CHAPTER 106

SCALE EFFECTS IN RUBBLE-MOUND BREAKWATERS

by

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ABSTRACT

In conducting model tests of wave transmission through permeable rubble-mound breakwaters, it is impossible to satisfy simultaneously the Froude and Reynolds criteria for dynamic similarity. The common practice has been to scale the wave parameters and breakwater dimensions in accordance with the Froude Number, and to use large models.

This study represents an attempt to develop theoretical expressions for the coefficients of reflection and transmission as functions of the effective porosity of the breakwater structure, as influenced by the Reynolds-dependent boundary layer growth on the pores. These expressions use linear wave theory and boundary layer theory to estimate the effective decrease in pore diameter due to growth of the displacement boundary layer thickness in the pore.

The theoretical expressions were compared with experimental results from a series of three model tests with breakwaters having vertical faces and using gravel with diameters of 1.37 in., 0.762 in., and 0.324 in. respectively. The prototype to model ratios (using the largest model as the prototype) were 1/1.80 and 1/4.23 respectively.

The experimental results show clearly the existence of scale effects in both coefficients of reflection and transmission. The theoretical expressions were found to overestimate the scale effect in reflection and to underestimate it in transmission.

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THEORETICAL DEVELOPMENT

This estimation of the scale effects for reflection and transmission proceeds in two steps: First, the establishment of a simple theory for the wave reflection and transmission coefficients depending only on the porosity of the structure (it being assumed, for the moment, that all other factors scale with the Froude Number, and may therefore be neglected); and second, the calculation of the Reynolds-dependent boundary layer growth inside the pores of the structure, and the resulting effects on the apparent porosity and thus the reflection and transmission coefficients.

Reflection-Transmission Theory

This theory describes wave reflection and transmission into the structure (H_r and H_{ti}, Fig. 1), it being assumed that $H_t \propto H_{ti}$ for the present purposes. The two basic assumptions are that energy is conserved; for a constant group velocity, this reduces to

$$E_i = E_r + E_{ti} \tag{1}$$

where E_i is the wave energy corresponding to the incident wave, H_i, given by linear wave theory as $\gamma\ H_i/8$, etc.; and that the water surface elevation is continuous across the face of the structure,

$$\eta_i + \eta_r = \eta_{ti} \tag{2}$$

where η_i is the total instantaneous water surface elevation associated with the incident wave, etc. Again, using linear wave theory,

$$\eta_i = a_i \cos\ (kx - \sigma_t) \tag{3}$$

where the incident wave amplitude, $a_i = H_i/2$; x is the horizontal coordinate in the direction of wave advance; σ is the radian wave frequency, $2\pi/T$; T is the wave period; and t is time.

Assuming that the energy flux of the wave being transmitted through the structure is the same as that of the same wave in open water, but reduced by the fraction of the water volume occupied by rock;

$$C_g\ E_{ti} = C_g\ m\ \frac{\gamma\ A_{ti}^2}{2} \tag{4}$$

where m is the porosity and C_g the group velocity.

With the introduction of the appropriate additional expressions from linear wave theory, the above assumptions yield the following expressions for the wave reflection and "transmission" coefficients:

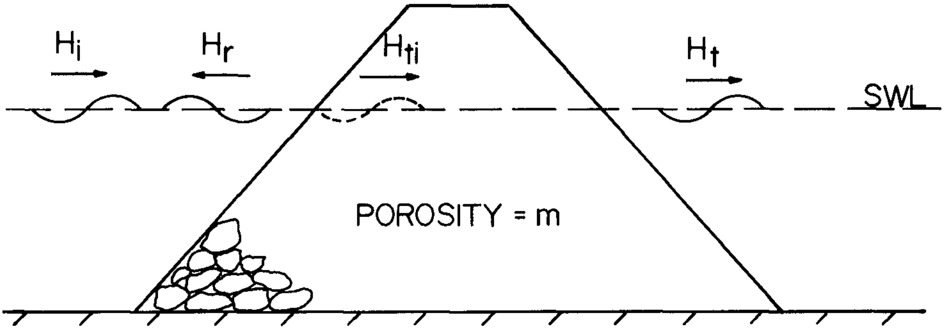


FIGURE I. WAVE REFLECTION AND TRANSMISSION

$$K_r = H_r/H_i = \frac{1-m}{1+m} \tag{5}$$

$$K_{ti} = H_{ti}/H_i = \frac{2}{1+m} \tag{6}$$

The above development is essentially identical to that for the reflection and transmission of waves encountering a contraction in width in a channel of constant depth.

Boundary Layer Effects

The growth of the boundary layer near the leading edge of a flat plate, or inside a short tube, represents the retardation of the flow velocity due to the presence of the solid boundary. In a short tube, this results in a reduction in the total flow discharge through the tube. One way of describing this discharge reduction is to calculate the displacement boundary layer thickness, δ^*, and to reduce the tube radius by this amount. From the definition of δ^*, then, the discharge obtained from the partly retarded flow velocity distribution in the original tube (Fig. 2). If we characterize the pores in the structure as tubes, this reduction in size of the tubes represents a reduction in porosity that can be applied to Eqs 5a & 5b to obtain the effect on K_r and K_{ti}.

First, the dimensions of the tube equivalent to the interstitial pores must be found. The tube length, ℓ, was arbitrarily chosen as 1/4 D, where D is the rock diameter. The tube diameter, D_p, was calculated from the hydraulic radius, R_h, as for a pipe: $D_p = 4\ R_h = 4\ A/P$. Here, A is the cross-section area of the pore, and P the wetted perimeter. Both these parameters are hard to measure; however, if both A and P are multiplied by a length (for example, D), they become proportional to the porosity, m, and the particle surface area per unit volume of structure, (1-M)S, where S is the specific surface, or surface area of the stones per unit volume of solids. For spheres, S=6/D; thus

$$D_p = \frac{4m}{1-m} \quad \frac{D}{6} \tag{7}$$

To calculate the boundary layer growth, the flow velocity must be known. For the present purpose, the linear theory flow velocity at the water surface corresponding to H_{ti}, averaged over half a wave period, is taken as characteristic:

$$U = \frac{2H_i\ K_{ti}}{T\ \tanh\ kh} \tag{8}$$

where h is the water depth.

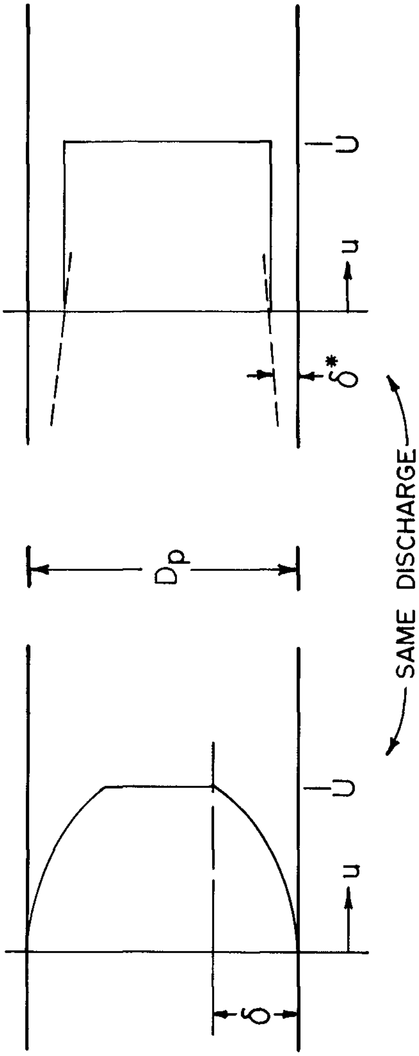


FIGURE 2. DISPLACED BOUNDARY LAYER EFFECT

For a laminar flow, the boundary layer thickness δ and the displacement thickness δ^* are given by

$$\delta = \frac{5x}{R_x^{1/2}} \tag{9}$$

and

$$\delta^* = \frac{1.73x}{R_x^{1/2}} \tag{10}$$

For the tube, x becomes ℓ, and the Reynold's number R_ℓ is given by $U\ell/\gamma$, where γ is the kinematic viscosity of the water. The maximum value of δ in the tube is limited to $D_p/2$, so that

$$\delta^* \max = 1.173\ D_p \tag{11}$$

The reduced porosity, m_r, is calculated as the reduction in cross-section area of the tube

$$m_r = m[1 - \frac{2\ \delta^*}{D_p}]^2 \tag{12}$$

To summarize, the following calculations are performed for both model and prototype:

1. The wave conditions, water depth and structure porosity are known for model and prototype.

2. K_{ti} is estimated from eq. 5b, and used to find U from Eq 7. D_p and ℓ are calculated using eq. 6 for D_p.

3. δ^* and m_r are calculated, using Eq 10 for m_r and Eqs 8b or 9 for δ^* (or empirical relations for turbulent boundary layers if R_ℓ exceeds 300,000).

4. K_r and K_{ti} are obtained from m_r using Eqs 5a and 5b. The estimate in step 2 is checked, and the calculations are revised as necessary.

5. The model-to-model prototype ratios for K_r and K_{ti} are obtained to find the scale effect.

Sample Calculations

Calculations of the scale effects were made for a hypothetical breakwater under the following prototype conditions:

Water Depth	$h = 40$ ft
Rock Diameter	$D = 3$ ft
Porosity	$m = 0.42$
Wave Period	$T = 8$ sec
Wave Heights	$H_i = 2$ and 10 ft

The results are presented as the reflection coefficient, $K_r = H_r/H_i$, and the transmission coefficient, equal to $K_{ti} = H_{ti}/H_i$, in the ratios of model to prototype (it being understood that the model values are first scaled up by the Froude laws).

TABLE I - Calculated Example

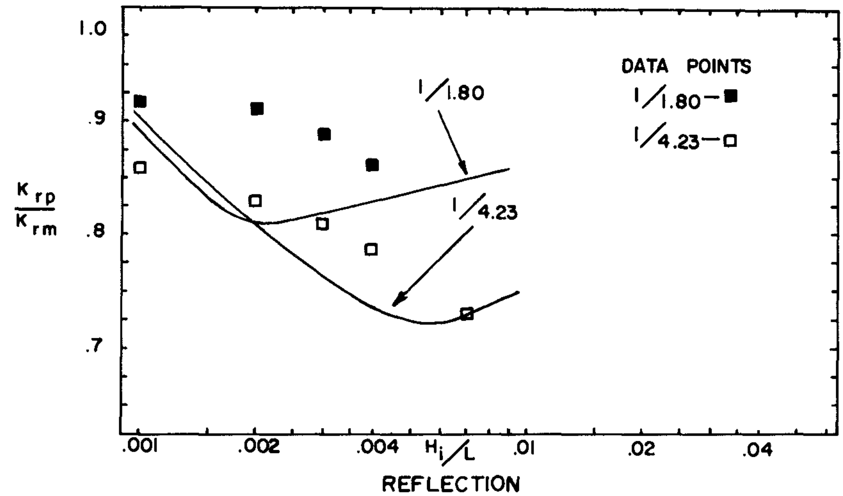
$H_i = 2.0$ Ft

Scale Ratio	Rock Dia., Inches	Reduced Porosity	Scale Ratios Trans.	Scale Ratios Reflec.
1/1	36	.408	1.00	1.00
1/5	7.2	.385	0.99	1.05
1/10	3.6	.362	0.97	1.12
1/20	1.8	.325	0.95	1.21
1/50	0.72	.234	0.87	1.28
1/100	0.36	.177	0.79	1.44

$H_i = 10.0$ Ft

Scale Ratio	Rock Dia., Inches	Reduced Porosity	Scale Ratios Trans.	Scale Ratios Reflec.
1/1	36	.411	1.00	1.00
1/5	7.2	.404	0.99	1.00
1/10	3.6	.393	0.98	1.02
1/20	1.8	.372	0.98	1.10
1/50	0.72	.331	0.95	1.19
1/100	0.36	.288	0.91	1.32

As expected, the transmission is reduced and the reflection increased in the smaller models. Assuming a 5% accuracy of laboratory wave measurements, the scale ratios where Reynold's scale effects become detectable are 1/5 for the 2-ft wave, and perhaps 1/15 for the 10-ft wave. A 1/5 model of a 40-ft high breakwater is still 8 ft high, and an expensive proposition.



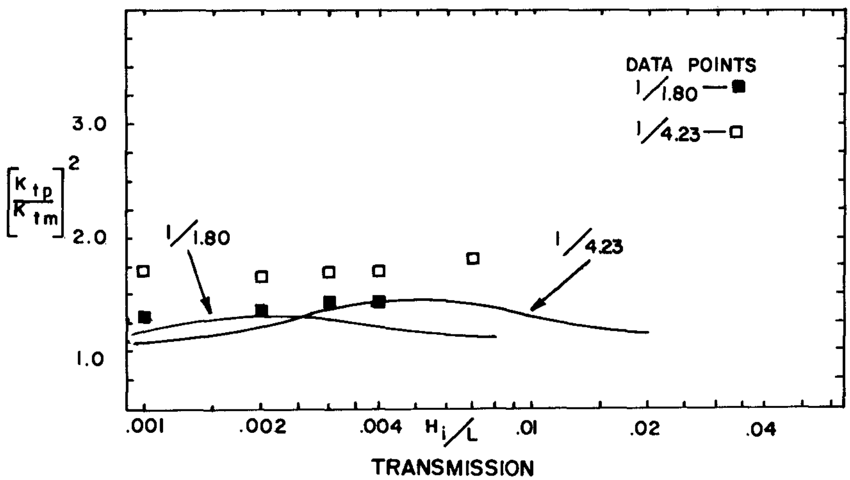


FIGURE 3. REFLECTION AND TRANSMISSION kh = 0.5

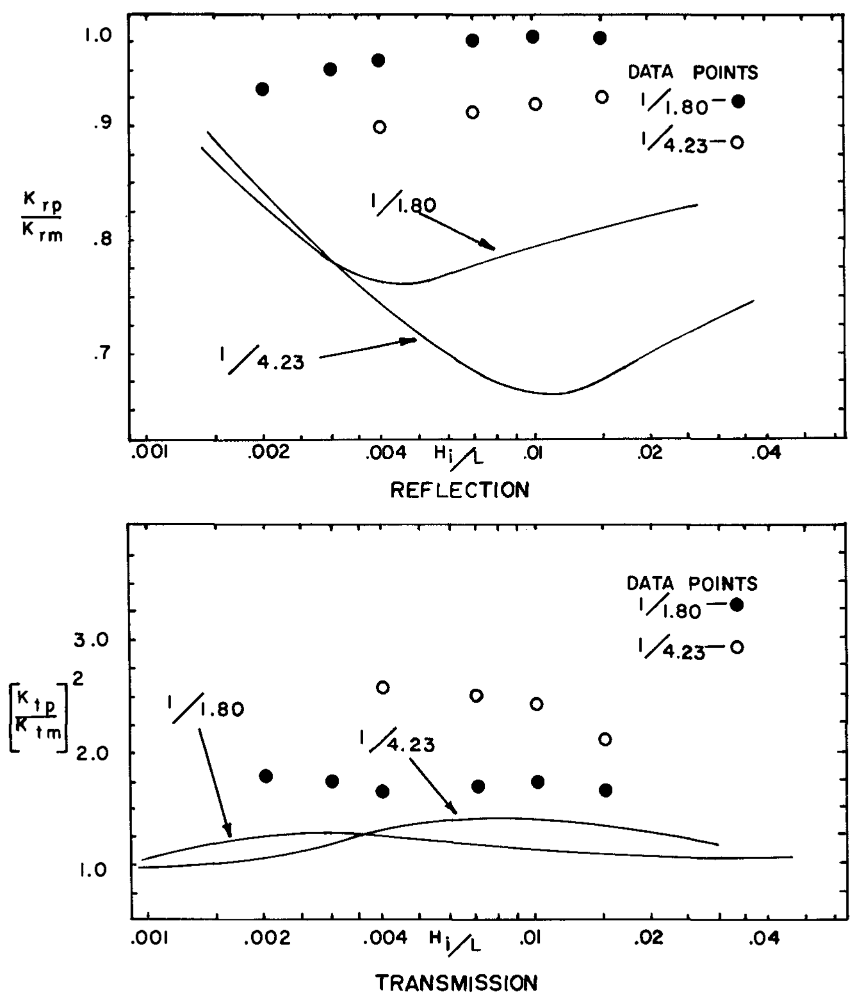


FIGURE 4. REFLECTION AND TRANSMISSION kh=1.0

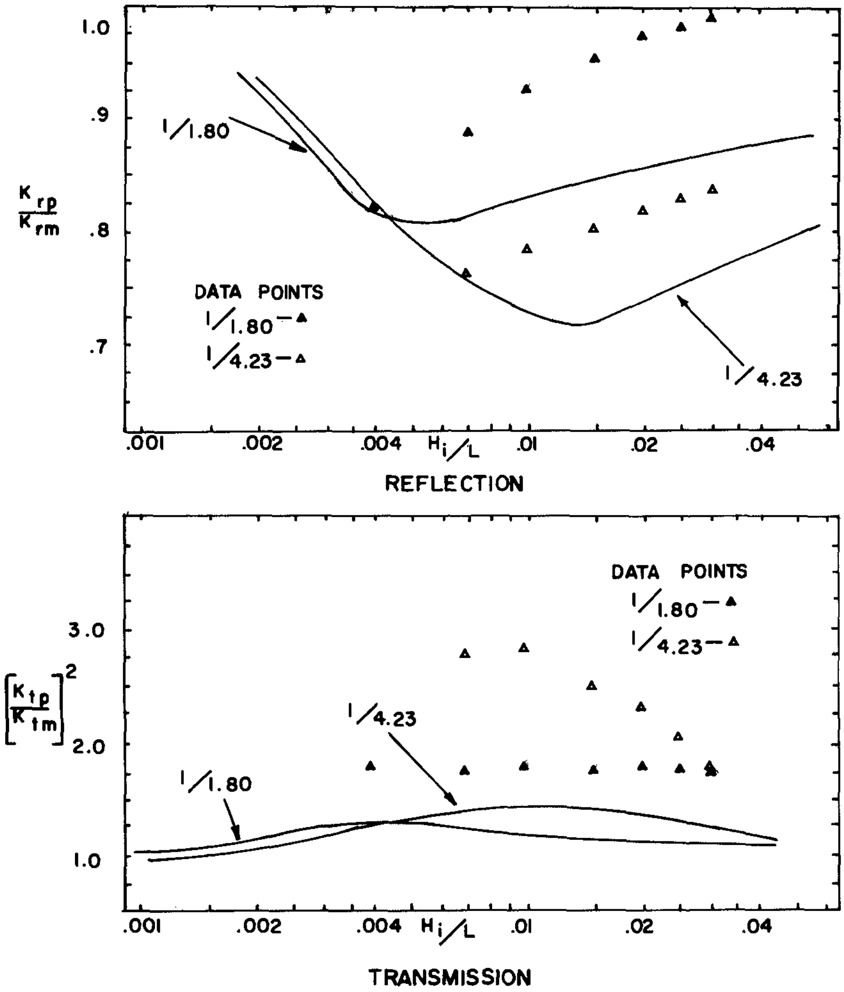


FIGURE 5. REFLECTION AND TRANSMISSION kh= 1.5

EXPERIMENTS

Several series of experiments were run in the large wave tank in the Hydrodynamics Laboratory at M.I.T. Crushed stone was graded, and placed in rectangular wire baskets to represent breakwaters of rectangular cross-section. The stone size (equivalent sphere diameter) was obtained from the number of particles, their weight and specific gravity, and the porosity in place from the basket size, stone weight and specific gravity. The three sizes used were as follows:

Stone	Diameter	Porosity	Scale Ratio
Large	1.37 inches	.437	1/1
Medium	.762	.411	1/1.80
Small	.324	.428	1/4.23

Wave heights (H_i, H_r, and H_t) were measured with parallel-wire resistance gages, with the gage on the "seaward" side mounted on a carriage to obtain the envelope of the partial reflection, and thus the height of the incident and reflected waves and the coefficient.

Results (as prototype/model ratios) for kh = 0.5, 1.0 and 1.5 are shown in Figs 3, 4 & 5. The agreement between the theoretical curves and the data is not particularly good. This may be due to several causes:

1. The assumption that H_t is proportional to H_{ti} (and thus $K_{ti} = K_t$) is particularly open to question; a better relationship, including scale effects, is needed.

2. Throughout the calculations, the "characteristic" dimensions and quantities used were chosen for convenience, with the implicit assumption that using a "characteristic" value instead of, say, a physically meaningful average value, was justified by similarity. Moreover, several phenomena such as energy losses at the seaward face were neglected, or assumed to scale by Froude laws. It is doubtful whether these are strictly correct, as these values and assumptions are used in computing Reynold's effects, which do not scale by Froude similarity.

3. The experiments are not really representative of a typical model=to-prototype relationship. Referring to the calculated example presented earlier, the experiments represent scales of 1/20 to 1/100, compared to a reasonable prototype. In this range, Reynold's effects are extensive, and, as the boundary layer nearly fills the assumed tube, the flat plate expressions used become doubtful.

CONCLUSIONS

The above calculations represent a very simplified "first approximation" at what is obviously a more complex phenomenon. Some large-scale data is needed to provide a proper evaluation of this level of analysis. The next level of sophistication in refining the analysis should be to incorporate a proper relationship between K_{ti} and K_t.

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