CHAPTER 19

TRANSFORMATION OF WAVES PASSING A SUBMERGED BAR

By

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ABSTRACT

This study investigates the transformation of nonbreaking waves that pass over a submerged longshore bar. An envelope for the occurrence of transformation of nonbreaking waves is established and compared with existing information. Power spectral analyses are made of both the incident and transformed waves to determine quantitatively the change in wave characteristics. The conclusion is made that the transformation process is a nonlinear phenomenon occurring over the bar, but, in the deeper water in the lee of the bar, a combination of linear wave forms results.

INTRODUCTION

Winter and hurricane weather conditions in the Gulf of Mexico produce waves of considerable height and steepness. As they reach the shore, these waves remove material from the beaches and deposit it some distance from the shoreline. Deposition of this eroded material soon creates longshore bars. The typical bar will have a cross section with the following properties: a gently sloping face toward the sea, a rounded crest, and a steep slope downward into a trough in the lee of the crest. These bar formations are usually transient both in shape and in location.

Water depths over these bar formations are often reduced to the point that waves passing over the bar begin to shoal and their steepness increases. Leeward of the bar, in the deeper water of the trough, the waves decrease in steepness and transmit shoreward the energy that was not reflected or expended in passage over the bar. Often the waves appearing in the lee of the bar have noticeably shorter period components superposed on the fundamental wave form.

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The characteristics of these modified wave forms in the lee of the bar are of interest both in the area of structural design and in the study of beach processes. The object of the research described herein was to establish limits for the occurrence of modified wave forms and to quantitatively define some of the characteristics of these transformed waves in the lee of the bar.

A REVIEW OF THE LITERATURE

It was noted in studies of the passage of waves over submerged bars, breakwaters, and docks that transformation of waves occurs in the lee of the obstruction in certain instances.

Mason and Keulegan (10) in 1944 investigated the profiles of waves passing over three different configurations of rectangular reefs. Their study developed a criterion for the "regular" passage of waves over the reef—regular meaning without forming multiple crests. Their criterion for regular transmission is given as

$$\left(H_i L_i\right)^{1/2} \leq 2C_b$$

where H_i and L_i are the incident wave height and length, respectively, and C_b is the water depth over the reef.

Horikawa and Wiegel (4) verified the criterion of Mason and Keulegan (10) for regular transmission in similar experiments conducted in 1959. In addition to these verifying tests, investigations were made in a channel containing no bottom discontinuities. A wave filter was used in the generator end of the channel, and a wave absorber was utilized in the other end. Various wave characteristics and channel depths were investigated for possible multiple wave crest formation. Some multiple crests did appear and their appearance agreed closely with a theoretically derived criterion for the limit applicability of the second order profile of the free surface developed by Miche (11).

Jolas (6) also studied the characteristics of wave encountering a submerged rectangular bar. These studies were primarily for the determination of reflection and transmission coefficients and their comparison with theoretical results. However, his analysis of wave profiles indicated the increasing formation of harmonics of the incident wave frequency as the depth over the bar to the incident wave length decreased.

Williams (13) investigated wave transformation occurring over a submerged dock. The waves on either side of the dock were "deepwater" waves (water-depth-to-wavelength ratio greater than 0.5) and the top of the dock was submerged less than twenty percent of the water depth. He showed that the nonlinear wave forms appearing over the dock reduce to a system of linear harmonic waves, the fundamental frequency of which is identical to that of the incident waves, in the deep water past the bar. Of note is his conclusion that the amplitudes of the harmonics are affected more by the dock length than the end conditions.

At Texas A&M, McNair (8) in investigating the characteristics of monochromatic waves breaking over a representative submerged bar noted that the broken waves reformed into a complex wave form. Applying power spectral analysis methods to his wave records, he found that the reformed wave had a fundamental period equal to the period of the incident waves. In addition, energy was present at frequencies harmonic to the fundamental wave. He also noted that these complex waves resulted from steep waves that passed over the bar without breaking.

In 1969, Byrne (2) studied the field occurrences of multiple wave formed in passage over a naturally occuring submerged bar. He found that the number of waves appearing past the bar was double the number of waves encountering the bar. The wave conditions for these occurrences were bar-depth-to-wavelength ratio (C_b/L_b) of 0.045 and wave-height-to-depth ratios (H_b/C_b) of 0.4 to 0.5. The ratio of bar depth to trough depth was approximately 0.6.

Madsen and Mei (9) turned their attention to the transformation of a solitary wave over an uneven bottom both numerically and experimentally for height-to-depth ratios (H_i/d) of 0.1 to 0.2 and depth-to-wavelength (d/L_i) ratios of 0.03 to 0.05. Of particular interest is their description of the nature of the transformation: when the wave, distorted by climbing a mild (1:20) slope, passes onto a following shelf, a hump of smaller height appears at the rear and gradually trails behind the main wave. Profiles are illustrated and a comparison with Byrne's (2) field observations is made. Madsen and Mei (9) further note that some of the transformed waves may more closely resemble cnoidal or even possibly sinusoidal waves rather than solitary waves.

Galvin (3) found that finite amplitude, periodic, sinusoidal waves generated in constant depth shallow water breakdown into two or more waves traveling at speeds dependent on their height. Galvin (3) calls these waves "solitons" after an analogous phenomenon in plasma physics. One of the more interesting characteristics of solitons occur when a larger wave temporarily decreases during the resulting interaction. This decrease in amplitude is explained by the acceleration and subsequent spreading of the larger soliton when its leading edge reaches the deeper water of the smaller soliton. After passing through the smaller wave, the larger wave regains its initial amplitude. In Galvin's (3) study, the interaction of solitons periodically reproduced the initial wave form if followed far enough down the wave tank.

He ran tests over a very wide range of depth-to-wave length (d/L_i) and height-to-depth (H_i/d) ratios to define areas of soliton appearance. The results of his definition of soliton production are shown in Fig. 5. As can be seen, varying numbers of solitons will be produced for a given $d/L_i < 0.09$ and $H_i/d > 0.05$.

The attention of Galvin (3) is also applied to the previously cited works of Mason and Keulegan (10), Byrne (2), and Horikawa and Wiegel (4) and it is shown that much of their work was in the area of soliton genration.

THE LABORATORY STUDY

A laboratory experiment was designed to investigate the characteristics of gravity water waves that had passed over a submerged longshore bar. A model simulating a longshore bar formation was installed in a two-dimensional wave channel of the Hydromechanics Laboratories of Texas A&M University. Monochromatic waves were generated toward the submerged bar. Time histories of the water surface were recorded both windward and leeward of the bar. Analysis of these recordings allowed comparison of the characteristics of the modified waves with the characteristics of the incident waves.

The Model.--The model simulating the bar formation is the same as used by McNair (8). The profile of the model is shown in Fig. 1.

The Flume.--The wave channel used in this study was a 2-ft wide by 3-ft deep by 120-ft long flume equipped with a mechanical wave generator (see Fig. 2). The flume walls were of glass so that tests might be observed visually and photographically. A pendulum type wave generator was used to generate the incident waves. A wire mesh wave filter was utilized near the generator to attenuate the higher frequency wave (noise) present in the generated wave. At the opposite end of the flume, a semipermeable wave absorber was used to minimize reflection of the transformed waves back into the area of the bar. Prior to installation of the bar, a variety of waves with periods and heights representing the range of this study were generated at the wave absorber. Reflection measurements showed negligible reflection for the wave conditions investigated in this study.

Measuring Equipment.--Water surface time histories were obtained through the use of an ultrasonic profiler. The choice of an ultrasonic profiler was dictated by its excellent linearity characteristics. In operation, the sonar transceiver "reads" the surface thirty times a second. The resulting signal was amplified and supplied to a strip chart recorder. A second transceiver was used to monitor the relative position of the bar. The profiler was securely mounted to a movable carriage fitted to rails on either side of the channel. The movable carriage allowed records to be taken at any point in the channel.

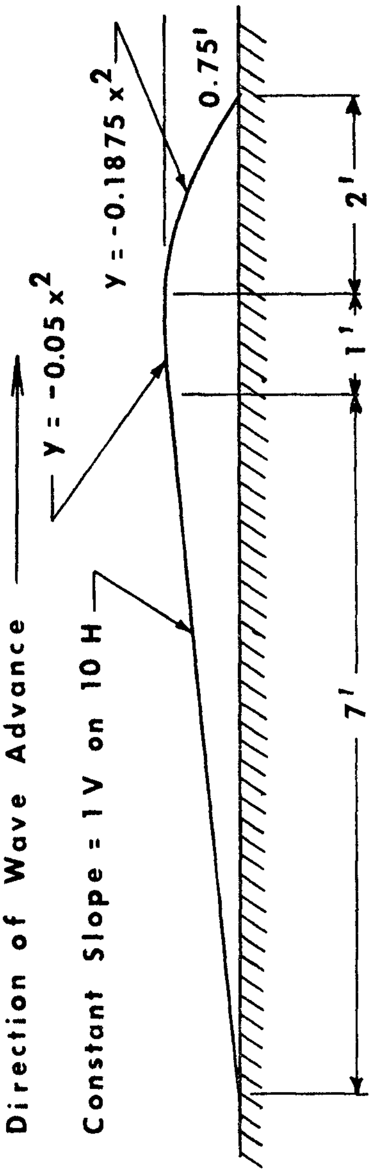

FIG. 1. - MODEL OF THE LONGSHORE BAR FORMATION

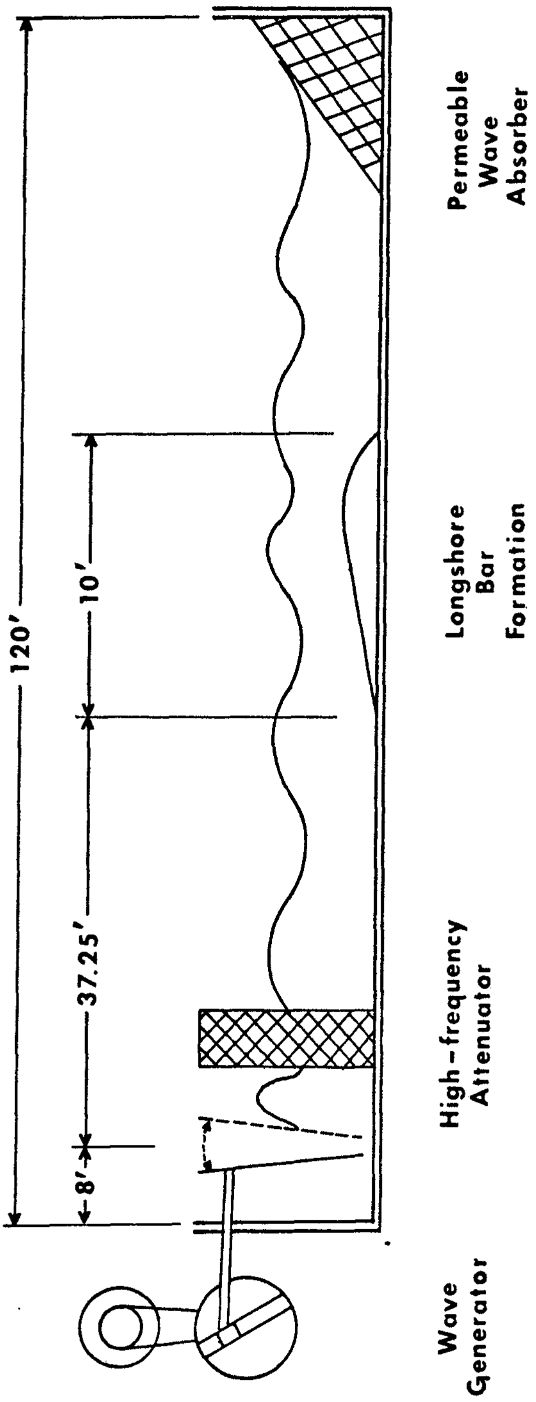

FIG. 2. - THE WAVE FLUME

Measurement of depth and dynamic calibration of the recording system were accomplished through the use of point gages.

Scope of the Study.--Waves of a range of periods (0.7 to 2.0 seconds) and heights (0.5 to 6.0 inches) were generated at the bar for several water depth (3.0 to 15.0 inches) over the bar. To reduce the limits of this investigation, primarily those waves which are transformed without breaking in passage over the bar were considered. However, several breaking conditions as well as the condition of waves passing the bar relatively unaffected were utilized in the envelope study.

It should be noted that a naturally occurring longshore bar is a steeper discontinuity in an already sloping beach. However, the effects of this beach slope are not accounted for in this study as the flume floor or representative ocean bottom was at a constant elevation. In addition, the axis of the bar was perpendicular to the direction of wave advance, so refractive effects would not be present in the wave forms.

EXPERIMENTAL PROCEDURE

With the water in the flume completely stilled, the recorder was switched on long enough to record a still-water level. Next, the wave generator was started and the excitation within the tank was allowed to come to a steady state. The recorder was switched on, and the profiler was slowly moved down the flume before the bar using a movable carriage. This procedure yielded a record for determining reflection coefficients and the height of the incident wave. These characteristics were determined using the envelope method of the linear wave theory. The period of the wave was determined by by using a stopwatch and a revolution counter on the generator flywheel. Any transformation was observed either visually or by utilizing the profiler/recorder system.

For the data necessary for power spectral density calculations, the above procedure was repeated. Once the incident wave height had been determined the profiler was located at a point in the flume having that wave height. Varying lengths of record were then made. Leeward of the bar, the profiler was stopped in several places, and records were again run at these locations. No specific locations were used as in the case of the records before the bar, as preliminary runs without the bar had indicated negligible reflection from the permeable wave absorber for the range of wave periods considered. These leeward records were also used to check for any wave "setup" that might be present.

EXPERIMENTAL RESULTS

The data obtained were in the form of strip chart records of the water-surface time-histories for locations before and after the bar. Samples of the wave profiles are shown in Fig. 3.

The Transformation Envelope.--For a given period, depth of water, and incident wave height derived from the analysis of the incident wave record, various relations were calculated. To eliminate problems of scale, only nondimensional relations were computed. Evidence of transformation was confirmed visually or by analysis of the leeward wave record.

The nondimensional relations and information on transformation were plotted in various ways to determine any functional relationships and to develop an envelope for the occurrence of transformation. The various plots showed that a wave-height-to-depth (H_i/C_b) ratio versus a depth-to-wavelength (C_b/L_i) ratio best defined a transformation envelope (see Fig. 4). This fact is in good agreement with Galvin (3).

Incident wave characteristics were used rather than wave conditions over the bar for two reasons: Incident wave characteristics can be more easily measured in the prototype; and there is difficulty in determining wave characteristics over the crest of the bar if the transformation process has already begun.

In Fig. 4, the demarcation between transformation and regular transmission for (C_b/L_i) approximately equal to 0.13 is not very distinct. The magnitude of the secondary waves in this region becomes so small as to be barely distinguishable. Thus, this boundary should probably be labeled the limit of measurable transformation. The same description should be applied to the boundary occurring for (H_i/C_b) of approximately 0.10.

Power Spectral Analysis.--The digital program used to evaluate power spectra in this study is based on procedures detailed in Blackman and Tukey (1). Additional description of their method applied to ocean waves is contained in Kinsman (7) and Robinson et al. (12).

In this study, a frequency bandwidth capable of evaluating third harmonics was used for maximum accuracy. For a given test, a frequency increment for energy computation was chosen such that the frequency interval was about a tenth of the fundamental frequency. This condtion allows very sharp definition of the energy peaks and their corresponding frequencies. Selection of the above values determined the sampling rate and the length of the record required. With this information, wave records from before and after the bar for nine different tests were digitized and supplied to the IBM 360/65 for analysis.

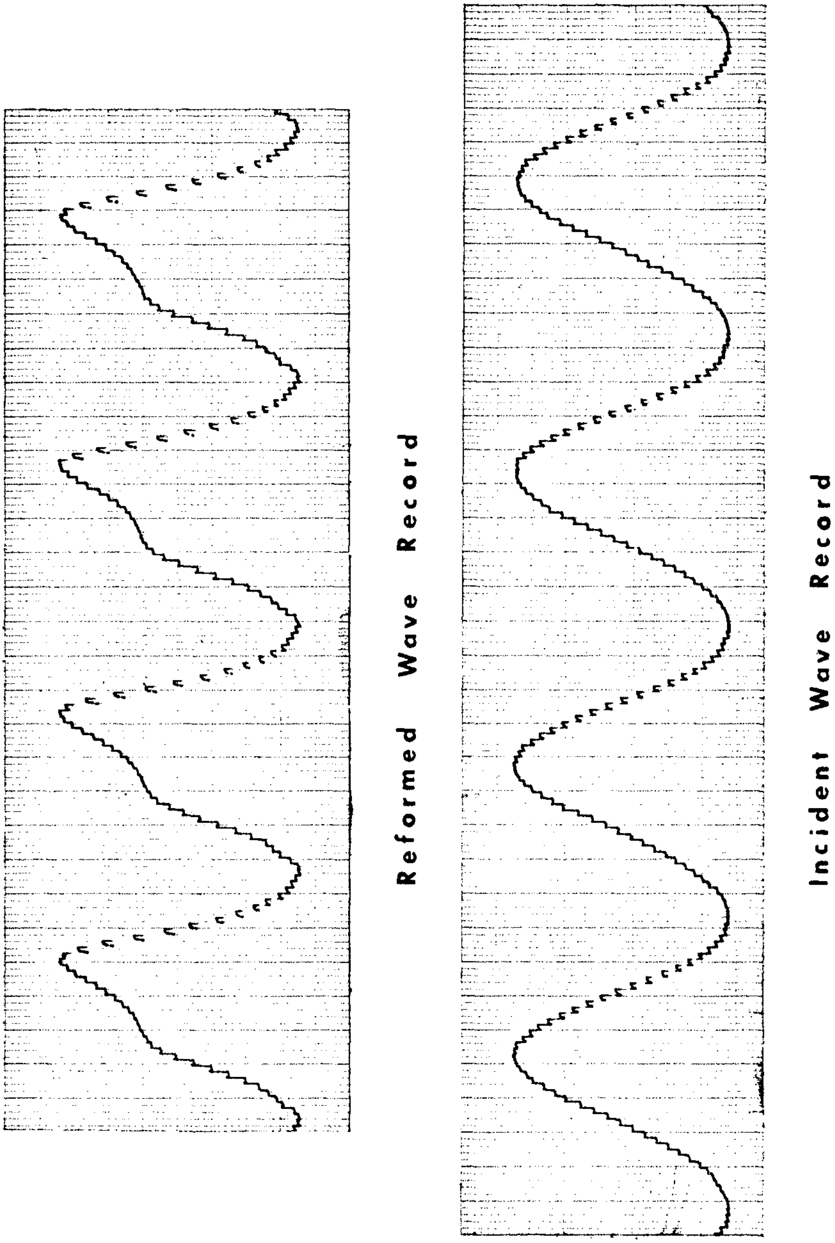

FIG. 3. - TYPICAL WAVE RECORDS

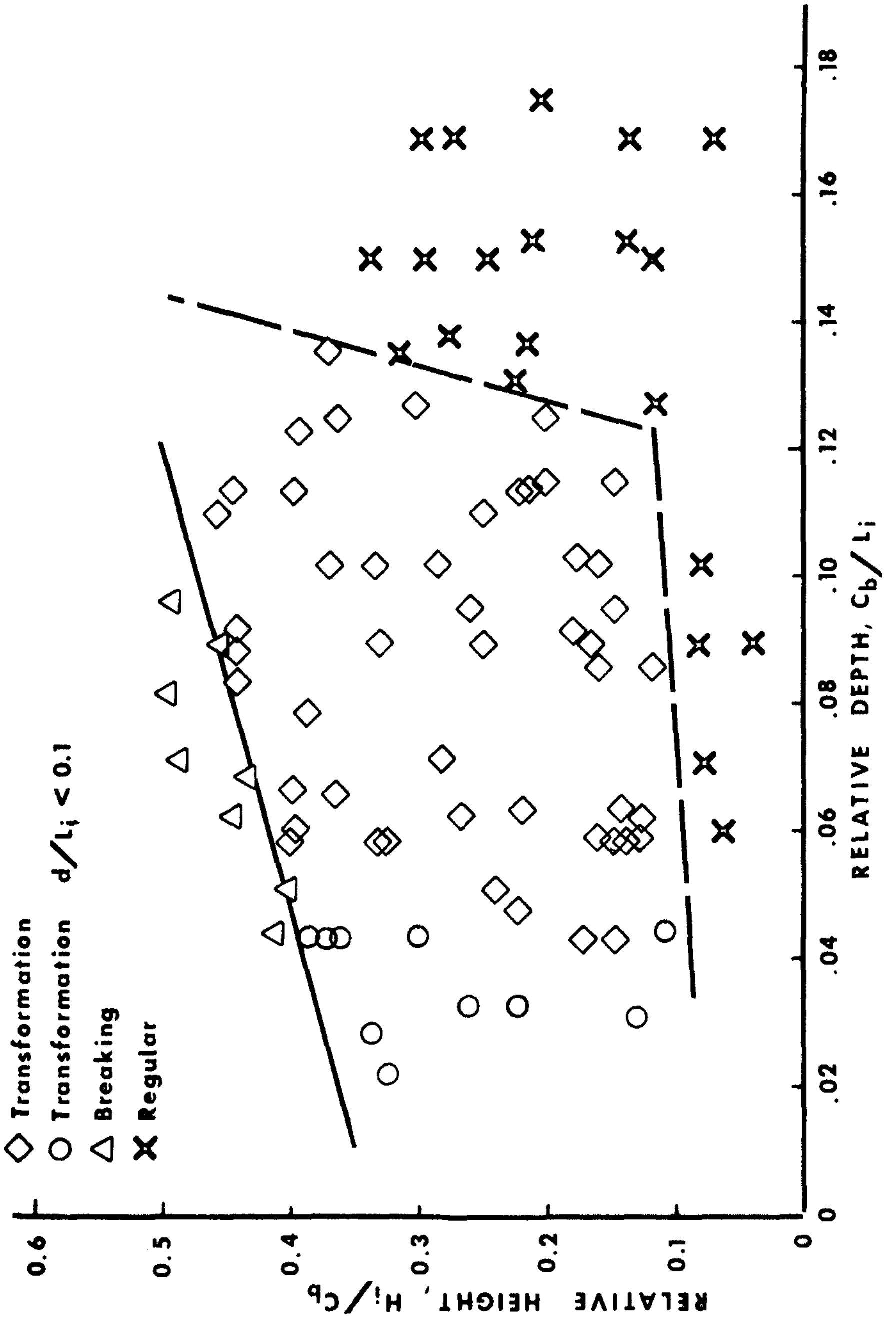

FIG. 4. - THE ENVELOPE OF TRANSFORMATION

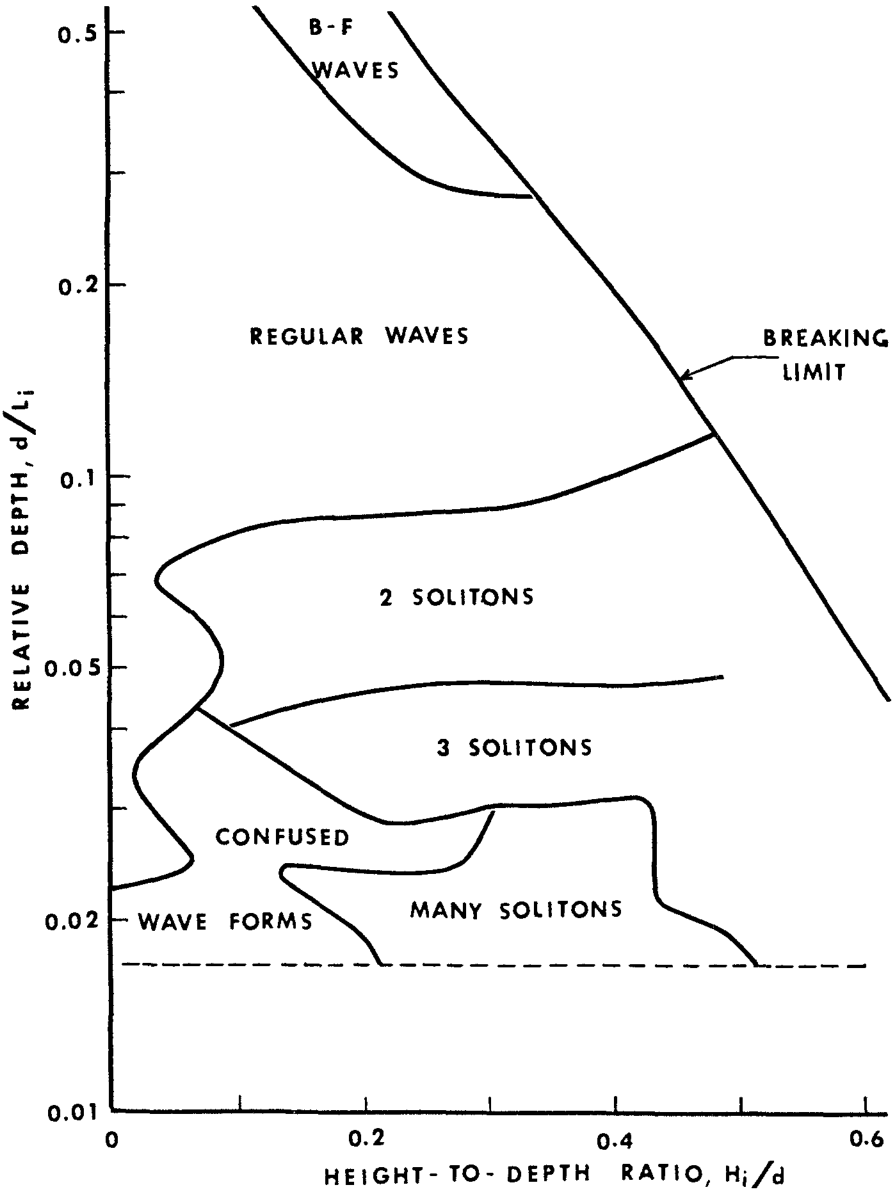

FIG. 5. - WAVE FORMS PRODUCED IN CONSTANT DEPTH BY A PERIODIC WAVE GENERATOR - AFTER GALVIN (3)

The digital program was verified using several combinations of sine waves. In each case, the output matched the input exactly. As a further check, the obtained power spectra of the incident wave was checked against the input conditions, again with excellent agreement for the nine tests used.

The power spectra show the reformed waves are definitely harmonic in composition. The amount of energy present in the various harmonics are tabulated in Table 1. The ratio of the harmonic energy to that of the fundamental wave $\left(E_h/E_{f_1}\right)$ is plotted in Fig. 6. Analysis of this energy transfer shows that the relative energy of the second harmonic increases with increasing $\left(H_i/C_b\right)$ for a given $\left(C_b/L_i\right)$, and the relative energy decreases with increasing $\left(C_b/L_i\right)$ for a given $\left(H_i/C_b\right)$.

The Transformed Wave.—The components of the transformed wave in the lee of the bar exhibit small-amplitude-theory characteristics of linear superposition and propagation velocities proportional to their respective periods. This lack of coupling between the various components was visually observed and was confirmed by the wave records. A typical series of wave profiles measured at foot increments leeward of the bar (Fig. 7) shows the phasing and superposition of the transformed wave elements very graphically. Sighting horizontally along the crests or troughs indicates the effects of these properties.

Fig. 8 shows the alteration of the incident wave as it passes over the bar. This alteration agrees closely with experiments by Madsen and Mei (9) on the transformation of solitary waves encountering a sloping bottom.

CONCLUSIONS

The experimental results clearly defined an envelope for transformation (Fig. 4). This envelope was determined using parameters involving incident wave characteristics and the depth over the bar.

The envelope for transformation seems to be somewhat similar to that of soliton wave appearance in a flat channel defined by Galvin (3) and reproduced in Fig. 5. The one significant difference in the depth-to-wavelength limit for regular transmission of waves which seems to be significantly higher. This difference is probably accounted for by the presence of the bar formation.

Although short period waves very close to the breaking limit were not analyzed spectrally because of difficulties previously explained in obtaining good profiles, the spectral analyses performed indicated that the secondary waves possed as much as six-tenths of the energy of the total

TABLE I

POWER SPECTRA CHARACTERISTICS

PARAMETER	Power Spectral Density Run								
	1	2	3	4	5	6	7	8	9
T in seconds	1.0	1.8	2.0	1.2	2.0	2.0	1.2	1.4	1.4
L_i in feet	4.9	10.5	12.8	6.6	12.8	12.8	6.6	8.2	8.2
H_i in feet	0.203	0.112	0.112	0.163	0.246	0.329	0.335	0.342	0.136
H_i/L_i	0.0379	0.0106	0.00872	0.0169	0.0192	0.0238	0.0508	0.0417	0.0166
C_b in feet	0.50	0.50	0.75	0.75	0.75	0.75	0.75	0.75	0.75
d in feet	1.25	1.25	1.50	1.50	1.50	1.50	1.50	1.50	1.50
H_i/C_b	0.372	0.223	0.149	0.218	0.328	0.405	0.447	0.456	0.181
C_b/L_i	0.104	0.0476	0.0586	0.1136	0.0586	0.0586	0.1136	0.0915	0.0915
E_h/E_{f1}	0.092	0.281	0.065	0.040	0.633	0.603	0.121	0.085	0.045
E_{f1} in inches^2	0.468	0.140	0.185	0.403	0.67	0.998	1.63	2.12	0.311
E_{f2} in inches^2	0.403	0.0327	0.012	0.016	0.413	0.552	0.197	0.14	0.014
E_{f3} in inches^2	------	0.0066	------	------	0.011	------	------	0.04	------
E_T in inches^2	0.511	0.179	0.197	0.419	1.094	1.60	1.84	2.30	0.325
E_I in inches^2	0.591	0.230	0.2185	0.524	1.17	1.74	2.31	2.31	0.351
$C_{TRANS} = E_T/E_i$	0.865	0.780	0.90	0.80	0.935	0.920	0.797	0.995	0.926
E_i in inches^2	0.620	0.225	0.225	0.500	1.15	1.67	2.02	2.10	0.333
C_{REFL}	0.077	0.313	0.231	0.074	0.186	0.096	0.040	0.050	0.060

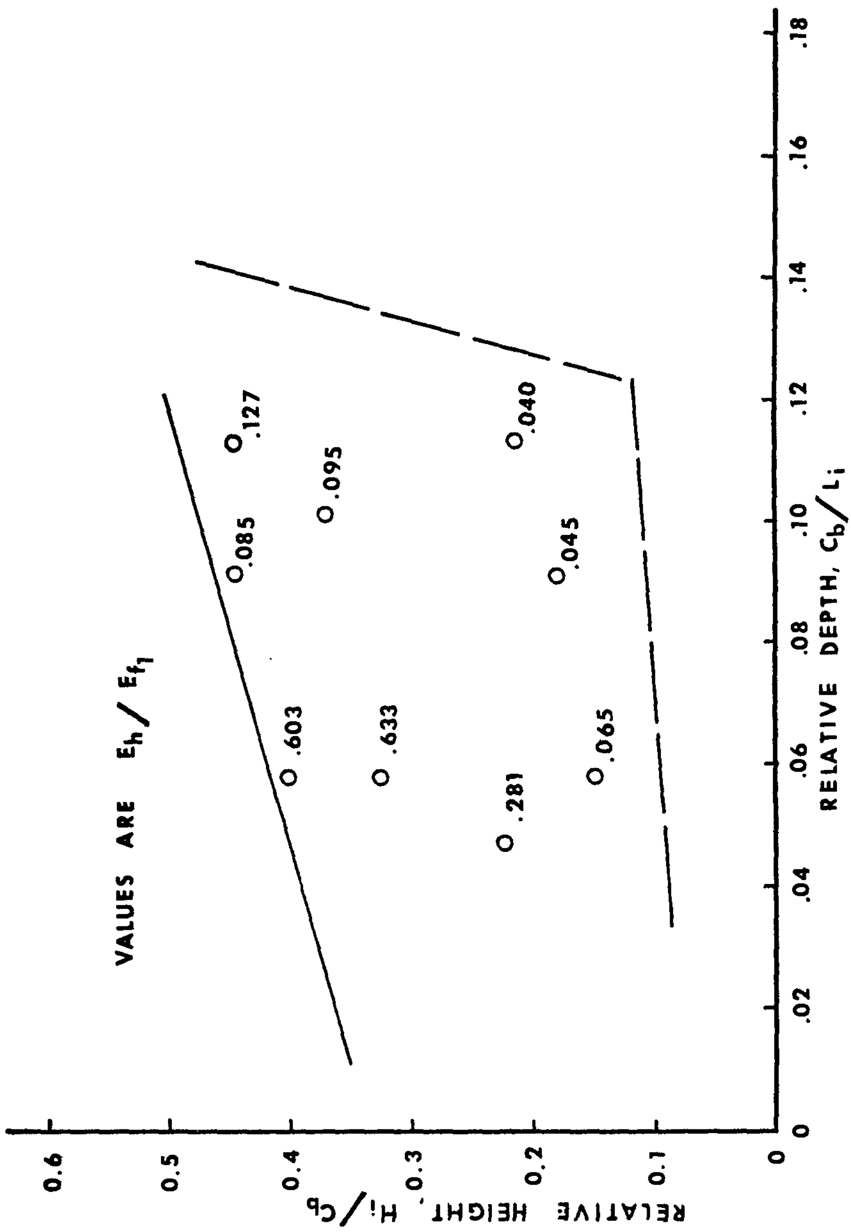

FIG. 6. - RELATIVE ENERGY TRANSFORMED TO HIGHER FREQUENCIES

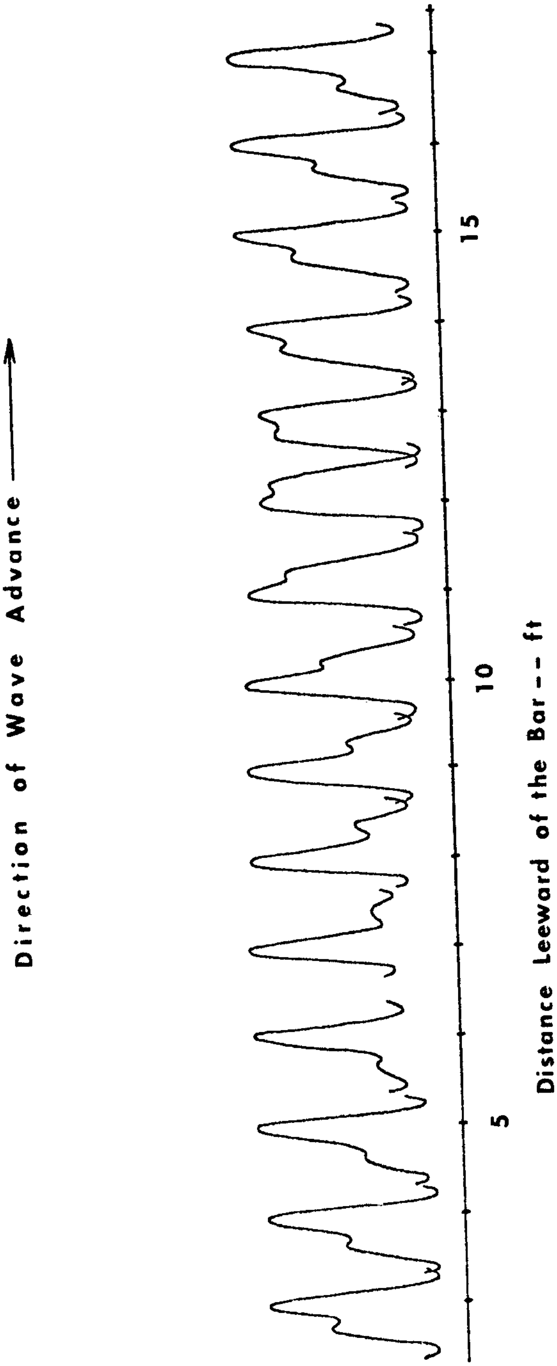

FIG. 7. - TYPICAL WAVE PROFILES LEEWARD OF THE BAR

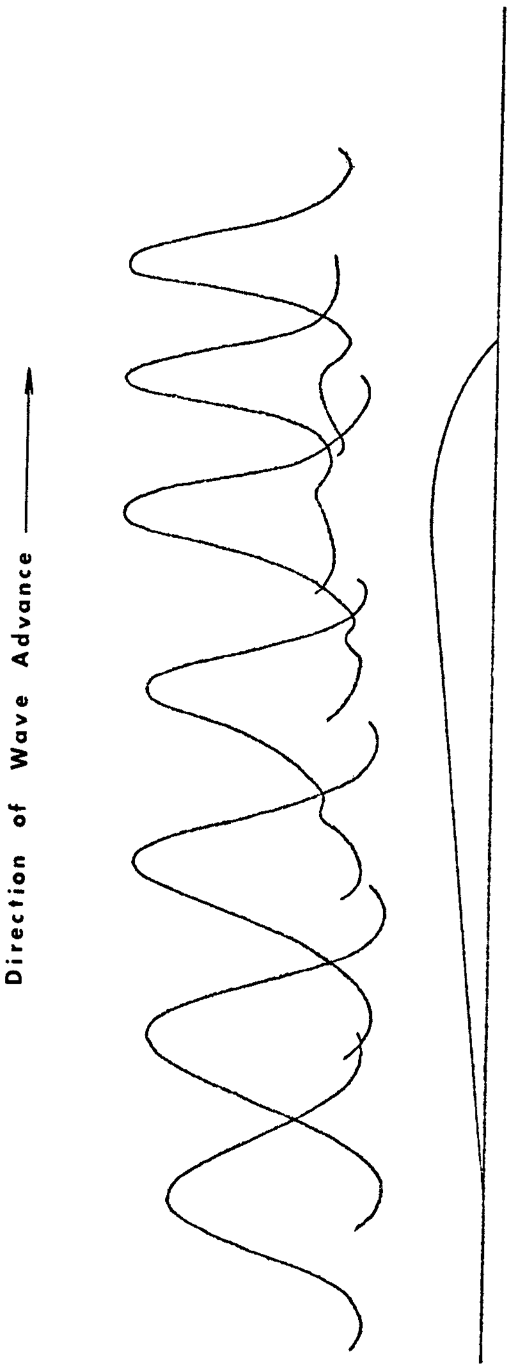

FIG. 8. - TRANSFORMATION OF A WAVE OVER THE BAR

energy of the transformed wave. The power spectra also show that this secondary crest has a frequency twice that of the primary wave.

Assuming that the depth-to-wavelength ratio in the channel before the bar precludes the formation of solitons, it is felt that the phenomenon of wave transformation is due to the generation of nonlinear wave forms in passage of the incident wave over the bar. In the deeper water in the lee of the bar, the nonlinear wave is altered into a sinusoidal form secondary to the main wave propagating with a speed proportional to its period as shown in Fig. 6. The actual alteration of the wave profile over the bar as shown in Fig. 7 agrees closely with experiments by Madsen and Mei (9) on the transformation of solitary waves moving up a sloping bottom.

This study has left many questions unanswered due to the limited scope of the experiments. In addition, this effort has probably generated further questions. Williams' (13) conclusion that the length of the submerged object has the greatest effect on transformation would suggest that several different bar formations, including a vertical plate, should be investigated. Further, power spectral analysis should be performed for a wide variety of conditions to determine functional relationships for the energies of the secondary waves.

NOTATIONS

The following symbols are used in this paper:

C_b = Water depth over the bar crest measured from still-water level.

C_{REFL} = Coefficient of reflection using small amplitude theory.

C_{TRANS} = Coefficient of transmission.

d = Water depth in the flume measured from still-water level.

E_{f_1} = Energy of fundamental harmonic of reformed wave.

E_{f_2} = Energy of second harmonic of reformed wave.

E_{f_3} = Energy of third harmonic of reformed wave.

E_h = Total energy of harmonics.

E_I = Energy of incident wave.

E_i = Energy of incident wave using small amplitude theory $= H_i^2/8$.

E_T = Total energy of reformed wave.

H_b = Wave height measured over the bar.

H_i = Incident wave height.

L_b = Wavelength over the bar.

L_i = Incident wavelength.

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