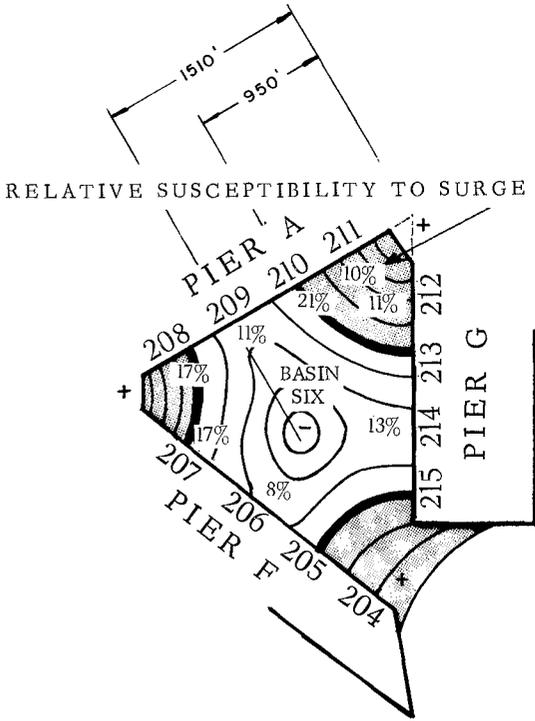


FIG. 18 - EXPECTED "CLOVER-LEAF" MODE OF OSCILLATION IN BASIN SIX, SOUTHEAST BASIN, LONG BEACH HARBOR, IN RELATION TO BERTH SUSCEPTIBILITY TO SURGE

Taking the cloverleaf oscillation to be most intense along the northeast bisector in accord with Fig. 18, the distance from the vertex to the node is 950 ft., yielding a value for k of $950k = 2.41$ or $k = 2.54 \times 10^{-3}$. Adopting $h = 45$ ft., Eq. (24) then yields the period of oscillation

$$T = 2\pi/\sigma = 1.08 \text{ mins} \quad (25)$$

The negative antinode of -42 , relative to a normalized value of 100 at the northeast corner of Basin Six, would occur at $kx = 3.83$ or $x = 1510$ ft. This distance agrees satisfactorily with the anticipated stem of the cloverleaf oscillation (Fig. 18).

This quantitative conclusion thus supports the inference of the last section, and its deduction, from statistical information of berth susceptibility to medium surge action suggests that the 1.08 oscillation is a potent factor in the surging of ships, supporting conclusions arrived at by several investigators in other parts of the world (1, 6, 7, 22, 26-30, 35).

Spectrum Analyses of Tide Gage Records in the Southeast Basin

What might be called the first good factual data on long period disturbances experienced in the Southeast Basin were obtained as marigrams from two special tide gages located at Berth 211 in Basin Six and at Berth 232 in the slip of the Southeast Basin (Fig. 9). These marigrams, for the occasion of May 8, 1971, shown as curves a and A in Fig. 19, had sufficiently open time scales (1 inch = 15 mins) to permit of digitization of the records for numerical spectrum analysis.

In the first instance the digitized data were high-pass filtered for the elimination of frequencies below 0.025 cycles per hour (periods greater than 40 mins) leaving the residual traces b and B in Fig. 19. These residuals, covering a record length of $9\frac{1}{2}$ hours, or 2000 data points at time increments of 0.285 min, were then analyzed numerically for their wave energy spectra at 400 lags, or a frequency resolution of 0.0044 cycles/min. The resulting general spectra are shown in Figs 20(a) and 21(a), which show, typically, dominant peaks of energy suggestive of prominent oscillations at particular frequencies.

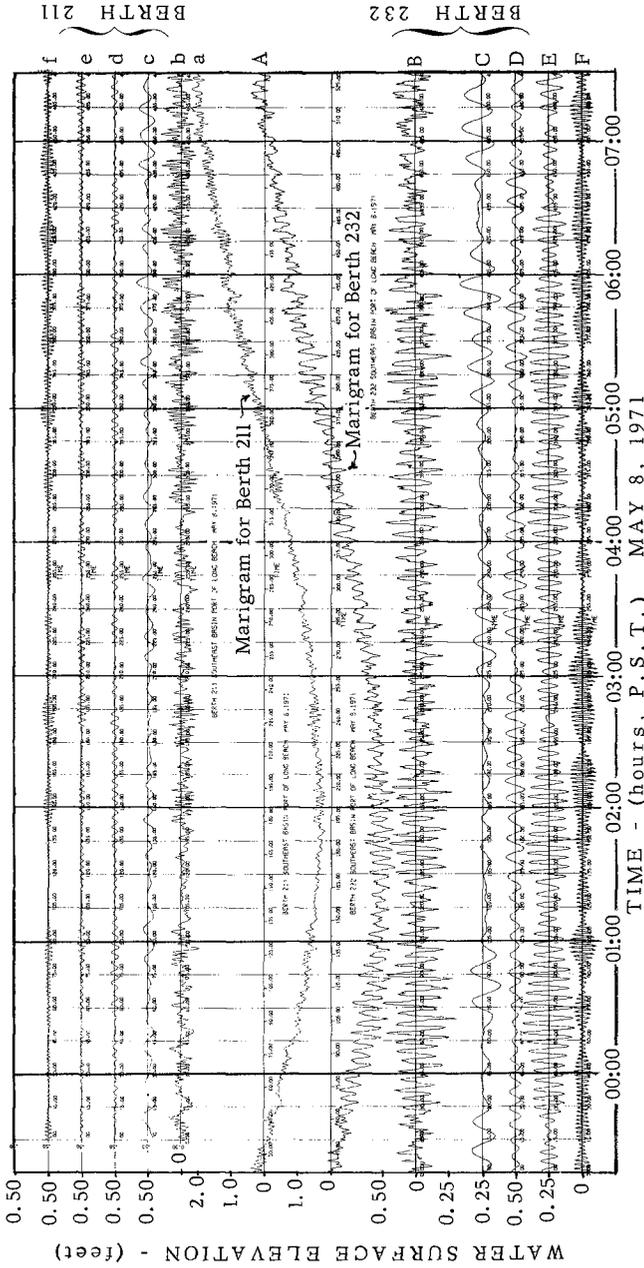


FIG. 19 - WAVE FORMS OF PRINCIPAL COMPONENTS IN MARIGRAMS OF MAY 8, 1971, FOR BERTHS 211 AND 232, SOUTHEAST BASIN, PORT OF LONG BEACH: (a) Marigram for Berth 211; (b) Residual of (a), filtered of tide; (c), (d), (e), (f), Components at periods $T \approx 13.3, 4.3, 2.9, 1.7$ mins, respectively; (A) Marigram for Berth 232; (B) Residual of (A), filtered of tide; (C), (D), (E), (F), Components at periods, $T \approx 13.3, 8.9, 4.4, 1.9$ mins, respectively

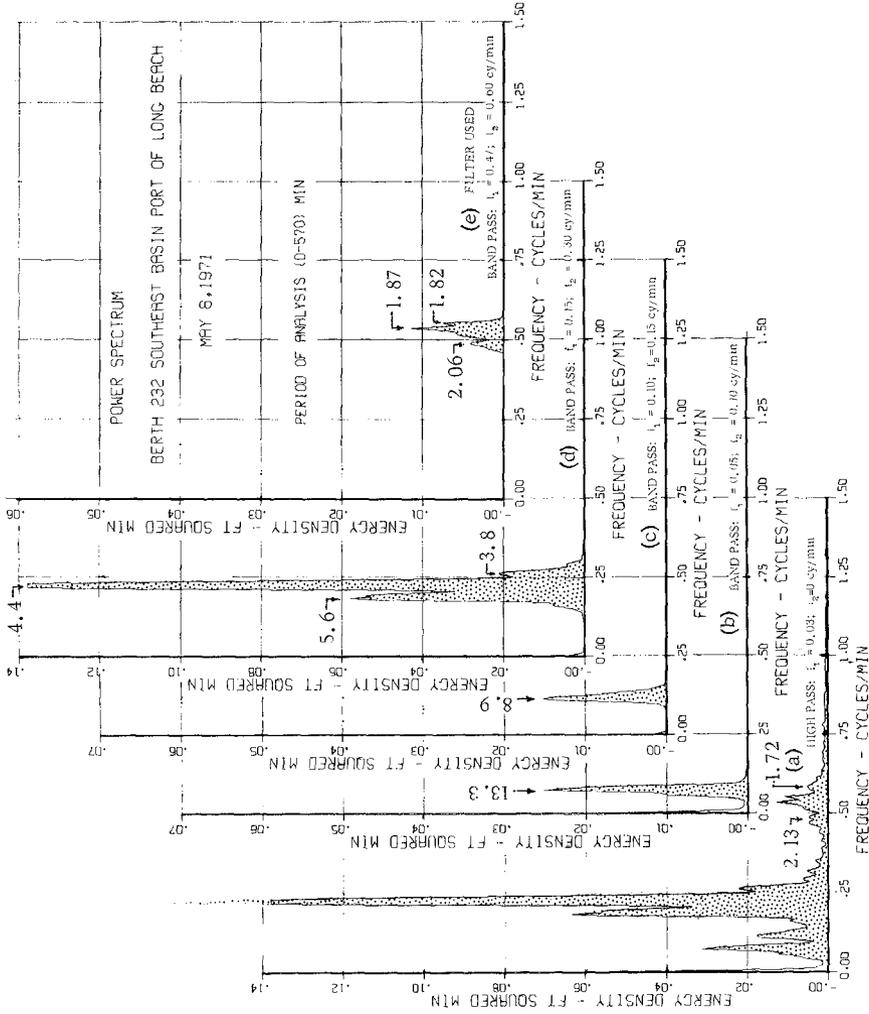


FIG. 21 - POWER SPECTRA FOR BERTH 232 MARICRAM, MAY 8, 1971; (a) General Spectrum, prefiltered of tide; (b) to (e), band pass filtered spectra for isolation of principal components. (Figures at peaks are periods in mins)

The remaining wave energy spectra, given as Figs 20(b) to (e) and Figs 21(b) to (e), are the result of isolating parts of the spectra by selective band-pass filtering of the residual traces *b* and *B* in Fig. 19. As examples, by filtering the data of Fig. 19(b) through the numerical band-pass having frequency limits of 0.05 and 0.15 cycles/min, the isolated wave form (c) in Fig. 19 is found corresponding to the spectral energy of Fig. 20(b): similarly by filtering the data of Fig. 19(B) through a band-pass with frequency limits of 0.05 and 0.10 cycles/min, the resulting wave form (C) in Fig. 19 corresponds to the limited spectrum (b) in Fig. 21.

Although the trace (c) in Fig. 19 is composite of at least three frequencies (of periods 18.2, 13.3 and 8.3 mins), whereas trace (C) is essentially a pure wave form of 13.3 mins period, there are many in-phase similarities between (c) and (C), suggesting that an oscillation of 13.3 mins period was common to Basin Six (Berth 211) and the Southeast Basin slip (Berth 232).

The information of Figs 19, 20 and 21 is most readily compared with the results of calculations, given in the first part of this paper, by assembly in Table 2. Here, in the first four columns are set out the computed modal periods of the Southeast Basin, as given previously in Table 1 and Eq. (22). In the next five columns are observational results derived from the marigrams (a) and (A) of Fig. 19. Periods corresponding to spectrum peaks are tabulated in columns (5) and (7) and maximum wave heights, identified from the wave forms in Fig. 19, are recorded in columns (6) and (8). Column (9) contains periods identified in Fig. 19(A) by use of Chrystal's method of residuation (3). The choice of rows for listing the observational data in columns (5) to (9), has been made in accord with the approximate agreement of periods with calculated modal periods; there is, of course, no positive way of identifying mode numbers with observed periods. Figures underlined denote oscillations of greatest strength. Because the tide gages were damped against registration of short period perturbations, no perceptible wave energy was recorded at periods below those given in columns (5) to (8), or, if it was, it was eliminated by hand-smoothing to permit digitization of the data. Column (9) shows that in the marigram for Berth 232, a periodicity of the order of 0.9 min was in fact present in the original tide gage trace.

TABLE 2 - SURGE OSCILLATIONS IN THE SOUTHEAST BASIN, LONG BEACH HARBOR: COMPARISON (VIA DIFFERENT TECHNIQUES) BETWEEN THEORY AND OBSERVATION (May 8, 1971)

THEORY				OBSERVATION (Analysis of Marigrams for May 8, 1971)				
Numerical Matrix Solution			Neumann's Impedance Method (approx. only)	Numerical Spectrum Analysis				Chrystal's Method of Residuation (approx. only) Berth 232
Mode No.	Basin as of 1968	Basin as of 1971		Berth 211		Berth 232		
N (1)	Period T (mins) (2)	Period T (mins) (3)	Period T (mins) (4)	Period T (mins) (5)	Max. Height H (ft.) (6)	Period T (mins) (7)	Max. Height H (ft.) (8)	Period T (mins) (9)
				18.2	0.24			
1	14.2	11.0	12.7	<u>13.3*</u>	<u>0.33</u>	<u>13.3</u>	<u>0.30</u>	12.0
2	7.0	7.4	5.2	8.3	0.12	8.9	0.21	7.5
3	3.7	3.7	4.3	4.9 4.3	0.10 0.13	<u>5.6</u> <u>4.4</u> <u>3.8</u>	<u>0.42</u> <u>0.68</u> <u>0.25</u>	<u>4.5</u> <u>3.9</u>
4	2.8	2.5	2.8	2.9	0.15	2.13 2.06	0.12 0.12	
5	2.0	1.80	2.2	1.87	0.14	<u>1.87</u>	<u>0.23</u>	
6	1.80	1.65	1.60	<u>1.72</u>	<u>0.20</u>	<u>1.82</u>	<u>0.22</u>	
7	1.76	1.58	1.48			1.72	0.13	
8	1.58	1.44	1.42					1.5
9	1.54	1.20	1.41					
10	1.44	1.18	1.07					0.9

* Figures underlined define strongest oscillations

Comparison of the theoretical and observational results in Table 2 suggests that the oscillations recorded on May 8, 1971, were probably indicative of several of the natural modes of response of the Southeast Basin. The 18.2 min period in column (5), however, is largely unexplained. Its presence in the Berth 211 record and absence at Berth 232 suggests that it may be fictitious and some sort of digitization error; any true oscillation of that period would have to pervade the basins as a whole. The periods at 13.3 and 8.3 - 8.9 mins in columns (5) and (7) or 12.0 and 7.5 mins in column (9) are probably evidence of the first and second modes of oscillation of the Southeast Basin system. Exact agreement of values is hardly to be expected because of real differences between the actual harbor and the mathematical modelling of it, as for example in water depths and in non-vertical basin side and end walls.

In the third mode row of Table 2, we find two periods listed in column (5), three in column (7) and two in column (9) against just one period for the comparable calculation in column (3). Any explanation for this has to be conjectural, but it may be assumed that some of the additional periods found in the May 8, 1971, data reflect the independent first modes of oscillation of Basin Six (for the Berth 211 record) and of the slip (for the Berth 232 record), as if these basins were entities in their own right. Reference to Fig. 11(a) suggests that at a slightly longer period than 3.70 mins the node for Basin Six as also for the slip would lie exactly at the mouths of these basins and would correspond to the uninodal oscillations for these open-mouth basins. In the case of the slip and the eastern portion of the Southeast Basin, it is also possible to suppose that this part of the total basin could also function as an independent open-mouth basin for which the fundamental mode oscillation would be one having a node at the approximate position of the -25 contour in Fig. 10(b). The period for such a mode would lie between the periods of Figs 10(b) and 11(a), that is, between 7.4 and 3.7 mins and might well account for the 5.6 min peak of energy recorded in column (7) of Table 2.

It becomes increasingly more difficult to identify positive association between observed and calculated periods at the higher modes. Nevertheless numerous near coincidences suggest modal correlations such as shown in Table 2.

PRELIMINARY OBSERVATIONS OF SHIP BEHAVIOR IN THE
SOUTHEAST BASIN

The occurrence of surge activity in the Southeast Basin on May 8, 1971, found the harbor empty of ships because of the prolonged west coast shipping strike, so that observation of ship behavior at that time was not possible. In fact because of this and other factors, occasions for obtaining significant information on ship response to surge activity have been extremely few. In this section nevertheless are given the first measurements that were made of ship movements in correlation with sea disturbances on September 11, 1970, soon after the tide gage at Berth 211 had been placed in Basin Six, but before the tide gage at Berth 232 in the slip had become operative.

Figs 22 and 23 record the components of motion in surge and sway of the centers of gravity (midships) of two ships, respectively one of 10,000 DWT at Berth 206 in Basin Six, the other of 11,000 DWT at Berth 245 along Pier J of the Southeast Basin (see insets). The simultaneous trace from the marigram of the tide gage is also recorded in each figure and shows that 18 sec swells with a height in excess of 6 ins were penetrating into Basin Six along with longer period oscillations whose presence in the trace is seemingly not very apparent until identified by the median dash-dot curve.

Some degree of influence of the 18 sec swells is seen to pervade the surge and sway motions of the ships in the two locations, but the dominant motions in surge and sway are obviously being dictated by much longer period effects. In the case of the "Eastern Cherry" at Berth 206 in Basin Six (Fig. 22), surge motion is conforming to an approximate period of 1.5 mins. Reference to Fig. 13(b) suggests that the nodal current for this oscillation could be influencing the ship. In sway motion the "Eastern Cherry" was responding dominantly to a 1.7 min oscillation and Fig. 12(b) suggests that this could be the influence of the sixth mode oscillation for the Southeast Basin which is transverse for Basin Six and would induce current on the beam to a ship occupying Berth 206. There is also a detectable small influence of a period of about 1 minute on the ship motions in Fig. 22 and this could have relation to an oscillation in Basin Six of the type of Fig. 18.

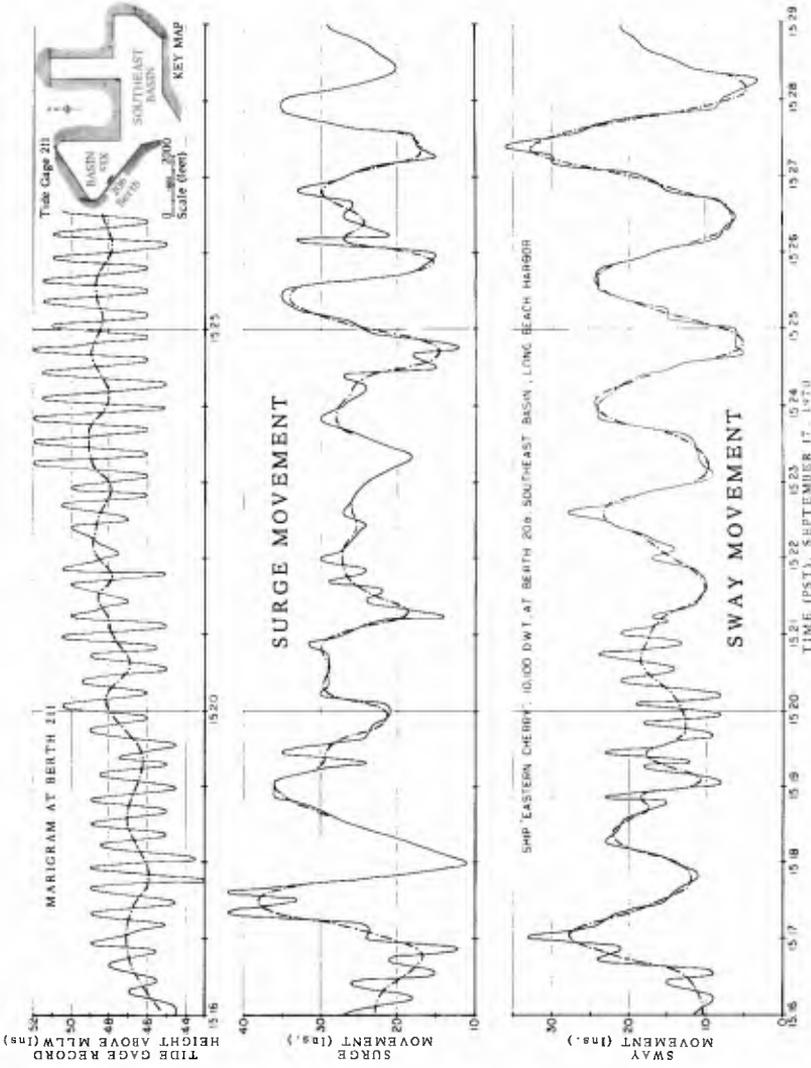


FIG. 22 - SURGE AND SWAY MOVEMENTS OF SS. "EASTERN CHERRY", 10,100 DWT, MOORED AT BERTH 206, BASIN SIX, SOUTHEAST BASIN, LONG BEACH HARBOR, IN RELATION TO SEA DISTURBANCE AT BERTH 211 ON SEPTEMBER 11, 1970

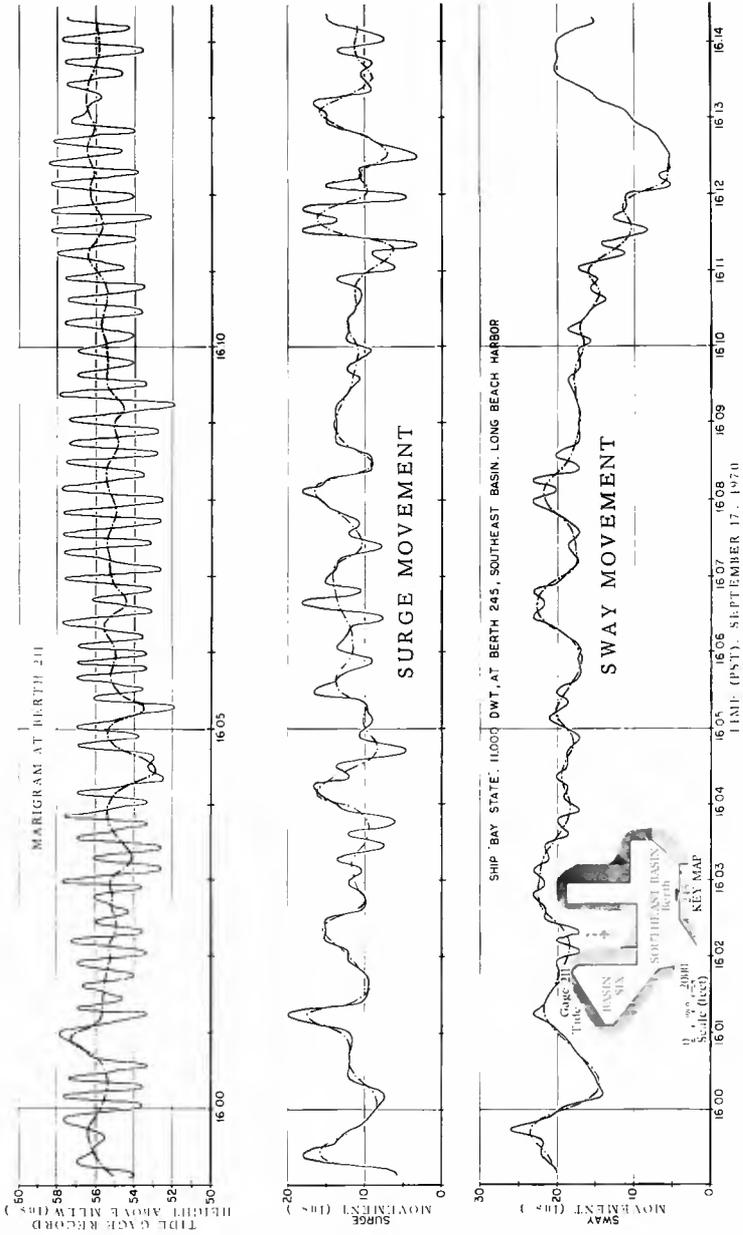


FIG. 23 - SURGE AND SWAY MOVEMENTS OF SS. "BAY STATE", 11,000 DWT, MOORED AT BERTH 245, SOUTHEAST BASIN, LONG BEACH HARBOR, IN RELATION TO SEA DISTURBANCE AT BERTH 211, BASIN SIX, ON SEPTEMBER 11, 1970

The motions of the "Bay State" at Berth 245, Southeast Basin are much less pronounced. Here again 18 sec swells are exerting strong influence at times on ship surging, but the underlying surge behavior appears to be composite of 1.5 and 1.0 minute oscillations. Here too Fig. 13(b), for the eighth mode of response of the Southeast Basin, could explain this effect because oscillatory current direction at $T \approx 1.4$ mins would favor ship motion along the quay. In sway motion, according to Fig. 23, movements appear to accord with periods of 1.2 to 1.5 mins. Fig. 14(a) then suggests that the ship was under the influence of the transverse mode of oscillation ($k=9$), producing oscillatory currents normal to the ship at Berth 245.

SUMMARY AND CONCLUSIONS

The numerical "matrix" method of solving the hydrodynamic long wave equations for the modes of free oscillation of a basin appears to be one of the few that can successfully cater to a completely arbitrary shape and bathymetry of the basin. Its accuracy is limited only by the memory capacity of the computer needed to handle the $N \times N$ matrix deriving from use of an N -point mesh to define the boundaries and intermediate depths of the basin. The method described uses a variable grid mesh which has the advantage that grid points can be made to fall precisely on the boundaries. Boundaries can be vertical walled, sloping, or opening upon larger bodies of water. In most cases of harbors and narrow mouth bays the nodal boundary condition at the mouth can be satisfactorily specified by the assumption of the nodal position. In bays having very wide mouths the assumption of a nodal position would call for a high degree of experience and judgement on the part of the operator, but, more accurately, such cases would need the extension of the mesh outside the bay as far as the continental shelf edge, where, along the sharp depth discontinuity, a nodal condition could reasonably be assumed again.

In application to the Southeast Basin of Long Beach Harbor the matrix method yields the modal periods, water surface elevation and oscillatory current patterns for the basin shape and depth in its condition of 1967-68 and for its present shape of 1971-72. To a reasonable degree of accuracy the eigen-periods for the

1967-68 shape have been checked by use of the "impedance" method of Neumann, and by the marching time-step method of Leendertse. They have also been confirmed in this paper to an acceptable extent with actual measurement of sea disturbances in the Southeast Basin, by correlation of the dominant frequencies uncovered by spectrum analysis of the records with the calculated modal periods. In another sense the rationality of the computed results has been demonstrated by the fact that the modal shapes of the basin oscillations accord with the expected manner in which the compartments of the Southeast Basin would naturally tend to oscillate. Intrinsically the Hamiltonian principle is satisfied by the results which show that particular modes of oscillation are frequently modal in character for a part of the total basin at the same time they are modal for the entire basin.

As against other systems of numerical calculation of the responses of basins of arbitrary shape to long period wave excitation, the method of this paper suffers from the disadvantage that it predicts only normalized eigen-responses (elevations and currents) and not the dynamic amplifications in relation to an external stimulation. However, it gives more readily the relative responses over the entire area of a basin and it treats more accurately variability of depth which is usually taken constant in other procedures. Dynamic amplifications over a basin area as a function of the frequency of an external excitation can always be obtained by use of an adjunct numerical procedure, such as Leendertse's (14), which introduces friction effects and allows for non-uniform depths.

Observations of the motions of two cargo ships moored in the Southeast Basin during an occurrence of long wave activity indicate that motions of surge and sway are plainly affected to a certain extent by 18 sec swell agitation in the harbor. However, major movements occur also at periods which appear to be combinations of 1.0, 1.2, 1.5 and 1.7 min periodicities. These periods accord with calculated and observed periods of oscillation of the Southeast Basin and the modal current directions are consistent with the surge and sway motions of the ships. It should be pointed out, nevertheless, that recent (unpublished) numerical calculations of the general motions of a moored ship in regular waves, by one

of the writers (Wilson), has shown that subharmonic effects are induced in which long period motions occur at periods several times that of the incident wave. The same effect in sway motion, in particular, has been observed and reported by Lean (10). The possibilities for grave resonance between these subharmonic motions and the stimulation of the longer period harbor oscillations is at once apparent.

ACKNOWLEDGEMENT

The main elements of the theoretical part of this paper are based upon work done for the U. S. Army Los Angeles District, Corps of Engineers under contract No. DACW 09-67-C-0065. The Port of Long Beach has provided the opportunity and assistance for acquisition of observational data on sea disturbances and ship motions in the Southeast Basin reported in this paper. The Chicago Bridge and Iron Co., Plainfield, Illinois, kindly provided assistance for the digitization of records and the numerical spectrum analyses. To all these authorities the writers express their sincere appreciation for permission to collect, correlate and publish the results. Finally credit is due to H. Soot for programming and processing the computer calculations and plot-outs.

APPENDIX - REFERENCES

1. Am. Soc. C. E. , "Proceedings of the NATO Advanced Study Institute on Analytical Treatment of Problems of Berthing and Mooring Ships", ASCE, New York, N. Y. , 1971, 344 pp.
2. Carr, J. H. , "Long Period Waves or Surges in Harbors", Proc. Amer. Soc. Civil Eng. , v. 78, Separate No. 123, 1952; also, Trans. Amer. Society Civil Eng, v. 118, 1953, pp. 588-603
3. Chrystal, G. , "Investigation of the Seiches of Loch Earn by the Scottish Loch Survey. Parts I and II", Trans. Roy. Soc. Edinburgh, v. 45(2), 1906, pp. 361-396
4. Defant, A. , "Physical Oceanography", Pergamon, Oxford, v. II, 1960
5. Hwang, L-S and Tuck, E. O. , "On the Oscillation of Harbors of Arbitrary Shape", J. Fluid Mech. , v. 42(3), 1970, pp. 447-464
6. Joosting, W. C. Q. , "Investigation into Long Period Waves in Ports", Proc. XIXth International Navigation Congress, Sect. II, Communication 1, London, 1957, pp. 205-227
7. Kilner, F. A. , "Model Tests on the Motion of Moored Ships Placed on Long Waves", Proc. 7th Conf. on Coastal Eng'g, (The Hague, Netherlands, Aug. 1960), Council on Wave Research, Univ. California, Berkeley, 1961, pp. 723-745
8. Knapp, R. T. and Vanoni, V. A. , "Wave and Surge Study for the Naval Operating Base, Terminal Island, California", Tech. Rep. Hydraul. Struct. Lab. , California Institute of Technology, Pasadena, Ca. , 1945
9. Lamb, H. , "Hydrodynamics", Dover, New York, 1945, (1932 ed.)
10. Lean, G. H. , "Sub Harmonic Motions of Moored Ships Subjected to Wave Action", Proc. Royal Institution of Naval Architects, Paper WI, 1971, 12 pp.

11. Lee, J. J. , "Wave-induced Oscillations in Harbors of Arbitrary Shape", Tech. Rep. No. KH-R-20, 1969, W. M. Keck Lab. Hydraul. Water Resour. , California Institute of Technology, Pasadena, Ca.
12. Lee, J. J. , and Raichlen, F. , "Wave Induced Oscillations in Harbors with Connected Basins", Tech. Rep. No. KH-R-26, 1971, California Institute of Technology, Pasadena, California
13. Lee, J. J. , and Raichlen, F. , "Oscillations in Harbors with Connected Basins", Proc. ASCE, v. 98(WW3), Aug. 1972, pp. 311-332
14. Leendertse, J. J. , "Aspects of a Computational Model for Long-period Water Wave Propagation", Memo RM-5294-PR, 1971, Rand Corp., Santa Monica, California
15. Leybold, H. , "California Seiches and Philippine Typhoons", U. S. Nav. Inst. Proc. , v. 63(6), 1937, pp. 775-788
16. Muñoz, A. C. , "Surge Action in the Outer Harbor, Los Angeles, 1910-13", Letter of C. G. Muñoz, Report of Lt. Col. C. H. McKinstrey, Corps of Engineers, Document No. 896, House of Representatives, 63rd Congress, 2nd Session, Washington, D. C. , Apr. 14, 1914
17. Neumann, G. , "Die Impedanz Mechanischer Schwingungs-systeme und Ihre Anwendung auf die Theorie der Seiches", Ann. Hydraul. Mar. Met. 72. 1944, pp. 65-76
18. Neumann, G. , "On Resonance Oscillations of Bights and the Mouth Correction for Seiches", Deut. Hydrogr. Z. 1, 1948, pp. 79-101
19. O'Brien, J. T. , "Seiches and Other Causes of Motion of Ships Already Moored", in "Analytical Treatment of Problems of Berthing and Mooring Ships", ASCE, New York, N. Y. , 1971, pp. 20-26
20. Raichlen, F. , "Long Period Oscillations in Basins of Arbitrary Shapes", Proc. Amer. Soc. Civil Eng., Spec. Conf. Coastal Eng. , 1965, Chap. 7, pp. 115-145

21. Rayleigh, Lord, "The Theory of Sound", Dover, New York, 1945, (1877 Edn.)
22. Russell, R. C. H., "A Study of the Movement of Ships Subjected to Wave Action", Proc. Inst. Civ. Engrs (London), v. 12, 1959, pp. 379
23. Sommet, J., "The Motion of a Ship under the Action of Seiche", Proc. Symp. "Behavior of Ships in a Seaway", Wageningen, Netherlands, 1957, v. 1, pp. 354-373
24. Stoker, J. J., "Water Waves", Wiley (Interscience), New York, 1957
25. Taylor, C., Patil, B. S. and Zienkiewicz, O. C., "Harbor Oscillation: A Numerical Treatment for undamped Natural Modes", Proc. Inst. Civil Eng., v. 43, 1969, pp. 141-155
26. Wilson, B. W., "Ship Response to Range Action in Harbor Basins", Proc. Amer. Soc. Civil Eng., v. 76, 1950, Separate No. 41; also, Trans. Amer. Soc. Civil Eng., v. 116, 1951, pp. 1129-1157
27. Wilson, B. W., "Origin and Effects of Long Period Waves in Ports", Proc. 19th Int. Navigation Congr., Sect. II, Commun. 1, London, 1957, pp. 13-61
28. Wilson, B. W., "The Energy Problem in the Mooring of Ships Exposed to Waves", Proc. Princeton Univ. Conf. Berthing Cargo Handling Exposed Locations, 1958, pp. 1-67; also, Bull. Int. Navigation Congr., (PIANC), 50 (1959)
29. Wilson, B. W., "Research and Model Studies on Range Action in Table Bay Harbour, Cape Town", Trans. S. African Inst. Civil Eng., v. 1(6), 1959, pp. 131-148; (7), pp. 153-177
30. Wilson, B. W., "Threshold of Surge Damage for Moored Ships", Proc. Inst. Civil Eng., v. 38, 1967, pp. 107-134; v. 40, 1968, pp. 363-382
31. Wilson, B. W., "Tsunami Responses of San Pedro Bay and Shelf, Ca.", Proc. Amer. Soc. Civil Eng. Conf. Civil Eng. Oceans, 1969, pp. 1099-1133; also, J. Waterways, Harbors Coastal Eng. Div., Proc. Amer. Soc. Civil Eng., v. 97(WW2), 1970, pp. 239-258

32. Wilson, B. W., "Harbor Oscillation: A Numerical Treatment for Undamped Natural Modes", [Discussion on (25)], Proc. Inst. Civ. Engrs (London), v. 44, 1970, pp. 203-210
33. Wilson, B. W., "Seiches", in "Advances in Hydroscience" (V. T. Chow, editor), Academic Press, New York, v. 8, 1972, pp. 1-94
34. Wilson, B. W., Hendrickson, J. A. and Kilmer, R. E., "Feasibility Study for a Surge-action Model of Monterey Harbor, California", Contract Rep. No. 2-136, 1965, Waterways Exp. Sta., U.S. Army Corps of Engineers, Vicksburg, Mississippi
35. Wilson, B. W., Jen, Y., Hendrickson, J. A. and Soot, H., "Wave and Surge Action Study for Los Angeles-Long Beach Harbors", Tech. Report to Los Angeles District Corps of Engineers, U.S. Army, Science Engineering Associates, San Marino, California, July 1968, (unpublished)