CHAPTER 61

ON THE GEOMETRICALLY SIMILAR REPRODUCTION OF DUNES IN A

TIDAL MODEL WITH MOVABLE BED

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ABSTRACT

A model design method is suggested which is based on the principles of the theory of dimensions and which is evaluated by experimental relations for the geometric properties of dunes. The model is distorted and, in general, non Froudian. The geometrically similar model dunes are reduced in vertical model scale. The scale of the flow velocity is derived from a generalised friction equation which takes into account the influence of both skin friction and form drag. The application of the method is illustrated by a numerical example. This example indicates that a practicable set of scales is obtained if the model bed is formed by a light weight material.

INTRODUCTION

When carrying out model tests with drilling structures, buried pipelines or any other objects which are in contact with a movable bed a major consideration is that the model dunes should be geometrically similar to their counterparts in the prototype. Indeed the functioning of the structures mentioned depends to a considerable extent on how their geometry compares with the geometry of dunes around them, so that a reliable prediction of the performance of these structures can only be made if the model and prototype dunes are geometrically similar.

At present no systematic information on dunes formed by tidal flows is available. On the other hand the measurements carried out in the North Sea, Liverpool Bay, Outer Thames and Sandettie

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(in English Channel) indicate that the dunes produced by tidal currents should be very similar in their size and shape to the dunes produced by the equivalent unidirectional flows.^{*} This similarity between the tidal and unidirectional dunes is, in fact, not surprising, for the formation of dunes by tidal currents takes place mainly in those parts of the periodic cycle when the whole body of the fluid moves in one direction, that is when the tidal flow becomes virtually a unidirectional flow. Considering this (fortunate) similarity between the tidal and unidirectional dunes, the present method is developed by using the existing information for unidirectional dunes.

SOME QUANTITATIVE ASPECTS OF RIPPLES AND DUNES

It can be shown that the dimensionless combinations (variables):

$$X = \frac{v_{\star}D}{v}$$
; $Y = \frac{\rho v_{\star}^2}{\gamma_S D}$; $Z = \frac{h}{D}$ *** (1)

are sufficient in order to express any dimensionless property of sand waves formed by a unidirectional tranquil (Fr < 1) flow (i.e. of ripples and dunes).

From the analysis of a large number of data it follows that the relative length Λ/h of dunes is a function of two variables :

$$\frac{\Lambda}{h} = f(X, Z)$$
(2)

Furthermore, the same data reveals that for $X > \approx 25$ and/or $Z > \approx 5000$ the function f (X, Z) reduces into a constant (2 π):

$$\frac{\Lambda}{h} = 2\pi \tag{3}$$

*** See "List of Symbols" at the end of the paper.

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^{*} The author is grateful to R.C.H. Russell and A.W. Price (HRS - Wallingford) for this information.

^{**} Extensive information on all the aspects summarised in this section can be found in Chapter VII of Ref []].

In natural estuaries, the values of X and Z are almost always large enough, as to assume that the length of dunes is given by the proportionality (3).

For the height \times of dunes the following expression is valid

$$\frac{\Delta}{\Lambda} = \Phi \left(\frac{Y}{Y_{cr}}, X, Z \right)$$
(4)

where the subscript cr signifies the "critical stage" (initiation of sediment transport), while the fundtion Φ characterises the dune steepness. Here the most important variable is Y/Y_{cr} ; the variations in Φ induced by X and Z are comparatively small. Accordingly the relation (4) can be replaced by its approximate equivalent:

$$\frac{\Delta}{\Lambda} \approx \Phi \left(\frac{Y}{Y_{cr}} \right)$$
 (5)

If $Z > \approx 1000$, then ripples and dunes exist in the regions shown in Fig. 1. When Z decreases from ≈ 1000 to ≈ 200 the point X_2 moves towards the fixed point $X_1 \approx 5$, as to reduce the <u>interval</u> X_2X_1 to the point $X_2 = X_1 \approx 5$, for all Z smaller than ≈ 200 . Hence, for $Z < \approx 200$ the simultaneous occurrence of ripples and dunes (in the form of ripples superposed on dunes) becomes impossible; the sand waves are <u>either</u> ripples <u>or</u> dunes. This is only natural, for ripples can be superposed on dunes only if their size $\Lambda_r = \text{const. D}$ is much smaller than the size Λ of dunes. Observe, however, that

$$\frac{\Lambda}{\Lambda}$$
 is proportional to $\frac{h}{D} = Z$

and thus that Λ_{r} cannot be "much smaller" than Λ if Z is not sufficiently large.

It follows that the presence of ripples on model dunes will certainly be avoided if the model value of X is selected as to be larger than ≈ 25 . (Note that this requirement coincides with one of the requirements for the validity of the proportionality (3)).

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SCALE RELATIONS

Let A' and A" be the prototype and model values of a quantity A , and $\lambda_{\rm A}$ = A"/A' be the scale of A .

From Eqn (5) it follows that the steepness of model and prototype dunes will be approximately the same, i.e. the approximate scale relation

$$\frac{\lambda_{\Delta}}{\lambda_{\Lambda}} \approx 1$$
 (6)

will be valid if the condition

$$\lambda_{\rm Y} = \lambda_{\rm Ycr} \tag{7}$$

is fulfilled.

Consider now the X; Z plane shown in Fig. 2. From the text preceding Eqn (3) it follows that if the model and prototype "points" M" (X", Z") and M' (X', Z') are both outside the shaded region, then

$$\lambda_{\Lambda} = \lambda_{h}$$

The present method is an attempt to achieve the similarity of dunes, i.e. the scale relation:

$$\lambda_{\Lambda} \approx \lambda_{\Lambda} \approx \lambda_{h} \tag{8}$$

by satisfying (7), and by selecting M'' outside the shaded part of the X; Z plane (assuming, of course, that the prototype point M' is also outside it).

A certain prototype corresponds to a certain set of the values of the variables X, Y, Z and Y_{cr} . Knowing the prototype values X', Y', Z' and Y'_{cr} , one determines the model values X", Y", Z" and Y'_{cr} from the relations:

$$X'' = \lambda_{\chi} X' ; Y'' = \lambda_{\gamma} Y' ; Z'' = \lambda_{\zeta} Z' ; Y''_{cr} = \lambda_{\gamma} Y'_{cr}$$
(9)

where

$$\lambda_{\chi} = \frac{\lambda_{y} \lambda_{D}}{\lambda_{x}^{\frac{1}{2}}} \qquad \lambda_{\gamma} = \frac{\lambda_{y}^{2}}{\lambda_{x} \lambda_{\gamma} s} \lambda_{D} \qquad \lambda_{Z} = \frac{\lambda_{y}}{\lambda_{D}}$$
(10)

In the relations above, λ_{χ} and $\lambda_{\dot{y}}$ are horizontal and vertical model scales respectively. These relations follow directly from the set (1) by substituting

$$\lambda_{g} = \lambda_{\rho} = \lambda_{v} = 1 \tag{11}$$

(model operating with water) and

 $\lambda_{v_{\star}}^{2} = \lambda_{h} \lambda_{s} \quad ; \quad \lambda_{h} = \lambda_{y} \quad ; \quad \lambda_{s} = \lambda_{y} / \lambda_{x}$ (12)

The following procedure can be suggested for determining the model scales.

- (i) Choose vertical scale and the model bed material (i.e. choose λ_y , λ_{γ_c} and λ_D);
- (ii) Knowing λ_y and λ_D , determine λ_Z and thus Z'' = λ_7, Z' ;
- (iii) Knowing $\gamma_{S}'' = \lambda_{\gamma_{S}} \gamma_{S}'$ and $D'' = \lambda_{D}D'$, determine Y''_{Cr}

(from the Shields curve);

- (iv) Knowing Y', Y'_{cr} and Y", determine $\lambda_{Y} = Y''_{cr}/Y'_{cr}$ and Y" = $\lambda_{Y}Y'$;
- (v) Knowing λ_{γ} , determine λ_{χ} (from second eqn of (10));
- (vi) Knowing λ_y , λ_x , and λ_D , determine λ_χ (first eqn of (10)) and the distortion $\lambda_y/\lambda_x = n$;
- (vii) Check whether the position of the point M"(X", Z") on the X; Z plane and the value of n are acceptable. If not repeat the procedure for another set of values of λ_y , λ_{γ_s} and λ_D .

Nothing has been said so far on how the flow velocity scale λ_v must be determined, and it is intended now to consider this relevant aspect of the model design.

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GENERALISED FRICTION FORMULA; VELOCITY SCALE

Let ${\rm S}_k$ and ${\rm S}_\Delta$ be the pure friction and form drag components of the total slope (free surface slope) S

$$s = s_k + s_{\Lambda}$$
(16)

Suppose the granular material is (almost) uniform. In this case, the "skin roughness" $k_{\rm S} \approx {\rm D}$ and the pure friction slope ${\rm S}_{\rm k}$ can be expressed as

$$S_{k} = \frac{Fr}{\left[\frac{1}{\kappa}\ln(A\frac{h}{k_{s}})\right]^{2}} \approx \frac{Fr}{\left[\frac{1}{\kappa}\ln(AZ)\right]^{2}}$$
(17)

while the form drag slope S_{Λ} can be given by

$$S_{\Delta} = \frac{1}{2} \cdot \frac{\Delta^2}{\Lambda h} \cdot Fr$$
 (18)

Hence:

$$\frac{S}{F_{r}} = \frac{1}{\left[\frac{1}{\kappa} \ln(AZ)\right]^{2}} + \frac{1}{2} \frac{\Delta^{2}}{\Lambda h}$$
(19)

Observe, that since \mbox{Fr} = $v^2/gh\,$ the eqn (19) can be written in the Chezy form

$$v = c\sqrt{gSh}$$
 (20)

where the generalised dimensionless Chezy coefficient c reflects the influence of the skin roughness $(k_s \approx D)$ as well as of the sand waves (Δ, Λ) as follows:

$$c = \left[\frac{1}{\left[\frac{1}{\kappa}\ln(AZ)\right]^2} + \frac{1}{2}\frac{\Delta^2}{\Lambda h}\right]^{-1/2}$$
(21)

(Note, that if the sand waves are not present (flat bed: $\Delta \equiv 0$) then the expression above reduces into the familiar form $c = (1/\kappa).ln(AZ)$).

* See e.g. Ref [2] and observe that S_k/Fr implies $[v_*)_k/v]^2$. The comment on the values of A is in the last footnote of the text. ** Refs [3], [4], [5]. The Eqn (19) gives for the scales

$$\frac{\lambda_{s}}{\lambda_{Fr}} = \frac{\left[\frac{1}{\kappa} \ln(AZ^{*})\right]^{-2} + \frac{1}{2} \frac{\Delta^{*2}}{\Lambda^{*}h^{*}}}{\left[\frac{1}{\kappa} \ln(AZ^{*})\right]^{-2} + \frac{1}{2} \frac{\Delta^{*2}}{\Lambda^{*}h^{*}}}$$

and thus

$$\frac{\lambda_{s}}{\lambda_{Fr}} = \frac{\left[1 + \frac{\ln \lambda_{Z}}{\ln(AZ')}\right]^{-2} + N'(\lambda_{\Delta}^{2}\lambda_{\Lambda}^{-1}\lambda_{h}^{-1})}{1 + N'}$$
(22)

where

$$N' = \frac{1}{2} \frac{{\Delta'}^2}{{\Lambda'}h^*} \left[\frac{1}{\kappa} \ln(AZ') \right]^2 .$$
 (23)

Since $\lambda_s = \lambda_y / \lambda_x = n$ and since the dunes are geometrically similar, i.e. $\lambda_A^2 \ \lambda_A^{-1} \ \lambda_b^{-1} \equiv 1$

the Eqn (22) can be written as

$$\frac{n}{\lambda_{\rm Fr}} = \frac{\left[1 + \frac{\ln \lambda_Z}{\ln(AZ')}\right]^2 + N'}{1 + N'}$$
(24)

which gives immediately

$$\lambda_{v} = \xi \sqrt{\lambda_{y}} \quad \text{with} \quad \xi = \left[\frac{1 \text{ n } \lambda_{Z}}{\frac{1 + \frac{1 \text{ n} (AZ')}{1 \text{ n} (AZ')}}{1 + N'}} \right] + N'$$
(25)

If the scales involved in the procedure explained at the end of the preceding section are determined, then λ_y and all of the terms that appear in the expression of the multiplier ξ are known, and the value of the scale λ_y can be computed from (25).

Note, that the multiplier ξ reflects the deviation from a "Froudian model". Indeed the model becomes Froudian ($\lambda_v = \sqrt{\lambda_v}$) only

^{*} Note that Eqns (19) and (22) are the generalised versions of the expressions given in Ref [6].

if $\xi = 1$, i.e. only if n, λ_Z , Z' and N' are interrelated as follows:

$$n = \frac{\left[1 + \frac{\ln \lambda_Z}{\ln(AZ')}\right] + N'}{1 + N'}$$
(26)

Usually it is very difficult to select such λ_{Z} and n which can satisfy the condition (26) and which can, at the same time, be regarded as "reasonable" (Z' and N' being determined by the prototype). In such cases it may be wiser to relax the condition (26) and thus to allow a deviation from a Froudian model (especially if the phenomenon forming the subject of model tests is related to the vicinity of the bed rather than to that of the free surface). *

NUMERICAL EXAMPLE

Consider the prototype specified by the following characteristics: **

h' = 27 m (
$$\approx$$
 90 ft)
D' = 0.2 mm
L' = 700 km
H' = 7 m
Y'_s/Y' = 1.65
(27)
(v = 10⁻⁶ m²/s, g = 9.81 m/s²)

Using Eqns (1) and the Shields' curve one determines for this prototype the following values of the dimensionless variables

X' = 10.3; Y' = 0.82; $Z' = 1.35.10^5$; $Y'_{cr} = 0.052$ (28)

If $\lambda_y = 1/45$, then h" = 27/45 = 0.6 m which is reasonable. Adopting this value of λ_y and using it for various combinations of λ_{γ_s} and γ_s

*	See more on	the	importance	of	the	Froude	number	in	Ref	[7]
	(Introducti	on).								

** Due to R.C.H. Russell (HRS - Wallingford).

 $^{\lambda}{}_{\rm D}$ one arrives (by applying the procedure described and the Eqn (25)) at the results shown in the table below. *

Model Bed Material	λ _γ s	λ _D	^λ у	λ _x	$n = \frac{\lambda y}{\lambda x}$	λ _χ	λy	λz	ξ
polystyrene (_Y " = 0.05) 3.0 mm	$\frac{1}{33}$	15	<u>1</u> 45	<u>1</u> 532	11.85	7.70	1 1.73	<u>1</u> 675	2.50
polystyrene (_Y " = 0.05 2.0 mm	$\frac{1}{33}$	10	1 45	$\frac{1}{343}$	7.64	4.12	<u>1</u> 1.79	$\frac{1}{450}$	2.05
polystyrene (_Y " = 0.05) 1.35 mm	$\frac{1}{33}$	6.75	$\frac{1}{45}$	1 239	5.32	2.32	$\frac{1}{1.73}$	<u>1</u> 304	1.77
perspex (_Y " = 0.19) 1.5 mm	$\frac{1}{8.7}$	7.5	$\frac{1}{45}$	1 209	4.65	2.41	$\frac{1}{1.85}$	<u>1</u> 338	1.64
sand (Y ["] _S = 1.65 0.12 mm	1	1 1.67	$\frac{1}{45}$	1 1775	39.5	$\frac{1}{1.78}$	1.46	$\frac{1}{27}$	5.64
sand (Y"s = 1.65 0.10 mm	1	$\frac{1}{2}$	$\frac{1}{45}$	1 1745	38.80	$\frac{1}{2.15}$	1.70	22.5	5.53

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* The values of N' were computed by adopting A = 15.00. The quantity A is related to B of Ref [2] (Chapter XX) by A = $e^{\partial eB}$. Using Fig. 20.18 of Ref [3] one can show that the value of A = $f(v \star k_S / v)$ varies within a relatively narrow interval 16.50 > A > 11.00 (for all $v \star k_S / v > 10$). Hence the reason for a constant (average) value A = 15.00.

Note, that sand models can hardly be regarded as acceptable (at least not for the prototype under consideration), as they require very large distortions (n) while their velocities deviate too much from the Froudian velocities (large ξ). Conversely, light weight materials yield acceptable values for n and ξ , and among them 1.35 mm - polystyrene and 1.5 mm - perspex appear to be as most favorable.

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LIST OF SYMBOLS

h	flow depth
S	free surface slope
v	average velocity representative values
۷*	shear velocity
ρ	fluid density
p _s	grain density
Ŷs	specific weight of grains in fluid

v kinematic viscosity	
D typical grain size (usually D ₅₀)	
$Fr = v^2/gh$ Froude number	
X,Y,Z,W dimensionless variables of the two phase motion	
as defined by Eqn (1)	
a' and a" prototype and model values respectively of a quantity	/ a
$\lambda_a = a''/a'$ scale of a	
x,y horizontal and vertical coordinates	
$n = \lambda_x / \lambda_y$ distortion	
ξ ratio of the non Froudian model velocity to the	
Froudian model velocity	



