## CHAPTER 119

DENSITY CURRENTS AND TURBULENT DIFFUSION IN LOCKS
by
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ABSTRACT

The paper describes a study of density-currents in locks where there is a net inflow of water into the dock The velocity profiles occurring are different than might be expected by simply superimposing the net flow onto a normal density-current profile The differences are shown to be due to the non-uniform salinity profiles occurring in the docks and a semp-theoretical study is presented which illustrates the importance of this salinity profile

The accretion in the docks is explained by the transport of material into the docks during the levelling period The quantitative agreement between estimates based on measurements during the levelling period and the dredged quantities from the dock are good The locking operation is also shown to be an important factor in certain cases

An attempt to describe the flow of silt into the dock using the onedimensional diffusion equation has shown that allowance must be made for the pick up of silt off the bed of the lock and the settling of silt (as the velocities drop) if quantitative agreement is to be obtained

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## INTRODUCTION

It is common practice in some docks on the Mersey Estuary (and elsewhere) to have a "levelling" period of about one hour (before high water) This is achieved, of course, by opening the gates at both ends of the lock communicating with the estuary, the water level in the dock having been run down (if necessary) in order to meet the rising tide level in the estuary Thus, water can be impounded in the dock by inward flow, vessels can enter or leave without delay, and those longer than the lock itself can come and go at this time

In the last few years, the problem of siltation in some of the Mersey docks has been investigated Velocities, silt-concentrations and salinities have all been measured during the levelling periods During this period, there is an influx of water into the dock, as the tide rises, but there is also a density-current between the estuary and the less-dense dock water

The observations in two particular docks are discussed - Gladstone and Bromborough In the case of Gladstone the water is impounded into the dock system by means of the levelling process and also by pumping At Bromborough dock there is a fresh water discharge into the dock from a small river, which along with the levelling process provides the impounded water Another important difference between the two docks is that the lock at Gladstone is long enough to cope with all the ships using the dock and therefore the levelling process is not essential (although of course it may be desirable) whereas the Bromborough lock is relatively small and the levelling period provides the only means for some ships to use the dock

## MEASUREMENTS

The measurements made in each of the locks have been similar in type, but partly because the sizes of the locks are very different the detall
execution of these measurements has been different at Bromborough than at Gladstone In each case vertical traverses have been made of velocity, silt, salinity and temperature using in-situ measurement with directreading instruments These traverses have been made throughout the levelling period and also for different tides ( 1 e spring/neap) and different season ( 1 e summer/winter) In addition to these, measurements of the silt in suspension near to the river entrance of the lock were made These were made using the same type of silt-meter as for the traverses, based on the light-extinction method, along with a recorder These allowed a continuous record of silt in suspension near the lock entrance to be made

Bromborough dock is relatively small, the lock is some 70ft wide, 160ft long and 30ft deep The vertical traverses in this case were made using a cantilever truss fixed to a dock transport bogie, the observationbogie was driven up to the edge of the lock, the truss being of sufficient length to reach the centre of the lock Two sets of instruments were suspended from the truss so that traverses could be made at above 25 ft and loft from the lock wall Each unit consisted of silt and salinity/ temperature probes attached to a current meter and the unit was lowered and raised by hand Readings were made at approximately six or seven levels and each traverse took about twelve minutes to complete in addition to these, measurements were sometimes taken at fixed levels above the bed so that an almost continuous record of velocity etc was obtained throughout the levelling period at these positions This particular rig had the advantage of cheapness and more important still, it allowed the instruments to be removed from the lock when necessary, for example the passage of ships in and out of the lock

Gladstone lock is some 130 ft wide, 600 ft long and 50 ft deep and this meant that measurements were somewhat more difficult in this situation
than at Bromborouyh In general, traverses were made by lowering the instruments from a vessel in the lock However, some traverses were made at about the middle of the lock and on the centre line of the lock by raising and lowering the instruments from a swinging foot-bridge across the lock

## DENSITY-CURRENT PROFILES

Although there is a considerable amount of literature published on density exchange flow the authors are not aware of any which deal with the situation where there is a net influx of water across the section In the case of the Gladstone lock the influx of velocity at the start of the level (when the tide is still rising at approximately 5 ft per hour) is considerably greater than the density current At high water, of course, there is no net influx of water and the velocities are entirely due to the differences in density In Bromborough lock the density differences are greater than at Gladstone, while the influx velocities are about the same as those at Gladstone

When the lock gates are opened to the estuary, at say $1 \frac{1}{2}$ hours before high water, water enters the dock through the lock due to the rising tide this velocity is termed continulty velocity and is defined as

$$
\begin{equation*}
\bar{V}=\frac{\text { plan area of dock } \times \text { rate of rise of } t 1 \text { de }}{\text { cross-sectionaT area of lock }} \tag{1}
\end{equation*}
$$

Superimposed upon this continuity velocity there is a density-current between the estuary and the less-dense lock water which produces an outward flow at the surface and an inward flow near the bed Figure 1 shows some examples of the vertical velocity profile occurring at the centre of the Bromborough lock during the levelling period on a spring tide The importance of the continuity velocity at the start of the level can be seen by comparing figure $1(a)$ with $1(c)$

The average influx velocity is defincd as

$$
\begin{equation*}
\frac{1}{H} \int_{0}^{H} u d y \tag{2}
\end{equation*}
$$

where $u$ is velocity into dock at depth $y$,
$H$ is the depth of water

The velocity profile due to density differences alone is seen around the high water period in figure l(c) when the average influx velocity is almost zero (and the continuity velocity is zero) The zero velocity point occurs at a level above the mid-depth position (approximately $40 \%$ depth) and the velocities out of the dock (in the upper surface layers) are considerably greater than those entering the dock (in the lower layers) It is immediately apparent therefore that the velocity profile due to the density-current alone is not that occurring in the situation illustrated in case (A), Figure 2, where the zero velocity point occurs at $50 \%$ depth and the overflow velocity is approximately $125 \times$ the underflow velocity (see refs 1 and 2)

Measurements of salinity within the Bromborough dock on a number of occasions have shown that the salinity profile within the dock is in fact far from uniform Two examples are given in Figure (3), and it is clear that the starting conditions are more likely to be similar to the situation illustrated in case (B) of Figure (2) than those of case (A) Some model tests have been made of both cases $A$ and $B$ and these have indicated the velocity profiles for the two cases are indeed different as illustrated in Figure (2) The model tests are still continuing and the results cannot therefore be discussed in detail in this paper but the profile indicated for case ( $B$ ) in Figure (2) is clearly very close to that measured in the docks on a number of occasions and in particular to that given in Figure 1 (c)

The underflow velocity occurring in the situation illustrated in case A ( $f$ ig 2) is given theoretically by

$$
\begin{equation*}
V_{u}=05 V_{\Delta} \tag{3}
\end{equation*}
$$

and has been assessed experimentally by a number of researchers (e g ref 1) as

$$
\begin{equation*}
v_{u}=0465 \mathrm{~V}_{\Delta} \tag{4}
\end{equation*}
$$

where

$$
V_{\Delta}=\left(\frac{\Delta \rho}{\rho_{\mathrm{m}}} g H\right)^{\frac{1}{2}}
$$

$g=$ the acceleration due to gravity
$\rho_{\mathrm{m}}=$ the average density of the two liquids
$\Delta \rho=$ the density difference between the two liquids
$H=$ the depth of the lock

Consider the general case for the situation illustrated in case B (fig 2), let the freshwater occupy the depth $\alpha H$ before the exchange flow starts Assume that the exchange flow is rectangular block flow as illustrated in the sketch below and that the crossover point (or zero velocity) is at depth KH Let the underflow and overflow velocities be denoted by $V_{u}$ and $V_{0}$ respectively Then at time $\Delta t$ after the barrier is removed the fronts will be in the positions illustrated in the sketch


The continuity equation gives

$$
\begin{equation*}
V_{0}=\left(\frac{1-K}{K}\right) V_{u} \tag{5}
\end{equation*}
$$

The gain in kinetic energy is
$\frac{1}{2} \rho\left\{K H\left(V_{0}+V_{u}\right) \Delta t\right\} V_{o}^{2}+\frac{1}{2}(\rho+\Delta \rho)\left\{(1-K) H\left(V_{0}+V_{u}\right) \Delta t\right\} V_{u}^{2}$
so substituting equation (5) into equation (6) and assuming $\Delta \rho$ is small compared to $\rho$

$$
\begin{equation*}
\text { gain in kinetic energy }=\frac{\frac{1}{2}}{2} \rho_{m} V_{u}^{3} \text { at } H \quad\left(\frac{1-K}{k^{2}}\right) \tag{7}
\end{equation*}
$$

The loss of potential energy is

$$
\begin{align*}
\Delta \rho\left(V_{0} \Delta t\right) g K H & (\alpha-05 K) H \\
& -\Delta \rho\left(V_{u} \Delta t\right) g(\alpha-K) H\{05(\alpha-K) H\} \tag{8}
\end{align*}
$$

or

$$
\begin{equation*}
\Delta \rho g \Delta t H^{2} V_{u}\left[\alpha-05 \alpha^{2}-05 k\right] \tag{9}
\end{equation*}
$$

Equating the gain in kinetic energy to the loss in potential energy gives

$$
\begin{equation*}
v_{u}^{2}=\left\{\frac{2 K^{2}\left(\alpha-05 \alpha^{2}-05 K\right)}{(1-K)}\right\} v_{\Delta}^{2} \tag{10}
\end{equation*}
$$

Case 1

$$
\alpha=10 \text { and } K=05 \text { equation (10) and (5) yields }
$$

$$
\begin{equation*}
v_{u}=05 v_{\Delta} \text { and } v_{0}=v_{u} \tag{11}
\end{equation*}
$$

which is the theoretical result for Case $A$ and confirms equation (3)
Case 2 $\alpha=05$ and $K=04$ equations (10) and (5) yield

$$
\begin{equation*}
v_{u}=033 v_{\Delta} \text { and } v_{0}=15 \mathrm{~V}_{u} \tag{12}
\end{equation*}
$$

and this represents the model test results of Case B (fig 2)
Case $3 \quad \alpha=02$ and $K=015$ equations (10) and (5) yield

$$
\begin{equation*}
V_{u}=0075 \mathrm{~V}_{\Delta} \text { and } V_{0}=567 \mathrm{~V} \tag{13}
\end{equation*}
$$

Equation (10) shows the importance of the salinity profiles occurring Within the docks. The overflow velocity may not alter very much with changes in salinity profile within the dock, but the underflow velocity is greatly reduced compared with that given by equations (3) or (4)

If the density difference is expressed in terms of a difference of salinity then for the 51 tuations in Bromborough lock ( $H=32 \mathrm{ft}$ approximately) and Gladstone lock ( $H=50 f t$ approximately); the underflow velocity according to equation (4) can be expressed as

$$
\begin{align*}
& v_{u}=0403(\Delta S)^{\frac{1}{2}} \text { at 'Bromborough } \\
& v_{u}=0507(\Delta S)^{\frac{1}{2}} \text { at Gladstone } \tag{14}
\end{align*}
$$

Using equation (12) the corresponding relationships are

$$
\begin{align*}
& v_{u}=027(\Delta S)^{\frac{1}{2}} \\
& v_{u}=034(\Delta S)^{\frac{1}{2}} \tag{75}
\end{align*}
$$

(allowing for the theoretical value to be reduced by 093 in a similar way to equation (3))

The velocities measured at Bromborough and Gladstone compare very well with those obtained using equation (1\%) whereas the values obtained using equation (14) which is based on equation (4) are considerably in error The profile is also more closely described by the modified block flow illustrated in the sketch and the surface velocity is close to the value given in equation (12)

Fig (4) shows the velocities measured at a fixed level above the bed These confirm that the resulting velocity during the levelling process is the simple addition of the average influx velocity (or continutty velocity) and the density-current The density-current is almost constant throughout the level period and equal in this case to $075 \mathrm{ft} / \mathrm{sec}$ The maximum underflow velocity due to the density difference 15 about $10 \mathrm{ft} / \mathrm{sec}$ (occurring at a depth of about $65 \%$ ) and compares very closely with the value obtained using equation (15) - the value of $\Delta S$ being approximately $12 \%$

## ACCRETION IN THE DOCKS

The main practical interest of the work was centred on the dredging requirements for the docks and the possible means of reducing them The transport of silt through the lock into the dock was determined from the measurements of velocity and silt For example the variation of silt concentration with depth for the traverse corresponding to $\operatorname{Fig} \mathrm{l}(\mathrm{a})$ is given in Fig 6(a) and the resultant transport of silt is proportional to (velocity $x$ silt concentration) This figure immediately shows the silt burdened estuary water entering the dock in the lower layers and the relatively clear dock water leaving in the upper surfaces By integrating traverses such as these the variation throughout the level period of silt transport into the docks has been determined (see for example Fig 6(a)) and thence the total amount of material entering during the level period

In the case of Bromborough, measurements on the 10th December 1969 a spring tide - showed that a total amount of 350 tons dry weight of silt entered the dock during the levelling period The amount of solids in the material dredged from the docks depends on the type of material and the consolidation occurring The density of the material in the hoppers of the dredgers has been measured on a number of occasions and has indicated that the percentage of solids is usually less than $40 \%$ and that a reasonable figure is $36 \%$ so that
> dredged tonnage $=28 \times$ (tons of dry silt transported into the dock)

This figure also implies that one cubic yard of dredged material weighs approximately one ton, assuming that the specific gravity of the solids is 27 Using this value the amount of material entering the dock in the level period during a spring tide in winter conditions is approximately 1000 dredged tons

Before any attempt can be made to calculate the total amount of silt
entering the docks throughout the year the variation with tide and season of the silt in suspension in the estuary near the entrance is required The silt concentration varies with tide range and with season and the silt pattern at any position can be very different from that occurring relatively short distances away (see ref 3) Continuous silt measurements were therefore made near the entrance to the locks and the results from these instruments were used to allow for the variations throughout the year

When this was done for Bromborough Oock the amount entering the dock during the levelling processes was estimated as 270,000 dredged tons (or cubic yards) per annum The average amount of material dredged during the period 1964-68 was 300,000 dredged tons per annum The estimated figure therefore compares very well with the actual amount, however there is another mechanism by which a significant amount of material can enter the dock - the locking procedure or operation

During a locking operation the water in the lock is almost always of different density than that in the estuary At Bromborough the water is usually less saline in the lock (due to the freshwater inflow into the dock) and the estuary water is also carrying a high suspended load which further increases the density difference When the outer gates are opened therefore a density exchange flow starts with the heavier estuary water intruding into the lock near the bed and eventually, if the gates are open long enough, replacing almost all the lock water (ref 2) If the water in the lock is more dense than that in the estuary, for example near the period of low water, then the heavier lock water leaves the lock in the lower layers bringing the estuary water into the lock in the upper layers Whichever mechanism occurs silt is brought into the lock from the estuary during the locking operation When the inner gates (into the dock) are next opened, then a similar mechanism carries the water and
silt from the lock into the dock so that all the silt carried into the lock during the locking procedure is eventually deposited on the bed of the dock

The relatively small size of Bromborough lock and the large density differences mean that the time needed for say $80 \%$ of the lock water to be replaced by estuary water is not very great (of the order of 10 minutes) At Gladstone the lock is relatively large and the density differences are small so that the time needed for $80 \%$ of the lock water to be replaced by estuary water may be quite large (of the order of an hour) Obviously, therefore, the locking procedure may be a significant factor for Bromborough, but is unlikely to be so in the case of Gladstone Estimates for Bromborough, allowing for half the volume of water in the lock to be exchanged during each locking operation, indicate that approximately 75,000 dredged tons per annum enter the dock through this mechanism The estimated total quantity of material entering the Bromborough dock is therefore approximately 350,000 dredged tons per annum and this, considering the various assumptions made, compares very well with the actual figure of 300,000 dredged tons per annum (averaged)

A number of estimates have been made for the Gladstone lock and these have all shown that the average annual dredging figure of 265,000 hopper tons (1966-68) is accounted for to within a few percent by the levelling process (ref 4) and that the locking procedure does not contribute significantly in this situation

As a result of this work the levelling process at Gladstone dock has been stopped - the water now being impounded entirely by pumping - and at Bromborough (where the levelling period provides the only means of some ships using the dock) the levelling period has been reduced to a minimum There has been immediate reductions in the quantities dredged at both docks

ONE-DIMENSIONAL TURBULENT DIFFUSION EQUATION

The one-dimensional convective diffusion equation for a soluble substance in turbulent flow is (see ref 5)

$$
\frac{\partial c}{\partial t}+u \frac{\partial c}{\partial x}=\frac{\partial}{\delta x} \quad\left(D_{t x} \frac{\partial c}{\delta x}\right)
$$

where $U$ is the average flow velocity across the section
$D_{t x}$ is the turbulent diffusion coefficient
$c$ is the concentration of the soluble substance

If a point injection of material is made into the flow at position $x=0$ and time $t=0$ then the concentration at time $t$ and position $x$ is given by Taylor as

$$
c=\frac{M}{A \rho\left(4 \pi D_{t} t\right)^{\frac{1}{2}}} e^{-\frac{(x-U t)^{2}}{4 D_{t} t}}
$$

where $\quad \begin{aligned} & M \text { is the mass of material introduced } \\ & A \text { is the cross-sectional area of flow } \\ & \rho \text { is the density of the fluid }\end{aligned}$

The solution for a series of point injections can obviously be obtained by superimposing the solutions for each separate injection This type of solution is well suited to the digital computer and a computer program has been written (ref 4) incorporating the Taylor solution for finite source injections The program allows for the possibility of injections at various points along the $x$-axis ( 1 e at points other than $x=0$ ) and this allows various initial boundary conditions to be imposed upon the system, 1 e at $t=0, c=c(x)$ is simulated by a series of injections of different strengths at various points along the $x$-axis

Silt measurements in the estuary at the entrance to the Gladstone lock presented the attractive possibility of comparing the measured dis-
tributions of silt at stations along the lock with computed values using the computer program The vertical distribution of silt was simulated by dividing the lock into five layers with injections being made into each layer, the magnitude of each injection being arranged to produce the measured vertical distribution of silt occurring at the time near the river entrance to the lock The technique for determining a suitable dispersion coefficient for each layer was empirical and to some extent arbitary (ref 4) Eventually after consulting the various literature a value of approximately $100 \mathrm{ft}^{2} / \mathrm{sec}$ was used

Comparison of the field measurements taken with the results obtained from the computer solution showed that there were large differences in magnitude between the two sets of curves However, the basic patterns exhibited by corresponding curves were similar The computed concentration results showed that the times of the peaks occurring in the $90 \%$ and $70 \%$ layers agreed reasonably with the observed results The average difference in concentration over the levelling period between the computed and observed results were determined for each layer and these suggested that material was being picked up from the bed and being diffused upwards into each layer, the quantity becoming successively smaller towards the surface layer This process is almost certain to be occurring since on the day in question a layer of fluid mud was present on the lock sill at most times during the levelling period

Close to high water the observed concentration in the lock decreased especially in the $70 \%$ and $90 \%$ layers, whereas the computed concentrations were still slowly increasing This is thought to be due to the occurrence of flocculation which in turn causes rapid settling of the suspended silt If quantitative agreement is to be obtained therefore it would seem that allowance must be made for both pick-up and settling of silt
(1) When there is a net influx of water superimposed upon a densitycurrent situation the resulting velocity distribution is given (to a first approximation) by superimposing a uniform velocity throughout the depth(equal to the influx flow) upon the density-current profile
(2) The density-current profile is considerably influenced by the salinity profiles occurring within the docks
(3) The practice of levelling is responsible for most of the accretion occurring in the docks examined in the Mersey Estuary The locking operations account for some material and of course if water is impounded by pumping this may also be an important contribution
(4) Comparison of field measurements of suspended solids in the lock, with results obtained using a computer solution incorporating the Taylor solution for a finite source injection (and dividing the depth of flow in the lock into layers), show that the general pattern of silt transport into the dock can be qualitively described by the sem-theoretical results However, quantitative agreement is poor and this is considered to be due to
(a) the pick-up of silt off the bed of the lock
(b) the settling of silt (when flocculation occurs)
neither of these processes being allowed for in the computer solution

## ACKNOWLEDGEMENTS

The field measurements described were sponsored by the Mersey Docks and Harbour Board and Unilever Merseyside Ltd, and the helpful cooperation of both these organisations is acknowledged The authors are particularly grateful to Mr J D Littler of the University Civil Engineering Department, for his considerable help with many of the measurements taken

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FIG 1 DENSITY-CURRENT PROFILES WITH NET INFLUX-
VELOCITIES ( Bromborough Lock 10~12~69)


FIG 2 COMPARISON OF VELOCITY PROFILES DURING EXCHANGE FLOW FOR DIFFERENT STARTING CONDITIONS (MODEL TESTS)


FIG 3 EXAMPLES OF SALINITY PROFILES IN BROMBOROUGH DOCK (MERSEY ESTUARY)


FIG 4 VELOCITY INTO DOCK MEASURED AT A POSITION If:FT ABOVE DOCK CILL (1e at approx $56 \%$ depth ) BROMBOROUGH LOCK $10-12 \sim 69$


FIG 5 VARIATION OF SILT CONCENTRATION AT JETTY (NEAR BED) AND TIDE LEVEL


FIG 6(a) VARIATION WITH DEPTH OF SILT CONCENTRATION AND INFLUX OF SILT INTO THE DOCK AT APPROX 47 MINS BEFORE HIGH WATER ( Corresponding velocity distribution is given in Fig 1 (a))


FIG 6(b) TRANSPORT OF SILT INTO DOCK
(BROMBOROUGH DOCK 10-12-69)

