CHAPTER 19

HORIZONTAL WATER PARTICLE VELOCITY OF FINITE AMPLITUDE WAVES

Yuichi Iwagaki
Professor
and
Tetsuo Sakai
Postgraduate

Department of Civil Engineering
Kyoto University
Kyoto, Japan

ABSTRACT

This paper firstly describes two methods to measure vertical distribution and time variation of horizontal water particle velocity induced by surface waves in a wave tank These two methods consist of tracing hydrogen bubbles and using hot film anemometers, respectively

Secondly, the experimental results by the two methods are presented with the theoretical curves derived from the small amplitude wave theory, Stokes wave theory of 3rd order, and the hyperbolic wave theory as an approximate expression of the cnoidal wave theory

Finally, based on the comparison of the experimental data with the theoretical curves, the applicability of the finite amplitude wave theories, which has been studied for the wave profile, wave velocity, wave length and wave crest height, is discussed from view point of the water particle velocity

INTRODUCTION

The water particle velocity induced by surface waves is one of the most important factors to solve the wave breaking mechanism, wave forces acting on submerged structures, mechanism of suspension and diffusion of materials by waves, and so on However, there have been very little experimental data[1,2,3,4], because of difficulty of the measurement Among the laboratory experiments of the water particle velocity, the data taken by Le Méhauté and others[4] using tracers of neutrally buoyant particles show by the comparison with the predicted values from the various small amplitude and finite amplitude wave theories that no theory is uniformly valid This conclusion makes us confused in applying finite amplitude wave theories to practical problems It is necessary, therefore, to check the validity of the conclusion

Recently, a new method to measure the water particle velocity was proposed, which is by using hydrogen bubbles as tracers, that is, tracing motions of hydrogen bubbles generated in water by electrolysis every very short period, to measure the water particle velocity induced by waves[5,6]. This method is a very useful tool to measure vertical distriburions of the water particle velocity at a particular phase of waves. On the other hand, in order to measure time variation of the water particle velocity, the hydrogen bubbles method is not so useful, but the use of a hot film anemometer is suitable, which was primarily developed for turbulence measurements in water[7]. These two methods were applied in this study to measure the water particle velocity induced by waves in a wave tank. As mentioned above, vertical distribution of the horizontal water particle velocity under the wave crest was measured by the method of tracing hydrogen bubbles, and time variation during one wave period at a definite height from the tank bottom was measured by a hot film anemometer.

This paper presents the experimental results of the horizontal water particle velocity obtained by these two methods, and comparison with theoretical values calculated from the small amplitude wave theory, Stokes wave theory of 3rd order[8], and the hyperbolic wave theory[9] as an approximate expression of the cnoidal wave theory of 2nd approximation[10]. Based on the comparison between the theories and the experiments, the applicability of the finite amplitude wave theories for the water particle velocity is discussed.

EXPERIMENTAL APPARATUS AND PROCEDURE

EXPERIMENTAL APPARATUS

The wave tank at Department of Civil Engineering, Kyoto University which has a wave generator of piston type was used in the experiments.

Method of tracing hydrogen bubbles. The experimental apparatus to measure the water particle velocity by this method is shown in Fig. 1. A platinum wire of 0.05mm in diameter was used as a negative pole to generate hydrogen bubbles in water. One end of the wire was attached to the tank bottom and stretched vertically, and the other end was connected with the pointgauge set a few centimeters high above the wave crest height. Four pieces of copper plates(15cm × 37cm), which were put on both side wall glasses of the wave tank arround the negative pole, were used as positive terminals. One wave gauge was mounted side by side with the negative pole in the transverse direction of the wave tank, while the other wave gauge was mounted as far as about 2m from it. Two wave gauges were connected to the recorder. The equipment to load pulse voltages on the terminal can supply output voltages of 400V, with the pulse period of 4 ∿ 700ms and the pulse width of 0.4 ∿ 70ms. Photographs of hydrogen bubbles generated along the platinum wire and moving with flow were taken with a camera through the side wall glass.

Method of using hot film anemometer. Two hot film anemometers of type 55D05 and their probes of type 55D85 made by DISA were used. The anemometer amplifies electrically heat convected from the probe which is one of bridge resistances. The probe, as shown in Fig. 2, consists of a thin metal film as electrically heated resistance and its support. This anemometer is of

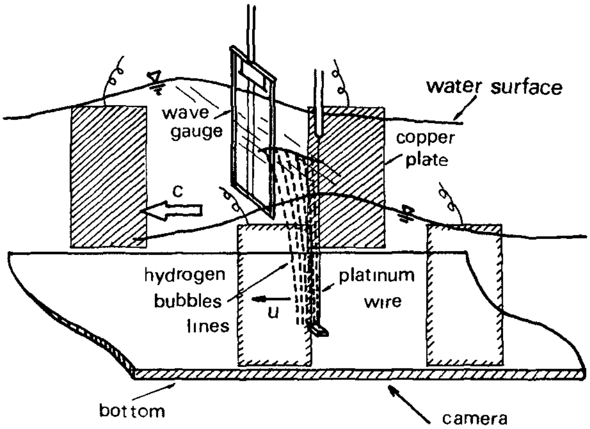

Fig 1 Schematic figure for measurement of water particle velocity by method of tracing hydrogen bubbles

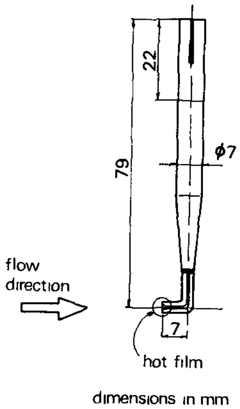

Fig 2 Probe of hot film anemometer

constant-temperature type designed for measuring energy required to keep always probe resistance(therefore probe temperature) constant(operating resistance)

When fluid flows as shown in Fig 2, the probe can detect its current velocity, but not its opposite current velocity Therefore, if the probe is mounted contronting waves as shown in Fig 3(1), the record of the anemometer is reliable when the water level is higher than the still water level(full line), but not reliable when the water level is lower than the still water level and the flow direction becomes opposite In addition, the probe can not detect opposite change of the flow direction On the other hand, if the probe is mounted following waves as shown in Fig 3(2), the record is reliable even when water level is lower than the still water level(full line), but the other part of the record is not reliable

In the experiments, two probes and one wave gauge were mounted side by side in the transverse direction Especially, two probes of the hot film were set at the same height, confronting each other(see Fig 4) The other wave gauge was mounted as far as about 2m from the probes All of two hot film anemometers and two wave gauges were connected to the recorder

EXPERIMENTAL PROCEDURE

Method of tracing hydrogen bubbles Among the waves generated by the wave generator, one wave which became fully stable and was not influenced by the reflected waves from another end of the tank was selected Just before the crest of this wave passed over the negative pole, pulse voltage of a proper period Δt began to be loaded Lines of hydrogen bubbles generated along the platinum wire are transported by the flow in the direction of wave propagation under the wave crest The photograph in this state was taken by a 35mm camera At the same time, the records of waves at the wire position and as far as about 2m from the wire were taken When pulse voltage is loaded on the terminal, the wave gauge can detect it and the disturbance appears in the record Therefore, the time t_0 when the first pulse was loaded can be detected (if the time when the crest of a selected wave passes over the wire is put as $t=0$, $t_0<0$)

The m th and (m+1) th hydrogen bubble lines were selected on the film, so that the arithmetic average $t_0+\{m-(1/2)\}\ \Delta t$ of $t_0+(m-1)\ \Delta t$ and $t_0+m\ \Delta t$ becomes the smallest(see Photo 1) Reading the distance between these m th and (m+1) th lines and dividing it by Δt, the horizontal water particle velocity u at the phase of $t/T=[t_0+\{m-(1/2)\}\ \Delta t]/T$ was obtained approximately(T the wave period) This process was repeated at different heights above the tank bottom, and the vertical distribution of the horizontal water particle velocity was obtained

Test conditions are shown in Table 1, Where h is the water depth, H the wave height, and p the pulse width The opening and shutter speed of the camera were always 1 4 and 1/125sec

Method of using hot film anemometer After the value of operating resistance of two hot film anemometers was set to be 1 00 $\sim$ 1 15 time of cold resistance value, the calibration curves of two anemometers were determined by

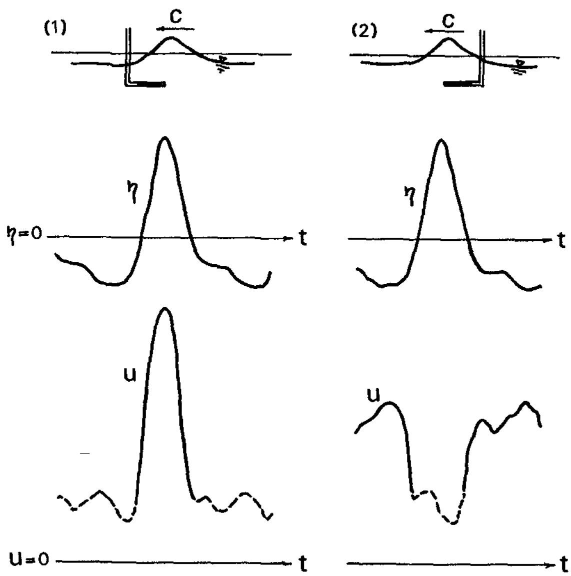

Fig 3 Relation between direction of probe and record of hot film anemometer

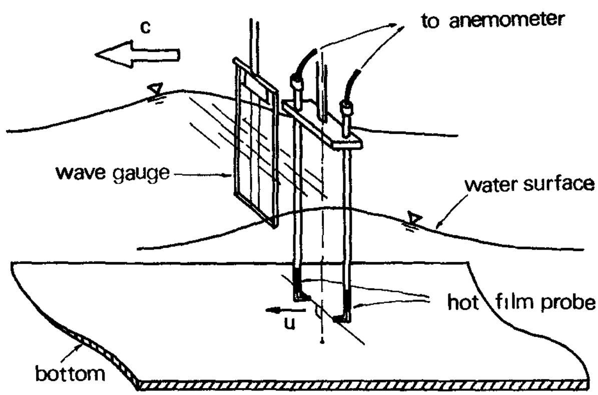

Fig 4 Schematic figure for measurement of water particle velocity by hot film anemometer

Photo 1 Hydrogen bubble tracer

Table 1 Conditions of experiments by method of tracing hydrogen bubbles

h (cm)	T (sec)	H (cm)	Δt (msec)	p (msec)	$T\sqrt{g/h}$	H/h
30.5	0.93	9.08	40	20	5.3	0.298
20.9	0.96	7.22	20	10	6.6	0.345
16.0	0.94	5.22	20	10	7.4	0.326
13.0	0.95	3.98	20	10	8.3	0.306
16.0	1.10	4.91	20	10	8.6	0.307
13.0	1.10	4.13	20	10	9.6	0.318
29.8	1.74	7.12	50	20	10.0	0.239
21.0	1.74	6.47	30	15	11.9	0.308
21.0	1.76	6.44	20	5	12.0	0.306
21.0	1.89	6.24	20	10	12.9	0.297
16.0	1.73	5.18	20	5	13.6	0.324
13.0	1.93	3.66	20	7	16.8	0.282

moving two probes with the carrier The records of two confronting anemometers were read out using two calibration curves The full lines in Fig 3(1) and (2) were selected, which are the part of the record of the anemometer confronting waves and that of the other anemometer following waves These two curves were drawn up side down each other on the same figure, matching the phases of the waves, as shown in Fig 5 In general, as mentioned above, two records do not represent zero values but overlap where the real values of the velocity are zero In order to obtain the time variation of the horizontal water particle velocity during one wave period, the records were modified assuming that the phase of u=0 was located where the absolute value of the upper record was equal to that of lower one, as shown in Fig 5

In the experiments, the water depth h was constant, and the wave period T and the wave height H were variable Test conditions are shown in Table 2, where z is the ordinate taken upwards from the still water level, and z_p is the depth of the probe, so that $z_p+h(z_p<0)$ means the height of the probe above the tank bottom As shown in Table 2, the value of $(z_p+h)/h$ was always 0 05

EXPERIMENTAL RESULTS

Fig 6(1)∿(12) show the experimental results of the horizontal water particle velocity at the phase of the wave crest measured by the method of tracing hydrogen bubbles under the conditions of Table 1 $u/\sqrt{gh}$ as dimensionless expression of u(g the gravity accerelation) is taken as an abscissa, and (z+h)/h as dimensionless expression of the height from the bottom taken as an ordinate, with parameters of $T\sqrt{g/h}$, H/h and the wave phase t/T In this figure, the experimental data are denoted by circles, and, for the same values of $T\sqrt{g/h}$, H/h and t/T, the vertical distributions based on the small amplitude wave theory, Stokes wave theory of 3rd order[8] and the hyperbolic wave theory proposed by Iwagaki[9] as an approximate expression of the cnoidal wave theory of 2nd approximation[10] are shown for comparison with full, broken and chain lines, respectively

The thick full lines in Fig 7(1)∿(12) show the profiles of the waves measured simultaneouly with the vertical distribution of the velocity As in Fig 6, the wave profiles based on the three theories are also shown for comparison in the figure

The thick full lines in Fig 8(1)∿(10) show the continuous records of time variation of the horizontal water particle velocity during one wave period at $(z_p+h)/h$=0 05 measured by the hot film anemometers under the condition of Table 2 and the wave profiles recorded simultaneouly Other three thin lines show the theoretical curves as in Fig 7

DISCUSSION

The error of the velocity measured by the method of tracing hydrogen bubbles, which results mainly from the error of reading films and the error of pulse period, will be approximately 3% Also, the error of the velocity measured by the hot film anemometer occurs in the stage of determining the calibration curve, which will be approximately 3% too

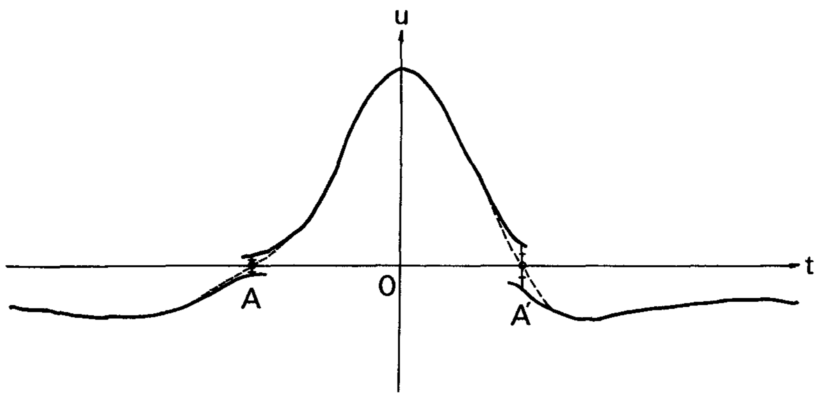

Fig 5 Schematic diagram for modification of record of water particle velocity by hot film anemometer

Table 2 Conditions of experiments by using hot film anemometer

h (cm)	T (sec)	H (cm)	z_p+h (cm)	$T\sqrt{g/h}$	H/h	(z_p+h)/h
16 0	1 03	6 72	0 80	8 1	0 420	0 05
16 0	1 30	6 86	0 80	10 2	0 429	0 05
16 0	1 58	6 04	0 80	12 4	0 378	0 05
16 0	1 83	7 21	0 80	14 3	0 448	0 05
16 0	2 08	7 05	0 80	16 3	0 441	0 05
16 0	2 39	4 89	0 80	18 7	0 306	0 05
16 0	2 63	5 06	0 80	20 6	0 316	0 05
16 0	2 84	4 13	0 80	22 2	0 258	0 05
16 0	3 07	4 11	0 80	24 0	0 257	0 05
16 0	3 40	3 66	0 80	26 6	0 229	0 05

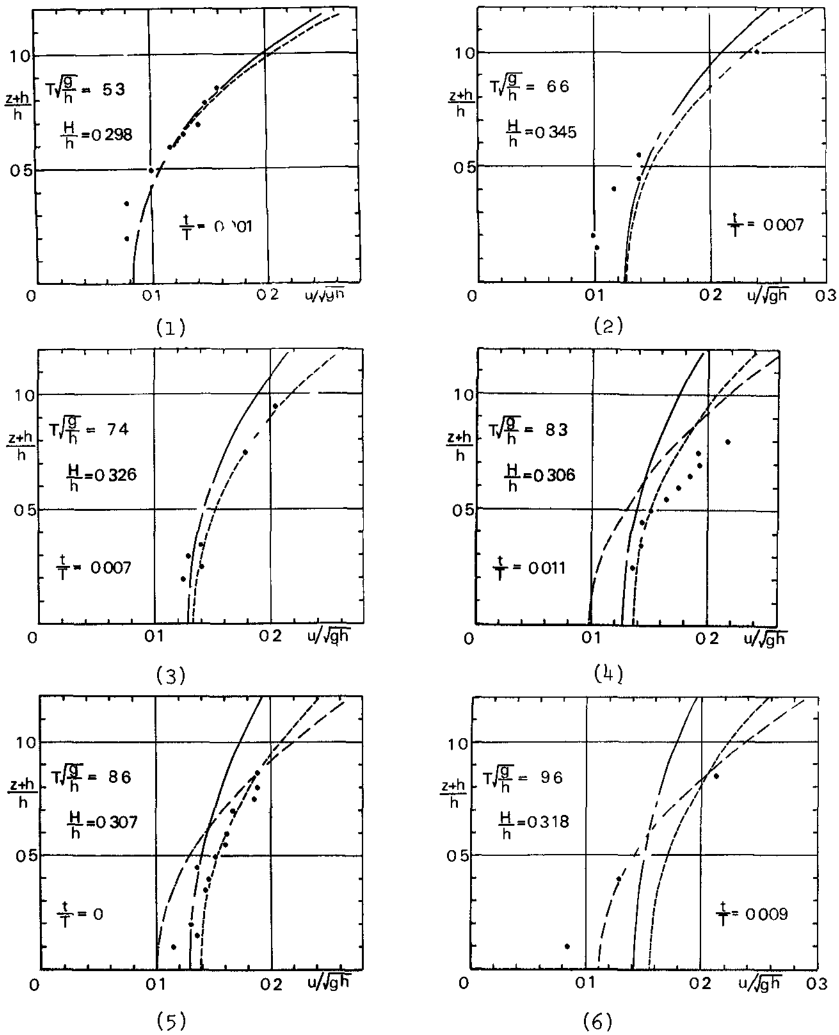

Fig 6(1)~(6) Vertical distribution of horizontal water particle velocity at phase of wave crest(full line small amplitude waves, broken line Stokes waves, chain line hyperbolic waves)

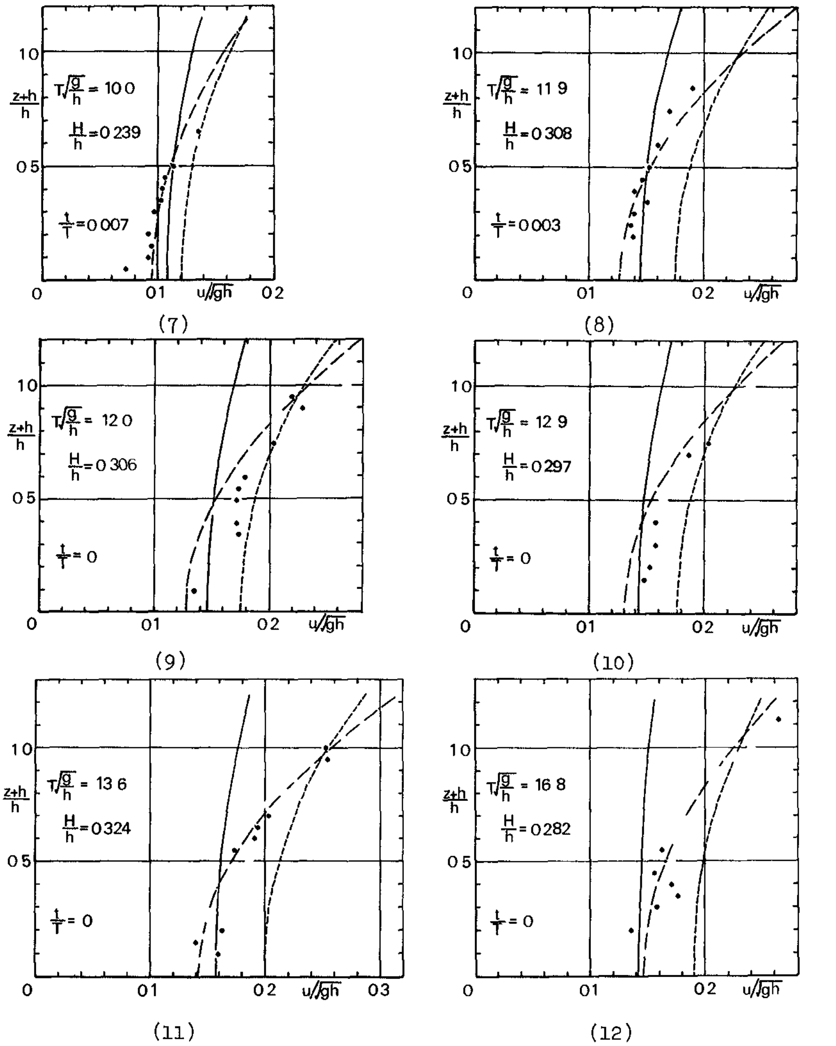

Fig 6(7)∿(12) Vertical distribution of horizontal water particle velocity at phase of wave crest (full line small amplitude waves, broken line Stokes waves, chain line hyperbolic waves)

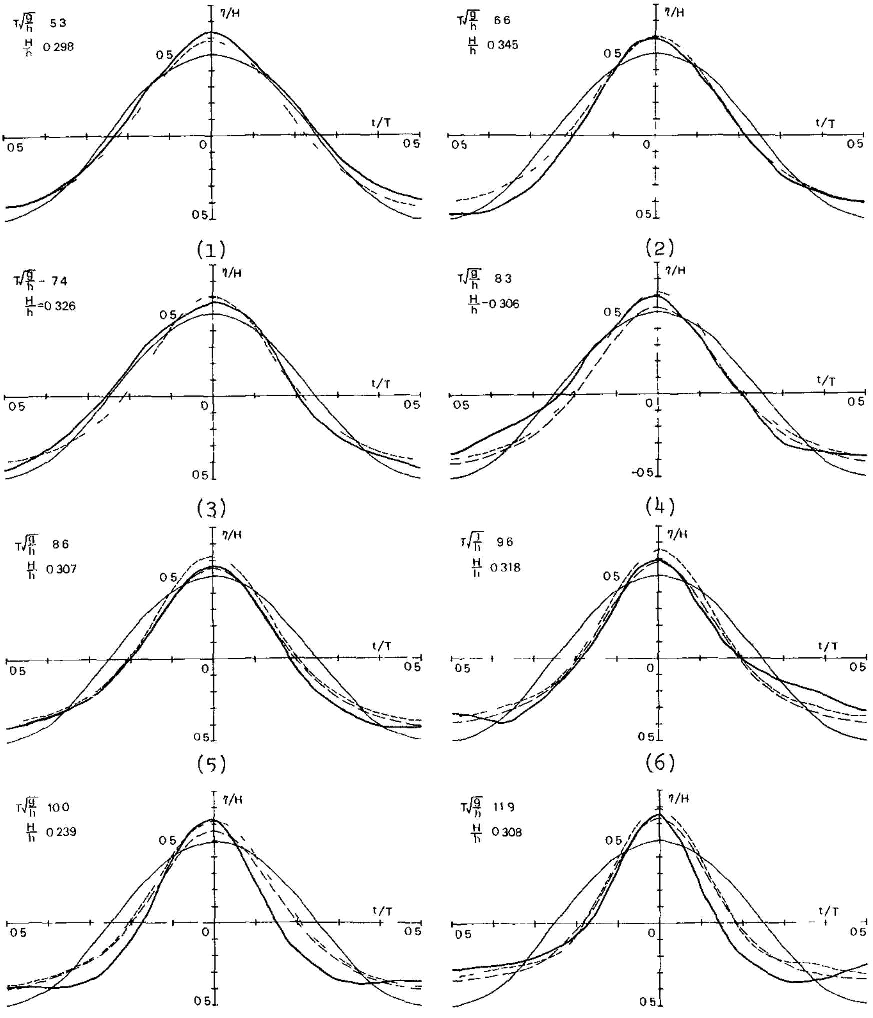

Fig 7(1)∿(8) Time variation of water surface corresponding to Fig 6(thick full line experiment, thin full line small amplitude waves, thin broken line Stokes waves, thin chain line hyperbolic waves)

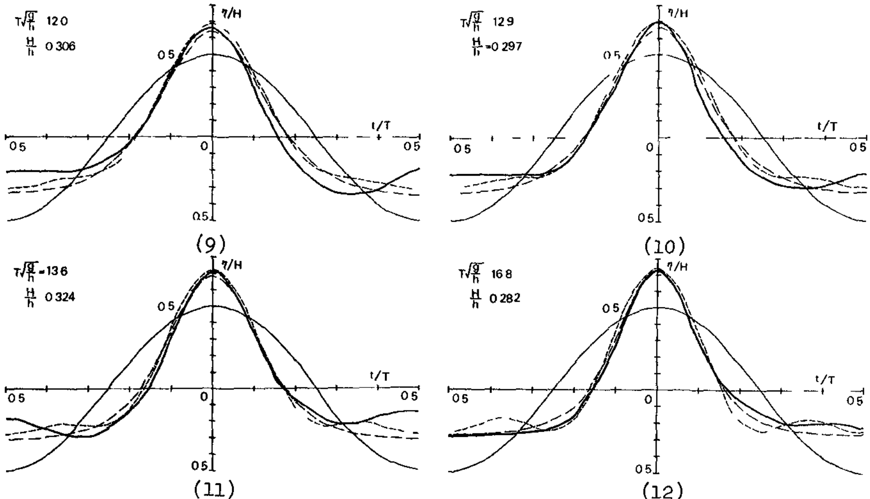

Fig 7(9)∿(12) Time variation of water surface corresponding to Fig 6(thick full line experiment, thin full line small amplitude waves, thin broken line Stokes waves, thin chain line hyperbolic waves)

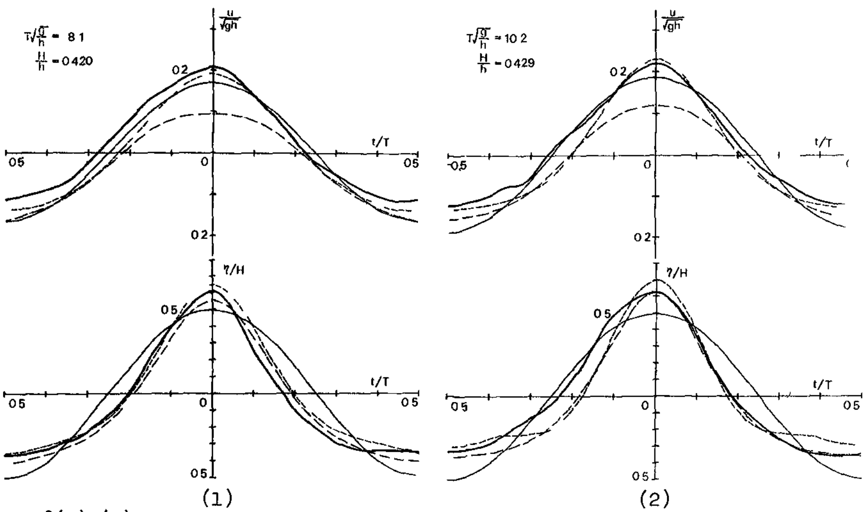

Fig 8(1),(2) Time variation of horizontal water particle velocity and corresponding water surface(thick full line experiment, thin full line small amplitude waves, thin broken line Stokes waves, thin chain line hyperbolic waves)

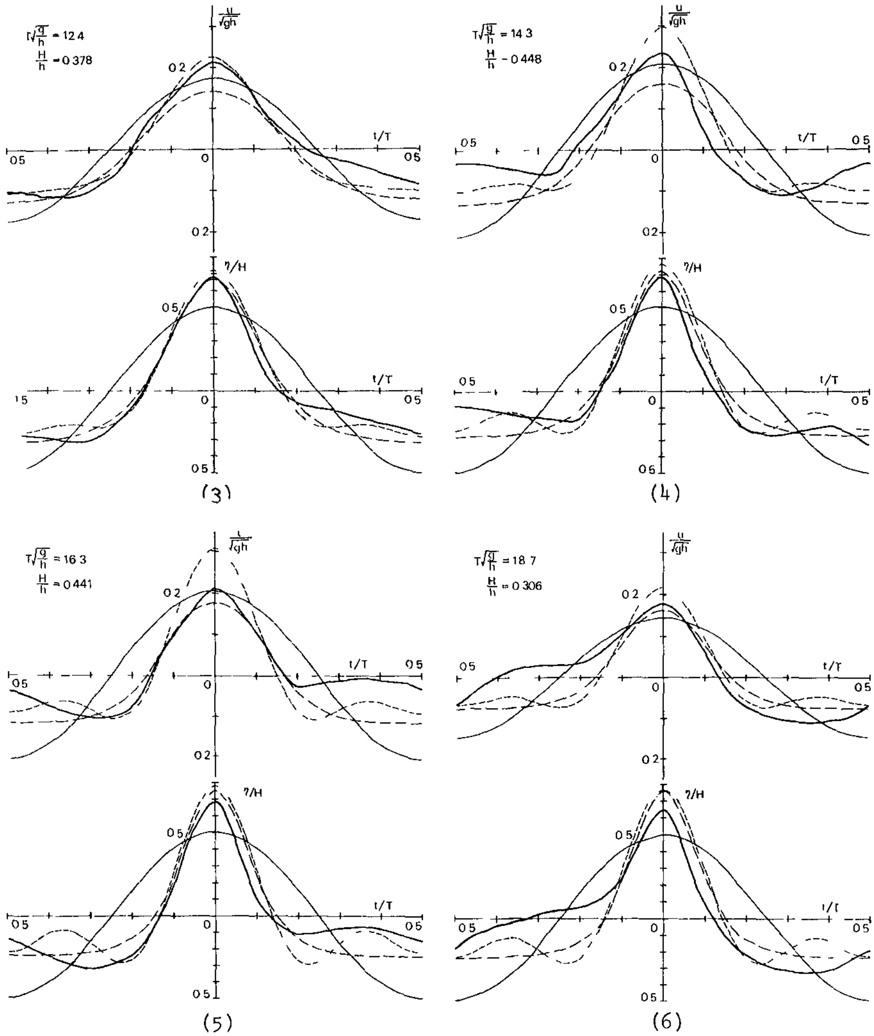

Fig 8(3)∼(6) Time variation of horizontal water particle velocity and corresponding water surface (thick full line experiment, thin full line small amplitude waves, thin broken line Stokes waves, thin chain line hyperbolic waves)

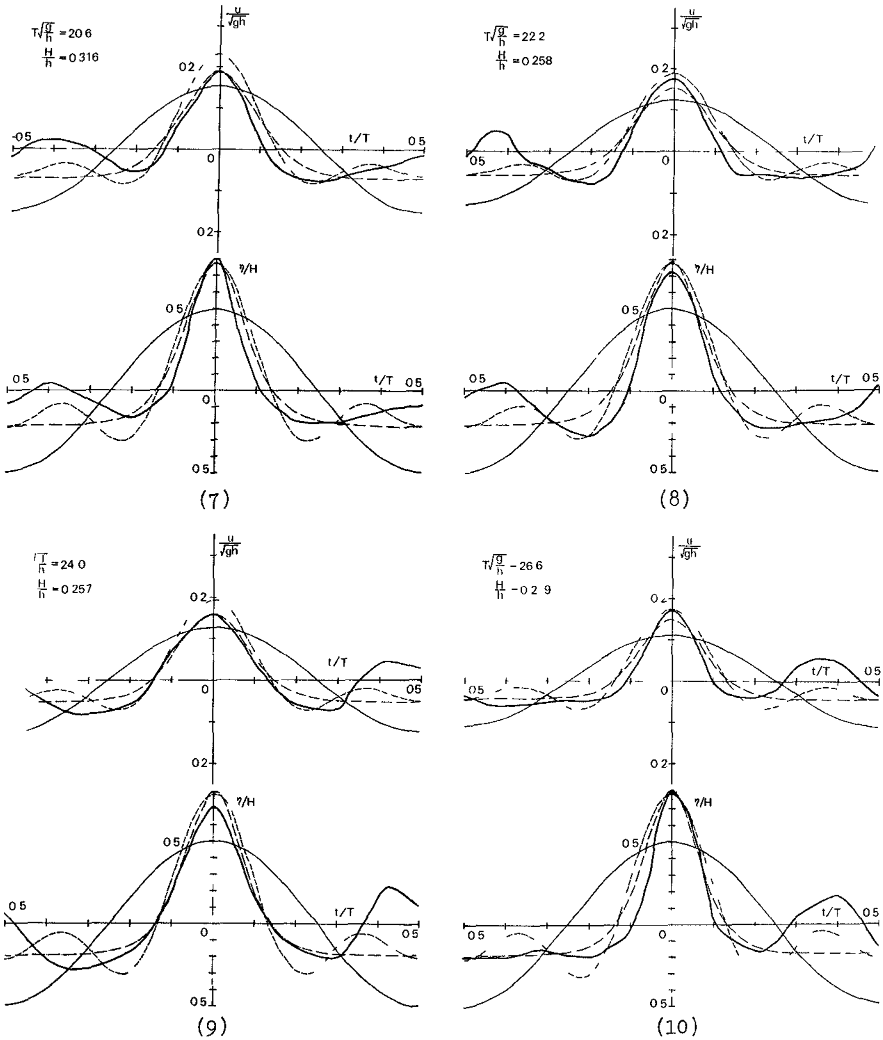

Fig 8(7)∿(10) Time variation of horizontal water particle velocity and corresponding water surface(thick full line experiment thin full line small amplitude waves, thin broken line Stokes waves, thin chain line hyperbolic waves)

In Fig 6, the values of wave phase t/T are not always zero, but the absolute values exist between 0 and 0 01, which means that the data taken are regarded as at the phase of the wave crest Generally speaking, among the theoretical curves of vertical distributions of the horizontal velocity, those of the small amplitude wave theory are steepest and the curves of the hyperbolic wave theory are most gentle The value of the velocity based on the small amplitude wave theory is smaller all over the depth than that of Stokes wave theory, and the difference between them becomes largest at the water surface On the other hand, the value based on the hyperbolic wave theory, in most cases, is smaller than that of the small amplitude wave theory near the bottom, but larger than that of Stokes wave theory near the water surface

The experimental data were obtained generally at every height of 5% of the water depth from the bottom However, in the region where $(z+h)/h<0$ 1, the velocity field induced by waves was disturbed by the obstacle of 5mm high to attach the platinum wire to the bottom, and in the region where $(z+h)/h>1$ 0 the velocity could not be measured because of insufficient generation of bubbles In the case when $T\sqrt{g/h}=5$ 3, the experimental values have the same trend as two theoretical distributions based on the small amplitude wave and Stokes wave theories In the cases when $T\sqrt{g/h}=6$ 6 and 7 4, the experimental values agree fairly with those of Stokes waves In the case when $T\sqrt{g/h}\geqq 8$ 3, the vertical distributions of the velocity based on the hyperbolic wave theory are also presented In cases when $T\sqrt{g/h}>8$ 6, the experimental data are plotted between the theoretical curves of Stokes and hyperbolic waves In general, when $T\sqrt{g/h}\geqq 10$, the vertical distribution of the velocity under the wave crest can be explained well by the hyperbolic wave theory as an approximate expression of the cnoidal wave theory rather than Stokes wave theory

Fig 7(1)∿(12) show the comparisons between theories and experiments of the wave profile corresponding to each case of Fig 6 as mentioned above It is evident that the experimental results of the wave profile are close to those of two finite amplitude wave theories rather than that of the small amplitude wave theory However, it is very difficult to decide which finite amplitude wave theory is fit to the experiment on the wave profile from the figures In other words, the difference between two finite amplitude wave theories is not clear in the case of the wave profile, but very clear for the vertical distribution of the velocity It should be noticed that this fact is very significant when the wave force acting on submerged structures is estimated by the theory Especially remarkable differences of the velocity between Stokes wave and hyperbolic wave theories appear near the bottom and above the still water level

The probes of the hot film anemometer were mounted as high as 5% of the water depth above the bottom rather than near the water surface, because of avoiding risk of damage by exposing the probe in air Tendencies of three theoretical curves of time variation of the velocity shown in Fig 8 are that the theoretical curves of Stokes and hyperbolic waves are sharper near the phase of the wave crest and flatter near the phase of the wave trough than that of the small amplitude wave theory, in the same manner as in the wave profile However, in the cases when $T\sqrt{g/h}=8$ 1 and 10 2, the velocity of hyperbolic waves at the trough is greater than that at the crest This trend, which is of the cnoidal wave theory itself, occurs near the bottom only when

$T\sqrt{g/h}$ <12 0 and the value of H/h is large Under the wave crest, the theoretical value of $u/\sqrt{gh}$ of Stokes waves is largest and that of hyperbolic waves is smallest near the bottom when $T\sqrt{g/h} \leqq 16$ 3, and the value of hyperbolic waves shifts in the middle of small amplitude waves and Stokes waves when $T\sqrt{g/h} \geqq 18$ 7

The experimental results of time variation of the horizontal velocity during one wave period show that, in general, the wave crest is sharper and the wave trough is flatter than those of small amplitude waves, in the same manner as the theoretical curves of finite amplitude waves It is seemed that the experimental curves near the crest are close to the theoretical values of Stokes waves when $T\sqrt{g/h} \leqq 12$ 4 and to those of hyperbolic waves when $T\sqrt{g/h} \geqq$ 16 3, although secondary waves appear at the wave trough In the vertical distributions of the velocity shown in Fig 6, as mentioned above, the experimental values agree fairly with the hyperbolic wave theory when $T\sqrt{g/h} \geqq$ 10 However, in the experimental results of time variation of the velocity, the experimental values agree roughly with the theoretical curves of Stokes waves at least when $T\sqrt{g/h} \leqq 12$ 4 One of reasons of the discrepancy may be the difference of the values of H/h as shown in Table 1 and 2, that is, when $T\sqrt{g/h}$ $\leqq 16$ 3 in Fig 8, values of H/h are greater than 0 4 in most cases, and considerably large compared with those in Fig 6

In Fig 8, measured wave profiles are close to theoretical profiles of finite amplitude waves rather than those of small amplitude waves However, differences between measured profiles and computed profiles based on two finite amplitude wave theories are not so clear as in the horizontal velocity

In all cases of the experiments by the hot film anemometer, Reynolds numbers defined as

$$N_R=\sqrt{\pi/\nu T}\quad H/\sinh(2\pi h/L)$$

were calculated(ν the kinematic viscosity) The maximum value among them is smaller than the critical Reynolds number N_R=160 for transition from laminar to turbulent boundary layer by waves[11] Also, it was confirmed that the calculated thickness of the laminar boundary layer in this case, which is assumed to be the height from the bottom where the velocity becomes 99% of the velocity at the outside of the boundary layer, was 4 8mm at maximum and smaller than the height of the probes from the bottom, 8mm

CONCLUSION

The authors described two methods to measure the horizontal water particle velocity induced by waves in the wave tank, the method of tracing hydrogen bubbles and the method of using a hot film anemometer Experimental results of the vertical distribution of the velocity under the wave crest measured by the former method and time variation of the velocity during one wave period at a definite height from the bottom by the latter method were presented Discuccions were made compared with the theoretical values based on the small amplitude wave theory, Stokes wave theory of 3rd order and the hyperbolic wave theory as an approximate expression of the cnoidal wave theory of 2nd approximation

After all, the following conclusions were obtained The region, where the hyperbolic wave theory should be applied to the horizontal water particle velocity rather than Stokes wave theory, is $T\sqrt{g/h} \geqq 10$, which was found for its vertical distribution under the wave crest, and $T\sqrt{g/h} \geqq 14$ for its time variation during one wave period It should be noted that, in computing the wave force acting on submerged structures, Stokes wave theory may give too small values of the horizontal water particle velocity at the water surface in the region $T\sqrt{g/h} \geqq 10$

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REFERENCES

1) Morison, J R and R C Crooke The Mechanics of Deep Water, Shallow Water, and Breaking Waves, U S Army, Corps of Engineers, B E B , Tech Memo No 40, March, 1953, pp 1-14
2) Elliott, John G Interim Report, Hydrodynamics Laboratory, Calif Inst of Tech , Contract Noy-12561, U S Navy, Bureau Yards and Docks, July, 1953
3) Goda, Y Wave Forces on a Vertical Circular Cylinder, Experiments and a Proposed Method of Wave Force Computation, Port and Harbour Technical Research Institute, Ministry of Transportation, Japan, Repert No 8, 1964
4) Le Méhauté, B , D Divoky and A Lin Shallow Water Waves A Comparison of Theories and Experiments, Proc 11th Conference on Coastal Engineering, 1968, pp 86-107
5) Schraub, F A , S J Kline, J Henry, P W Runstadler, Jr and A Littell Use of Hydrogen Bubbles for Quantitative Determenation of Time-Dependent Velocity Fields in Low-Speed Water Flows, Trans ASME, Jour of Basic Engineering, Vol 87, 1965, pp 429-444
6) Horikawa, K and A Watanabe Laboratory Study on Oscillatory Boundary Layer Flow, Proc 11th Conference on Coastal Engineering, 1968, pp 467-486
7) Richardson, E V and R S McQuivey Measurement of Turbulence in Water, Jour of the Hydraulics Division, ASCE, Vol 94, No HY2, Proc Paper 5855, March, 1968, pp 411-430
8) Skjelbreia, L Gravity Waves, Stokes' Third Order Approximation, Table of Function, Council on Wave Research, The Engineering Foundation, 1959, pp 1-337
9) Iwagaki, Y Hyperbolic Waves and Their Shoaling, Proc 11th Conference on Coastal Engineering, 1968, pp 124-144
10) Laitone, E V The Second Approximation to Cnoidal and Solitary Waves, Jour of Fluid Mechanics, Vol 9, 1961, pp 430-444
11) Collins, J I Inception of Turbulence at the Bed under Periodic Gravity Waves, Jour of Geophysical Research, Vol 68, No 21, 1963, pp 6007-6014