CHAPTER 49

SEDIMENT TRANSPORT BY WAVE ACTION

by

H. A. Einstein
Emeritus, Professor of Hydraulic Engineering
University of California
Berkeley, California

This paper summarizes the results of a continuing study at the hydraulic laboratory at the University of California at Berkeley on this subject, which is supported by the Coastal Engineering Research Center (CERC) and which has resulted over the years in the Theses of Huon Li (1954), M. Manohar (1955), G. Kalkanis (1957, 1963), M. M. Abou-Seida (1965), M. M. Das (1968) and is at this time being continued by T. C. Mac Donald. All these researchers have greatly contributed to the success of this work while the author was mostly responsible for the continuity of the study.

The aim of the study was to see if it is possible to establish for the description and prediction of sediment transport by waves a general system of approach similar to that which the author published in 1950 under the title "The Bed-Load Function for Sediment Transport in Open Channel Flows".[3] It was hoped at the time that many of the basic steps of such a description may at least be similar to those used for uni-directional flow. It became apparent that such similarities of approach were quite feasible; but another difficulty became apparent from the beginning. While in the uni-directional flow many details of the flow, such as velocity distributions, boundary layers and turbulence had been studied and described previously, such knowledge was almost entirely lacking for wave motion. The first part of the study consisted entirely of hydraulic measurements and of their analysis. In order to determine the necessary scope of such hydraulic studies, the analogy with sediment transport in uni-directional flow was used. Some of the principles governing uni-directional flow transport are:

1. Sediment motion can be divided into bed-load motion or surface creep and suspension.

2. While moving as bed load, the particle weight is to a large part transmitted directly to the nonmoving bed, not to the flow.

3. The rate of bed-load motion is defined by the equilibrium exchange of sediment between the bed-load and the nonmoving bed.

4. This equilibrium gives a direct relationship between the sediment rate and the flow conditions near the bed, including the turbulence.

5. The flow condition near the bed can be predicted for a uni-directional boundary layer as a function of the bed shear and the bed roughness, only.

6. The bed particles moving as bed load in the rather thin bed layer define in this layer a sediment concentration.

7. Above the bed layer, bed particles move in suspension, i.e., continuously transmitting their weight to the surrounding water.

8. The distribution of the concentration in a suspension is defined by the equation of equilibrium exchange of particles through the various horizontal planes, with the bed-load concentration in the bed layer as boundary condition.

All these eight principles are assumed (and seem) to apply equally to the problem of sediment transport by wave action except No. 5. All unidirectional equilibrium flows moving sediment on a bed of the same kind of particles are friction controlled; they represent in their totality a boundary layer. The basic variable describing such a flow is, therefore, the rate of friction or of shear stress at the boundary. A wave motion, on the other hand, is mainly an exchange between potential and kinetic energy and may in the first approximation be described as a frictionless motion--it does not by itself define a shear stress at the boundary. Such a shear stress can be derived only if the additional condition of zero-velocity at the boundary is introduced. That shear stress and the resulting boundary layer are caused by the wave motion and take their energy from the wave energy, but are in deep water of negligible magnitude compared with the total wave energy. For our purpose of predicting the sediment motion near the bed the description of this boundary layer is of prime importance, because it defines the shear forces on the bed and the velocity distribution in its vicinity from which the bed-load motion may be derived. It is also the only source of turbulence from which the suspension may be derived, except where the waves become very steep and begin to break, which case is excluded from this present treatment. In case of wave breaking this additional source of turbulence must be introduced separately and its effect on the sediment transport must be combined with that of the bottom friction.

Like any other boundary layer, also this reversing layer may be either laminar or turbulent. The laminar boundary layer between an oscillating fluid and a still boundary, or between a still fluid and an oscillating boundary has been known for a long time (see for instance (8)). Kalkanis (6) showed that from the viewpoint of the boundary layer the two cases are exactly equivalent, even if there is a minor difference with respect to the ability of the two to move sediment.

If an extended plain boundary moves parallel to itself at constant speed with respect to a large mass of still fluid, the resulting boundary layer motion always becomes turbulent eventually. In contrast, if this same boundary undergoes an oscillatory motion parallel to itself, the resulting boundary layer may remain permanently laminar. If the amplitude or period of this motion is properly changed, the motion in the boundary layer becomes turbulent and the entire flow condition changes drastically. Lamb in the above-quoted reference gives the solution of the laminar boundary layer, but no source could be found which defines the limiting conditions at which the flow becomes turbulent. It was thus decided to define this limiting flow condition experimentally as the first step of the entire study.

THE CRITICAL REYNOLDS NUMBER OF THE BOUNDARY LAYER

The turbulence in a frictional flow is caused by an instability of the laminar flow. Its stability may be tested by comparing the magnitude of the inertia forces with that of the frictional forces; instability will occur when the inertia forces become large compared with the friction forces, or when the Reynolds Number which expresses the ratio of the two types of forces becomes larger than a certain limiting value. The Reynolds Number is obtained by multiplication of a characteristic length with a characeristic velocity and by division of the product by the kinematic viscosity of the fluid. Which characteristic values must be chosen can often be decided by logical arguments, but if too many variables of the same dimension are involved it may become necessary to determine the proper variables empirically. The critical value of the Reynolds Number must always be found by experiment.

If the wave theory predicts near the bed at a given location the flow velocity u

$$u = u_o \sin (\omega t) \qquad -1-$$

with ω the angular velocity of the motion $\omega = 2\pi/T$ and T the wave period. The constant $u_o = a\omega$ where a is the amplitude or half excursion of the motion. For thin boundary layers one may neglect all gradients in the x-direction parallel to the horizontal bottom as compared with the corresponding gradients in the vertical direction z. In that case Eq. 1 may be interpreted as the oscillatory motion of a long horizontal bottom under an infinite body of still water, and one obtained in both cases a velocity distribution of

$$u = u_o e^{-\beta z} \sin (\omega t - \beta z) \qquad -2-$$

with respect to the fluid outside the boundary layer (at infinity) where β has the value $\sqrt{\omega/2\nu}$. $1/\beta$ is the scale of the distance z above the bottom. Equation 2 is the description of the boundary layer for the laminar case (8) and may be written in the more general form

$$u = u_o f_1(z) \sin (\omega t - f_2(z)) \qquad -2a-$$

which will be used for the turbulent boundary layer, too. In the laminar case β = constant in Eq. 2. The motion may be interpreted as a shear wave moving away from the bottom with the constant speed $(\omega/\beta) = (\omega\nu)^{\frac{1}{2}}$, simultaneously reducing its amplitude exponentially.

In order to describe the effect of a wave motion over a still bottom, the time history of the velocity in a given cross section is found by a simple transformation of the coordinates in the form

$$u = u_o \{\sin (\omega t) - e^{-\beta z} \sin (\omega t - \beta z)\} \qquad -3-$$

or again in more general terms

$$u = u_o \{\sin(\omega t) - f_1(z) \sin(\omega t - f_2(z))\} \qquad -3a-$$

Here u is a function of z and t, but the two f-functions depend only on z. Since both motions of Eqs.3 and 3a are sine-functions of the same frequency, it is possible to describe the composite motion in the form of an Eq. 2 or 2a. The two motions have different phases and, therefore, must be combined vectorially. This is done for Eq. 3a in Eq. 4

$$\left.\begin{aligned} u &= u_o f_1^*(z) \sin\{\omega t - f_2^*(z)\} \\ f_1^* &= \{1 + f_1^2(z) - 2 f_1(z) \cos f_2(z)\}^{\frac{1}{2}} \\ f_2^* &= \tan^{-1}\left\{\frac{f_1(z) \sin(f_2(z))}{1 - f_1(z) \cos(f_2(z))}\right\} \end{aligned}\right\} \qquad -4-$$

The thickness of the layer affected by friction may be estimated as the distance z from the smooth bed at which the friction-induced motion is reduced to 10% of its original value, i.e., where $e^{-\beta z}$ becomes 0.1. For a kinematic viscosity of $\nu = 10^{-5}$ ft^2/sec and a period T = 15 sec the thickness of the layer becomes 1/4 inch. This shows that the boundary layer is actually thin, as previously assumed.

The limiting flow condition at which the laminar boundary layer becomes unstable, as well as the turbulent velocity distribution, was determined empirically; the latter showed different solutions for smooth, two- and three-dimensional roughness investigated.

THE EXPERIMENTAL EQUIPMENT

After the decision was made to restrict the study to conditions of long wave periods it became possible to simplify the equipment by moving the bottom harmonically under still water. The velocity in the laminar boundary layer is then described by Eq. 2, and Eq. 4 may be used to derive from it the solution of the wave problem. In setting up the differential equations for both cases it is easily seen that the same considerations also apply to turbulent boundary layer. However, there is one acceleration term different in the two cases when the motion of particles with a density different from that of the fluid is described. This term contains the acceleration of the wave motion which is believed small compared to local accelerations of the flow near particles and due to turbulence. It was thus decided to incur this small mistake for the sake of a much smaller, simpler, and more flexible instrumentation.

The tank in which the measurements and observations were taken, contains a horizontal platform about half the length of the tank located 4" above the tank bottom, moving first on fixed rollers, later sliding on two rails, pulled by an endless steel tape. The tape followed the bottom and aorund wheels along the two end plates at the inside. This design avoided seals for the drive. A sled moving on rails above the tank was rigidly connected to tape loop and directly driven by a crank mechanism with variable speed and amplitude. The tank was 12 ft long, 3 ft deep and 1 ft wide. A pair of displacement cylinders were arranged, one at each end, and connected by a cable such that they would make opposite vertical movements. They were driven by the same drive at such adjustable amplitude and phase as to compensate for the standing wave excited by the motion of the platform. The higher harmonics of the standing wave were negligible without compensation.

The amplitude of the platform was directly measured as the eccentricity of the crank. The period was measured by stop watch or by recording electric pulses from a switch at the crank. Turbulence was detected by introduction of dye from the free surface. Velocities were measured with a sensitive pitot device directly connected to a diaphragm, the motion of which was recorded by means of a capacitance gage.

THE CRITICAL FLOW CONDITION

The smooth case may be idealized by a half-space filled with the fluid at rest which has the kinematic viscosity ν, bounded by an infinite plane oscillating in its own plane with the angular velocity ω and the amplitude a. The total displacement of the plane $d_1 = 2a$ at the onset of turbulence was plotted in Fig. 1 against ω/ν in a log-log graph and gave a straight line of slope -1/2 except at the highest values of ω where vibrations of the system may have caused somewhat more unstable conditions. This means that $d_1^2\ \omega/\nu$ = const. = $6.8 \cdot 10^5$. This combination has the character of a Reynolds Number and is composed of the only parameters of the problem. In terms of amplitude one may write for the critical case

$$N_R = a^2\ \omega/\nu = 1.7 \cdot 10^5 \qquad -5-$$

That the value of the critical Reynolds Number is much higher than that for pipe flow has no significance since it describes the stability of an entirely different flow geometry. It is of the same order as that of the steady flow boundary layer.

Next rough surfaces were investigated, both with two- and three-dimensional roughness. For the two-dimensional roughness cylindrical rods of a given diameter were arranged side by side on the platform with their axes cross-wise. To reduce the displacement of the moving platform half-cylinders were used for the large diameters.

For each roughness a graph of the type of Fig. 4 was plotted indicating that the combination $d_1\ \omega/\nu$ = const. held. This value changed from roughness, to roughness, however, One would not expect such a parameter with the dimension of L^{-1} to be constant. From dimensional considerations one would expect the Reynolds Number $\varepsilon d_1\ \omega/\nu$ to become constant if ε is the size of the roughness. ε was plotted against a ω/ν and it was found that both for two- and three-dimensional roughness lines resulted with the slope -1. in a log-log graph. They define the critical Reynolds Numbers for rough surface as

$$\varepsilon\ a\ \omega/\nu = 640 \text{ for two-dimensional roughness} \qquad -5a-$$

$$\varepsilon\ \acute{a}\ \omega\ \nu = 104 \text{ for three-dimensional roughness} \qquad -5b-$$

These values apply for

$$\frac{a}{\varepsilon} < \begin{cases} 266 \text{ for two-dimensional roughness} \\ 1630 \text{ for three-dimensional roughness} \end{cases}$$

All other cases behave hydraulically smooth.

TURBULENT VELOCITY DISTRIBUTION NEAR AN OSCILLATING SMOOTH WALL

It was first unsuccessfully attempted to describe the velocity distribution using the known methods of vorticity and momentum exchange which are both successful in unidirectional flow, but it was soon found that momentum exchange was not applicable in any case, and that vorticity exchange could describe the velocity distribution along a smooth wall, but was not applicable along rough walls. It was thus decided to use Eqs. 2a, 3a and 4 and to describe f_1 and f_2 empirically as well as could be done by power and exponential functions. A double-sided symmetric Pitot tube was used exclusively and proved to be very reliable.

The results are given in Fig. 2 in which the velocity amplitude is given as a function of the distance from the wall. In order to obtain a unique relationship for all different flow conditions, the two variables were plotted in dimensionless form. The distance z from the wall was measured in units of $(\nu/\omega)^{\frac{1}{2}}$ and the velocity u was measured in units of u_o, the velocity amplitude of the movable bottom (or of the wave motion near the bottom in the wave case). The dashed curve gives the predicted velocity distribution of the laminar boundary layer for comparison. The velocity u here is the velocity amplitude at various distances from the wall of the sinusoidal velocity component with the same frequency as that of the bed motion (or of the wave). From this one may derive the function f_1 for the smooth boundary

$$f_1(z) = 0.3\, e^{-75\, z/a} \qquad -6-$$

with a the amplitude (half excursion) of the moving bed (or of the particle motion as predicted by wave theory).

Figure 3 gives the corresponding values for the phase shift ωt in terms of the dimensionless distance $z/(\nu/\omega)^{\frac{1}{2}}$. Again the deviation from the laminar prediction is significant. The function f_2 for the smooth boundary may be derived from this

$$f_2(z) = 1.55\, (z/\sqrt{2\nu/\omega})^{\frac{1}{3}} \qquad -7-$$

The two functions $f_1(z)$ and $f_2(z)$ are purely empirical. They may be used in connection with Eqs. 2a, 3a, and 4.

TURBULENT VELOCITY DISTRIBUTION NEAR AN OSCILLATING ROUGH WALL

For the two-dimensional roughness half-cylinders of diameter D were used and arranged with their axes at right angle to the motion. The three-dimensional roughness were sand and gravel of sieve diameter D glued to the movable bottom. Only uniform roughnesses fully covering the bottom were considered. The following descriptions were found:

For two-dimensional roughness

$$f_1(z) = e^{-1000 \frac{z/\sqrt{2\nu/\omega}}{a\,D}} \qquad -8-$$

$$f_2(z) = 0.5\ (z/\sqrt{2\nu/\omega})^{\frac{2}{3}} \qquad -9-$$

For three-dimensional roughness

$$f_1(z) = e^{-\frac{133\ z/\sqrt{2\nu/\omega}}{a\,D}} \qquad -10-$$

$$f_2(z) = 0.5\ (z/\sqrt{2\nu/\omega})^{\frac{2}{3}} \qquad -11-$$

These expressions which are strictly empirical have been obtained by experiments covering the following ranges:

$$\left.\begin{array}{l} 0.0039\ \text{ft} \le D \le 0.104\ \text{ft} \\ 0.104\ \ \text{ft} \le a \le 2.00\ \text{ft} \\ 0.174\ \text{rad/sec} \le \omega \le 10.4\ \text{rad/sec} \end{array}\right\}\ \text{Two-dimensional roughness}$$

$$\left.\begin{array}{l} 0.0009\ \text{ft} \le D \le 0.0453\ \text{ft} \\ 0.104\ \ \text{ft} \le a \le 2.00\ \text{ft} \\ 0.169\ \text{rad/sec} \le \omega \le 6.82\ \text{rad/sec} \end{array}\right\}\ \text{Three-dimensional roughness}$$

All experiments used water between 66° and 75° F. The scatter of the measured points is about the same as in Figs. 2 and 3. All the above expressions may be applied in connection with Eq. 2a for moving bottom and to Eqs. 3a and 4 for wave motion over a still bottom. The system of reference coordinates is standing still in both cases.

It is interesting to observe how much faster two-dimensional roughness elements permit the velocity amplitude $f_1(z)$ to reduce than the three-dimensional roughness while the phase shift for both is the same.

THE TRANSPORT OF BED PARTICLES BY WAVE MOTION

A strictly symmetric oscillation, as the wave motion near the bed has been assumed to be, can only result in an equally symmetric motion of the sediment particles. It cannot cause any net motion of sediment. Its most important function is that of keeping continuously a greater or smaller number of particles per unit bed area in motion. These particles move with the water near the bed and will follow the water in any direction in which it flows. If the water has, in addition to the wave motion, a small additional one-directional velocity, such as littoral current caused by the angular incidence of the waves or a secondary circulation caused by the geometry of the beach, then one must expect a systematic transport of the moving particles parallel or normal to the shoreline, respectively.

In an attempt of predicting such a transport one may thus first find the number of particles which a given wave motion keeps in motion. This mass is then subjected to the systematic motion. In this study no attempt was made to study the small systematic motion since that should best be done in the prototype. However, the particles available for transport by being in motion at a given time by wave action is easily studied in the laboratory, using the same installation as for the velocity measurements. The platform was for this purpose covered with loose sand. At half-length a tray was built into the platform covering the full width of the sand bed. Its sides were level with the bed as to not hinder any motion of sand into it. When the platform was moving towards the left sand from the left would move into the tray, during motion towards the right sand would enter from the right. The total sand Q_s collected in the tray after T seconds of wave motion over the width B permitted the determination of a specific sediment rate q_s.

$$q_s = \frac{Q_s}{BT} \qquad -12-$$

which is a measure for the rate of transport independent of direction. If the velocity of the particles composing q_s is v_q, then the amount of sediment in motion per unit of bed area S_o is

$$S_o = \frac{q_s}{v_q} \qquad -13-$$

which is the value that was to be found. The two values q_s and v_q must be determined separately: q_s by measurement and by available sediment theory, v_q by calculation from the boundary layer description.

CALCULATION OF THE SEDIMENT LOAD

Before one can begin to describe successfully the transport of sediment by water, it is necessary to observe the motion of the particles. This can be done easily in a laboratory flume, but observation at the beach or in a river shows exactly the same picture. Some particles seem to slide and roll along the nonmoving bottom. At higher rates of motion an entire sheet of moving particles appears to cover the bed. Even within this layer the particles roll, giving the layer an aspect of being continuously mixed. In this layer the particles' weights are still supported by the bed even if the flow exerts a lifting force on the top particles of the bed and thus initiates their motion from time to time. This type of motion is usually called bed-load motion and has been described in the past (1) as an equilibrium between the rate at which bed-load particles are deposited on the bed and that at which bed particles are picked up by the flow and made part of the bed load.

One factor simplified the description of the motion. Observation shows that most beach sands are well sorted, i.e., contain only a narrow range of grain sizes. The derivation will thus be made for uniform material. The experimental sands were also well sorted. The following derivation will essentially follow the procedure of reference (3) at least up to the point where the effective velocity near the particles is introduced to find the force on a nonmoving particle in the top layer of the bed. Reference (7) gives more detail; only the main steps are given here except for the new points which are particular to the application of the transport description in wave motion.

In order to describe the rate at which particles are settled out on the bed, an empirical piece of information is used which was obtained about 35 years ago by the author: every particle moving as bed load will--as an average--travel a distance of 100 diameters until it finds another suitable location for deposit. If at that spot the local instantaneous lift force due to flow conditions prevents deposition, another 100 diameter step is added and so on. In the following all constants of proportionality are omitted as they are in the end determined empirically.

The rate at which particles are deposited per unit bed area and per unit of time is proportional to

$$\sim \frac{q_s}{L\,D^3\,\gamma_s} \qquad -14-$$

if q_s is the sediment rate in weight per unit of width, L the length of the average step, D the diameter and γ_s the specific weight of the particles. The value of the average step L can be expressed as

$$L = 100\,D/(1-p) \approx D/(1-p) \qquad -15-$$

if p is the fraction of the bed at which at a given time, or the fraction of time at which at a given point, the lift force on a particle of size D and of specific weight γ_s is larger than its weight. With this the rate of deposition per unit area of bed becomes proportional to

$$\frac{q_s\,(1-p)}{D^4\,\gamma_s} \qquad -16-$$

The rate at which particles are eroded from the bed and which is equal to the rate of deposition in Eq. 16 may be expressed by dividing the number of particles per unit area, which is proportional to D^{-2} through the time t_1 necessary for one exchange of a particle and by multiplication with p as defined above, t_1 may be estimated as

$$t_1 \approx \frac{D}{v_s} \qquad -17-$$

Being a characteristic of the grain, t_1 should be a function of other grain characteristics such as the diameter and the settling velocity in still water. If the turbulent settling velocity is used, one obtains

$$v_s \approx \sqrt{\frac{g\,(\gamma_s - \gamma_f)\,D}{\gamma_f}} \qquad \text{-18-}$$

and the rate of scour becomes

$$\approx \frac{\sqrt{\frac{g\,(\gamma_s - \gamma_f)\,D}{\gamma_f}}}{D^2\;D}\;p \qquad \text{-19-}$$

By combining expressions Eqs. 16 and 19 and by introduction of the constant A_* of proportionality, the following expression is obtained

$$\frac{p}{1-p} = A_* \left[\frac{q_s}{\gamma_s}\sqrt{\frac{\gamma_s}{g(\gamma_s - \gamma_f)}}\;D^{-\frac{3}{2}}\right] = A_*\,\Phi \qquad \text{-20-}$$

which may be solved for p

$$p = \frac{A_*\,\Phi}{1 + A_*\,\Phi} \qquad \text{-21-}$$

Now we must express that p is the probability for an exchange to take place at a given place of the bed. This probability, however, expresses basically only the fraction of time during which the lift on a particle of the bed surface is larger than its weight. The weight is constant and is submerged proportional to

$$W \approx (\gamma_s - \gamma_f)\,D^3 \qquad \text{-22-}$$

while the lift force is porportional to

$$L \approx \gamma_f\,\frac{u^2}{g}\,D^2 \qquad \text{-23-}$$

The ratio $\frac{W}{L}$ may thus be written

$$\frac{W}{L} = B_* \left[\frac{\gamma_s - \gamma_f}{\gamma_f}\;\frac{Dg}{u^2}\right] \qquad \text{-24-}$$

Reference (4) shows that in a boundary layer the instantaneous lift force may be derived from the instantaneous velocity at a distance 0.35 D above the theoretical boundary, that the turbulent component of the force follows statistically the normal error law and that its root-mean-square value is proportional to the corresponding lift derived from the average velocity. In the case of the wave-induced transport the problem is complicated by the fact that the lift due to the main motion also varies with time, but in contrast to the turbulent part periodically. The problem of finding the resulting probability may best be explained by Fig. 4. This isometric sketch shows three axes: towards the right that of the turbulent lift, towards the rear that of the phase ωt, and vertically that of the probability for a given L' to occur. If we focus our interest on a quarter period, the time which is covered in Fig. 4 by Gauss-curves, the total time is given by the volume under these probability-curves. The time of motion is given by the condition $W - \bar{L} < L'$ which is demonstrated in the figure as follows: the weight W is plotted as a line parallel to the ωt-axis. From this value of L' the sinusoidal $\bar{L}$ was subtracted. All values $L' > W - \bar{L}$ are to the right of this line $W - \bar{L}$ and the cross-hatched part of the time volume under the probability curves represents the "favorable" part of the time. The ratio of the favorable time to the total time is equal to p.

This may be written mathematically as

$$p = \frac{2}{\pi\sqrt{2\pi}} \int_0^{\pi/2} \int_{B_*\psi - 1/\eta_o}^{\infty} e^{-\frac{m^2}{2}} \, dm \, d(\omega t) \qquad \text{-25-}$$

if

$$\psi = \frac{\gamma_s - \gamma_f}{\gamma_f} \, \frac{Dg}{u_a^2} \qquad \text{-26-}$$

where u_a is the velocity amplitude at 0.35 D from the boundary, η_o is the scale ratio between $\bar{L}$ and L' and m is a variable of integration. Elimination of p between Eq. 21 and Eq. 25 produces the bed-load equation.

$$\frac{A_* \Phi}{1 + A_* \Phi} = \frac{2}{\pi\sqrt{2\pi}} \int_0^{\pi/2} \int_{B_*\psi\xi - 1/\eta_o}^{\infty} e^{-\frac{m^2}{2}} \, dm \, d(\omega t) \qquad \text{-27-}$$

For this equation A_*, B_*, and η_o must be determined empirically. This was done in Fig. 5. First the four theoretical curves according to Eq. 27 and $A_* = B_* = 1$ and for $\frac{1}{\eta_o} = 1.0$, 1.5, 2.0, and 2.5 were calculated and plotted. Then, for all transportation experiments the values Φ and ψ were calculated and also plotted in Fig. 5. As the plotting is done on a log-log graph, the constants A_* and B_* represent only a parallel shift in the Φ and ψ direction, respectively. A reasonable fit was achieved with the values

$$\left.\begin{aligned} A_* &= 13.3 \\ B_* &= 6.0 \\ 1/\eta_o &= 2.0 \end{aligned}\right\} \qquad \text{-28-}$$

which define the bed-load equation for the bed-load movement of uniform sediment by wave motion in deep water.

The correction factor ξ for the parameter ψ is the analogous correction which had been introduced as "hiding factor" in unidirectional flow (3). There, it indicates the ability of small sizes of a sediment mixture to hide between the larger grains and also the ability of small grains to hide in the viscous sublayer. The first case is not applicable here because of the assumption of uniform sands. The second application, on the other hand, is very important and has been studied by Abou-Seida (1), who ran special sets of experiments in the flume with the movable botton with especially fine sands (D_{50} = 0.000475 ft and D_{50} = 0.000984 ft diameter) to study this effect. The results are given in Fig. 6 as a graph of ξ against $\frac{D}{1.39\ \delta^*}$. Herein D is the grain size and δ^* the displacement thickness of the sublayer. Both, the curve of Fig. 6 and the definition of δ^* are different from those for unidirectional flow. In unidirectional flow the velocity distribution and the sublayer thickness are expressed in terms of boundary shear or u_*. In the boundary layer due to wave action the shear stress could not be expressed. The sublayer thickness is thus determined according to Einstein and Li (5) using for the sublayer a critical Reynolds Number $N_R = 4\ \delta^*\ U/\nu = 1100$, where U is the amplitude of the velocity near the bottom. This represents the minimum thickness of the sublayer over the wave period, not a constant value as in unidirectional flow, giving rise to an entirely different curve (Fig. 6). Figure 5 includes all points which were obtained by Abou-Seida (1) in addition to the measurements of Kalkanis (7).

Already Kalkanis had proposed to apply this theory to sediment transport by surface waves in wave channels. The main wave motion sets the bed particles in motion while the superimposed "mass transport current" of the same wave pattern provided the systematic velocity causing a unidirectional transport. Abou-Seida performed a set of such experiments and attempted the analysis of their results, as well as those of similar experiments performed by Vincent (12). Herein he expressed the mass transport velocity according to Longuet-Higgins (10) multiplied it with the average concentration of bed particles per unit of bed area in motion due to the oscillating wave motion. This concentration was obtained by division of the oscillating transport rate q_s of Eq. 26 by the layer thickness 2D and the average flow velocity in that layer.

Comparison of the measured with the predicted sediment rates showed very significant deviations with the measured transport. The measured values were consistently high. He explained the deviation by the fact that the theory was derived for waves of long period and low height while the experimental waves were all of short length and period and steep. In order to test this explanation he argued that in steep and short waves acceleration effects could not be neglected compared with the velocity effects on the grain. He introduced, therefore, the parameter $u^2/(\partial u/\partial t)\ D$ as an indicator for the acceleration effects. He plotted a correction factor M by which the measured transport is higher than the predicted against this acceleration parameter and found systematically decreasing deviations M for increasing acceleration parameters or, with other words, for decreasing acceleration effects.

One may conclude that for the prototype conditions of long-period waves in deep water the theory must apply satisfactorily without this acceleration correction.

SUSPENSION IN THE OSCILLATING BOUNDARY LAYER

The study of suspension due to wave action which is being pursued actively at this time is restricted to the particle motion caused by turbulence created in the boundary layer. Any turbulence created by breaking of the waves is for the time being excluded. Like in the description of the bed load it is attempted to follow the same thoughts which led to a solution in unidirectional flow, but, unfortunately, some of the most basic assumptions in the derivation of the unidirectional suspension do not apply here. In unidirectional flow the turbulence, which is responsible for the suspension is described by the shear stress at the boundary. In wave action it was not possible to find a usable expression for the shear stress; boundary layer has been shown to be very thin. The suspension, on the other hand, was found to be covering a much larger range of elevations. It appears to be impossible to derive the particle exchange in wave action from the momentum or vorticity exchange of the velocity distribution.

It was decided to attack the suspension problem in a strictly empirical way, i.e., to try to establish an approximate description of the suspended concentration distribution empirically with sufficient accuracy to complement the bed-load motion, which cannot be expected to include any suspension. Unfortunately, this work has not been advanced sufficiently to permit any results to be presented here.

The work on the suspension by the boundary layer under wave action was mostly hampered by the utter lack of any observation of the turbulence of this flow, as well as the nonexistence of the proper instrumentation to obtain such information. A method of measuring turbulent velocity components in turbulence without the presence of an average velocity was developed for this purpose by Das (2), who also developed an optical method of measuring the sediment concentrations which makes the measurement without any interference with the flow. It is hoped that soon results will be available that will permit to estimate this effect with sufficient accuracy to complement the bed-load predictions.

SUMMARY AND CONCLUSIONS

In order to describe the sediment transport due to wave action near the bed, the velocities were divided into

1. A symmetric wave motion assumed to be a horizontal sine motion with the frequency of the waves,
2. A boundary layer motion caused by the friction along the bed, including turbulence, and
3. A systematic motion, which may or may not be connected with the wave motion, but has a time-average velocity different from zero.

Of these velocities 1. and 2. are assumed to be sufficiently large to cause the motion of the bed particles; 3. may or may not be of that strength. From the velocity components 1. and 2. an average amount of sediment is derived which is at any time in motion without being permanently displaced by these motions. While they are in motion these particles are assumed to be also under the influence of 3. which displaces them permanently and causes the sediment transport. The motion as bed load along the bed seems to be satisfactorily developed; the resulting suspension is still under investigation.

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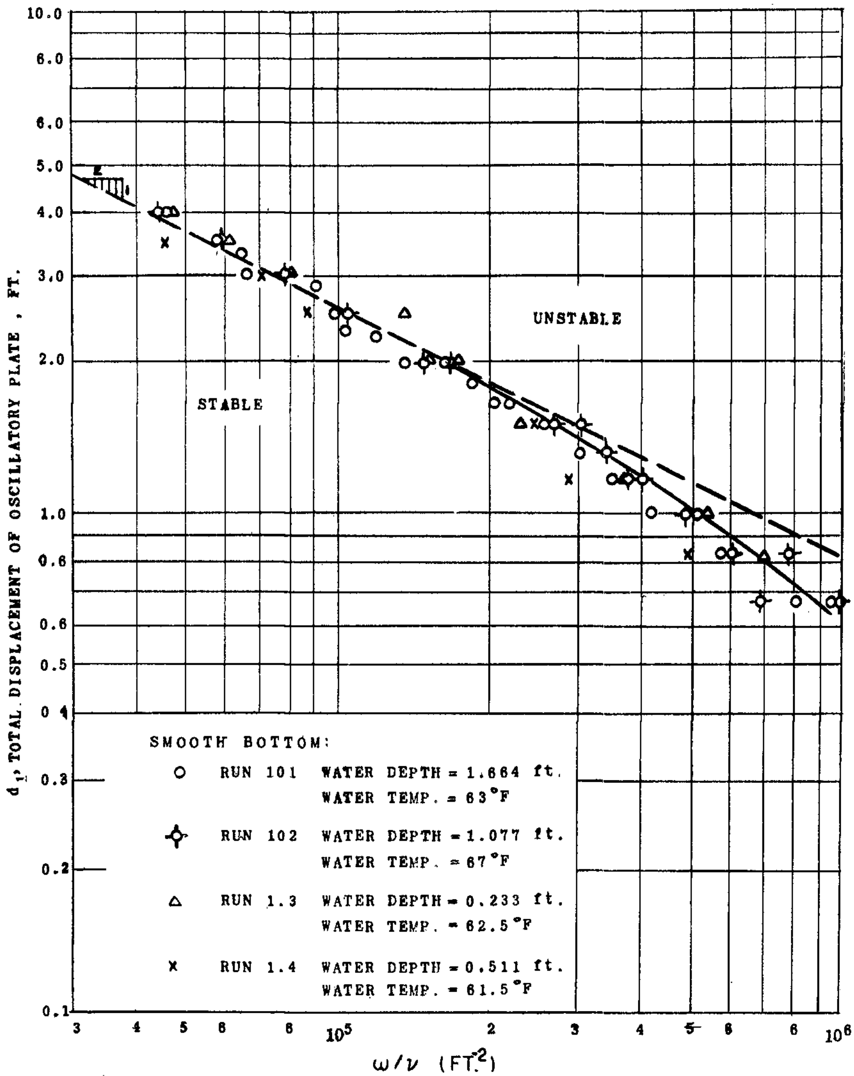

EXPERIMENTAL RESULTS OF TRANSITION INVESTIGATION OF OSCILLATORY MOTION

SMOOTH BOTTOM

FIGURE I

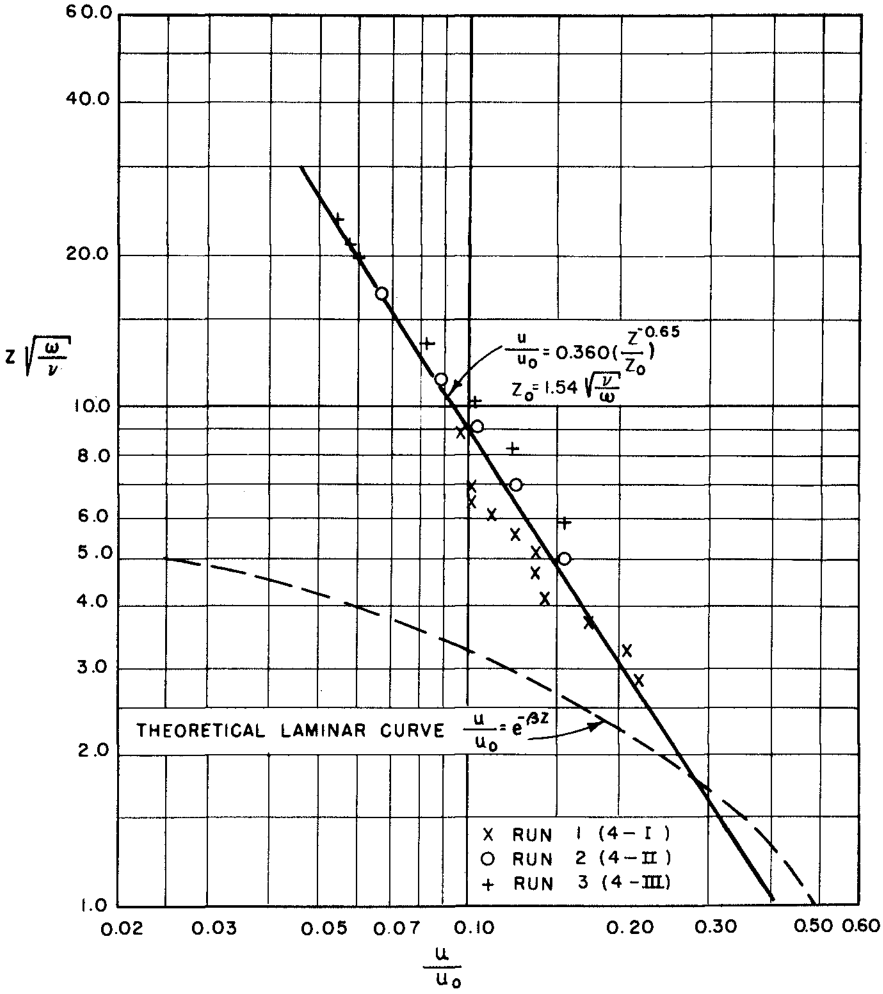

FIG. 2. PLOTTING OF EXPERIMENTAL DATA OF VELOCITY MEASUREMENT (SMOOTH BOTTOM)

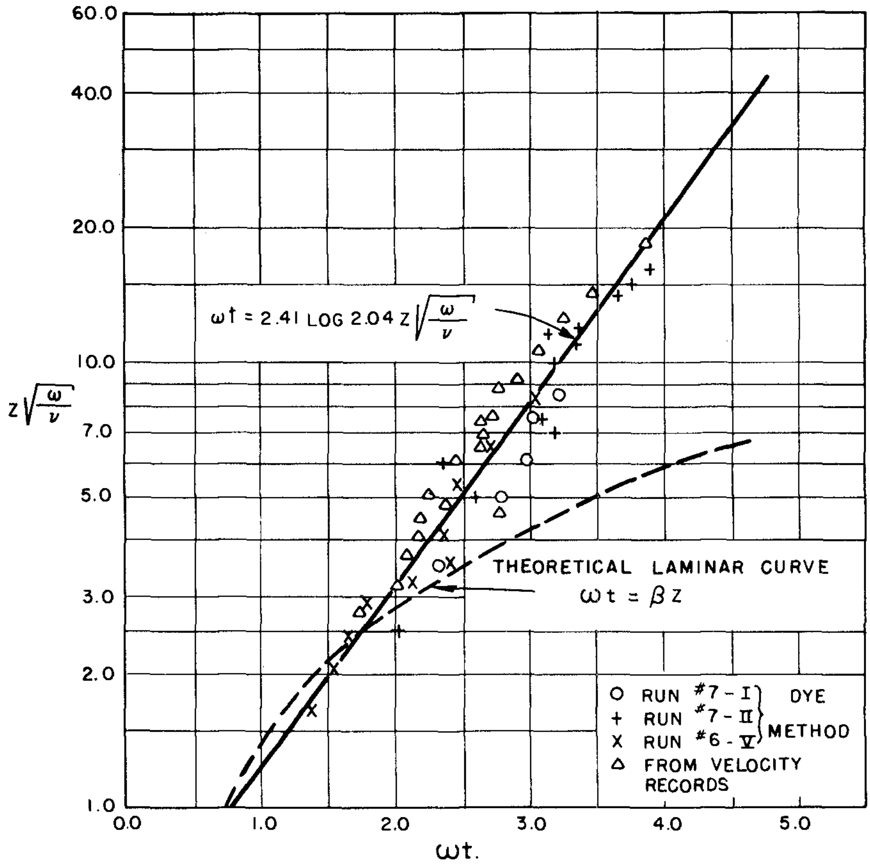

FIG. 3. PLOTTING OF EXPERIMENTAL DATA OF PHASE SHIFT MEASUREMENT (SMOOTH BOTTOM)

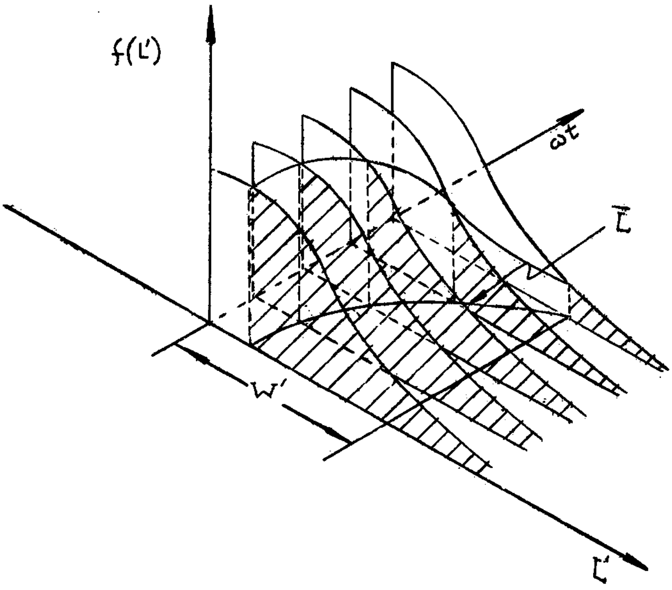

FIG. 4.

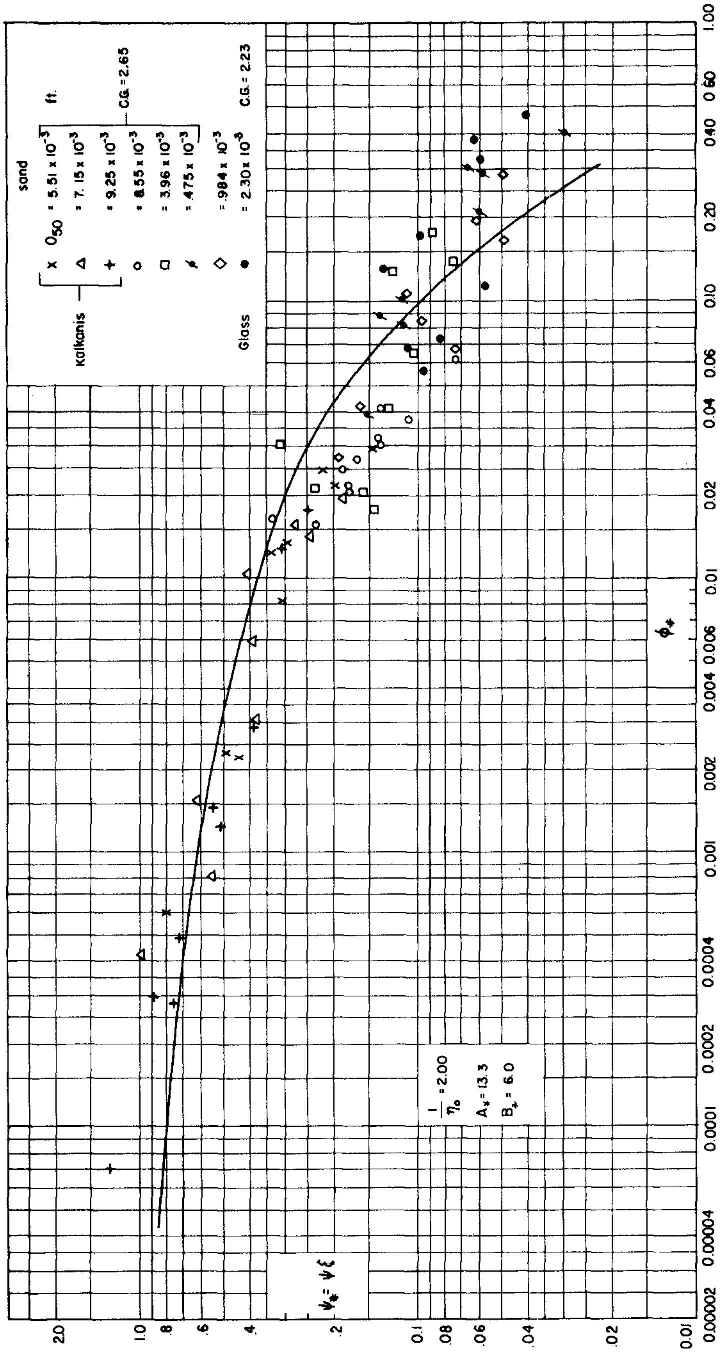

FIG. 5. GRAPHICAL REPRESENTATION OF THE BED LOAD EQUATION, $\phi_* - \psi_*$ CURVE

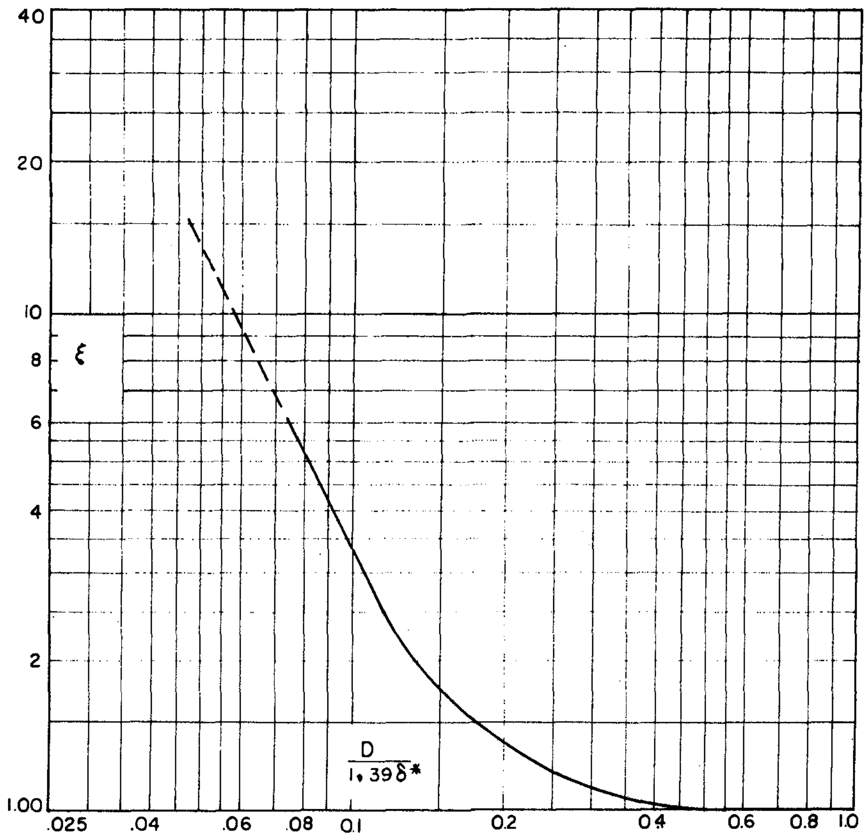

FIG. 6. PRESSURE REDUCTION IN THE SUBLAYER