CHAPTER 4

SOME RESULTS OF A DIRECTIONAL WAVE RECORDING STATION

by

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ABSTRACT

An experimental deep water wave recording station has been operated in Lake Ontario, Canada, from 1969 to 1971. The station included a triangular array with four wave sensors and a stable platform. The aim of the study was to obtain a better understanding of the usually rather vaguely defined wavelength parameter of a wind-generated sea and to record the direction of wave propagation directly in the wave generation field.

The station has also been used to test newly developed wave sensors, or to check and calibrate some of the existing wave recording systems, including staff gauges, pressure cells and accelerometers.

The paper describes briefly the mooring system of the floating stable platform and the triangular array. A comparison is made between various wave recorders, mainly by using spectral analysis, and some results of the wavelength and wave direction study are discussed.

INTRODUCTION

A general wave climate study of the Great Lakes and the Gulf of St. Lawrence for the Canadian Ministry of Transport in the late sixties resulted in some 20,000 wave frequency spectra. These wave data were presented in a report, which listed the significant wave heights and frequencies of the peak of the spectra as the two basic parameters defining the recorded sea state. ("Wave Climate Study - Great Lakes and Gulf of St. Lawrence", J. Ploeg, May 1971, NRC Report MH-107A or T- and R-Bulletin 2-17, Society of Naval Architects and Marine Engineers.) A number of other parameters was derived from the two basic parameters, such as the wavelength, which was calculated from the frequency of the peak of the spectrum, using the first order, small amplitude wave theory, $L = \dfrac{g \times 2\pi}{\omega^2}$.

However, it was felt that no information was available on the physical meaning of this computed wavelength parameter. In the past, the mean wave period has often been used to calculate the wavelength, mainly because spectral

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analysis was not readily available.

Just as the original significant wave height defini-
tion was chosen to correspond to the wave height estimated
visually by an experienced observer, the wavelength should
perhaps also be defined as a physically more meaningful
parameter, possibly corresponding to the obvious periodicity
of the sea, which can be observed on so many aerial photo-
graphs.

More directional information of wind generated seas
in areas of limited fetch lengths, such as the Great Lakes,
was also desirable.

Lake Ontario was chosen to install an experimental
wave recording station, from which the above mentioned
studies could be conducted. The development of new wave
recorders and the calibration of some of the existing
instruments was a natural extension of the main study.
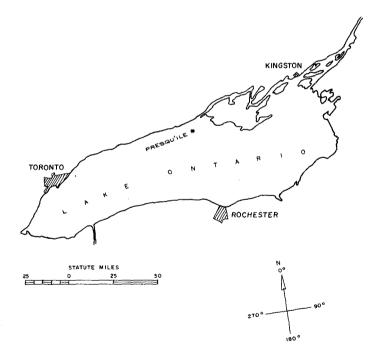
Fig. 1.   Location of Wave Direction Study Station

MOORING SYSTEM OF THE STATION

The station was installed near Presqu'ile Point, Ont.,
about 7 miles offshore in 155 ft of water, guaranteeing the
recording of true deep water wave conditions. The maximum
fetch was approximately 110 miles from the S.W. (Fig. 1).
The station consisted of a stable floating platform, built
up from 8 inch diameter tubular steel, with an 8 ft triang-
ular cross-section and an overall length of 100 ft. The
10,000 lbs positive buoyancy was supplied by two steel
tanks, housed within the triangular framework; the vertical
stability of the platform was excellent. Two buffered guy
wires prevented rotational movements of the platform. On
top of the platform, which protruded 12 ft above the mean
water level, a 60 ft high fibreglass mast was mounted to
install the meteorological sensors (Fig. 2). All power
supply and recording equipment was mounted in waterproof
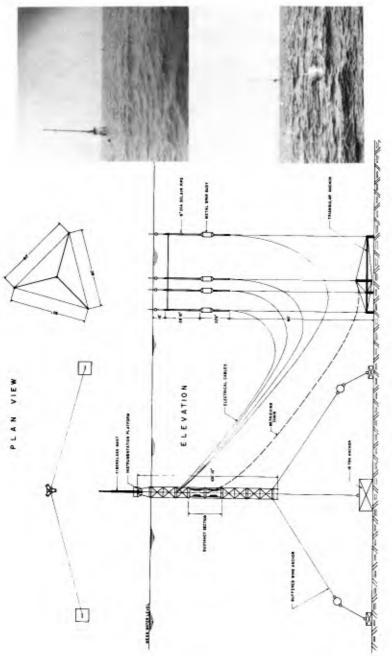cabinets on top of the platform.

The array of wave sensors was moored to a 60 ft triang-
ular anchor, built up from tubular steel and guy wires. It
was placed about 100 ft away from the platform. Four, 60 ft
long slender spar buoys, made up of steel and plastic pipes,
with 600 lbs buoyancy each, were suspended from the three
apexes of the triangle and from the centroid. Pressure type
wave sensors were installed on top of the buoys, 15 ft below
the mean water level. A small triangular framework was
attached to the buoys just below the pressure cells, to
guarantee the exact geometry of the array. The 60 ft
dimension of the array was chosen as half the most frequ-
ently occurring wavelength on Lake Ontario.

INSTRUMENTATION AND ANALYSIS

The platform was instrumented with a step resistance
type wave staff gauge, a transmission line type wave staff
gauge and an absolute and differential pressure cell type
wave sensor. The array was equipped with four absolute
pressure cells. All these instruments were developed and
built at the National Research Council. Two commercial
accelerometer type wave recorders were moored a short
distance away from the platform, namely a B.I. wave re-
corder made in Canada and a Waverider, made in the
Netherlands.

The step resistance type staff gauge and the pressure
cells were of fairly standard design and no details are
given here. The transmission line gauge used a relatively
new approach and is briefly described next.

The gauge consisted of two concentric metal tubes, the
outside one 2 inches in diameter, the inside one 1 inch in
diameter. The outside tube was perforated to allow the

Fig. 2. Mooring Diagram of Offshore Wave Recording Station

water level inside the tube to follow the wave profile. The
two tubes formed a coaxial transmission line. An electro-
magnetic wave generated at the top of the gauge and guided
by this transmission line, reflected upon the water surface,
where the permittivity changed by a ratio of 1:81 (air/water).
The reflected wave travelled back to the top of the gauge,
where it was used to generate a new wave, but of opposite
polarity. The result, of course', was a square wave oscilla-
tion with the period linearly proportional to the wave height.
The frequency of the oscillation was typically several mega-
hertz and had to be scaled down to be recorded on tape.

     Output signals of all wave recording instruments were
transmitted by cables to the recorders on the platform to
eliminate noise problems. The pulse frequency or period
modulated signals were simultaneously recorded on a multi-
channel tape recorder. Only the Waverider system was trans-
mitted to a shore station and recorded separately as an
analog signal on an F.M. tape recorder.

     The analysis of the recorded data was done on a small
digital computer, which had an extensive analog to digital
interface. A special programme was written, which allowed
the analysis to be carried out as a one-pass computation,
with a minimum of manual intervention from the operator.
The programme allowed a simultaneous input of up to four
channels. Demodulation of the signal into an analog voltage
preceded the power spectral density analysis, which used the
Fast Fourier Transform technique. The spectra were displayed
to enable the operator to select frequency limits, if so
desired, to compensate the "pressure spectra" using the Airy
theory to obtain wave height spectra. Filtering of the data
to specific bands of frequency could be done next. The
computation of the RMS values of the four records was used
to normalize the data, before the cross-correlation functions
were computed.

     The various calculations were done for the spectra as a
whole as well as for different filtered portions of the spectra,
with the filter limits selecting the frequency bandwidth defined
by the location of peaks in the spectra. The bandwidths of the
filtered spectra were typically 0.1 to 0.2 rad/sec. The time
delays, resulting from the 6 cross-correlation functions
were then used to compute the direction of wave progagation
and the wave celerity. The wavelength was obtained by multiply-
ing this wave celerity with the period corresponding to the
frequency of the peak of the unfiltered or filtered spectrum.
A flow chart of the programme is shown in Fig. 3.

                    COMPARISON OF WAVE RECORDERS

     Although a comparison of wave recorders was originally
not the reason for this study, it certainly turned out to be

CALIBRATION FACTORS PSI / Hz $\left.\right\}$ N, ΔT, S P

C
X ⎯ ⎯ ⎯
Y ⎯ ⎯ ⎯
Z ⎯ ⎯ ⎯

4-CHANNEL DEMODULATION → FAV FOR C,X,Y,& Z IN H:

CONVERSION FROM REAL TO INTEGER

W_e
W_L
W_h

POWER SPECTRAL DENSITY
ANALYSIS VIA FFT → RMS ∞

CALIBRATION FACTOR
FREQUENCY AT ATMOSPHERIC PRESSURE
ACTUAL BAROMETRIC PRESSURE
DEPTH OF WATER
CUT-OFF FREQUENCY

TRANSDUCER COMPENSATION
FROM PSI → ft → RMSA

FILTER BAND LIMITS

DATA FILTERING OF
ALL 4 CHANNELS

RECORD LIMITS IN SECONDS

DATA SEPARATION INTO
3 SUBGROUPS

RMS-CALCULATIONS AS
REQUIRED BY X-CORRELATION
ANALYSIS → RMS ∞

MAXIMUM SHIFT M = 25

CROSS-CORRELATION & NORMALIZATIONS
FOR XC, XY, YC, YZ, ZC, & ZX
FOR EACH SUBGROUP

DIRECTION CALCULATIONS

1. DETERMINE TIME-LAGS FOR SHORT LEGS
2. ESTIMATE ANGLE OF WAVE PROPAGATION
3. ESTIMATE TIME LAGS FOR LONG LEGS
4. DETERMINE TIME LAGS FOR LONG LEGS,
   CLOSEST TO ESTIMATES
5. CALCULATE ALL ANGLES

— CROSS-CORRELATION COEFFICIENTS
— WAVE CELERITIES
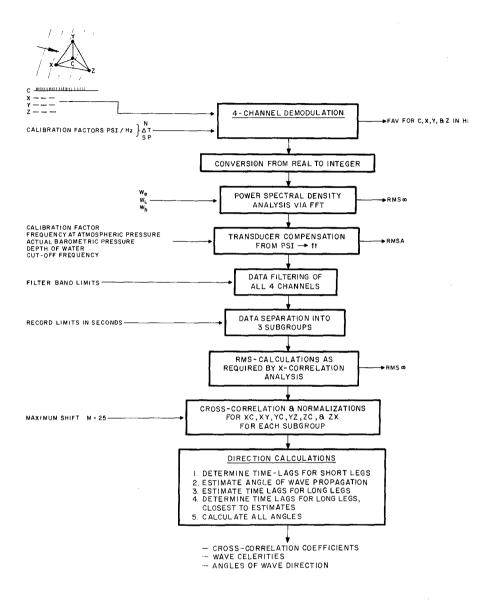— ANGLES OF WAVE DIRECTION

Fig. 3.   Flow Chart of Directional Wave Analysis

an integral part of the programme. The entire facility lent
itself pre-eminently for this purpose and the results were
rather interesting.

    The two different types of staff gauges yielded nearly
always identical results in the power spectral densities, as
well as the demodulated analog signal of the wave heights.
Figure 4 is a typical example of the excellent comparisons
obtained between the two instruments. The distance between
the two gauges on the platform was approximately 4 ft. The
transmission line wave staff gauge will be developed further
by the National Research Council within the next year, be-
cause the cost and maintenance requirements turned out to be
much less than any of the known staff gauge type wave recorders.

    The differential pressure cell showed generally also
good agreement with the wave staffs, except for frequencies
higher than 0.3 Hz. The time history of wave heights from
a pressure cell record was obtained by doing a Fourier trans-
form on the recorded pressures, a compensation and conversion
from pressures to amplitudes and a subsequent inverse Fourier
transform to give wave height (Fig. 5).

    The absolute pressure cell was slightly less accurate
than the differential cell, probably because of the depth of
submergence term, which had to be supplied from estimates of
the mean water level. The differential cells, were of course
not sensitive to the long term changes of the mean water level.
Figure 6 shows an example of the typical results of a compari-
son between an absolute pressure cell and a wave staff gauge.

    The B.I. wave recorder is not available anymore, but
since it had been used extensively in the earlier quoted
Wave Climate Study, this evaluation was extremely useful
and a number of corrections were indeed applied to the
original data as a result of this study.

    The Waverider system could not be recorded on the same
tape recorder with the other instruments and a direct
comparison was therefore not possible. However, spectra
recorded over the same time intervals compared reasonably
well, both in spectral shape and in RMS values, the latter
mostly within 10% of those of the wave staffs. It should be
remembered that Lake Ontario does not have any high currents,
which may in other locations affect some of these floating
and moored wave recording systems severely.

                RESULTS OF THE DIRECTIONAL ANALYSIS

    The principal aim of the study was to determine a
realistic wavelength parameter in a wind generated sea.
The flow chart in Fig. 3 showed briefly how the outputs of
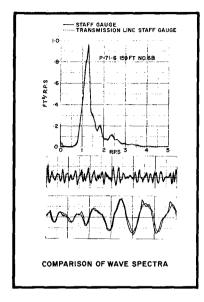the four sensors of the triangular array were used to

Fig. 4. Comparison of Step
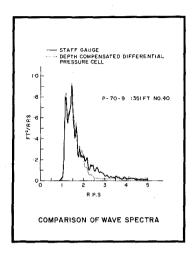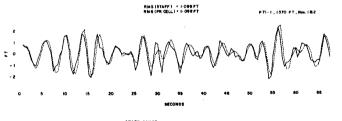Resistance and Transmission
Line Gauges

Fig. 6. Comparison of Staff
Gauge and Pressure Cell

Fig. 5. Wave Record from Pressure Cell Through
Inverse Fourier Transform

calculate the wave celerity and the direction of wave propaga-
tion.

In all cases the wavelengths resulting from this calcula-
tion turned out to be very close to the wavelength parameters
obtained from the frequency of the peak of the unfiltered or
filtered spectra and the small amplitude wave formula
$L = \frac{2\pi \cdot g}{\omega^2}$. An interesting observation was that the devia-
tions which occurred always indicated the wavelengths observed
with the array to be longer than those calculated from the
standard linear formula. The maximum difference was of the
order of 10%. This discrepancy might be real, or might have
been caused by a slight movement of the mooring system of the
array. A further survey will be done in the near future to
examine this in detail.

It appears reasonable to use the frequency of the peak
of the spectrum to calculate a wavelength parameter, using
the linear formula. This wavelength will indeed have a very
real physical meaning, since it almost certainly corresponds
to the length dimension between the crests of successive
waves, which is so obvious when flying over a wind generated
sea. It might be useful to refer to this wavelength para-
meter in the future as the "dominant wavelength".

As far as directional information is concerned, the
results of the analysis indicated that the propagation of
waves with frequencies close to the peak of the spectrum was
parallel to the average wind direction, after the wind had
been blowing sufficiently long from the same direction.
There was some doubt about the accuracy of the observed wind
direction by the meteorological mast on the platform, due
to a possible rotational movement of the platform over ±10°.

The triangular array in Lake Ontario was not very
sensitive for the higher frequency components of the spectra.
Directional and wavelength resolution beyond 0.3 Hz was not
very good, although improvements in the results can be
obtained by a more sophisticated analysis technique, which
is presently being programmed.

From the results so far it appears that no definite
trend in the spread of the wave direction occurred over the
various bands of frequency of the spectra. It is expected
that next year's study will resolve this question.

However, it should be noted that Lake Ontario has
limited fetch lengths and swell does not develop. Only
locally wind generated waves are observed at a recording
station. In the oceans, swell will often start reaching a
wave recording station long before the actual storm arrives
which causes the locally generated waves. The low frequency

components of an ocean wave spectrum (the swell) will usually
have a directional component different from the direction of
the wind driven sea.   Therefore, less spread should be
expected in the Lake Ontario spectra than in the ocean
spectra reported in the literature.
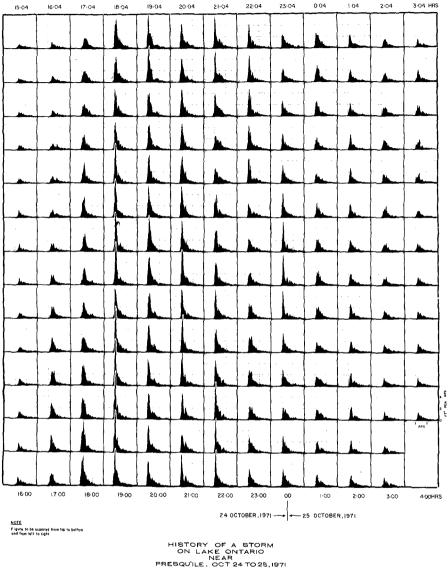
SOME OTHER ANALYSIS RESULTS

A number of other analyses were performed on the wave
data from the Lake Ontario station and some of the prelim-
inary results are briefly reported here because of their
general interest.

Wave and meteorological data were observed and recorded
continuously during an average storm on October 24 and 25,
1971.   The record of a wave staff gauge was used to carry
out some 200 spectral analyses over 16 minute intervals,
with the starting points only 4 minutes apart, covering a
total of about 13 hours of the storm.   Figure 7 illustrates
the spectral history of the storm, showing the familiar
pattern of the build-up to a fully developed sea state with
the peak of the spectrum continuously shifting to the lower
frequencies and increasing its power spectral density value.

One of the conclusions to be drawn from this presenta-
tion pertains to the degree of stationarity of the input
signal in the various frequency bands.   Figure 8 is a plot
of the power spectral density values at 13 different freq-
uencies versus time.   Typically the analysis had 20 degrees
of freedom, giving a spread in the estimates of the spectral
density values of about ±50%, using the 90% confidence limits
of the Chi-square distribution.   Figure 8 reveals that at
most frequencies the spectral density values are within the
±50% band and therefore the process can be considered to be
stationary.   Only at the lower frequencies does the non-
stationarity of the record become obvious and conclusions
from spectral analysis techniques should thus be drawn with
the necessary care.

The final figure, Fig. 9, is a plot of the RMS values,
the peak frequencies and the meteorological data versus time
for the length of this particular analysis.   The RMS values
over the 16 minute periods at 4 minute intervals show ex-
cellent continuity and give added confidence in the use of
the RMS value for the estimation of wave heights even if the
process is somewhat non-stationary as demonstrated earlier.
Evidently, spectral analysis techniques are often employed
to obtain RMS values because of compensation requirements.

The peak frequency curve seems to show a tendency to
jump between discrete frequencies.   No satisfactory physical
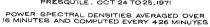interpretation has been found so far.

NOTE
Figure to be scanned from top to bottom
and from left to right.

HISTORY OF A STORM
ON LAKE ONTARIO
NEAR
PRESQU'ILE. OCT 24 TO 25, 1971

POWER SPECTRAL DENSITIES AVERAGED OVER
16 MINUTES AND COMPUTED EVERY 4·26 MINUTES

Fig. 7. History of a Storm as Power Spectral Density Curves

HISTORY OF A STORM ON LAKE ONTARIO
NEAR PRESQU'ILE OCT 24 TO 25, 1971

POWER SPECTRAL DENSITY VALUES
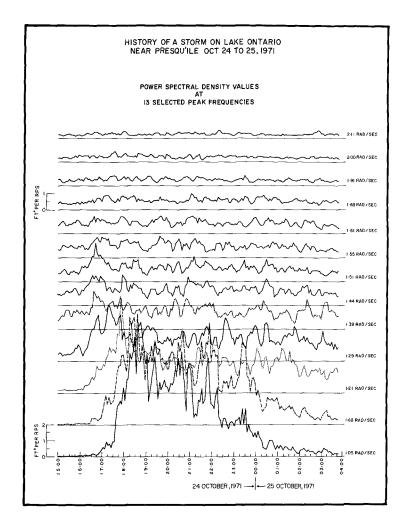AT
13 SELECTED PEAK FREQUENCIES

Fig. 8.   Power Spectral Density Values at
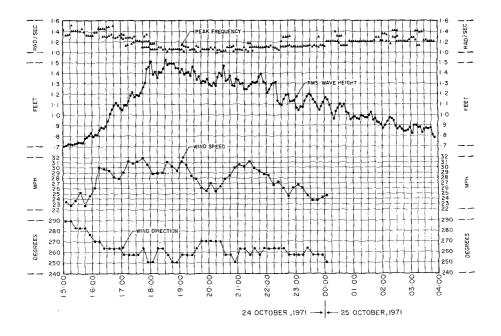   Different Frequencies During a Storm

Fig. 9. Wave and Wind Parameters of a
Lake Ontario Storm

The wind data were recorded 10 m above the mean water
level; no attempt has been made yet to correlate the wave
and wind records.

CONCLUSIONS

The study has probably up to now presented more problems
than it has solved, although the question of a definition of
the wavelength appears resolved. It is intended to improve
on the mooring and recording aspects of the station in the
coming years so that the many other questions which have
arisen may also be answered, such as the up to 10% difference
between measured and calculated wavelengths, the spread in
the direction of wave propagation in a wind generated sea,
the possibility of the occurrence of discrete frequencies in
a wave spectrum and also, a further correlation between the
wind and wave records.