CHAPTER 65

SIMILARITY OF EQUILIBRIUM BEACH PROFILES

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ABSTRACT

In the design of mobile bed coastal models it is inherently assumed that prototype beach processes may be modelled using lightweight sediment. At the Queen's University Coastal Engineering Research Laboratory, a long range project is currently in progress to determine scaling laws and scale effect for mobile bed coastal models. A large portion of this program is directly concerned with beach profiles and in this paper preliminary work is reported, in which a comparison is made between two dimensional laboratory beach profiles obtained from controlled "prototype", undistorted model and some distorted model tests.

INTRODUCTION

A recent publication by Nayak (1970) provides a bibliography of the work on beach profiles to date, and summarises the conclusions drawn by several investigators. Fundamental differences in definition and experimental conditions make it extremely difficult to compare the data accumulated, and in previous work very little attempt has been made to achieve model similarity for the physical processes of beach formation which is required before a valid comparison can be made with prototype data.

This paper concerns itself with the derivation and use of reasonable scale laws for modelling equilibrium beaches. In practical cases, it is seldom possible to adhere strictly to all scale laws. This causes errors between the prototype and the model, which are a direct result of non-adherance to certain scale laws. These errors

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are called "scale effect" in this paper and some causes and types of scale effect are discussed.

DIMENSIONAL ANALYSIS

Two phase phenomena in the vicinity of a mobile bed may be expressed as a function of

$$\rho,\ \nu,\ \gamma_s,\ D,\ \rho_s,\ \ell,\ v_*$$

where ρ is the fluid density, ν the kinematic viscosity of the fluid γ_s the submerged unit weight of the sediment, D a general length parameter which specifies the overall particle geometry, ρ_s the sediment density, ℓ a general length parameter describing the fluid motion at the bed, and v_* a typical shear velocity. For more detail see Yalin (1971) and Kamphuis (1972).

For beach profiles formed by short waves ℓ may be replaced by the wave orbital amplitude at the bed, a_b . The shear velocity v_* is time varying and is characterised here by $\hat{v}_*$, the maximum shear velocity during the wave cycle. Particle geometry cannot be adequately described by a limited number of parameters, however, in order to draw attention to the most important ones, the general length parameter D is defined as

$$D = f\,(D_{50},\ \sigma_\Phi,\ \alpha)$$

where D_{50} is the sediment median grain size, σ_Φ the standard deviation of its Φ size distribution, and α the natural angle of repose. The beach profile P , may now be expressed as

$$P = f\,(\rho,\ \nu,\ \gamma_s,\ D_{50},\ \sigma_\Phi,\ \alpha,\ \rho_s,\ a_b,\ \hat{v}_*,\ i_o,\ ) \qquad (1)$$

where i_o is the initial beach slope. Equation 1 may be rewritten in the dimensionless form

$$\Pi_p = \Phi\left(\frac{D_{50}\hat{v}_*}{\nu}\ ,\ \frac{\rho\hat{v}_*^{\,2}}{\gamma_s D_{50}}\ ,\ \sigma_\Phi,\ \alpha\ ,\ \frac{\rho_s}{\rho}\ ,\ \frac{a_b}{D_{50}}\ ,\ i_o\ \right) \qquad (2)$$

In determining practical model scale laws, it is desirable to

eliminate a_b and $\hat{v}_*$ since they vary with location and are almost impossible to measure.

Equation 2 may be rewritten - Paul (1972) - as

$$\Pi_p = \Phi \left(\frac{D_{50}\sqrt{gH_o}}{\nu}, \frac{\rho g H_o}{\gamma_s D_{50}}, \sigma_\Phi, \alpha, \frac{\rho_s}{\rho}, \frac{D_{50}}{L_o}, \frac{H_o}{L_o}, \frac{d}{L_o}, i_o \right) \qquad (3)$$

and the maximum shear velocity $\hat{v}_*$ has effectively been replaced by $\sqrt{gH_o}$.

SCALE LAWS

Since in practice it is necessary to use water in both model and prototype $n_\rho = n_\nu = n_g = 1$, where n is the scale expressed as prototype value over model value. From Equation 3 the following scale laws may be derived

$$n_{D_{50}} = n_{H_o}^{-\frac{1}{2}} \qquad (4) \qquad n_{H_o} = n_{\gamma_s} n_{D_{50}} \qquad (5)$$

$$n_{\rho_s} = n_\rho = 1 \qquad (6) \qquad n_{D_{50}} = n_{L_o} \qquad (7)$$

$$n_{H_o} = n_{L_o} = n_d = n_z \qquad (8) \qquad n_{\sigma_\Phi} = n_\alpha = n_{i_o} = 1 \qquad (9)$$

Equations 4 and 5 yield conflicting values for $n_{D_{50}}$ since 4 is essentially derived from a Reynolds number while 5 is derived from a Froude number (a type of Shields parameter). This familiar situation is discussed in detail in Kamphuis (1972). One case which has a simple solution occurs when the particle Reynolds number is large enough to ensure turbulent flow around the grains in both model and prototype. Then the influence of fluid viscosity is negligible and Equation 4 may be ignored. Equation 6 implies that lightweight sediments cannot be used, while Equation 7 indicates that particle sizes must be scaled down geometrically. The non-similarity of Equation 4 is then considered to cause scale effect. This type of model is very limited because Equation 7 quickly results in model particles in the silt and clay range, subject to suspension and electro-chemical charges.

If the particle Reynolds number is not in the turbulent range, one expedient often used is to neglect similarity of ρ_s/ρ and D_{50}/L_o, i.e.

lightweight sediment is used (Fan and Le Mehaute, 1969; Yalin, 1971; Kamphuis 1972). Thus Equations 4, 5, 8 and 9 form a basis for the design of practical scale models using lightweight sediments while the non-similarity of ρ_s/ρ and D_{50}/L_o is considered to cause "scale effect".

For a laboratory beach model, the only water depth d, over which control can be maintained is the limiting depth at the toe of the beach d_T. Thus of necessity Equation 8 has been restricted to $n_{d_T} = n_z$. This equation and $n_i = 1$, often are not satisfied when different laboratory tests are compared. Since these scale laws may be important (Paul, 1972) it is difficult to make a valid comparison between different beach profiles reported in the literature. It is generally also difficult to exercise any real control over the variable α, and thus $n_\alpha \neq 1$ contributes to "scale effect". The requirement $n_{\sigma_\phi} = 1$ is a rough indication that the shape of the grain size distribution curves for the model and prototype sediment should be similar. This can be achieved by proper mixing of materials.

EXPERIMENTAL INVESTIGATION

In order to determine the validity of the scale laws derived and the scale effects introduced by non-similarity of the various dimensionless variables, several series of two dimensional equilibrium beach profiles were tested using both lightweight and natural beach materials. Coarse sand, coarse bakelite, fine sand, fine bakelite were used and the largest beach which reasonably could be constructed in the laboratory wave flume using coarse sand, was defined as the prototype. Waves of constant period, $T_1 = 1.71$ sec, were generated in a water depth $d_T = 0.68$ m, corresponding to a fixed ratio $d_T/L_o = 0.15$. These waves impinged on an initial beach slope of 1:10, for sufficient time to form an equilibrium profile. Separate tests were undertaken at fixed values of deep water wave steepness H_o/L_o ranging from 0.016 to 0.040, the upper limit being controlled by wave stability in the flume.

The test series, using coarse bakelite and fine sand were defined as models of the coarse sand prototype. The lightweight bakelite models followed the scale laws expressed by Equations 4, 5, 8 and 9, which agree with those proposed by Fan and Le Mehaute (1969) and Yalin (1971) for a distortion, N = 1; the fine sand models were based on the

assumption that the grain size Reynolds number was in the turbulent range and thus followed Equations 5 to 9 . One additional series of tests was completed in which fine bakelite, modelled a fine sand prototype, just as the coarse bakelite modelled the coarse sand prototype. This was intended to provide an additional check on the validity of the scale laws proposed for the use of lightweight sediments. The variables involved are listed in Table 1, while the resulting values of the Reynolds numbers and Shields parameters for the various tests are listed in Table 2.

All tests were carried out in a 50 m long x 1,20 m deep x 0,60 m wide wave flume at the Queen's University Coastal Engineering Research Laboratory. A flap type wave generator was used which could be adjusted to produce waves of any desired height compatible with the practical range of wave periods, 0.6 - 2.0 secs. Wave heights were measured immediately seaward of the toe of the beach slope, using a capacitance type proximity probe. They were recorded on chart paper, at the beginning and immediately before the completion of each test. An average wave height H_1, allowing for wave reflection, was determined by moving the wave gauge, mounted on a level trolley system, along the flume centreline for a distance of at least one half a wave length at a speed of approximately 20 mm/sec. Since the waves were generated in shallow water (d_T/L_o = 0.15) allowance was made for shoaling in the determination of a deep water wave height H_o from the measured wave height H_1.

The sand beach profiles reached equilibrium within 24 hours, but were run for at least 36 hours, the bakelite beach profiles reached equilibrium within 6 hours, and were run for 12 hours. Each beach profile was measured with a blunt point gauge (10 mm dia tip) along the flume centreline. Ripples covered a portion of the beach profile following most tests. Where these were particularly prominent their heights and lengths were measured. Otherwise the profile recorded was an envelope of the ripple crests. The beach profile data were plotted in the dimensionless form x/L_o versus z/L_o, where x is the horizontal distance measured from an intersection between the initial profile and mean water level, and z is the vertical distance above mean water level.

In Figures 1, 2 and 3, the different model beach profiles are compared with their corresponding prototype beach profiles. For both coarse and fine lightweight bakelite models (Figures 1 and 2) a

TABLE 1

Comparison between variables for the different beach profile test series

Variable	Coarse Sand (Series 4)	Coarse Bakelite (Series 7)	Fine Sand (Series 5)	Fine Bakelite (Series 6)
γ_s (kg/m^3)	1710	600	1670	600
D_{50} (mm)	0.63	0.91	0.357	0.525
d_T (m)	0.68	0.34	0.38	0.19
T_1 (sec)	1.71	1.20	1.29	0.91
L_o (m)	4.54	2.26	2.57	1.28
Av. water temp. (^{o}C)	19.5	9.9	18.7	20.4
ν(cm^2/sec; stokes)	1020	1300	1040	1000
ρ_s/ρ	2.71	1.60	2.67	1.60
$(D_{50}/L_o)x10^5$	13.8	40.0	13.8	40.7
d_T/L_o	0.15	0.15	0.15	0.15
σ_Φ	0.667	0.653	0.458	0.550
α(in water)	33^{o}20'	37^{o}40'	32^{o}50'	37^{o}50'
i_o	0.10	0.10	0.10	0.10

Table 2

Variation of the dimensionless variables $X_2 = \frac{\rho g H_o}{\gamma_s D_{50}}$ and $X_1 = \frac{D_{50}\sqrt{gH_o}}{\nu}$ with wave steepness H_o/L_o

H_o/L_o	Coarse Sand (Series 4)			Coarse Bakelite (Series 7)			Fine Sand (Series 5)			Fine Bakelite (Series 6)		
	H_o(ft)	X_2	X_1	H_o(ft)	X_2	X_1	H_o(ft)	X_2	X_1	H_o(ft)	X_2	X_1
0.016	0.240	67.9	523				0.136	68.1	219	0.068	65.6	236
0.020	0.300	84.9	593	0.149	83.3	466	0.169	86.7	244	0.085	82.0	264
0.025	0.375	106.1	653	0.187	104.2	521	0.212	108.3	273	0.106	102.4	295
0.030	0.450	127.4	715	0.224	125.0	571	0.254	130.0	299	0.127	122.9	324
0.040	0.600	169.8	832	0.299	166.7	660	0.339	173.3	346	0.169	163.9	374
0.050				0.373	208.3	737						
0.060				0.448	249.9	808						

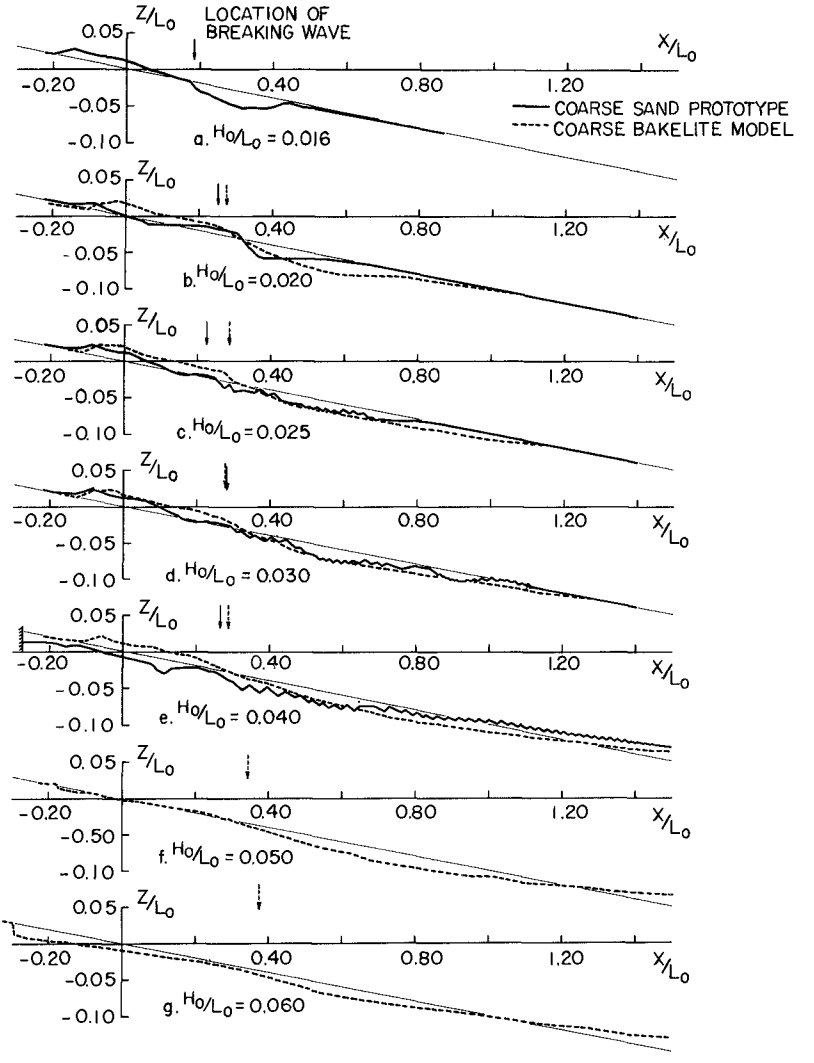

FIGURE 1 : DIMENSIONLESS COMPARISON BETWEEN PROFILES IN COARSE SAND AND COARSE BAKELITE

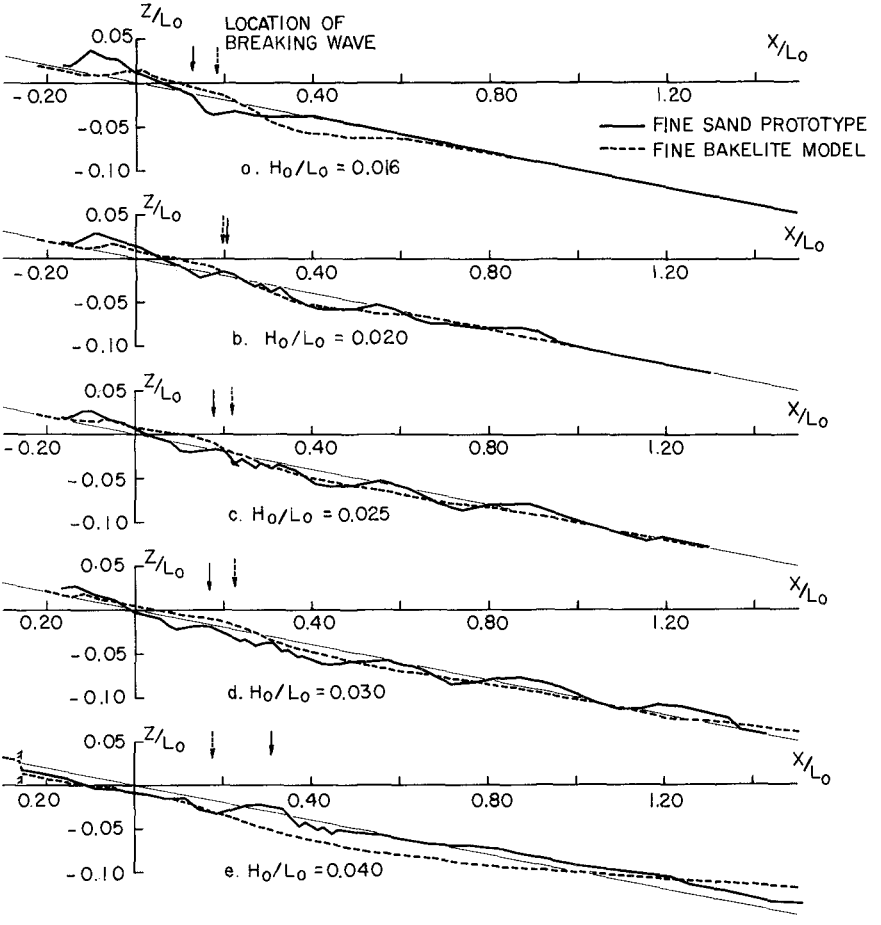

FIGURE 2 : DIMENSIONLESS COMPARISON BETWEEN BEACH PROFILES IN FINE SAND AND FINE BAKELITE.

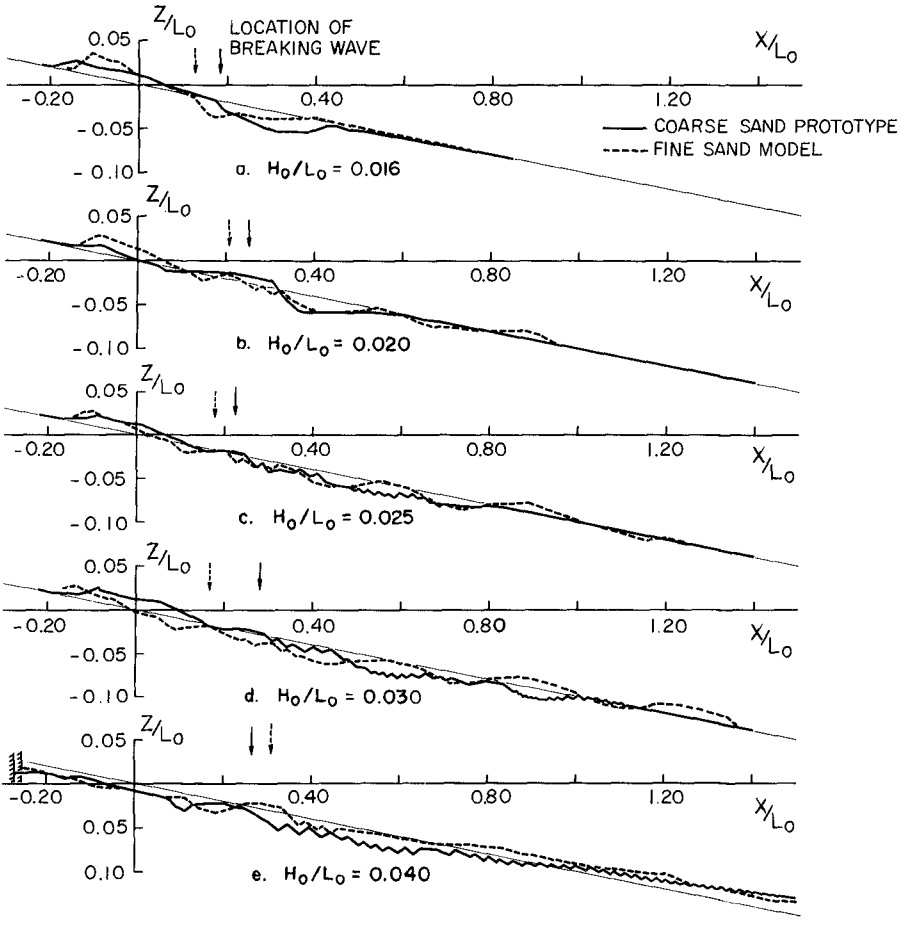

FIGURE 3 : DIMENSIONLESS COMPARISON BETWEEN BEACH PROFILES IN COARSE AND FINE SAND

considerable difference was observed between the profiles.

For all wave steepnesses (H_o/L_o) greater than 0.025 the natural sand beaches formed a "storm" profile with an offshore bar whereas the bakelite beaches showed no evidence of an offshore bar even at a wave steepness H_o/L_o = 0.060. Nayak (1970) summarised several different criteria for the generation of offshore bars (Figure 4). Superimposed on Figure 4 is the range covered by the present test data. The range is limited and therefore no general conclusions can be drawn as to the validity of the previously published criteria. However, for those tests using natural sand, transition from a "summer" to a "winter" profile occurred at wave steepnesses slightly less than those predicted by Johnson (1949) and Iwagaki and Noda (1963). On the other hand, for lightweight bakelite, no "winter" profile was evident, even for the highest values of H_o/L_o tested. This data throws considerable suspicion on the validity of any generalized criterion, such as proposed by Nayak, for the generation of an offshore bar. Instead it would appear that a separate criterion must exist for each different material. Most probably the criterion is a function of the dimensionless variable ρ_s/ρ , as well as of the reflection off the beach. It was noted that longshore bars for the sand profiles occurred at the antinodes on the reflection envelope. The bakelite beaches were much more porous in nature, and exhibited substantially smaller reflection coefficients and no appreciable offshore bars. Ripples, however, were noted for both materials at wave steepnesses exceeding 0.025.

From all profile comparisons it was evident that a greater proportion of the crushed bakelite, rather than the natural sand, had been transported shorewards. This resulted in the bakelite beach, shorewards of the wave breaking zone, being invariably higher than the corresponding natural sand beach, and the shoreline being further to sea-wards leaving the impression that the bakelite model beach was considerably steeper. Initially it was sought to explain this difference simply in terms of a "natural" distortion phenomenon. Different slope parameters similar to those postulated by Waters (1939) and Bagnold (1940) were used to characterise the beach slopes θ within the wave breaking zone. These yielded estimates of beach slope scales n_θ varying from 0.64 - 1.55, the lower values being associated with higher values of wave steepness H_o/L_o.

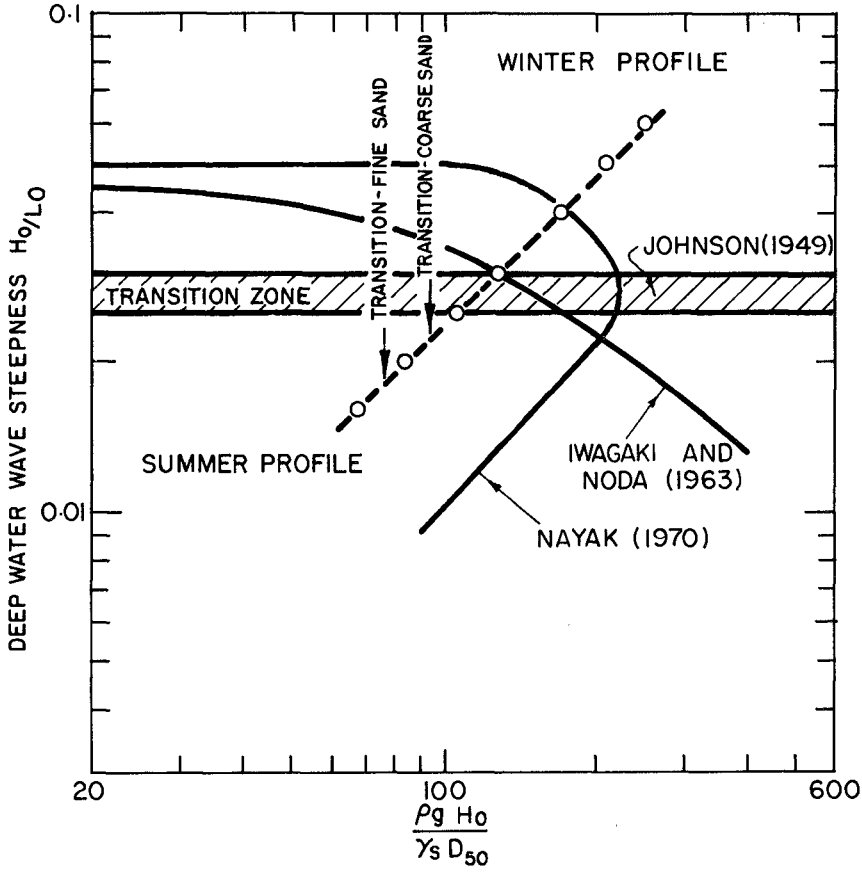

FIGURE 4 : CRITERIA FOR GENERATION OF AN OFFSHORE BAR — AFTER NAYAK (1970)

The comparison between test results for coarse and fine sand (figure 3) indicated a closer similarity between model and prototype. Offshore bars formed at corresponding values of H_o/L_o, and ripples dominated the offshore topography in both profiles, for wave steepnesses $0.025 \leq H_o/L_o \leq 0.030$. In spite of this improved similarity significant differences still existed between comparable profiles, and therefore it was concluded that fluid viscosity affected the model results, i.e. the model design was not acceptable because equation 9 (i) was not satisfied. Another possible source of difference between the sand prototype and sand model could be the replacement of v_* by $\sqrt{gH_o}$ in the scaling process. This would of course also affect the bakelite models and will be discussed later.

DISTORTED MODELS

Figure 5 indicates that for tests at wave steepnesses $H_o/L_o = 0.020$ and 0.040 a change in the initial beach slope i_o, for a lightweight sediment model, is sufficient to alter the final beach profile significantly. Although the basic shape of the beach profile in the foreshore region, from beach crest to wave breaking zone, is not greatly altered, Figure 5 shows that the position of the final shoreline, and the size of the foreshore berm are very much a function of the initial slope.

From Figures 1 and 2 it is apparent that an undistorted model beach, using lightweight bakelite invariably produced a profile in which the foreshore region was more prominent and the shoreline further to seaward than the corresponding prototype profile. Combining these two observations would indicate that distortion of the initial prototype slope in a lightweight sediment model, will reduce the beach berm, cut back the shoreline, and produce closer similarity between the model and prototype, except for the fact that offshore bars are not formed in the model at $H_o/L_o \geq 0.025$.

For a wave steepness of 0.020, a matching of the profiles and a comparison of the measured beach slope parameters indicate that if n_{i_o} were 1.2 to 1.6, the best similarity with respect to foreshore berm and shoreline would be obtained. From the earlier reference to n_θ, the beach slope scale, it may be seen that θ is very dependent upon its definition and that n_θ does not differ a great deal from 1. It is often argued that a model should be distorted so that

$$N = n_\theta \tag{10}$$

where N is the model distortion. Since n_θ has an average value of approximately 1 this would indicate all models should be undistorted. However,

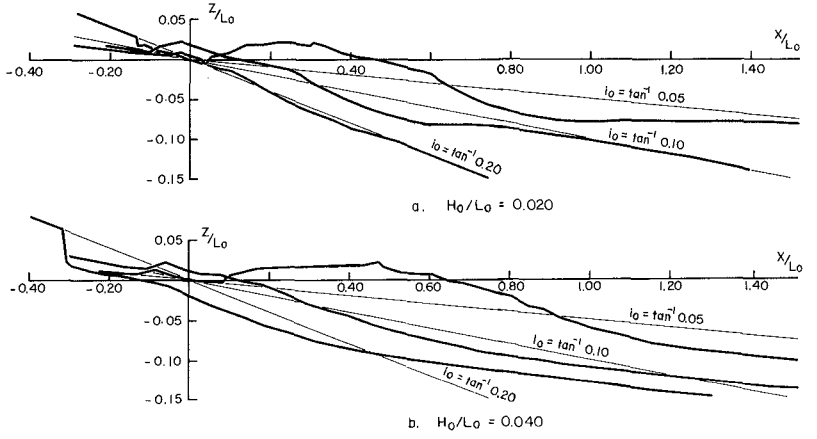

FIGURE 5 : DIMENSIONLESS COMPARISON BETWEEN PROFILES IN COARSE BAKELITE, FOR DIFFERENT INITIAL SLOPES, i_o .

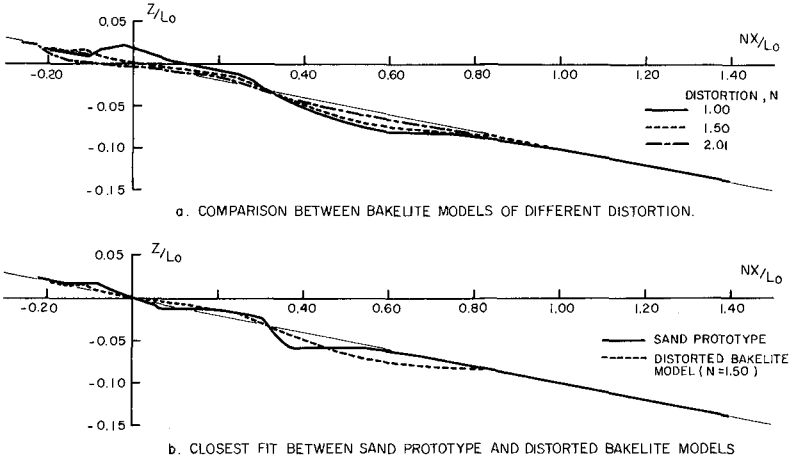

FIGURE 6: DIMENSIONLESS COMPARISON BETWEEN BEACH PROFILES USING MODEL DISTORTION, FOR TESTS AT A WAVE STEEPNESS H_o/L_o = 0.020

undistorted models do not reproduce the foreshore and shoreline correctly. This can be achieved by distorting i_o, as mentioned above. Thus the reason for model distortion is not Equation 10, but the matching of the onshore zone and shoreline.

Yalin (1971, p. 235) derives another expression for distortion

$$N = \eta_z^{\frac{1}{2}} \qquad (11)$$

which, yields a value of N = 1.42 for the lightweight bakelite model.

To determine the validity of model distortion, tests were carried out with the same coarse bakelite sediment, a constant wave steepness H_o/L_o = 0.020, and N = 1.50 and 2.01. Increasing the distortion decreases the dimensionless variable $\frac{\rho g H_o}{\gamma_s D_{50}}$, and thereby reduces the mobility of the sediment particles.

In Figure 6 a) the coarse bakelite beach profiles obtained for distortions N = 1.0, 1.50, and 2.01 are compared by plotting Nx/L_o against z/L_o. In Figure 6 b) it may be observed that the closest fit to the sand prototype data, in the foreshore region, was obtained for a distortion N = 1.50. In the offshore region beyond the wave breaking zone the fit between profiles still was not particularly good.

SCALE EFFECT IN THE ONSHORE ZONE

The equation of motion for a solid particle moving through a fluid shows that the particle trajectory is dependent on the particle specific gravity ρ_s/ρ. Only for a situation where the solid particle acceleration $du_s/dt \to 0$, does the particle motion become independent of ρ_s/ρ. For the undistorted lightweight sediment models investigated the trajectory of a model bakelite particle will be proportionally higher and longer than for a corresponding prototype sand particle. This difference in particle trajectory, caused by non-similarity of ρ_s/ρ, is a possible explanation for the observed difference between model and prototype equilibrium beach profiles, particularly in the portions of the bed where high fluid accelerations are present such as the foreshore and wave breaking zones. It seems reasonable to attribute non-similarity of the beach profile in this region primarily to non-similarity of ρ_s/ρ.

SCALE EFFECT IN THE OFFSHORE ZONE

In the comparison between beach profiles it is apparent that, beyond the breaking zone, the bakelite model sediment was moved at a proportionally greater depth than the sand prototype sediment. This is

particularly obvious for the "summer" profiles formed by low steepness waves - Figures 1 (b) and 2 (a). This "scale effect" may be explained in terms of non-similarity of a_b/D_{50} and this hypothesis was examined in some detail. Exaggerated model scour may be predicted by using the concepts developed by Riedel et al (1972). Figure 4 of their paper reproduces the experimentally determined relationship between wave friction factor, $f_w = 2(\hat{v}_*/\hat{U}_b)^2$, the maximum amplitude Reynolds Number for sinusoidal motion $RE = \hat{U}_b a_b/\nu$, and a_b/k_s the relative roughness. Here $\hat{U}_b$ is the maximum wave orbital velocity near the bed, and k_s is the equivalent sand grain roughness.

If k_s is assumed equal to D_{50} the relative roughness becomes a_b/D_{50}. Using small amplitude wave theory, it is possible to calculate values of a_b/D_{50} and $\hat{U}_b a_b/\nu$ for equivalent locations in the model and prototype, and determine corresponding values of f_w. For the test illustrated in Figure 1 b) where $H_o/L_o = 0.020$, and $d/L_o = 0.058$ at the point of scour in the prototype.

$$f_{w_{proto.}} = 2\,(\hat{v}_*/\hat{U}_b)^2_{proto.} = 0.025 \qquad (12)$$

If the model scour depth were the same, i.e. $(d/L_o)_{model} = 0.058$

$$f_{w_{model}} = 2(\hat{v}_*/\hat{U}_b)^2_{model} = 0.057 \qquad (13)$$

and $f_{w_{model}} > f_{w_{proto.}}$ for similar scour depths. More specifically

$$n_{\hat{v}_*} = \frac{n_{\hat{U}_b}}{1.5} \qquad (14)$$

for the particular depth $d/L_o = 0.058$ and the wave conditions considered. Equation 14 demonstrates that

$$n_{\hat{v}_*} < n_{\hat{U}_b} = n_{H_o}^{\frac{1}{2}} \qquad (15)$$

which is in conflict with the initial model design assumption that $v_* \propto \sqrt{gH_o}$. Since H_o/L_o and $D_{50}\sqrt{gH_o}/\nu$ are modelled correctly and d/L_o is constant in this particular example the inequality expressed in Equation 15 must be a direct result of non-similarity in D_{50}/L_o. The shear velocity and bottom shear stress for the model are exaggerated and additional scour will occur in the model. This is as observed, and the result is a deeper scour hole.

Kamphuis (1972) recognises that, for mobile bed short wave models, non-similarity of the variables ρ_s/ρ and a_b/D_{50} cause "scale effect". For a plain granular bed roughness distortion is simply equal to the particle size distortion and the following scale laws may be postulated - (Kamphuis (1972a, Eq. (31)):

$$n_{D_{50}} = n_z^{-5/16} \quad ; \quad n_{\gamma_s} = n_z^{15/16} \quad ; \quad n_\tau = n^{5/8} \qquad (16)$$

If these are used for model tests instead of Equations 4 and 5, then for the same model material ($n_{D_{50}} = 1/\sqrt{2}$), n_z would be equal to 3 instead of 2 and $n_\tau = 2$. With $n_{\hat{U}_b}$ now equal to $\sqrt{3}$, Equation 14 would become

$$n_{\hat{v}_*} = n_{\hat{U}_b} / 1.2 = n_{H_o}^{\frac{1}{2}} / 1.2 \qquad (17)$$

Thus modelling $v_* \alpha \sqrt{gH_o}$ would now give closer similarity between model and prototype in the offshore zone. In addition, the distortion is entered as a function of the fit of model and prototype profiles only, therefore the tests performed using Equations 16 may be representative of any distortion. Tests are in progress to assess the validity of Equations 16 and similar equations for rippled beds.

CONCLUSIONS

Two dimensional equilibrium beach tests, based on the scale laws developed in this paper, indicate that it is impossible to achieve exact similarity between a sand prototype beach profile and its lightweight sediment model. Closer correspondence between the profiles exists when exact geometric similarity is maintained, and sand is used in the model. For normal prototype sand beaches, however, where $D_{50} < 1.0$ mm, there is a severe limitation on the permissible scale reduction if these beaches are to be modelled using natural materials, because clay size model sediments must be avoided. This scale limitation renders these scale laws largely impractical. Practical circumstances require that water is used for the model, as well as the prototype. This invariably introduces "scale effect"

which is attributable to non-similarity of some of the dimensionless quantities since not all can be satisfied simultaneously.

For beach profile models using lightweight sediment in particular, it may be concluded that:

1. Non-similarity of the dimensionless variable ρ_s/ρ in a mobile bed short wave model causes non-similarity of the equilibrium beach profile. This scale effect is more pronounced in the foreshore and wave breaking zones where the influence of fluid accelerations is greatest.

2. For "summer" beach profiles, formed by low steepness waves ($H_0/L_0 < 0.025$), the above scale effect may be reduced by introducing a distortion N into the model design. This distortion is not equal to n_θ^{-1} since $n_\theta \simeq 1$. The distortion results from a desire to match the shoreline and foreshore in model and prototype.

3. For "winter" or "storm" beach profiles which are formed by high steepness waves ($H_0/L_0 > 0.025$) and exhibit substantial offshore bars, similarity of the beach profile cannot be achieved using lightweight sediments since the model profiles do not exhibit any bar structure. A generalised criterion for bar formation for all materials appears to be impossible to derive because of differences in specific gravity, porosity and angularity of the particles of the various materials.

4. For the offshore region, beyond the wave breaking zone, mainly non-similarity of the dimensionless variable a_δ/D_{50} causes exaggeration of the scour depths in the model.

5. For "summer" beach profiles, this "scale effect" will be reduced by revising the initial assumption that $v_* \propto \sqrt{gH_0}$, on the basis that

$$\frac{\hat{v}_*}{\sqrt{gH_0}} = f\left(\frac{a_\delta}{D_{50}}\right)$$

This leads to an increase in the vertical scale n_z, with respect to the scales n_{γ_s} and $n_{D_{50}}$, thereby decreasing both the mobility of the model sediment particles and the depth of scour. The scale laws recently proposed by Kamphuis (1972a) warrant investigation.

6. If the model beach particles have an angle of repose that is radically different from the prototype material, then this difference will significantly affect similarity of the model beach profile.

7. Natural sand beaches absorb less wave energy than those formed by lightweight bakelite. The resulting larger reflected wave may have a significant influence on the equilibrium beach profile and the formation of bars.

8. If similarity of the bottom profile is required then large distortions, permissible for rigid bed models, are not valid for mobile bed models of the coastal regime, because distortion is limited by the maximum angle of repose of the model beach material.

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