# CHAPTER 103 

# ON THE HYDRODYNAMIC FORCES 

OF TWIN-HULL VESSELS

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#### Abstract

ABSTRAC'T

The added mass and damping coefficients for semi- and fullysubmerged twin cylinders in vertical motion are determined as functions of the oscillation frequency, the cylinder spacing ratio, and the cylinder submergence ratio It has been found that resonance may occur in particular combinations of cylinder spacing and oscillation frequency at which the hydrodynamic inertial and damping characteristics deviate from the trend curves for the case of a single cylinder Justification of using the two-dimensional results to calculate motions of threedimensional twin-hull vessels is discussed It is suggested that, by means of $\operatorname{str}_{1 p}$ theory approach, these results can be used to estimate the hydrodynamic forces for catamaran type vessels in pitch and heave motions


## 1 INTRODUCTION

This paper presents a method to estimate the vertical hydrodynamic forces for twin-hull vessels, including the catamaran type surface ship which has two hulls floating on the free surface and a certain type of floating platforms which have two parallel and closely spaced hulls submerged under the free surface

Analytical calculation of the mono-hull ship motions generally follows the method of strip theory approach The validity of using the strip theory approach to calculate motions of twin-hull vessels has not been completely established Objections certainly may raise, if one considers the overall width of the vessel as the beam, which is relatively too large for the strip theory to apply However one may feel different of he considers it as though there were only one hull plus a wall effect

It is not the purpose of this paper to validify the strip theory for twinhull vessels, but through the availability of the twondimensional results for the twin-hull model, obtained from this and the previous analyses, investigations on motions of catamaran type vessels by means of strip theory approach may proceed

According to the strip theory, the hydrodynamic quantities, such as the added mass and damping coefficients, are estimated by making use of two-dimensional data of long cylinders For the present purpose, we consider a body having two identical, rigidly connected, circular cylinders As to the added mass and damping of the catamaran type surface vessels, the two cylinders are considered semi-submerged initially in the free surface Theoretical and expermental investigation of this problem has been given by Wang and Wahab [1] To complete the analysis, the present work considers the two cylinders being fully submerged The problem is formulated as a linearized boundary value problem in the theory of small amplitude waves Within this framework, a potential function is constructed by superimposing a series of various order singularities, and the solution is obtained through determining the singularity strength by means of satisfying the boundary conditions

Numerical results of the added mass and damping coefficients are presented For the convenience of discussion, the results of the semisubmerged case are also summarized and reviewed Finally, applications of these results to calculating forces on catamaran type vessels are discussed

## 2

## SUBMERGED TWIN CYLINDERS

## Formulation of the Problem

We consider two identical circular cylinders, each of radius a, rigidly connected with a spacing distance of 2 b between their center-line axes They are fully submerged and are forced to make small vertical harmonic oscillations about a mean level $f(>a)$ under the free surface The problem reduces to the special case of semi-submerged cylinders oscillation if $f=0$ The two cylinders are assumed to be infinitely long, and the resulting motion is two-dimensional

Taking the undisturbed free surface as the x~axis, a Cartesian coordinate system is defined as shown in Figure 1 The center~line axes of the two cylinders are then (b,f) and ( $-\mathrm{b}, \mathrm{f}$ ) Two sets of polar coordinates are employed with their origins located at the two cylinder centers, they are related to the Cartesian coordinates as follows

$$
\begin{align*}
& \mathrm{x}=\mathrm{b}+\mathrm{r} \sin \theta=-\mathrm{b}+\mathrm{r}^{\prime} \sin \theta^{\prime} \\
& \mathrm{y}=\mathrm{f}+\mathrm{r} \cos \theta=\mathrm{f}+\mathrm{r}^{\prime} \cos \theta^{\prime} \\
& \mathrm{r}=\left[(\mathrm{x}-\mathrm{b})^{2}+(\mathrm{y}-\mathrm{f})^{2}\right]^{1 / 2} \\
& \mathrm{r}^{\prime}=\left[(\mathrm{x}+\mathrm{b})^{2}+(\mathrm{y}-\mathrm{f})^{2}\right]^{1 / 2} \tag{1}
\end{align*}
$$



Figure 1 Coordinate systems

The fluid is assumed invisid and incompressible, then the problem reduces to seeking a potential function for the described motion Let the motion be simple harmonic of period $2 \pi / \omega$ When the motion amplitude $\eta_{0}$ is sufficiently small, the velocity potential $\Phi(x, y, t)$ may be described by a series of singular function $\varphi_{n}(x, y)$, linearly supermposed in the following form

$$
\begin{equation*}
\Phi(x, y, t)=a \omega \eta_{0} \operatorname{Re}\left[\sum_{r=0}^{\infty} A_{n} \varphi_{n}(x, y) e^{-1 \omega t}\right] \tag{2}
\end{equation*}
$$

The function $\varphi_{n}(x, y)$ satisfies
(A) $\nabla^{2} \varphi_{\mathrm{n}}=0$ in the fluid
(B) $K \varphi_{n}+\frac{\partial \varphi_{n}}{\partial y}=0$ on the free surface $\left(K=\frac{\omega^{2}}{g}\right.$ )
(C) $\frac{\partial \varphi_{n}}{\partial x}=0$ at $x=0$
(D) $\operatorname{Lim}\left(\frac{\partial \varphi_{n}}{\partial x} \mp{ }_{1 K} K \varphi_{n}\right)=0$ as $x \rightarrow \pm \infty$
(E) $\varphi_{\mathrm{n}}=0$ as $\mathrm{y} \rightarrow \infty$

The coefficient $A_{n}$ is a complex-valued number corresponding to the strength of the $n^{\text {th }}$ order singularity it is a function of the cylinder spacing $b$, submergence $f$ and frequency $\omega$ These coefficients are to be determined by satisfying the kinematic boundary conditions on the cylinder surface, which will be described later

## Method of Solution

The singular solution of the two dimensional Laplace equation (3) satisfying the boundary conditions (4), (6) and (7) may be written as a source function or its derivatives of any assigned order For a singularity located at ( $O$ f) the solutions have been given by Thorne [2] as follows

$$
\begin{align*}
G_{0}(x, y)= & \log \frac{\rho}{\rho_{1}}+\oint_{0}^{\infty} \frac{e^{-k(y+f)}}{K-k} \cos k x d k \\
& -12 \pi e^{-K(y+f)} \cos K x \tag{8}
\end{align*}
$$

$$
\begin{align*}
G_{n c}(x, y)= & \frac{\cos n \sigma}{\rho n}+\frac{(-1)^{n-1}}{(n-1)^{\prime}} \oint_{0}^{\infty} \frac{K+k}{K-k} k^{n-1} e^{-k(y+f)} \cos k x d k \\
& +1 \frac{(-1)^{n}}{(n-1)^{\prime}} 2 \pi K^{n} e^{-K(y+f)} \cos K x  \tag{9}\\
G_{n s}(x, y)= & \frac{\sin n \sigma}{\rho n}+\frac{(-1)^{n}}{(n-1)^{\prime}} \oint_{0}^{\infty} \frac{K+k}{K-k} k^{n-1} e^{-k(y+f)} \sin k x d k \\
& +1 \frac{(-1)^{n-1}}{(n-1)^{\prime}} 2 \pi K^{n} e^{-K(y+f)} \sin K x \tag{10}
\end{align*}
$$

Equations (8) and (9) are solutions for describing symmetrical motions, and Equation (10) for anti-symmetrical motion In these equations, $(\rho, \sigma)$ are the polar coordinates, measured from origin ( $o, f$ ) and $\rho_{1}$ is the radial distance from the image of $(0, f), 1$ e , (o, -f)

By expanding the integrals in Equations (9) and (10), one may show that any of the higher order potentials, $G_{n c}$ and $G_{n s}$, can be constructed by the following functions

$$
\left.\left.\begin{array}{rl}
G_{1 c}(x, y)= & \frac{\cos \sigma}{\rho}+K \oint_{0}^{\infty} \frac{e^{-k(v+f)}}{K-k} \cos k x d k \\
& -12 \pi K e^{-K(y+f)} \cos K x \\
G_{1 s}(x, y)= & \frac{\sin \rho}{\rho}-K \oint_{0}^{\infty} \frac{e^{-k(y+f)}}{K-k} \sin k x d k \\
& +12 \pi K e^{-K(y+f)} \sin K x \\
G_{n c}^{\prime}(x, y)= & \frac{\cos n \sigma}{\rho n}+\frac{K}{n-1} \frac{\cos (n-1) \sigma}{\rho n-1} \\
G_{n s}^{1}(x, v)= & \frac{\sin (n+1) \sigma}{\rho n+1}+\frac{K}{n} \frac{\sin n_{\sigma}}{\rho n} \tag{14}
\end{array}\right\} n=2,3\right)
$$

The functions $G_{l c}$ and $G_{l s}$ represent, respectively, the vertical and the horizontal dipoles, combining with appropriate wave function to satisfy the free surface condition (4) and the wave radiation condition (6) Each term in these equations satisfies the infinite depth condition (7) The functions $G_{n c}^{\prime}$ and $G_{n s}^{\prime}$ are wave-free potentials, they represent only local fluid motion which decays rapidiy at a distance and yields no waves at infinity

To derive the potential function $\varphi_{n}(x, y)$, one may construct it by using singular solutions corresponding to singularities located along the two cylinder axes, (b,f) a nd ( $-\mathrm{b}, \mathrm{f}$ ), so as to satısfy the symmetry condition, Equation (5) This may be obtained as follows

$$
\begin{array}{ll}
\omega_{0}(x, y)= & G_{0}(x-b, y)+G_{o}(x+b, y) \\
\omega_{1 c}(x, y)= & G_{1 c}(x-b, y)+G_{1 c}(x+b, y) \\
\omega_{1 s}(x, y)= & G_{1 s}(x-b, y)-G_{1 s}(x+b, y) \\
\omega_{n c}(x, y)= & G_{n c}^{\prime}(x-b, y)+G_{n c}^{1}(x+b, y) \\
\varphi_{n s}(x, y)= & G_{n s}^{\prime}(x-b, y)-G_{n s}^{\prime}(x+b, y) \tag{19}
\end{array}
$$

Substituting Equations (8), (11), (12), (13) and (14) into (15) through (19) and invoking the following identities [3]

$$
\begin{align*}
& \frac{\cos n \theta}{r^{n}}=\frac{1}{(n-1)^{\prime}} \int_{0}^{\infty} k^{n-1} e^{-k(y-f)} \cos (x-b) a h  \tag{20}\\
& \frac{\sin n \theta}{r^{n}}=\frac{1}{(n-1)^{\prime}} \int_{0}^{\infty} k^{n-1} e^{-k(y-f)} \sin (x-b) d k \tag{21}
\end{align*}
$$

we obtain

$$
\begin{align*}
D_{0}(x, y)= & \log \frac{r^{\prime} r^{\prime}}{r_{1} r^{\prime} 1}+4 \oint_{0}^{\infty} \frac{e^{-k(y+f)}}{K-k} \cos k b \cos k x d k \\
& -14 \pi e^{-K(y+f)} \cos K b \cos K x  \tag{22}\\
\mathscr{D}_{1 c}(x, y)= & 4\left\{\int_{0}^{\infty} e^{-k y} \sinh k f \cos k b \cos k x d k\right. \\
& \left.+K \oint_{0}^{\infty} \frac{e^{-k(y+f)}}{K-k} \cos k b \cos k x d k\right\} \\
& -14 \pi K e^{-K(y+f)} \cos K b \cos K x  \tag{23}\\
& -4\left\{\int_{0}^{\infty} e^{-k y} \cosh k f \sin k b \cos k x d k\right. \\
\omega_{1 s}(x, y)= & -K \oint_{0}^{\infty} \frac{e^{-k(y+f)}}{K-k} \sin k b \cos k x d k
\end{align*}
$$

$$
\begin{align*}
& \omega_{n c}(x, y)=\frac{4}{(n-1)^{\prime}} \int_{0}^{\infty}(k+K) k^{n-2} e^{-k y} \begin{array}{c}
\cosh k f \\
\sinh k f
\end{array} \cos k b \cos k x d k, n=\begin{array}{c}
\text { even } \\
\text { odd }
\end{array}  \tag{25}\\
& \omega_{n s}(x, y)=-\frac{4}{(n-1)^{\prime}} \int_{0}^{\infty}(k+K) h^{n-2} e^{-k y} \begin{array}{l}
\cosh k f \\
\sinh k f
\end{array} \sin k b \cos k x d k, \quad n=\begin{array}{c}
\text { odd } \\
\text { even }
\end{array} \tag{26}
\end{align*}
$$

If we put $A_{n} \varphi_{n}=A_{n c} \varphi_{n c}+A_{n s} \varphi_{n s}$, we May write Equation (2) in a different form as follows

$$
\begin{equation*}
\Phi(x, y, t)=a \omega \eta_{0} \operatorname{Re}\left[A_{0} \omega_{0} e^{-1 \omega t}+\sum_{n=1}^{\infty}\left(A_{n c} \omega_{n c}+A_{n s} \omega_{n s}\right) e^{-1 \omega t}\right] \tag{27}
\end{equation*}
$$

To determine the expansion coefficients the velocity potential shall be forced to satisfy the normal velocity on the body surface

$$
\begin{array}{ll}
\frac{\partial \Phi}{\partial r}=\omega \eta_{0} \cos \omega t \cos \theta & \text { on } r=a \\
\frac{\partial \Phi}{\partial r^{\prime}}=\omega \eta_{0} \cos \omega t \cos \theta^{\prime} & \text { on } r^{\prime}=a \tag{28b}
\end{array}
$$

assuming that the vertical velocity of the body is $\omega \eta_{0} \cos \omega t$
Since the two cylinders are identical, the boundary condition (28b) is equivalent to (28a) and need not be considered For the convenience of computation in this particular case, the potential functions (Equations (22) through (26)) are expanded into series about the cylinder axis (b f) The numerical computation then proceeds by formulating Equation (28a) into a set of simultaneous algebraic equations, and the expansion coefficients can be determined by a collocation technique

## Added Mass and Damping Coefficients

The steady state vertical force on a unit length of the cylinders is

$$
\begin{equation*}
F_{v}=-2 \rho a \int_{0}^{2 \pi} \frac{\partial \Phi}{\partial t} \cos \theta d \theta \tag{29}
\end{equation*}
$$

One may write $F_{v}$ in terms of $\eta(t) \quad\left(=\eta_{0} \sin \omega t\right)$ as

$$
\begin{equation*}
F_{v}=-M_{v} \eta(t)-N_{v} \eta(t) \tag{30}
\end{equation*}
$$

then the added mass coefficient $\alpha$ and the nondimensional damping coefficient $\delta$ is defined as follows

$$
\begin{align*}
& \alpha\left(K a \frac{b}{a} \frac{f}{a}\right)=\frac{M_{v}}{2 \pi \rho a^{2}}  \tag{31}\\
& \delta\left(K a \quad \frac{b}{a}, \frac{f}{a}\right)=\frac{N_{v}}{2 \pi \rho \omega a^{2}} \tag{32}
\end{align*}
$$

These coefficients are directly related to the potential function $\Phi$, using the expression given in Equation (2), one obtains

$$
\begin{align*}
& \alpha=\frac{1}{\pi} \operatorname{Re}{\underset{L}{n-0}}_{-\infty}^{L_{0}^{\infty}} \int_{0}^{2 \pi} A_{n}\left(K a, \frac{b}{a}, \frac{f}{a}\right) \omega_{n}\left(K a \frac{b}{a}, \frac{f}{a} \theta\right) \cos \theta d \theta  \tag{33}\\
& 0=\frac{1}{\pi} \operatorname{Im}\left[\sum_{n=0}^{\infty} \int_{0}^{\infty} A_{n}\left(K a, \frac{b}{a}, \frac{f}{a}\right) \omega_{n}\left(K a, \frac{b}{a}, \frac{f}{a}, \theta\right) \cos \theta d \theta\right. \tag{34}
\end{align*}
$$

The Horizontal Force
Because of the unsymmetrical flow over the cylinder surface, there is a horizontal force component induced between the two cylinders, exciting sideway oscillations For a unit length of the cylinder, this force is

$$
\begin{equation*}
F_{h}=-\rho a \int_{0}^{2 \pi} \frac{\partial \Phi}{\partial t} \sin \theta d \theta \tag{35}
\end{equation*}
$$

or

$$
\begin{equation*}
I_{h}=\sqrt{M_{h}^{2}+N_{h}^{2}} \sin (\omega t-\varepsilon) \tag{36}
\end{equation*}
$$

where

$$
\begin{align*}
& \left.M_{h}=\rho w^{2} \eta_{o} a \operatorname{Re} L_{n=0}^{\infty} \int_{0}^{2 \pi} A_{n}\left(K a \frac{b}{a}, \frac{f}{d}\right) r \rho_{n}\left(K a, \frac{b}{a}, \frac{f}{a}, \theta\right) \sin \theta d \theta\right] \\
& N_{h}=\rho w^{2} n_{0} a \operatorname{Im}\left[\sum_{n=0}^{-1} j_{0}^{2 \pi} A_{n}\left(K a, \frac{b}{a}, \frac{f}{a}\right) \varphi_{n}\left(K a, \frac{b}{a}, \frac{f}{a}, \theta\right) \sin \theta d \theta\right] \tag{37}
\end{align*}
$$

In terms of the maximum hydrostatic force variation during the oscillation a horizontal force coefficient is defined as

$$
\begin{equation*}
h=\frac{\sqrt{M_{h}^{2}+N_{h}^{2}}}{20 g a \eta o} \tag{39}
\end{equation*}
$$

and the phase angle by which the inward horizontal force falls behind the oscillating displacement is therefore

$$
\begin{equation*}
e=\tan ^{-1} \frac{\mathrm{~N}_{\mathrm{h}}}{\mathrm{M}_{\mathrm{h}}} \tag{40}
\end{equation*}
$$

## Added Mass and Damping Coefficients

(1) Semı-submerged twin cylinders

Before entering into discussion of the results for the fully submerged case, we shall first review and summarize the results for the semi-submerged case, which has been obtained in [1] The theoretical results of the added mass and damping coefficients for this case are shown in Figures 2 and 3 These results are presented as a function of nondimensional frequency Ka with the hull spacing ratio $\mathrm{b} / \mathrm{a}$ as a parameter

It is important to note that there exists a discrete set of characteristic frequencies at which the motion of the fluid between the two cylinders is strongly excited by the forcing oscillations These characteristic frequencies can be obtained from the following equation

$$
\begin{equation*}
K(b-a)=n \pi r \quad n=12, \tag{41}
\end{equation*}
$$

They bear a correspondence to the natural modes of the motion of fluid between two vertical walls of $2(b-a)$ apart

The first few characteristic numbers of $K a \operatorname{in}$ accordance with the normal modes for four cylinder space ratios are listed below

| $b / a$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 15 | $2 \pi$ | $4 \pi$ | $6 \pi$ | $8 \pi$ |
| 20 | $\pi$ | $2 \pi$ | $3 \pi$ | $4 \pi$ |
| 30 | $\pi / 2$ | $\pi$ | $3 \pi / 2$ | $2 \pi$ |
| 40 | $\pi / 3$ | $2 \pi / 3$ | $\pi$ | $4 \pi / 3$ |

Table I Characteristic frequencies as a function of hull spacing ratio


Figure 2 Added Mass Coefficient $\alpha$ as a Function of Ka Sem1-submerged Twin-cylinders, f/a $=0$


Figure 3 Nondimensional Damping Parameter $\delta$ as a Function of Ka Semi-submerged Twin-cylinders, $f / a=0$

From the graphs, it can be seen that the added mass and damping coefficients are markedly peaked at values of Ka close to those specified above, and as the oscillation passes across the characteristic frequency (a phase change taking place in the surface waves), the added mass becomes negative in a very narrow band of frequencies and the damping coefficient falls down to zero at a certain frequency

Besides these normal modes of resonance, one may find, from Figures 2 and 3 that there is another peak occurring between $\mathrm{Ka}=0$ and the first characteristic frequency This peak corresponds to the degenerate mode (or zero'th mode) at which the fluid displacement between the cylinders is approximately uniform and $180^{\circ}$ out of phase with that immediately outside the cylinders This peak occupies a rather wide range of frequencies, as compared to those which occur at highex modes

In regard to the resonance phenomena discussed above, it must be noted however that they are strictly two-dimensional characteristics All the resonance peaks, except that of the zero'th mode, would disappear if the hull beams vary along the length (regular catamaran hull) or if the band width of the input oscillation is large For a detailed discussion, one is referred to [1]
(2) Fully-submerged twin cylinders

The computation has been done for four cylinder spacing ratios, $\mathrm{b} / \mathrm{a}=15,2025$ and 30 and four submergence ratios, $f / a=15,20$, 25 and 30 To demonstrate the spacing effect which arises from the interference between the two cylinders values of $\alpha$ and $\delta$ for different spacing ratios are superamposed and presented in Figures 4 through 11

Each plot is given for one given submergence ratio As the frequency of interest for practical application is mostly in the neighborhood of Ka= 10 , the presentation is limited to a frequency range up to $\mathrm{Ka}=20$

As discussed in the preceding case when the two cylinders are semisubmerged, there are certain characteristic frequencies, around which in a very narrow frequency band width the added mass deviates from its normal trend and the damping coefficient falls to zero while the radiated wave changes phase When the two cylinders are fully submerged, although the spacing distance still has a tendency to amplify the surface waves between the two cylinders around those characteristic frequencies as given in Table I, there is no physical boundary on the free surface to characterize the wave length so that there is no equivalent resonant phenomenon as described for the semi-submerged case except that the radiated wave does change its phase around the neighborhood of those frequencies and that the damping falls to sero

Similar to the semi-submerged case, there is a peaked added mass and a zero damping occurring somewhere between $K a=0$ and the first characteristic frequency It must be noted that, for both semı- and fullysubmerged cases, these peaked added mass and deviated damping occupy a wide band of frequencies This is very important to the twin-hull vessels, as these peaks and deviations will not be completely removed either by the effect of three-dimensional hull form or by random input oscillations


Fig 5 The Added Mass Coefficient for Fully-
Submerged Cylinders, Submergence Ratio $f / a=20$

Fig 4 The Added Mass Coefficient for Fully-
Submerged Cylinders, Submergence Ratio $f / a=15$

Fig 7 The Added Mass Coefficient for Fully-
Submerged Cylinders, Submergence Ratio f/a $=30$

Fig 6 The Added Mass Coefficient for Fully-
Subme.ged Cylinders, Submergence Ratio f/a $=25$


Figure 9 The Damping Coefficient for Fully-submerged Cylinders, Submergence

Figure Il The Damping Coefficient for
Fully-submerged Cylinders, Submergence
Ratio f/a $=30$


[^0]
## The Horizontal Force

The horizontal force, as well as the vertical component, added mass and damping oscillates in the neighborhood of the characteristic frequencies It can be shown however, all the high frequency peaks are not larger than the first one But this is not the case, when $f / a=0$ (semı-submerged) The high frequency peaks for the sem1-submerged case may be much larger because of the surface wave resonance Nevertheless, this is strictly a two-dimensional phenomenon as discussed before and it occupies only a very narrow band of frequencies, the high frequency peaks, therefore, are of little importance to the practical applications

In Figure 12, the dimensionless peak amplitude of the horizontal force are given as a function of cylinder spacing All these values including those for the case $f / a=0$, refer to the lowest frequency peaks The hull spacing effect is clearly demonstrated in this figure for the semi-submerged case, the maxımum side force can increase by ten times when the hull spacing ratio $\mathrm{b} / \mathrm{a}$ is reduced from 30 to 15 For the fully submerged cases, the hull spacing effect is not as strong as that for the semi-submerged case, however, the submergence effect seems rather evident The maximum side force may reduce approximately $70 \%$ when the submergence depth is increased by one cylinder radius Based on the computed data, the maximum force coefficient $h$ for the fully submerged case, can be interpolated in terms of the hull spacing ratio $\mathrm{b} / \mathrm{a}$ and submergence ratio f/a as follows

$$
h=e^{2-0} 2 \frac{b}{a}-114 \frac{f}{a} \quad\left(\begin{array}{ll}
1 & 5<\frac{b}{a}<30  \tag{42}\\
1 & 5<\frac{f}{a}<3
\end{array}\right)
$$

## 4

## APPLICATION

A group of floating platforms (such as the Mohole) and a certain type of the novel high speed vessels (such as the Trisec [4]) consist of two parallel cylindrical type hulls to facilitate good maneuverability, as well as to provide buoyancy As to these kinds of vessels, the results of the present analysis on the fully submerged twin cylinders are to provide good hydrodynamic information and can be used directly for the vessel motion response estimation

As to the catamaran type surface vessels, the results obtained from the semi-submerged twin cylinders analysis can be used to approximate the added mass and damping in both heaving and pitching by means of strip integration technique As an example, the heave motion coefficients of an ASR (submarine rescue) catamaran are estimated The procedure of this estimation begins with calculating the two-dimensional section added mass and damping of one hull of the catamaran by considering it independent from the interaction of the other hull Then these values are correctedin considering the twin-hull interation effect by multiplying the following ratio


Figure 12 Effect of the Cylinder Spacing Ratio on the Maximum Horizontal Force Coefficient

$$
\begin{equation*}
R=\frac{a_{s}\left(\frac{b}{a}, K a\right)}{a_{0}(K a)} \tag{43}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{a}_{\mathrm{s}}= & \text { added mass or damping coefficient (whichever applies) } \\
& \text { for twin-cylander oscillation }
\end{aligned}, \begin{aligned}
& \text { added mass or damping coefficient (whichever applies) } \\
& \mathrm{a}_{\mathrm{o}}= \\
& \text { for single-cylinder oscillation }
\end{aligned}
$$

The ratio $R$ is a function of the cylinder spacing ratio $\frac{b}{a}$ and the frequency parameter Ka , where the cylinder radius a here 1 s assumed equivalent to and substituted by the local half-beam of one hull, and $b$ is one-half of the spacing between center to center of the two hulls This approximation implies that the effect of twin-hull interaction is not sensitive to the hull shape details

The principal dimensions and the hull lines of this catamaran are shown in Figure 13 A $1 / 1689$ scale model of this catamaran was tested by NSR DC for the pur pose of determining the added mass and damping coefficients in both pitching and heaving oscillations [5] The model tests included investigations of the speed effects, covering a speed range up to Froude No $=0316$ The estimated results of the added mass and damping coefficients are compared with the experimental results for the catamaran at zero speed and shown in Figure 14 The agreement is fairly good in regard to the negative added mass in the neighborhood of $\mu=25$, where $\mu$ is a nondimensional frequency defined in Figure 14 The experimental results of damping are rather scattered This is essentially due to the effect of resonance, which occurs in the neighborhood of $\mu=25$

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(b) nondimensional damping cocificseat $B_{33}=\frac{\mu \int_{\ell} \delta(x) s(x) d,}{\int_{\ell} s(2) d x}$

Figume 14 Hydrodynme cocificients of $A S R$ cotamax in mein heng motion


Compuiced
0


[^0]:    Figure 10 The Damping Coefficient for Fully-submerged Cylinders, Submergence
    Ratio $\mathrm{f} / \mathrm{a}=25$

