CHAPTER 134

TIDAL MOTION IN BAYS

by

O H Shemdin, Associate Professor
R M Forney, Research Assistant

Department of Coastal and Oceanographic Engineering
University of Florida, Gainesville, Florida

ABSTRACT

A method is proposed to investigate periodic tidal motion in single or multiple basins connected to the ocean by an inlet   Non-sinusoidal tidal motion in the ocean and square friction law in the inlet are both considered   The method is applied to Boca Raton inlet, Florida   The calculated tidal elevation and velocity in the inlet are found to be in reasonable agreement with measured values   The bottom shear friction coefficient is defined

$$\tau_{to} = \rho \, r \, u|u|$$

where ρ is the water density, τ_{to} is the shear stress, and u is the average velocity in the inlet   The results of the study yield $r = 0.0039$, and predict net transport of sediment into the bay

INTRODUCTION

The dynamics of tidal motion in a bay connected to the ocean by an inlet wat investigated by Brown (1928) who considered only sinusoidal tidal motion in the ocean and linear bottom friction in the inlet   Later, Keulegan (1951) treated the same problem but included tne square friction law in the inlet an predicted a non-sinusoidal oscillation in the bay elevation   Van de Kreeke (1968) developed a scheme which predicted tidal oscillations in bays in the presence of freshwater inflow by rainfall or streams   The above investigations were all limited to a sinusoidal tidal oscillation in the ocean and all neglected flow acceleration in the inlet

An extensive treatise on tides and tidal propagation is given by Dronkers (1964)   It is known that tidal motion is not simple but has many harmonic constituents   The principal components are shown in Table I

Table 1   Principal Tidal Components

Symbol	Component	Periods (hrs)
M2	Principal lunar semidiurnal	12 421
S2	Principal solar semidiurnal	12 000
N2	Lunar elliptic semidiurnal	12 658
K1	Lunisolar declinational diurnal	23 936
O1	Lunar declinational diurnal	25 819

The relative importance of the different constituents depends on geographical location and depth of water   The principal lunar semidiurnal component (M-2) has usually the highest amplitude

This study proposes a general method for analyzing tidal oscillation in bays connected to the ocean by inlets   It includes acceleration of flow in the inlet, square friction law, and a non-sinusoidal tidal oscillation in the ocean   The method utilizes the general equations of motion and approximates the periodic tidal oscillation by a series in circular functions   Use is made of complex variables to simplify computations   The method can be applied to multiple basins connected to each other and to the ocean   The present study is restricted to relatively deep bays in which negligible spacial variation in water elevation exist   The inlet area is assumed constant and equal to the mean area during one tidal cycle   The method can be applied, however, to shallow bays   The method has a resemblence to Dronker's (1964) harmonic method of tidal propagation although he did not specifically apply the method to inlets   A method proposed by Sidjabat (1970) for describing the nonlinear friction is employed in this study

THEORETICAL APPROACH

The theoretical development to follow describes the dynamics of flow in an inlet with a constant cross sectional area connected on one side to a basin with uniform water level and on the other side to an ocean which has harmonic but non-sinusoidal tidal oscillation   The flow in the inlet is assumed one-dimensional   Resonance and bottom friction in the bay are neglected   Quantitatively bays satisfying these conditions must be at least 20 feet deep when the longest dimension equals 5 miles

A   Single Bay Coupled to Ocean

A definition sketch is shown in Figure 1   The equation of motion in the x-direction (direction of inlet flow) is used to describe the inlet flow

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial T_v}{\partial z} \quad , \tag{1}$$

where u is the inlet velocity, p is the pressure, ρ is the water density, and T_v is the viscous shear stress   Denoting

$$u(x,y,z,t) = U_m(x,z,t) + u'(x,y,z,t)$$

$$w(x,y,z,t) = \qquad\qquad w'(x,y,z,t) \qquad\qquad ,$$

where U_m denotes the mean flow over a length of time, T, which is small compared to tidal period by long compared to time scale turbulence   The overbar is taken to denote averaging over period T

$$\bar{u} = \frac{1}{T} \int_0^T u\,dt = U_m$$

Equation (1) becomes

$$\frac{\partial U_m}{\partial t} + \frac{\partial}{\partial x} (U_m^2 + \overline{u'^2}) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial z}(\tau_v + \tau_t) \quad , \tag{2}$$

where τ_v is the turbulent Reynolds stress defined by

$$\tau_t = -\rho\ \overline{u'w'} = -\rho \int_0^T u'\,w'\,dt$$

Next, a vertical average over the depth h is specified   We define

$$U(x,t) = \frac{1}{h} \int_0^h U_m(x,z,t)\,dt \tag{3}$$

Integrating Equation (2) over depth and assuming hydrostatic pressure distribution

$$\frac{\partial U}{\partial t} + \alpha \frac{\partial}{\partial x} U^2 = -g\frac{\partial h}{\partial x} - \frac{1}{\rho h}(\tau_v + \tau_t)_o \qquad , \tag{4}$$

where

$$\alpha U^2 = \frac{1}{h} \int_0^h (U_m^2 + \overline{u'^2})\,dz \qquad ,$$

and the subscript o denotes bottom values (z = 0)   From open channel considerations α is the momentum factor and is of the order of 1 05   The value of 1 0 will be assumed sufficiently accurate   In inlets, the flow is mostly turbulent except when it approaches zero during a tidal reversal The bottom shear stress will be assumed to be primarily due to turbulence rather than viscosity (i e $\tau_v << \tau_t$) and to have the empirical square-law form

$$\tau_{to} = \rho r\, U|U| \qquad\qquad , \qquad\qquad\qquad (5)$$

where r is the friction coefficient   Equation (4) now becomes

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\, U^2 = -g\, \frac{\partial h}{\partial x} - \frac{r}{h}\, U|U| \qquad\qquad (6)$$

Equation (6) is applied to the inlet shown in Figure 1   Integration of Equation (6) along the x-direction yields an equation which relates the ocean level to that in the basin   Assuming the inlet is finite in length and the velocity accelerates towards the inlet but achieves a constant velocity, U_1, in the inlet, Equation (6) is integrated to yield

$$h_s = h_{b_1} + \frac{\partial U_1}{\partial t}\, \frac{L_1}{g} + (1 + \frac{r}{h_1}\, L_1)\, \frac{U_1|U_1|}{g} \qquad\qquad , \qquad (7)$$

where h_s, h_{b_1}, L_1 and h_1 are defined in Figure 1   The equation of mass conservation consistant with the above assumptions becomes

$$U_1 A_1 = A_{b_1}\, \frac{dh_{b_1}}{dt} \qquad , \qquad\qquad\qquad (8)$$

where A_1 and A_{b_1} are as defined in Figure 1   Equation (8) further assumes that the changes in A_{b_1} and A_1 due to depth changes are small over a tidal cycle   Eliminating U_1 in Equation (7) yields

$$h_s = h_{b_1} + (\frac{A_{b_1}}{A_1})\, \frac{L_1}{g}\, \frac{d^2 h_{b_1}}{dt^2} + (1 + \frac{r_1}{h_1}\, L_1)\, \frac{1}{g}\, (\frac{A_{b_1}}{A_1})^2\, \frac{dh_{b_1}}{dt}\, \left|\frac{dh_{b_1}}{dt}\right| \qquad (9)$$

1   The Linearized Solution for Simple Motion

The tidal motion in the ocean is described by complex variables to simplify computations

$$h_s(t) = \hat{h}_s\, e^{i\sigma t} \qquad\qquad , \qquad\qquad\qquad (10)$$

where $\hat{h}_s$ is a complex number and σ is the tidal frequency   Let

$$h_{b_1} = -i\, \hat{h}_{b_1}\, e^{i\sigma t} \qquad\qquad ,$$

where $\hat{h}_b$ is a real number representing the bay tidal amplitude   Linearization of the last term in Equation (9) implies

$$\frac{dh_{b_1}}{dt}|\frac{dh_{b_1}}{dt}| \simeq \frac{8}{3\pi} \sigma^2 h_{b_1} e^{1\sigma t} + \quad ,$$

which yields the solution

$$\hat{h}_s = \hat{h}_{b_1}[\varsigma \hat{h}_{b_1} - 1n] \quad , \tag{11}$$

where

$$\varsigma = \frac{8}{3\pi} (1 + \frac{r_1}{h_1} L_1) \frac{1}{g} (\frac{A_{b_1}}{A_1})^2 \sigma^2 \quad ,$$

and

$$n = [1 - (\frac{A_{b_1}}{A_1}) \frac{L_1}{9} \sigma^2]$$

The term $(\frac{A_{b_1}}{A_1}) \frac{L}{g} \sigma^2$ represents the flow acceleration and has been traditionally neglected It need not be always negligible compared to 1 0, it was found to be equal to 0 25 for Macquarie Harbor which was studied by Van de Kreeke (1968) The amplitude and phase shift of the ocean fluctuation are given respectively,

$$|\hat{h}_s| \simeq \hat{h}_{b_1} \sqrt{(\varsigma h_{b_1})^2 + n^2} \quad , \tag{12}$$

and

$$\tan \theta = - \frac{n}{\varsigma \hat{h}_b} \tag{13}$$

O'Brien (1969) proposed an empirical relationship between the tidal prism, P, and inlet area A_1

$$A_1 = CP \quad , \tag{14}$$

where C is a constant   The present analysis yields an analytical representation for C which can be used to test the empirical constant of O'Brien

$$C = \frac{(\frac{8}{3\pi}) (1 + \frac{r_1}{h_1} L_1) \frac{\sigma^2}{g} (\frac{A_{b_1}}{A_1})^2 \tan \theta}{2[(\frac{A_{b_1}}{A_1}) \frac{L_1}{g} \sigma^2 - 1]} \tag{15}$$

2   <u>Solution for Non-sinusoidal Motion</u>

The tidal motion in the ocean is not simple as indicated previously so that sinusoidal simplifications introduce errors   Since the bay motion is affected by nonlinear friction, motion is not simple even when the ocean motion is   The latter case was investigated in detail by Keulegan (1951) When the tidal motion in the ocean is periodic but not simple, the bay motion is even more complex since each ocean harmonic constituent generates its own harmonic as well as multiple harmonics due to nonlinear friction   Thus a linearized procedure becomes invalid, in fact it is unrealistic and suggests negative friction at the higher harmonics

In the following a method is proposed to describe the harmonic motion in both bay and ocean   The complex representation is employed

$$h_s = \sum_n \frac{1}{2} (r_n - i s_n) \, e^{in\sigma t} \qquad , \qquad (16)$$

and

$$h_{b_1} = \sum_n \frac{1}{2} (c_n - i d_n) \, e^{in\sigma t} \qquad , \qquad (17)$$

where the summation ranges over both positive and negative values of the index n which specifies   the principal tidal frequencies, linear combinations thereof, and higher harmonics   The terms corresponding to n = 0 reflect mean levels in ocean and bay   Since h_s and h_{b_1} are real functions, the following relationships must be satisfied (Lee (1967))

$$
\begin{array}{ll}
r_n = r_{-n} \qquad , & s_n = -s_{-n} \\[2mm]
c_n = c_{-n} \qquad , & d_n = -d_{-n}
\end{array}
\qquad (18)
$$

The average velocity is specified by

$$U(t) = \sum_n \frac{1}{2} (a_n - i b_n) \, e^{in\sigma t} \qquad (19)$$

where

$$a_n = a_{-n} \quad , \quad b_n = -b_{-n}$$

The representation of T_{to} in series form was considered by Sidjabat (1970) in conjunction with tidal propagation in wide shallow bays   His description of the nonlinear term is conveniently adopted   Equation (15) may be expressed

$$T_{to} = \rho r [\sum_\ell \sum_m \frac{1}{4} (a_\ell - i b_\ell)(a_m - i b_m) e^{i(\ell+m)\,t}]^{\frac{1}{2}} \sum_n (a_n - i b_n) e^{i\sigma t}$$

which is equivalent to

$$\tau_{to} = \rho r \{ [\sum_{\ell} \frac{1}{4}(a_{\ell}^2 + b_{\ell}^2) + \sum_{\ell} \sum_{\ell \neq m} \sum_{m} \frac{1}{4}(a_{\ell} - ib_{\ell})(a_m - ib_m)e^{i(\ell+m)t}]^{\frac{1}{2}}$$

$$\sum_{n} (a_n - ib_n)e^{in\sigma t} \} \qquad\qquad\qquad (20)$$

Denote

$$\lambda^2 = \sum_{\ell=-\infty}^{\infty} \frac{1}{4}(a_{\ell}^2 + b_{\ell}^2) = \frac{1}{2} \sum_{\ell=0}^{\infty} (a_{\ell}^2 + b_{\ell}^2) \qquad\qquad (21)$$

Then

$$\tau_{to} = \rho r \lambda (1+\epsilon)^{\frac{1}{2}} \sum_{n} (a_n - ib_n) e^{in\sigma t} \qquad , \qquad (22)$$

where

$$\epsilon = \frac{1}{\lambda^2} \sum_{\ell} \sum_{\ell \neq m} \sum_{m} \frac{1}{4}(a_{\ell} - ib_{\ell})(a_m - ib_m)e^{i(\ell+m)\sigma t}$$

It was found by Sidjabat (1970) that in areas where M2 is the dominant tide λ is determined primarily by the M-2 component and ε does not exceed the value 0 25   Equation (22) is approximated by

$$\tau_{to} \approx \rho r \lambda \sum_{n} \frac{1}{2}(a_n - ib_n)e^{in\sigma t} \qquad\qquad (23)$$

The error in the friction term is less than 12 5% and corresponds to smaller errors in computing tidal elevation   In studies where such an error is significant, it is possible to calculate ε and to include its effect in Equation (22)   The above method will be applied to Boca Raton inlet, the terms comprising ε will be neglected subject to comparison with measured tidal elevations

The non-linear term in Equation (9) becomes according to the above analysis (See Equation (17))

$$\left| \frac{dh_{b_1}}{dt} \right| \frac{dh_{b_1}}{dt} = \lambda_1 \sum_{n} \frac{1}{2}(in\sigma c_n + n\sigma d_n) e^{in\sigma t} \qquad , \qquad (24)$$

where

$$\lambda_1 = \frac{1}{\sqrt{2}} \ [\ \sum_{\ell=0}^{\infty} \ \sigma^2 \ell^2 \ (c_n \ + d_n \ )]^{\frac{1}{2}} \tag{25}$$

Substituting for h_s, h_{b_1}, and $\left| dh_{b_1}/dt \right|$ (dh_{b_1}/dt) and equating the real and imaginary coefficients, the following algebraic equations for the coefficients of different harmonic constituents are deduced

$$r_n = [1 - n^2\sigma^2 \ (\frac{A_{b_1}}{A_1}) \ \frac{L_1}{g} \ ] \ c_n + \frac{\lambda_1}{g} \ (1 + \frac{r}{h_1} L_1)(\frac{A_{b_1}}{A_1})^2 n\sigma d_n \qquad ,$$

$$\tag{26}$$

$$s_n = [1 - n^2\sigma^2(\frac{A_{b_1}}{A_1}) \ \frac{L_1}{g} \ ] \ d_n - \frac{\lambda_1}{g} \ (1 + \frac{r}{h_1} L_1) \ (\frac{A_{b_1}}{A_1})^2 n\sigma c_n$$

Equations (26) relate the bay motion to the ocean tidal oscillation   The non-linear friction is specified by the term λ_1 which also couples the different harmonic constituents of the ocean elevation

The procedure for solution depends on the available information   If the tidal elevations in both ocean and bay are measured, it is possible to evaluate the friction coefficient r   When only the motion in the bay is known, it is possible to predict the tidal motion in the ocean for any given r   However, when the tidal motion in the ocean is given the motion in the bay can only be computed by an iterative procedure for any given r

B˙   Two Bays Coupled to Ocean

A definition sketch for two bays coupled together and to the ocean is shown in Figure 2   The second bay is not connected to the ocean   The tidal elevations in the ocean and first bay are specified by Equations (16) and (17), respectively   The tidal motion in the second bay is specified by

$$h_{b_2} = \sum_n \frac{1}{2}(p_n - iq_n)e^{in\sigma t} \tag{27}$$

Subject to all assumptions stated previously, the equation which relates the motions in the two bays becomes

$$h_{b_1} = h_{b_2} + (\frac{A_{b_2}}{A_2}) \ \frac{L_2}{g} \ \frac{d^2 h_{b_2}}{dt^2} + (1 + \frac{r}{h_2} L_2) \ \frac{1}{g} \ (\frac{A_{b_2}}{A_2}) \ \frac{dh_{b_2}}{dt} \ \left| \frac{dh_{b_2}}{dt} \right| \ , \ (28)$$

where the equation of continuity in the second inlet

$$U_2 A_2 = A_{b_2} \ \frac{dh_{b_2}}{dt} \qquad , \tag{29}$$

was used   The resulting algebraic coefficient equations relating the two bays become

$$c_n = [1 - n^2\sigma^2 \left(\frac{A_{b2}}{A_2}\right) \frac{L_2}{g}] p_n + \frac{\lambda_2}{g} (1 + \frac{r}{h_2} L_2) \left(\frac{A_{b2}}{A_2}\right)^2 n\sigma q_n \quad ,$$

$$d_n = [1 - n^2\sigma^2 \left(\frac{A_{b2}}{A_2}\right) \frac{L_2}{g}] q_n - \frac{\lambda_2}{g} (1 + \frac{r}{h_2} L_2) \left(\frac{A_{b2}}{A_2}\right)^2 n\sigma p_n \quad ,$$

(30)

where

$$\lambda_2 = \frac{1}{\sqrt{2}} [\sum_{\ell=0}^{\infty} \sigma^2 \ell^2 \ (p_n{}^2 + q_n{}^2)]^{\frac{1}{2}}$$

(31)

The motion of the first bay may now be related to the motions of both ocean and second bay   Equation (7) remains valid for the first inlet, but the continuity equation now takes the form

$$U_1 A_1 = A_{b_1} \frac{dh_{b_1}}{dt} + U_2 A_2$$

Using Equation (29)

$$U_1 = \frac{A_{b_1}}{A_1} \frac{dh_{b_1}}{dt} + \frac{A_{b_2}}{A_1} \frac{dh_{b_2}}{dt} \qquad ,$$

(32)

and substituting for U_1 in Equation (7) the equation which relates the first bay to the ocean is obtained

$$h_s = h_b + \frac{L_1}{g} \left(\frac{A_{b_1}}{A_1}\right) \frac{d^2 h_{b_1}}{dt^2} + \frac{L_1}{g}\left(\frac{A_{b_2}}{A_2}\right) \frac{d^2 h_{b_2}}{dt^2}$$

$$+ (1 + \frac{r}{h_1} L_1) \frac{1}{g} \left(\frac{A_{b_1}}{A_1} \frac{dh_{b_1}}{dt} + \frac{A_{b_2}}{A_1} \frac{dh_{b_2}}{dt}\right) \left|\frac{A_{b_1}}{A_1} \frac{dh_{b_1}}{dt} + \frac{A_{b_2}}{A_1} \frac{dh_{b_2}}{dt}\right|$$

(33)

Using the representation for h_s, h_{b_1}, and h_{b_2}, given by Equation (16), (17) and (27), respectively, the following algebraic coefficient equations are deduced

$$r_n = c_n [1 - n^2\sigma^2 \frac{L_1}{g} \left(\frac{A_{b_1}}{A_1}\right)] - p_n [n^2\sigma^2 \frac{L_1}{g} \left(\frac{A_{b_2}}{A_1}\right)]$$

$$+ \frac{\lambda_3}{g} (1 + \frac{r}{h_1} L_1) [\left(\frac{A_{b_1}}{A_1}\right) n\sigma d_n + \left(\frac{A_{b_2}}{A_1}\right) n\sigma q_n]$$

(34)

$$s_n = d_n [1 - n^2\sigma^2 \frac{L_1}{g} \left(\frac{A_{b_1}}{A_1}\right)] - q_n [n^2\sigma^2 \frac{L_1}{g} \left(\frac{A_{b_2}}{A_1}\right)]$$

$$- \frac{\lambda_3}{g} (1 + \frac{r}{h_1} L_1) [\left(\frac{A_{b_1}}{A_1}\right) n\sigma c_n + \left(\frac{A_{b_2}}{A_1}\right) n\sigma p_n] \qquad ,$$

where

$$\lambda_3 = \frac{1}{\sqrt{2}} \left[\sum_{\ell=0}^{\infty} \{ (\frac{A_{b_1}}{A_1}) \, n\sigma d_n + (\frac{A_{b_2}}{A_1}) \, n\sigma q_n \}^2 + \{ (\frac{A_{b_1}}{A_1}) \, n\sigma c_n + (\frac{A_b}{A_1}) \, n\sigma p_n \}^2 \right]^{\frac{1}{2}} \quad (35)$$

The procedure for solution again depends on the available information   The friction factor can be determined when the motion in the ocean and both bays are known   The elevations in the first bay and ocean can be obtained deterministically for any given r when the motion in the second bay is known The motion in the bays can be determined by an interative scheme for any given r when the motion in the ocean is known

APPLICATION - BOCA RATON INLET

The theoretical method outlined above was applied to Boca Raton Inlet, Florida   The inlet connects the Atlantic Ocean to Boca Raton Lake which is also connected to Lake Wyman   A plan view of these lakes is shown in Figure 3 The intercoastal waterway which connects Lake Wyman and Boca Raton Lake extends to South Lake Worth Inlet north and Hillsboro Inlet south   Comparisons of tide records obtained at Boca Raton Lake and at a station on the Intercoastal Waterway south of Boca Raton indicated no possible flow to or from the Inter-coastal Waterway south of Boca Raton Lake   Boca Raton Inlet primarily influences Boca Raton Lake, Lake Wyman, and areas occupied by boating marinas north of Lake Wyman

Boca Raton Inlet was chosen for this study because tide records were available at the Inlet North Jetty (denoted by Station 1 in Figure 3), at Boca Raton Lake (denoted by Station 2), and at Lake Wyman (denoted by Station 3) Actually tide records at three stations around Boca Raton Lake were also available   These records indicated no special variation of tide elevation in Boca Raton Lake and verifies the representation of conservation of mass given by Equation (8)   The tide elevations recorded at Stations 1, 2 and 3 are shown in Figure 4   They indicate the magnitude and phase shift of tidal motion in the two bays relative to the ocean   Other available data included velocity measurements over a tidal cycle at Station 1 to be shown later in comparison with computed velocities

Since tidal records were available in both bays and at the inlet, the multiple bay analysis was used to determine the friction factor r for the system   The tidal record in Lake Wyman was used to predict the tidal elevation in Boca Raton Lake using Equation (30)   The computed elevation was then compared with the measured one   With the tidal elevation in Boca Raton Lake known, the ocean elevation in the ocean (Station 1) was computed using Equations (34) and the results compared with the measured values   The variances between the computed and measured elevations were computed at Stations 1 and 2 for different friction coefficients   The quantities which describe the bay system (see Figure 2) A_1, L_1, A_{b_1}, h_1, A_2, L_2, h_2 were all known   The area A_{b_2} was not known   The computed motion which gave the best fit to measured tidal elevations at Stations 1 and 2 corresponded to r = 0 0039 and $A_{b_2} = 23 \times 10^6$ ft^2 The latter area is much larger than Lake Wyman's area and suggests that the tidal motion extends to areas occupied by marinas north of Lake Wyman   Table 2 summarizes the physical properties of the above two-bay system

TABLE 2   Properties of the Two-Bay System

	Boca Raton Lake	Lake Wyman
Inlet area (ft^2)	1 40 x 10^3	1 68 x 10^3
Bay Area (ft^2)	3 74 x 10^6	23 00 x 10^6
Inlet depth (ft)	7 0	8 0
Inlet length (ft)	2 67 x 10^3	7 33 x 10^3
M-2 measured amplitude (ft)	1 57	1 40

The variances between computed and measured tidal elevations at Boca Raton Lake (station 2) and at the Ocean (station 1) were 1 9 x 10^{-2} and 2 0 x 10^{-2}, respectively for r = 0 0039   The difference in computed and measured phase shifts of the M-2 component were 2 degrees for Boca Raton Lake and 4 degrees for the ocean

SIGNIFICANCE OF NON-SINUSOIDAL TIDAL COMPUTATION

A comparison between the measured tide record at station 1 and the M-2 constituent is shown in Figure 5   The agreement is reasonable only over part of the tide cycle   A comparison between the measured tide elevation at station 1 and the computed elevation using six harmonics is shown in Figure 6 Better agreement is recognized over most of the tidal cycle   The importance of considering the different harmonics in tidal computations appears more dramatically in velocity computations, however   In Figure 7, the measured velocity at station 1 is compared to the computed velocity using the M-2 constituent only   Near the maximum three velocity peaks appear in the measured record but are absent in the sinusoidal computation   These peaks appear in computations which include different harmonics as shown in Figure 8 Since the contribution of each harmonic to velocity depends on frequency and amplitude of that harmonic the higher harmonic contributes significantly to velocity even when the amplitude is small

The velocity cycle has an important influence on sediment transport in the inlet   The sand trapping capability of an inlet may play an important role in shoreline stability in the vicinity of the inlet   The trapping capability of Boca Raton Inlet is investigated in what follows

The theoretical basis for describing movable beds is empirical   Many relationships exist that relate volume of sediment transport to bottom shear stress   While there is no one relationship which is far superior to others, the empirical result arrived at by Einstein (1942) is used

$$q_s' = 40 \; \rho F \frac{U_*}{[g(s_s - 1)]^{3/2}} \qquad , \qquad (36)$$

where q_s' = the weight rate (in water) of sediment transport per unit width, ρ = water density, U_* = shear velocity (=$_{to}/\rho$), g = gravitational acceleration,

s_s = specific gravity of sand, and F is given by

$$F = [\frac{2}{3} + \frac{36 \ \nu^2}{gd^3 \ (s_s - 1)}]^{\frac{1}{2}} - [\frac{36 \ \nu^2}{gd^3 \ (s_s - 1)}]^{\frac{1}{2}} \quad , \tag{37}$$

where ν = kinematic viscosity of water Equation (36) was used successfully by Shemdin (1970) in modeling of sediment flow in the coastal zone Equation (5) relates the average velocity to the shear velocity

$$U_\star = r^{\frac{1}{2}} \ U$$

For a wide channel with width B the weight rate of sediment transport, q_s, becomes

$$q_s = q_s' B = 40\rho FB \ \frac{r^3 \ U^6}{[g \ (s_s - 1) \ d]^{3/2}} \tag{38}$$

The net sediment transport into Boca Raton Inlet was computed from results similar to those shown in Figure 8 Tide records were found to fluctuate in amplitude and typical records were used for two different days The net transport into the inlet for the two days was calculated to be 15 and 47 (yd^3/day) which correspond to 5,400 and 17,000 yd^3/yr , respectively A recent dredging operation in Boca Raton inlet have been removing 30,000 - 40,000 cubic yards of sand per year from the inlet The computed transport in the inlet is of the same order of magnitude Fugure research on inlets may fruitfully include field measurements of sand transport

CONCLUSIONS

A non-linear coupled procedure is proposed to analyze tidal motion in inlets and bays The importance of the different tidal constituents is shown to be more important in velocity computation compared to surface elevation The exchange of sediment between bays and the ocean is dependent on the velocity variation over the tidal cycle and can only be computed accurately by considering different harmonic constituents in a tidal record The procedure is applied to Boca Raton Inlet and the result indicate that more sand transport occurs during the flood period compared to the ebb period The inlet consequently behaves like a sand trap

REFERENCES

1   Brown, E I   (1928), Inlets on Sandy Coasts, Proc ASCE, Vol IIV

2   Dronkers, J J (1964), Tidal Computations in Rivers and Coastal Waters, North - Holland Publishing Col, Amsterdam

3   Einstein, H A (1942), Formulas for the Transportation of Bed Load, Trans ASCE, Vol 107, pp 561-577

4   Keulegan, G H (1951), Water-level Fluctuations of Basins in Communication with Seas, Report No 1146, National Bureau of Standards, Washington, D C

5   Lee, Y W (1967), Statistical Theory of Communication, John Wiley & Sons,
     Inc , New York

6   O'Brien, M P (1969), Equilibrium Flow Areas of Inlets on Sandy Coasts,
     J Waterways and Harbors Div , Vol 95, No WWI, pp 43-52

7   Shemdin, O H (1970), River-Coast Interaction   Laboratory Simulation,
     J Waterways and Harbors Div , Vol 96, WW4

8   Sidjabat, M M (1970), The Numerical Modeling of Tides in a Shallow
     Semi-enclosed Basin by a Modified Elliptic Method, Ph D Dissertation
     Submitted to the University of Miami, Coral Gables, Florida

9   Van de Kreeke, J (1968), Water Level Fluctuations and Flow in Tidal
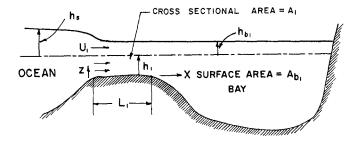   Inlets, J Waterways and Harbors Div , Vol 93, No WW4, pp 97 - 106

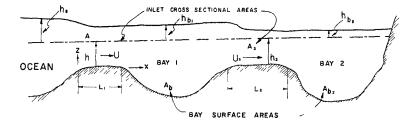Figure 1   Definition sketch for a single bay coupled to ocean

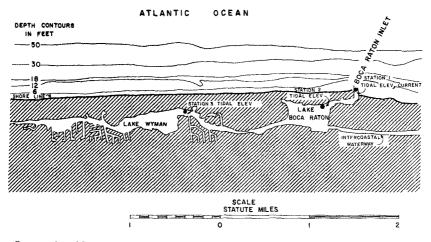Figure 2   Definition sketch for two bays coupled to ocean

ATLANTIC OCEAN

DEPTH CONTOURS
IN FEET
— 50
— 30
— 18
— 12
— 6
SHORE LINE

STATION 1
TIDAL ELEV, CURRENT
STATION 2
TIDAL ELEV
STATION 3 TIDAL ELEV
LAKE
BOCA RATON
LAKE WYMAN
INTERCOASTAL
WATERWAY

BOCA RATON INLET

SCALE
STATUTE MILES
1   0   1   2
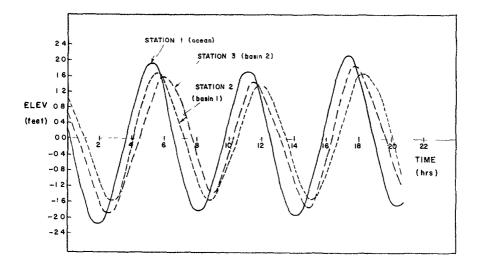
Figure 3   Plan view of Boca Raton Inlet and lake, and Lake Wyman
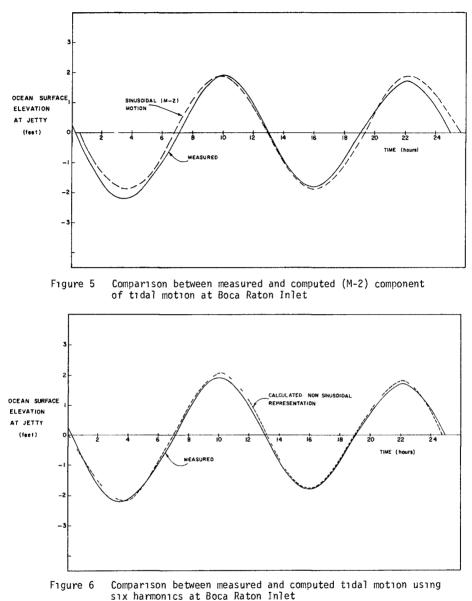
Figure 4   Tide elevations at Boca Raton Inlet, Boca Raton Lake, and Lake Wyman

Figure 5   Comparison between measured and computed (M-2) component
            of tidal motion at Boca Raton Inlet

Figure 6   Comparison between measured and computed tidal motion using
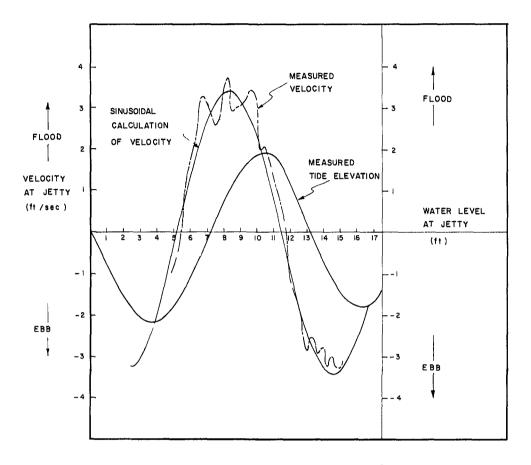   six harmonics at Boca Raton Inlet

Figure 7   Comparison between measured and computed velocity using
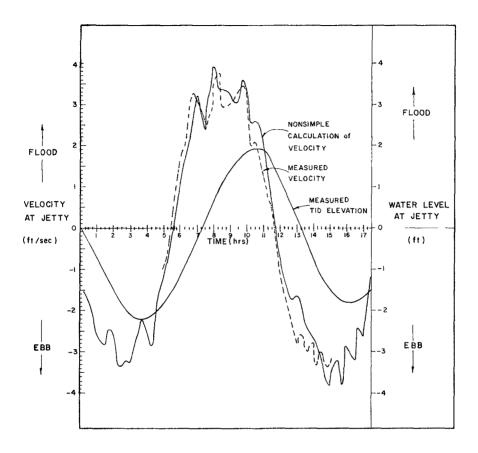(M-2) component only at Boca Raton Inlet

Figure 8   Comparison between measured and computed velocity using
            six harmonics at Boca Raton Inlet