CHAPTER 87

SUCCESSIVE SALTATION OF A SAND GRAIN BY WIND

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ABSTRACT

In order to establish the mechanics of sand transport in an air or water stream, the mechanics of saltation of sand grains should first be considered In sand storms, most of the saltating sand grains on a granular bed have successively continued the saltation motion In this paper, such a saltation motion is defined as successive saltation A theoretical approach to the saltation of a single sand grain on a fixed granular bed is proposed on the basis of the equations of motion for the saltation and the dynamic characteristics of collision between a saltating sand grain and bed sand grains Some experiments of the successive saltation of a single sand grain on a fixed granular bed were carried out to compare with the theoretical relationships It was verified from the comparison that the theoretical relationships of the height and distance of saltation of a sand grain are in fairly good agreement in substance with the results of experiment

INTRODUCTION

One of the crucial problems in the mechanics of sediment transport by wind is to establish the mechanics of the motion of sand grains near the bed In 1941, Bagnold published a famous book entitled "The physics of blown sand and desert dunes", and investigated the motion of sand by wind defined as saltation and surface creep Although many investigations have been conducted since then, the mechanics of the motion of sand grains have not yet been established completely In 1951, Kawamura proposed an excellent theory of sand movement by wind based on the equation of motion of a sand grain by applying the drag force acting on the sand grain to the equation of motion but neglecting the virtual mass force since the force is usually very small compared with the drag force Recently, Owen studied the mechanism of saltation of sand grains by wind to discover the velocity profile in a saltation layer and the rate of sediment transport

On the other hand, in the case of water streams, in 1964 Yalin first established a theory of saltation of a sand grain by taking into consideration the uplift force acting on a sand grain, as measured by Einstein and Sammi and by Chepil and proposed a formula for the rate of sediment transport And after that Kishi and Fukuoka recently carried out a basic experiment on the first saltation of a single spherical particle from the beginning of movement in a turbulent stream and modified Yalin's theory of saltation by taking into consideration the virtual mass force The authors also conducted the same experiments as those done by Kishi and Fukuoka to make clear the mechanism of successive

saltation of a single sand grain from the beginning of motion and other basic experiments on the motion of sand grains in bed load In addition the authors proposed a theoretical approach to the motion of sand grains, namely the sliding or rolling motion and the saltation motion, based on a different concept of the motion of a sand grain from the theories of Yalin and of Kishi and Fukuoka In the theory there are two types of motion of a sand grain which has begun to move from the rest condition The first one is defined as the rolling motion including the sliding one and the second the so-called saltation motion which the grain skips for a distance With regard to the transition from the rolling motion to saltation it was pointed out from the photographs and the direct observations of motion that the grain always begins saltation after rolling for a certain distance And it was concluded from the theory that the rolling distance is a function of the flow intensity and the ratio of the density of grain to that of fluid and that the distance dereases rapidly with the increase of flow intensity and of the density ratio With regard to the hydrodynamic forces acting on a sand grain of which the size is large, both the drag and virtual mass forces were applied to the establishment of the equation of motion of a sand grain, because the so-called uplift force is considered to be very small compared with the drag force as measured by Chepil and calculated by Iwagaki

In this paper, a modification of the theory is made based on the fact that the rolling and sliding motion do not exist in the case where sand grains are transported by wind and an application of the theory to the saltation of sand grains by wind is presented in comparison with some results of experiments on the successive saltation of a sand grain on a fixed granular bed

THEORY OF THE SALTATION OF A SAND GRAIN

(1) Equation of Motion

Since the Reynolds number becomes very high in the motion of a sand grain in general, the quadratic law for drag forces is applicable to the equation of motion It is assumed that the size of the grain is so large that the effect of turbulence on the motion is not taken into consideration Although the hodograph space can be used in establishing the equation of motion, the equations are assumed to be established in the vertical and horizontal directions respectively, because the saltation height is assumed not to be very high compared with the saltation distance Neglecting the Basset term which is one of the virtual mass forces, the equation of motion of a sand grain can be written as

$$\left.\begin{aligned} dW/dt &= \mp(3/4)C_D\, W^2/(\sigma/\rho+1/2)d-(\sigma/\rho-1)g/(\sigma/\rho+1/2) \\ dU/dt &= (3/4)C_D\,(u-U)^2/(\sigma/\rho+1/2)d \end{aligned}\right\} \qquad (1)$$

in the vertical and horizontal directions respectively, in which W is the vertical velocity component of the sand grain, U the horizontal component, C_D the drag coefficient, d the diameter of the grain, g the acceleration of gravity, t the time and σ and ρ the densities of the grain and fluid respectively And u in Eq (1) denotes the velocity in a saltation layer which is a function of the ordinate

Let the following dimensionless quantities be introduced into Eq (1)

$$\left.\begin{aligned} &U=U/u^*, \quad \bar{W}=W/u^* \quad K^2=(4/3)\{(\sigma/\rho-1)gd/u^* C_D\} \\ &\tau=\{C_D/(\sigma/\rho+1/2)\}(u^*t/d) \end{aligned}\right\} \tag{2}$$

in which u_* is the shear velocity and the solution under the initial condition that $\bar{W}=\bar{W}_0^*$ and $\bar{U}=\bar{U}_0$ at $\tau=0$ becomes

$$\left.\begin{aligned} &\bar{W}=K\{(\bar{W}_0/K)-\tan K\tau\}/\{1+(\bar{W}_0/K)\tan K\tau \\ &\bar{U}=u-(u-\bar{U}_0)/\{1+(u-\bar{U}_0)\tau\} \end{aligned}\right\} \tag{3}$$

in the upward motion of the sand grain And the solution for W in the downward motion under the initial condition that $\bar{W}=0$ at $\tau'=0$ becomes

$$\bar{W}=-K\tan(K\tau) \tag{4}$$

in which τ' is the same expression as τ Therefore, the velocity components $\bar{W}_1$ and $\bar{U}_1$ just before arriving at the bed can approximately

$$\bar{W}_1\simeq\bar{W}_0, \quad \bar{U}_1=u-(u-\bar{U}_0)/\{1+2(u-U_0)\bar{W}_0/K^2\} \tag{5}$$

In the case where the velocity in a saltation layer u is assumed to be constant because the saltation height is very small, further integration of Eq (3) under the initial condition that $\xi(=x/d)=\eta(=z/d)=0$ at $\tau=0$ yields

$$\left.\begin{aligned} &\bar{H}=(2/3)\{(\sigma/\rho+1/2)/C_D\}\log\{1+(\bar{W}_0/K)\}-(2/3)\{(\sigma/\rho+1/2)/C_D\}(\bar{W}_0/K) \\ &L=(4/3)\{(\sigma/\rho+1/2)/C_D\}\{2u\bar{W}_0/K-\log|2(u-\bar{U}_0)\bar{W}_0/K^2+1|\} \end{aligned}\right\} \tag{6}$$

in which H is the saltation height, L the distance, $\bar{H}=H/d$ and $\bar{L}=L/d$

(2) Collision and Rebound of a Saltating Sand Grain on Bed Grains

Fig 1 shows a schematic diagram for the collision and rebound of a sand grain in which V_1 and V_2 denote the velocity vectors of saltating sand grain and the other notations are shown in the figure Making some assumptions, the conservation law of momentum in the vertical and horizontal directions yields

$$\left.\begin{aligned} &-eV_1\cos(\gamma-\alpha)=V\cos(\pi-\beta-\gamma) \\ &V_1\sin(\gamma-\alpha)=V_2\sin(\pi-\beta-\gamma) \end{aligned}\right\} \tag{7}$$

in which e is the coefficient of rebound of a saltating sand grain Introducing the quantities

$$\left.\begin{aligned} &V_1\cos\alpha=U_1 \quad V_1\sin\alpha=W_1 \quad V\cos\beta=U_0 \\ &V\sin\beta=W_0 \quad U_1=-W_1\cot\alpha=\delta_1 W_1(\delta_1<0) \\ &U_0=W_0\cot\beta \quad \delta W_0(\delta>0) \\ &\bar{W}_1=W/u_* \quad \bar{U}_1=U_1/u_* \quad \bar{W}_0=W_0/u_* \\ &\bar{U}_0=U_0/u_* \quad \bar{U}_1=\delta_1\bar{W}_1 \quad \bar{U}_0=\delta\bar{W}_0 \end{aligned}\right\} \tag{8}$$

the relationships between the velocity components just before and after the

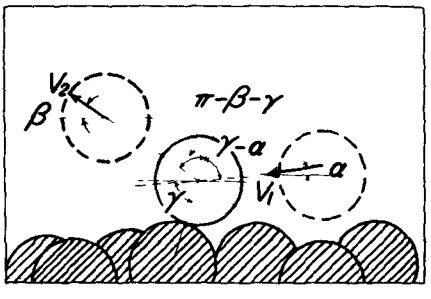


Fig 1 Schematic diagram for collision of sand grain

collision can be expressed approximately as

$$\left.\begin{aligned}\bar{U}_0 &= e\bar{U}_1 \\ \bar{W}_0 &= e\{b_1\bar{U}_1 + b_2\bar{W}_1\}/\{b_3 + b_1(\bar{W}_1/\bar{U}_1)\}\end{aligned}\right\} \qquad (9)$$

in which $b_1=(1+e)\tan\gamma$ $b_2=(1-e\tan^2\gamma)$ and $b_3=(\tan^2\gamma-e)$

(3) First Saltation

In this paper the first jumping motion of a sand grain from the rest condition is defined as first saltation In the sand movement by wind the rolling or sliding motion scarcely ever occurs The friction force in the equation of motion can be neglected because the sand grain always begins to saltate just after the collision on a neighboring sand grain Therefore the equation of motion of a sand grain for the motion from the rest condition to the saltation can be written as

$$dU'/dt=(3/4)\{C_D/(\sigma/\rho+1/2)d\}(u-U')^2 \qquad (10)$$

in which U' is the horizontal velocity of the grain and u' the velocity near the grain which may be affected by the velocity of fluctuation and assumed to be $\bar{u} = u'/u_*$ The integration of Eq (10) with the initial condition that U' = 0 at t = 0 yields

$$U/u^*=(3/4)u^2\tau/\{1+(3/4)u\tau\} \qquad (11)$$

From Eq (11) the relationship for the change of velocity with the distance under the assumption that u is very small can be expressed approximately as

$$U/u^*=u\sqrt{2(x/d)N}/\{1+\sqrt{2(x/d)N}\} \qquad (12)$$

in which $N=(3/4)\{C_D/(\sigma/\rho+1/2)\}$

As described already, making the assumption that the sand grain begins to saltate just after the collision on a neighboring sand grain from the rest condition, the value of x/d in Eq (12) is assumed to be that x/d $\approx$ 1 Since the initial velocity of the grain for the first saltation can be estimated, putting the velocity into an approximate expression of Eq (6) for $(W_0/K)^2 \ll 1$, the saltation height can be expressed approximately as

$$\bar{H}=(1/2)\beta^2A_r^2\{(\sigma/\rho+1/2)\}\{2N/(1+\sqrt{2N})^2\}\{u^{*2}/(\sigma/\rho-1)gd\} \qquad (13)$$

in which $\beta = W_0/U_0$ and $A_r = \bar{u}$ By the same means the calculation for the saltation distance can also be made

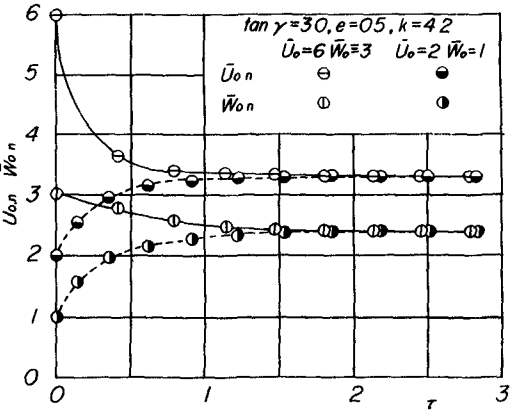


Fig 2 Changes of initial velocity of sand grain in successive saltation

(4) Successive Saltation

Although the successive

saltation can be calculated by Eqs (5), (6) and (9) as seen in Fig 2, the velocity components of the saltating sand grain become constant after several saltations Such a saltation is defined as the stationary saltation of which the velocity components in the vertical and horizontal directions are expressed by W_s and U_s respectively

Assuming that the sand grain has alternately taken saltation and rebound keeps the stationary velocity at the k-th step in saltation, the relationship can be obtained as

$$U_{k0}=P u \quad \bar{W}_{k0}=Q u \quad U_{k10}=U_{k0}=P u \quad \bar{W}_{k10}=W_{k0}=Q u \tag{14}$$

Transformation of Eq (14) using Eqs (5), (6) and (9) yields approximately

$$\left.\begin{aligned}\bar{W}_s&=\lambda[(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(u/K)\}}]u/2\\ \bar{U}_s&=[(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(u/K)^2\}}]u/2\end{aligned}\right\} \tag{15}$$

According to Eq (15), it is seen that the values of W_s and U_s are real, since e = 1 in general, and then the roots in the equation also are real As seen from Eq (15), the condition that $\bar{W}_s = \bar{U}_s = 0$ can be written as

$$u_*^2/(\sigma/\rho-1)gd=(2/3)(1/C_D)\{(1-e)/e\lambda u^2\} \tag{16}$$

which is generally different from the so-called critical flow intensity

It is concluded from the above description that the sand grain moving downstream, repeating saltation and rebound alternately, reaches a certain stationary velocity after several steps of successive saltation in the case where the flow intensity is larger than the critical one expressed by Eq (16) and that the initial velocity components in the vertical and horizontal directions in stationary saltation are expressed by Eq (15)

Although the height and distance of saltation of a sand grain are formulated by Eq (6) in connection with the initial velocities, without loss of generality, for simplicity, the following relationships can be used under the assumptions that $(\bar{W}_0/K)^2 \ll 1$ and $2(\bar{u}-\bar{U}_0)W_0/K^2 \ll 1$

$$\left.\begin{aligned}\bar{H}&\simeq(2/3)\{(\sigma/\rho+1/2)/C_D\}(\bar{W}_0/K)^2\\ \bar{L}&=(8/3)\{(\sigma/\rho+1/2)/C_D\}(\bar{U}_0\bar{W}_0/K^2)\end{aligned}\right\} \tag{17}$$

Putting Eq (15) into Eq (16), the relationship of the height and distance of saltation with the flow intensity and the condition of the granular bed can be written approximately as

$$\left.\begin{aligned}\bar{H}_m&=(1/6)\{(\sigma/\rho+1/2)/C_D\}\lambda[(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(u/K)\}}]^2(u/K)\\ \bar{L}_m&=(2/3)\{(\sigma/\rho+1/2)/C_D\}\lambda[(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(u/K)^2\}}]^2(u/K)^2\end{aligned}\right\} \tag{18}$$

in which H_m denotes the mean values of saltation height and L_m the mean values of saltation distance From Eq (18), the value of λ can be expressed as

$$\lambda=4\ (\bar{H}_m/L_m) \tag{19}$$

which is an empirical constant to be determined by the experimental results for the mean values of height and distance of saltation

(5) Distributions of Height and Distance of Saltation

Although the distribution characteristics of the saltation height and distance of a sand grain generally depend upon the characteristics of the velocity fluctuation of a sand grain just before and after its collision with bed grains, the dispersion characteristics of the angle of collision and the effect of turbulence on the motion of the grain, it is assumed that the distribution of height and distance of saltation is affected only by the characteristics of the velocity fluctuation

From this point of view, assuming that the density functions of the horizontal velocity of a sand grain both in first and stationary saltations $f_1(\bar{u})$ and $f_1(\bar{U})$ can be expressed respectively by the Gaussian distribution in the form

$$f_1(u) = (1/\sqrt{2\pi}\sigma_0) \exp\{-(u-u_m)^2/2\sigma_0^2\} \tag{20}$$

for first saltation and

$$f_2(\bar{U}) = (1/\sqrt{2\pi})(1/\sigma_U)\exp\{-(\bar{U}-U_s)^2/2\sigma_U^2\} \tag{21}$$

for stationary saltation, in which σ_0 and $\sigma_{\bar{U}}$ are the standard deviations of the dimensionless horizontal velocities $\bar{u}$ and $\bar{U}$ respectively and are assumed to be

$$\sigma_0 = \varepsilon u_m \qquad \sigma_U = \epsilon U \qquad \epsilon = \text{const} \tag{22}$$

and $\bar{u}_m$ is the mean value of $\bar{u}$, assuming that the relation between the horizontal and vertical velocities can be expressed generally by $\bar{W} = \lambda\bar{U}$, the density function of the saltation height can finally be written after some transformations as

$$f(\bar{H}) = (1/2\sqrt{2\pi})(1/\epsilon\sqrt{\bar{H}\bar{H}_m})\exp\{-(\sqrt{\bar{H}} - \sqrt{\bar{H}_m})^2/2\epsilon^2\bar{H}_m\} \tag{23}$$

for both first and stationary saltation Similarly the density function of the saltation distance can finally be expressed as

$$f(\bar{L}) = (1/2\sqrt{2\pi})(1/\epsilon\sqrt{\bar{L}\bar{L}_m}\exp\{-(\sqrt{\bar{L}} - \sqrt{\bar{L}_m})^2/2\epsilon^2\bar{L}_m\} \tag{24}$$

EXPERIMENTS OF SUCCESSIVE SALTATION OF A SAND GRAIN

(1) Experimental Apparatus and Procedure

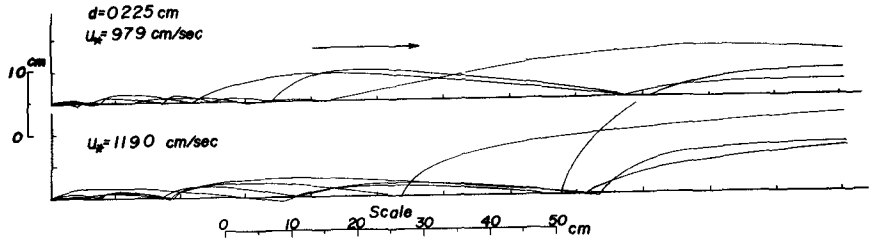
A wind tunnel, 21 6 m long, 0 75 m wide and 1 0 m deep was used in order to make clear the characteristics of successive saltation of a sand grain on a fixed granular bed Properties of grains used in the experiments are shown in Table 1, in which ρ_0 is the density of water Paths of saltation of a grain from the rest condition were photographed with a 16 mm high speed camera under various conditions of wind and the film was analyzed with a film motion analyzer Wind velocity profiles in the wind tunnel were measured with a hot wire anemometer and the shear velocity was estimated by the wind velocity profiles measured based on the logarithmic law of velocity profile

Table 1 Properties of grains used in the experiment

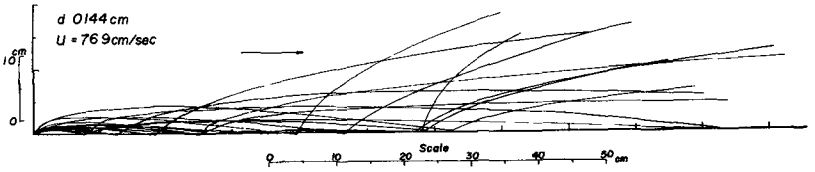
Kind of grains	Diameter d cm	Specific gravity σ/ρ_0
sand	0 225	2 624
	0 184	2 523
	0 144	2 474
seed	0 184	1 155
	0 144	1 155

(2) Results of Experiments

Fig 3 shows some examples of the successive saltation of sand grains and seeds respectively obtained in the experiment From the results of the experiment the saltation angle β shown in Fig 1 were measured Fig 4 describes the relationship between the angles in each step of saltation and the flow intensity in which β_1, β_2 and β_3 are the saltation angles in the first, second and third saltations respectively It can be found that the angle of the first saltation is mostly independent of the flow intensity and is approximately 40 degrees



(a) In the case where sand grains were used



(b) In the case where seeds were used

Fig 3 Some examples of paths of saltating grains in successive saltation

Fig 5 shows a comparison between the experimental values of the height of first saltation and the theoretical relationship for the first saltation obtained by Eq (13) in which A_r is assumed to be 10 5 including the effect of turbulence

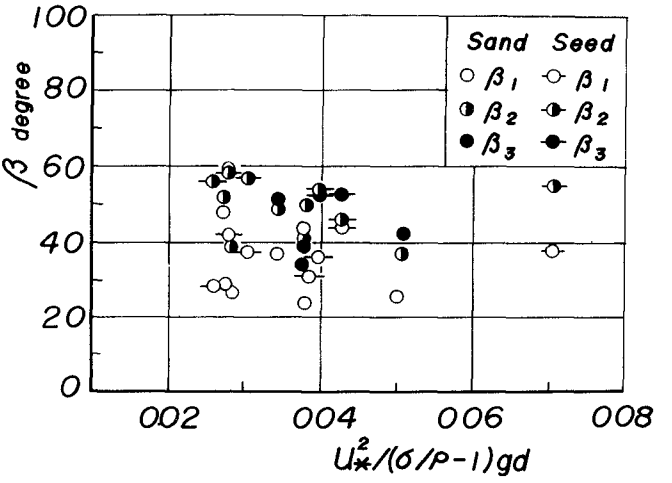


Fig 4 Variations of saltation angles with flow intensity

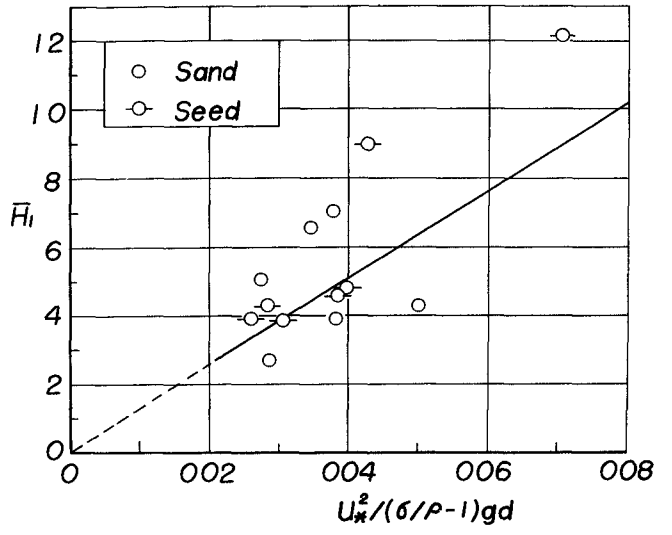


Fig 5 Comparison between theoretical relationship for the first saltation and experimental values

and β is determined by the best fit to the values of experiment which is estimated to be nearly 0 8 It is concluded from the comparison that the theoretical relationship for the first saltation is in good agreement with the experimental values though there is a large scatter

Fig 6 describes the variation of the value of λ which is the ratio of the saltation height to the distance with the increase of flow intensity It is seen that the value of λ is nearly constant and is estimated to be 0 23 The value is different from that in a water stream because of the difference of col-

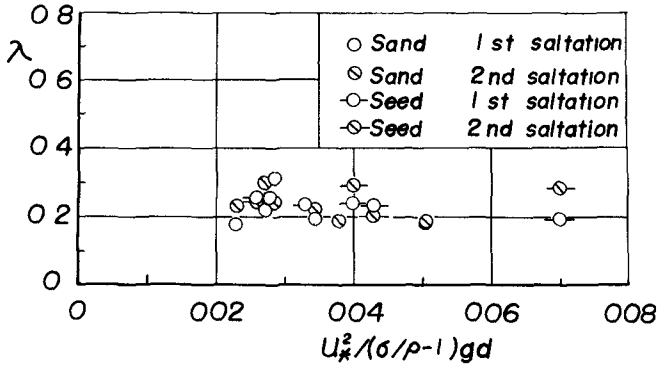


Fig 6 Variation of value of λ with flow intensity

lision characteristics

Figs 7 and 8 show the comparisons between the theoretical curves of the saltation height and distance in stationary saltation and the experimental values for the first, second and third saltations Experiments of more successive saltation than the third saltation could not be conducted due to the limitation of the experimental apparatus Therefore a complete comparison between the theory for the stationary successive saltation and the experiment cannot be made It is seen from the comparisons however that the experimental values in successive saltation tend to approach the theoretical curves for stationary saltation in which the value of A_r is assumed to be the value corresponding to H = 100 because the saltation height is not small and the effect of velocity profile on the saltation should be taken into consideration It is concluded that this theoretical approach to the saltation of a single sand grain by wind is in fairly good agreement with the results of experiments

CONCLUSION

Although the phenomena of saltation of a sand grain by wind are very complicated, there generally exists some kinds of saltation such as first saltation, successive saltation and stationary saltation as defined in this paper

A theory of successive saltation is established, based on the equations of motion of a sand grain and the dynamic relationship of the collision of a saltat-

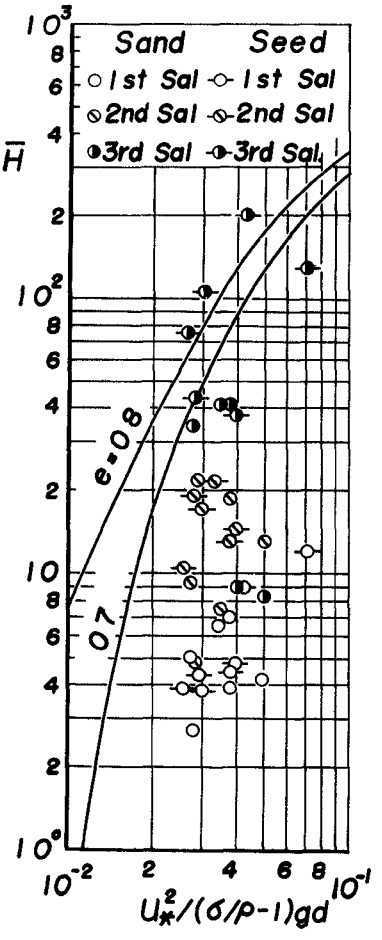


Fig 7 Comparison between theoretical curves of saltation height in stationary saltation and experimental values for first, second and third saltations

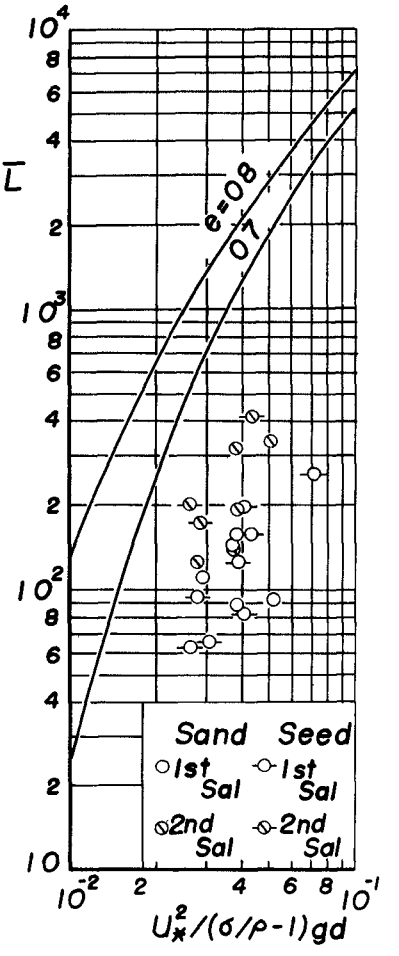


Fig 8 Comparison between theoretical curves of saltation distance in stationary saltation and experimental values for first, second and third saltations

ing sand grain with bed grains It is concluded that the theoretical relationships for the saltation are in fairly good agreement with the results of the experiment although further comparisons should be made

Further investigations on the saltation of sand grains will be conducted studying the saltation characteristics of sand grains in a water stream and comparing the results with the data of field observations in sand storms

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REFERENCES

Bagnold, R A (1941) The physics of blown sand and desert dunes Methuen & Co Ltd , London

Chepil, W S (1961) The use of spheres to measure lift and drag on wind-eroded soil grains Soil Sci Soc American Proc , Vol 25, pp 343-345

Einstein, H A and Sammi, E L (1949) Hydrodynamic forces on a rough wall Reviews of Modern Phys , Vol 21, No 3, pp 520-524

Iwagaki, Y (1956) Hydrodynamical study on the critical tractive force Trans JSCE, No 41, pp 1-21 (in Japanese)

Kawamura, R (1951) Study of sand movement by wind Report of Tech Res Inst , Univ of Tokyo, Vol 5, pp 95-112 (in Japanese)

Kishi, T and Fukuoka, S (1966) The mechanism of saltation of sand grains and the rate of sediment transport Proc 10th Conf on Hydraulics, JSCE, pp 59-64 (in Japanese)

Owen, P R (1964) Saltation of uniform grains in air Jour Fluid Mech , Vol 20, Part 2, pp 225-242

Tsuchiya, Y and Sumino, M (1967) Experiments on the mechanism of movement of a spherical sand particle on a granular bed in turbulent streams, Annuals, Disas Prev Res Inst , Kyoto Univ , No 10, pp 97-107 (in Japanese)

Tsuchiya, Y , Watado, K and Aoyama, T (1969) On the mechanism of saltation of a sand particle in a turbulent stream (1), Annuals, Disas Prev Res Inst Kyoto Univ , No 12B, pp 475-490 (in Japanese)

Tsuchiya, Y and Aoyama, T (1970) On the mechanism of saltation of a sand particle in a turbulent stream (2) -- On a theory of the successive saltation, Annuals, Disas Prev Res Inst , Kyoto Univ No 13B, pp 199-216 (in Japanese)

Yalin, M S (1963) An expression for bed load transportation, Proc ASCE, Vol 89, HY3, pp 221-250