CHAPTER 8

DIRECTIONAL SPECTRA FROM WAVE-GAGE ARRAYS

by

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SYNOPSIS

The ocean surface may be considered to be composed of many waves traveling at different directions with different frequencies A graphical plot showing the allocation of wave energy to the different component frequencies and directions is the directional spectrum Directional spectrum has many applications in Coastal Engineering Herein an analytical procedure is developed to obtain the directional spectrum from records of an array of wave gages The two methods developed are the "locked phase method" and the "random phase method " The locked phase method can be used to obtain the distribution of both phase as well as energy of the waves with respect to frequency and direction and is a deterministic approach The random phase analysis, on the other hand, is more suitable for wind waves in the ocean and yields just the distribution of energy alone as in most other procedures of spectrum analysis The procedures programmed for computers are checked using simulated data and laboratory data Wave records of the Pacific Ocean obtained off Point Mugu, California, on a 5-gage array were analyzed using the method developed and examples of the directional spectra obtained are presented

INTRODUCTION

When confronted with a design or operation in the ocean environment, an engineer invariably needs to know how high the waves are and from which direction they are coming Had ocean waves been single sinusoids, one could immediately obtain the height and direction of the waves But waves in the ocean do not look like sinusoids At best the ocean surface may be thought of being the result of adding together very many sinusoids of various frequencies traveling in various directions Therefore one has to specify the particular wave for which the height and direction are desired As frequency, i e , inverse of the period of the component wave, is the least easily changeable wave parameter, the component wave may be specified by its frequency So, the problem is to obtain the wave amplitude a_1 and direction θ_1 for the various component frequencies, f But to consider the ocean surface to be composed of a finite number of sinusoids is a poor approximation A better approximation would be obtained if one lets the number of component simple harmonic waves to approach infinity and the frequency interval, Δf , between them to approach zero The individual wave amplitudes, a_1 , in this case must approach zero in order for the overall wave heights to maintain a finite, mean square value However, in this limiting process the quantity $a_1^2/2\Delta f$ remains finite and therefore it can be plotted as a continuous function of frequency If the quantity $a_1^2/2\Delta f$ is plotted against frequency, the area under the curve within the frequency band of Δf is $a_1^2/2$, but this is proportional to the wave energy contributed by that frequency band because the energy of a simple harmonic wave of amplitude a_1 in a medium of unit weight γ is $\gamma a_1^2/2$ per unit width of crest, see, for example, Wiegel (44) In the same manner the number of component directions may be assumed to approach infinity to yield a continuous function of direction and the area under that curve for any angular width will be proportional to the energy of the waves traveling in those directions for a particular frequency Alternately, the above two plots can be combined into a two-dimensional plot which will show the distribution of energy with respect to frequency and direction This is the Directional Spectrum

USES OF DIRECTIONAL SPECTRA

Directional spectrum shows the distribution of wave energy against frequencies and directions Therefore, it specifies the wave climate more completely than any other way, ideally When only some particular information like the predominant wave period and height alone are needed, that can be obtained from the directional spectrum R M S wave height, for example, is the volume under the directional spectrum Scott (37) and Neumann and Pierson (34) provide equations for the significant wave height from R M S wave height assuming different distributions for the frequency spectrum As Wiegel (46) pointed out, there are still situations where the significant wave concept is useful to the design engineer, but it might be possible to obtain that information from the directional spectrum Nevertheless, there are many engineering problems where directional spectra are the necessary input data for the correct design or prediction For example, for the study of the diffraction of wind waves directional spectrum is necessary and, in fact, Wiegel and Mobarek (45) and Fan (19) successfully used it for the study of diffraction of laboratory wind waves Refraction of ocean waves is another problem where the directional spectrum is needed and Karlsson (24) used it for practical cases In the design of offshore towers and piles directional spectrum is useful for the analysis of vibration and three-dimensional analysis of structures with torsional loads. Malhotra and Penzien (29) have succeeded in developing the procedure for analyzing tower structures using the spectrum and indicated the need for the directional spectrum. For structures subjected to random forces, in general, a design by the method of simulation on a computer is highly suited and directional spectra are the suitable input data. Borgman (11) showed how this can be done for offshore pile structures by linearizing wave forces. Directional spectrum is necessary for the prediction of the response of ships and floating drilling vessels to sea conditions as the spectra of their motions can be obtained from the directional spectra of the sea. It is, in fact, used by naval architects (see for example, Abkowitz, Vassilopoulos and Sellars (1)). Even for the design of an ocean outfall sewer, directional spectrum may be useful because, as Wiegel (43) pointed out, mixing and wave spectra are related. Once a single wave model for longshore transport is established, directional spectrum may prove to be specifically cut out for the study of littoral transport because it provides at once the three crucial parameters of the problem, viz. the wave energy, frequency and direction. Another major use for directional spectrum is in wave forecasting and hindcasting. In hindcasting the location of the origin of storms and the path of swell can be deduced from the variation of the non-stationary directional spectra, see Munk et al. (32).

When directional spectra become available for desired locations, several other uses also may be found for them. The problem at present is its non-availability for almost any place. Here again, barring the huge expenses involved in the collection of necessary data, the main hurdle is the lack of a valid, dependable computational procedure to obtain directional spectra in a routine manner.

REVIEW OF METHODS USED

The most direct way of obtaining the directional spectrum is to obtain the sea surface elevations over an area by stereophotographs and to analyze these data to get the directional spectra. This was done in the Atlantic Ocean by W. J. Pierson and his group, see Cote et al. (17), and in a much more modest scale by Ijima et al. (23). From the story of the Stereo Wave Observation Project [Cote et al. (17)] one realizes how arduous and expensive this method is. Kinsman (25) therefore doubts whether it will ever become habit forming. Another successful method applicable for deep ocean is to take records of the elevation and tilt of a free floating buoy as used by Longuet-Higgins, Cartwright and Smith (27). From the water surface elevation and the slopes in two coordinate directions they computed the first five coefficients (i.e. two harmonics) of the Fourier series expansion of the directional spectrum. To remove the appearance of negative energy they used a smoothing function $\omega_2 = \frac{8}{3}\cos^4\frac{\varphi}{2}$. The net result of having a directional spectrum represented only up to two harmonics and then smoothing it is to get a very broad angular width for the spectrum. If more accuracy is needed, records of the water surface curvatures are to be obtained. Ewing (18) also obtained a sequence of ten records of the directional spectra from the motions of a floating buoy located in the North Atlantic

The use of an array of wave recorders to measure wave parameters and the computation of directional spectrum from it was tried by several workers Barber (3) was probably the first to suggest it The wave-gage array could be one-dimensional (line array) or two-dimensional If one can make sure that no wave comes from one of the sides of a line array, a line array can be used to obtain directional spectrum Barber and Doyle (4), described a procedure to get the directions of swells by using just two gages Stevens (39) described the procedure to obtain directional spectrum from a line array and used it for an array in Buzzards Bay Macovsky and Mechlin (28) described a possible method of using a line array of inverted acoustic fathometers mounted on the deck of a submarine Two-dimensional arrays, in general, would be more appropriate for a general situation Barber (6) discussed a general theory of gage arrays and suggested ways to compare the directional resolving power of different arrays Mobarek (31) found that the discrete energy method was the most successful of the methods for the estimation of directional spectra for laboratory wind waves Fan (19) used a 4-gage array in the form of a star and compared the Fourier transform method and Least Square Method by simulation technique to obtain the directional spectra He found that the Fourier Transform Method gave better results for higher frequency components and the Least Square Fitting Method gave better results for lower frequency components Two-dimensional arrays have also been used in the ocean Munk, Miller, Snodgrass and Barber (32) used three bottom pressure gages forming an equilateral triangle with sides about 900 ft in 330 ft of water and obtained the direction of long period swell from the data Bennett, Pittman and Austin (7) and Bennett (8) described a 6-gage array in the form of a Pentagon with one gage at the center, which was used off Panama City, Florida in the Gulf of Mexico at depths of 63 ft and 104 ft Bennett (8) essentially used the procedure of Munk et al (32) by fitting a single wave of a particular frequency to the cross-spectrum equations This may, perhaps, be sufficient to obtain the direction of long period swell But as Tukey (42) pointed out, in analyzing or thinking about a computational process involving several layers of approximations, or the propagation of sampling fluctuations through several layers of tranformations, step-by-step analysis is not likely to be enough and an analysis of the overall process is needed

A few other methods have also been reported Nagata (33) measured orbital motions with electromagnetic current meters and used it to obtain the directional spectra Ford, Timme and Trampus (20) used a triset sensor made up of three vertical surface-penetrating wave staffs located at the corners of a right triangle of side about 5 ft to obtain the three outputs, viz two components of wave slope and the average wave amplitude at the sensor From these they calculated the directional spectrum accurate up to 2 harmonics just in the same way as Longuet-Higgins et al (27) did Simpson (38) had a similar arrangement but the probes at the apices of the right triangle measured orbital velocities instead, and the side of the triangle was about 3 ft By this arrangement he could obtain the first 4 harmonics of the directional spectra Suzuki (40) proposed another method of determining the directional spectra of sea waves using a wave gage and a wave direction meter which can record X and Y component of wave force acting on a bottom mounted sphere

There may, perhaps, be many more ingenious ways of obtaining the directional spectra But it seems to the authors that wave gage arrays might be the most convenient arrangement for collecting data in a routine way to determine the directional spectra Hence a general theory for the determination of directional spectra from records of wave gage arrays and a computer program for it were developed A brief description of the theory and the results obtained are described below The equations for use when the wave gages measure surface elevations are presented here The detailed development of the general case will be given in a separate report

A THEORY FOR GAGE ARRAYS

The Modes of Analysis

Two modes of analysis are developed - the locked phase method and the random phase method The locked phase mode of analysis is essentially a deterministic approach where the phases of the component waves are assumed to be fixed Hence the analysis provides both the distribution of energy with frequency and direction as well as the distribution of phase angle with frequency and direction In the random phase method the phases of component waves are considered to be random and independent of each other, hence they average out in the analysis The locked phase mode of analysis is appropriate to situations where phase is locked to particular values, such as in a wave tank with flapper The random phase mode of analysis, on the other hand, is applicable to situations where phase changes randomly with time as in narrow band surf or wind waves

Locked Phase Mode of Analysis

The wave surface elevation, q, at a given instant of time t, is considered to be the result of superposition of a large number of simple harmonic waves each with its own frequency and direction Let the amplitudes of the component waves be a_0, a_1, a_2, a_m, a_M and frequencies f_0, f_1, f_2, f_m, f_M and let them propagate in all directions between $-\pi$ and π Let φ be the phase and θ the direction of wave Let the coordinates of gage j in an array be x_j and y_j and let the wave number be $k = 2\pi$/wave length Then the water surface elevation at gage j at time t can be written as

$$q_j(t) = \sum_{m=0}^{M} \int_{-\pi}^{\pi} a_m(\theta) \cos\,[kx_j \cos\theta + ky_j \sin\theta - 2\pi ft + \varphi_m(\theta)] \qquad (1)$$

The subsequent development will make use of the Fast Fourier-Transform technique [see Cooley & Tukey (16), Cochran et al (15), Bergland (9) or Bingham, Godfrey and Tukey (10)] Let the length in time of the water surface elevation record be T and let the discrete time interval of recording be Δt Let T and Δt be such that $N = \frac{T}{\Delta t}$ be a power of 2 One gets the complex amplitude spectrum A_m by taking the FFT of the surface elevation record q_n

$$A_m = \Delta t \sum_{n=0}^{N-1} q_n \, e^{-i2\pi mn/N} \tag{2}$$

Let the directional distribution be represented as a finite Fourier series in complex form as below

$$\begin{aligned} a_m(\theta) e^{i\varphi_m(\theta)} &= F(\theta) \\ &= \frac{a_o + ia_o'}{2} + \sum_{n=1}^{N} [(a_n + ia_n') \cos n\theta + (b_n + ib_n') \sin n\theta] \end{aligned} \tag{3}$$

To determine the directional spectrum one therefore has to evaluate the coefficients a_0, a_0', a_1, a_1', b_1, b_1' etc After going through some mathematical manipulations, one can come up with the following two equations for the real and imaginary parts of the FFT coefficients A_m of the surface elevation record for each gage

$$\begin{aligned} \Re(2A_m^{(j)}/\pi T) = a_0 A_{0j}^* - (a_1' A_{1j}^* + b_1' B_{1j}^*) - (a_2 A_{2j}^* + b_2 B_{2j}^*) \\ + (a_3' A_{3j}^* + b_3' B_{3j}^*) + (a_4 A_{4j}^* + b_4 B_{4j}^*) - \end{aligned} \tag{4}$$

$$\begin{aligned} \Im m(2A_m^{(j)}/\pi T) = a_0' A_{0j}^* + (a_1 A_{1j}^* + b_1 B_{1j}^*) - (a_2' A_{2j}^* + b_2' B_{2j}^*) \\ - (a_3 A_{3j}^* + b_3 B_{3j}^*) + (a_4' B_{4j}^* + b_4' B_{4j}^*) + \end{aligned} \tag{5}$$

In these equations,

$$A_{nj}^* = 2 \cos n\beta \, J_n(kD) \tag{6}$$

and

$$B_{nj}^* = 2 \sin n\beta \, J_n(kD) \tag{7}$$

where

β = angle of gage j from origin

$J_n(kD)$ = Bessel function of order n with argument kD in which k = wave number and D = distance of the gage from the origin

Once the coefficients a_n, a_n', b_n and b_n' are evaluated from the above equations up to the number of harmonics feasible with the number of gages, one may obtain the energy and phase by the following relationships

$$a_m^2(\theta) = |F(\theta)|^2 \tag{8}$$

$$\varphi_m(\theta) = \arg\,[F(\theta)] \tag{9}$$

Random Phase Analysis

Let $p(f,\theta)$ be the directional spectral density function valid for $f > 0$ and $-\pi \le \theta < \pi$ Then it can be shown, after Pierson and Marks (35), that the water surface elevation $q(x,y,t)$ at gage j at time t can be symbolically written as

$$q(x,y,t) = 2\int_0^\infty \int_{-\pi}^{\pi} \sqrt{p(f,\theta)dfd\theta}\,\cos\,(kx\cos\theta + ky\sin\theta - 2\pi ft + \varphi) \tag{10}$$

$q(x,y,t)$ here ends up with a Gaussian probability density for any fixed x,y and t, because of the normal convergence criterion [Brown (14), Loeve (26), Takano (41)] The cross covariance between water surface elevations at two gages therefore turns out to be independent of phase The directional spectrum may be represented as a finite Fourier series of the form

$$p(f,\theta) = \frac{a_o}{2} + \sum_{n=1}^{N} (a_n \cos n\theta + b_n \sin n\theta) \tag{11}$$

Here the Fourier series coefficients a_n and b_n are to be evaluated In terms of the co- and quad- spectrum for each pair of gages the following two equations can be written down

$$\text{Co-spectrum}_{j\ell} = \pi[a_0A^*_{0j\ell} - (a_2A^*_{2j\ell} + b_2B^*_{2j\ell}) + (a_4A^*_{4j\ell} + b_4B^*_{4j\ell}) - \tag{12}$$

$$\text{Quad-spectrum}_{j\ell} = \pi[(a_1A^*_{1j\ell} + b_1B^*_{1j\ell}) - (a_3A^*_{3j\ell} + b_3B^*_{3j\ell}) + (a_5A^*_{5j\ell} + b_5B^*_{5j\ell}) - \tag{13}$$

The co-spectrum and quad-spectrum can be calculated from the FFT coefficients on gage j and gage ℓ The quantities A^*_n and B^*_n are as below

$$A^*_n = 2\cos n\beta\, J_n(kD) \tag{14}$$

and

$$B^*_n = 2\sin n\beta\, J_n(kD) \tag{15}$$

where β = angle between gage j and gage ℓ

D = distance between the gages,

k = wave number = 2π/wave length and

J_n = Bessel function of order n

The unknowns are the Fourier series coefficients a_0, a_1, b_1, a_2, b_2 etc There are two equations for each pair of gages and two unknowns for every harmonic

Least square analysis is used to make coefficient estimates In both procedures, a unidirectional wave train will produce analytical results spread over an angular band width, because only a finite number of Fourier coefficients can be estimated

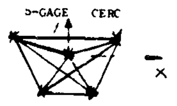
COMPUTATION OF DIRECTIONAL SPECTRA

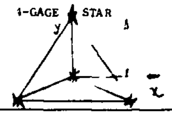
A very general computer program was developed to compute the directional spectrum, the details of which will be reported subsequently The analytical procedure and the scheme of computation was verified for their validity and workability by computation of known directional spectra using the scheme The directions obtained out of the computer checked very well with the known directions of the simulated wave as well as regular waves generated in the laboratory when the respective data were fed in However, there was considerable angular spread in the results obtained for directions These were due to leakage, finite length of data and the truncation of Fourier series representing directional spectra It was also noticed that there was considerable negative energy showing up in the spectra As the negative energy caused by the presence of one wave may foul up with the positive contribution from another wave in the system, the presence of negative energy may affect appreciably the directional resolution Hence this had to be cured For this a non-negative smoothing function $W_2(\varphi)$ was applied where

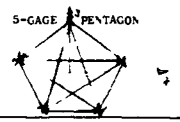
$$W_2(\varphi) = R_N \cos^{2N} (\frac{\varphi}{2}) \tag{16}$$

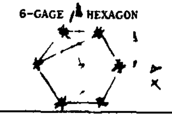
in which R_N is a coefficient to be obtained for each harmonic Borgman (12) has described the procedure to apply this smoothing The smoothing, however, broadens the angular spread of the directional spectrum and decreases the value of the spectral peak The problem of desmoothing the spectra seems to be very important and perhaps Medgyessey (30) may yield some clues

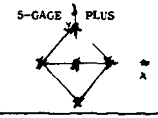
The scheme of computation developed was used to compare the directional resolving power of some two-dimensional arrays by simulating a single wave train and comparing the response to it from different gage arrays Figures 1 and 2 tabulate the relevant quantities for comparison The difference between the two tables is that the quantities in Table 1 are obtained without W_2 smoothing, whereas the quantities in Table 2 are smoothed A comparison shows that all the arrays considered give the direction correctly, but there is a difference in the angular spreads and the values of spectral peaks For a single wave train the spectrum should have been theoretically a Dirac delta function, i e a spike So, the narrower the angular spread and the higher the peak, the better the resolving power of the array By this token, out of the five gage arrays tested, the CERC array seems to be the best Figure 3 gives the plots of smoothed directional spectra obtained for different arrays for various input directions





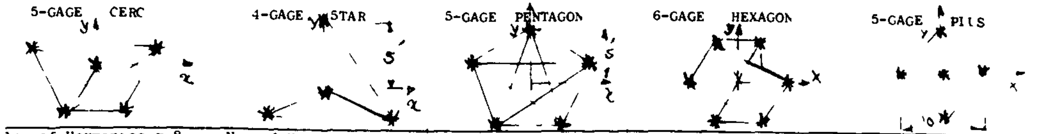






Input Wave Direction in Degree	5-GAGE CERC, No of Harmonics = 8: Peak Spectral Density	Maximum Sidelobe	Minimum - Sidelobe	4-GAGE STAR, No of Harmonics = 4: Peak Spectral Density	Maximum Sidelobe	Minimum - Sidelobe	5-GAGE PENTAGON, No of Harmonics = 4: Peak Spectral Density	Maximum + Sidelobe	Minimum - Sidelobe	6-GAGE HEXAGON, No of Harmonics = 4: Peak Spectral Density	Maximum + Sidelobe	Minimum - Sidelobe	5-GAGE PLUS, No of Harmonics = 2: Peak Spectral Density	Maximum + Sidelobe	Minimum - Sidelobe
-90	1 87311	0 25676	-0 41661	0 98057	0 14427	-0 22566	1 00077	0 15734	-0 23329	0 98637	0 14361	-0 22291	0 54966	0 10918	-0 14352
-80	1 87631	0 26657	-0 42757	0 98217	0 14733	-0 22034	0 98187	0 15297	-0 23493	0 98476	0 14447	-0 22289	0 54762	0 10757	-0 14163
-70	1 87116	0 25783	-0 41931	0 98653	0 14723	-0 22474	0 96929	0 13525	-0 21877	0 98363	0 14402	-0 22431	0 54479	0 10624	-0 13557
-60	1 86699	0 26044	-0 42718	0 98929	0 14320	-0 22070	0 99266	0 16019	-0 23962	0 98411	0 14437	-0 22384	0 54563	0 10953	-0 13522
-50	1 86571	0 24650	-0 41769	0 98653	0 14723	0 22474	0 99698	0 16117	-0 23935	0 98363	0 14402	-0 22431	0 54850	0 11436	-0 13532
-40	1 85183	0 21709	-0 41777	0 98217	0 14733	-0 22773	0 97211	0 11121	-0 22459	0 98476	0 14447	-0 22289	0 54850	0 11436	-0 13532
-30	1 85945	0 24925	-0 41255	0 98057	0 14427	-0 22566	0 97645	0 14736	-0 23020	0 98637	0 14361	-0 22291	0 54563	0 10953	-0 13522
-20	1 83773	0 23238	-0 40213	0 98217	0 14733	-0 22773	0 99980	0 16013	-0 23718	0 98476	0 14447	-0 22289	0 54479	0 10624	-0 13557
-10	1 85535	0 24408	-0 40648	0 98653	0 14723	-0 22474	0 98749	0 15734	-0 23820	0 98363	0 14402	-0 22431	0 54762	0 10757	-0 14163
0	1 83199	0 22419	-0 38835	0 98929	0 14320	-0 22069	0 96831	0 13020	-0 21345	0 98411	0 14437	-0 22384	0 54968	0 10918	-0 14352
10	1 [illegible]	0 [illegible]1108	-0 10618	0 98653	0 14723	-0 22171	0 98749	0 15734	-0 23820	0 98363	0 14402	-0 22431	0 54782	0 10757	-0 14163
20	1 83773	[illegible]	[illegible]	0 98217	0 14733	-0 [illegible]2773	0 99980	0 16013	-0 23718	0 98176	0 14417	-0 22289	0 54179	0 10621	-0 13557
30	1 8[illegible]41	0 [illegible]1925	-0 [illegible]	0 980[illegible]7	0 11127	-0 [illegible]566	0 97615	0 11736	-0 23020	0 98637	0 11361	-0 22291	0 51563	0 10953	-0 13522
40	1 8[illegible]183	0 [illegible]1701	-0 [illegible]	0 98217	0 11733	-0 [illegible]2773	0 97211	0 11121	-0 [illegible]2159	0 98176	0 11117	-0 22289	0 51850	0 11136	-0 13532
50	1 8[illegible]1	[illegible]	-0 [illegible]	0 98653	0 11723	-0 [illegible]171	0 99698	0 16117	-0 23935	0 98363	0 11102	0 22431	0 51850	0 11136	-0 13532
60	1 8[illegible]	[illegible]	-0 [illegible]718	0 989[illegible]9	0 113[illegible]0	-0 2[illegible]070	0 99[illegible]66	0 16019	-0 23962	0 98111	0 11137	-0 22384	0 51563	0 10953	-0 13522
70	1 8[illegible]11	0 [illegible]783	-0 [illegible]	0 98[illegible]3	0 11723	-0 22171	0 96929	0 13525	-0 21877	0 98363	0 11102	-0 22131	0 51179	0 10621	-0 13557
80	1 8[illegible]	[illegible]	[illegible]	0 98[illegible]17	0 11733	-0 [illegible]	0 98187	0 15297	-0 [illegible]3193	0 98176	0 11117	0 22289	0 51762	0 10757	-0 11163
90	1 8[illegible]	[illegible]	-0 [illegible]	0 980[illegible]	0 1112[illegible]	-0 [illegible]	1 00077	0 1[illegible]731	-0 23329	0 98637	0 11361	0 22291	0 51966	0 10918	-0 11352

FIG 1 DIRECTIONAL RESOLVING POWER OF DIFFERENT WAVE GAGE ARRAYS
DIRECTIONAL SPECTRA OBTAINED FOR AN INPUT WAVE OF AMPLITUDE
0 1' AND PERIOD 2 SEC WITHOUT SMOOTHING



Input Wave Direction in Degrees	5-GAGE CERC: No of Harmonics = 8, No Positive Sidelobe		4-GAGE STAR: No of Harmonics = 4, No Positive Sidelobe		5-GAGE PENTAGON: No of Harmonics = 1, No Positive Sidelobe		6-GAGE HEXAGON: No of Harmonics = 1, No Positive Sidelobe		5-GAGE PILS: No of Harmonics = 2, No Positive Sidelobe	
	Peak Direc Spectral Density	Maximum Sidelobe Value	Peak Direc Spectral Density	Maximum Sidelobe Value	Peak Direc Spectral Density	Maximum Sidelobe Value	Peak Direc Spectral Density	Maximum Sidelobe Value	Peak Direc Spectral Density	Maximum Sidelobe Value
-90	0 61539	-0 05835	0 39654	No Sidelobe	0 40019	-0 00023	0 40091	-0 00112	0 29052	-0 00314
-80	0 61539	-0 05834	0 39830	-0 00098	0 40005	-0 00046	0 40048	-0 00096	0 29063	-0 00285
-70	0 61540	-0 05835	0 40183	-0 00327	0 39995	-0 00048	0 39965	-0 00005	0 29119	-0 00117
-60	0 61542	-0 05837	0 40361	-0 00328	0 40013	-0 00052	0 39934	No Sidelobe	0 29232	No Sidelobe
-50	0 61540	-0 05834	0 40183	-0 00327	0 40017	-0 00038	0 39965	-0 00005	0 29332	No Sidelobe
-40	0 61541	-0 05836	0 39830	-0 00105	0 39997	-0 00048	0 40048	-0 00096	0 29332	No Sidelobe
-30	0 61539	-0 05834	0 39654	No Sidelobe	0 40000	-0 00047	0 40091	-0 00112	0 29232	No Sidelobe
-20	0 61537	-0 05832	0 39830	-0 00105	0 40019	-0 00032	0 10048	-0 00096	0 29119	-0 00117
-10	0 61538	-0 05833	0 40183	-0 00327	0 40009	-0 00045	0 39965	-0 00005	0 29063	-0 00285
0	0 61535	-0 05830	0 40361	-0 00328	0 39994	-0 00049	0 39924	No Sidelobe	0 29052	-0 00314
10	0 61538	-0 05833	0 40183	-0 00327	0 40009	-0 00045	0 39965	-0 00005	0 29063	-0 00285
20	0 61537	-0 05832	0 39830	-0 00105	0 40019	-0 00012	0 40048	-0 00096	0 29119	-0 00117
30	0 61539	-0 05834	0 39654	No Sidelobe	0 40000	-0 00047	0 40091	-0 00112	0 29232	No Sidelobe
40	0 61541	-0 05836	0 39830	-0 00105	0 39997	-0 00048	0 40048	-0 00096	0 29332	No Sidelobe
50	0 61540	-0 05834	0 40183	-0 00327	0 40017	-0 00038	0 39965	-0 00005	0 29332	No Sidelobe
60	0 61542	-0 05837	0 40361	-0 00328	0 40013	-0 00042	0 39934	No Sidelobe	0 29232	No Sidelobe
70	0 61540	-0 05835	0 40183	-0 00327	0 39995	-0 00048	0 39965	-0 00005	0 29119	-0 00117
80	0 61539	-0 05834	0 39830	-0 00105	0 40005	-0 00046	0 40048	-0 00096	0 29063	-0 00285
90	0 61539	-0 05835	0 39654	No Sidelobe	0 40019	-0 00023	0 40091	-0 00112	0 29052	-0 00314

FIG 2 DIRECTIONAL RESOLVING POWER OF DIFFERENT GAGE ARRAYS DIRECTIONAL SPECTRA OBTAINED FOR AN INPUT WAVE TRAIN OF PERIOD 2 SEC AND AMPLITUDE 0 1' WITH W_2 SMOOTHING

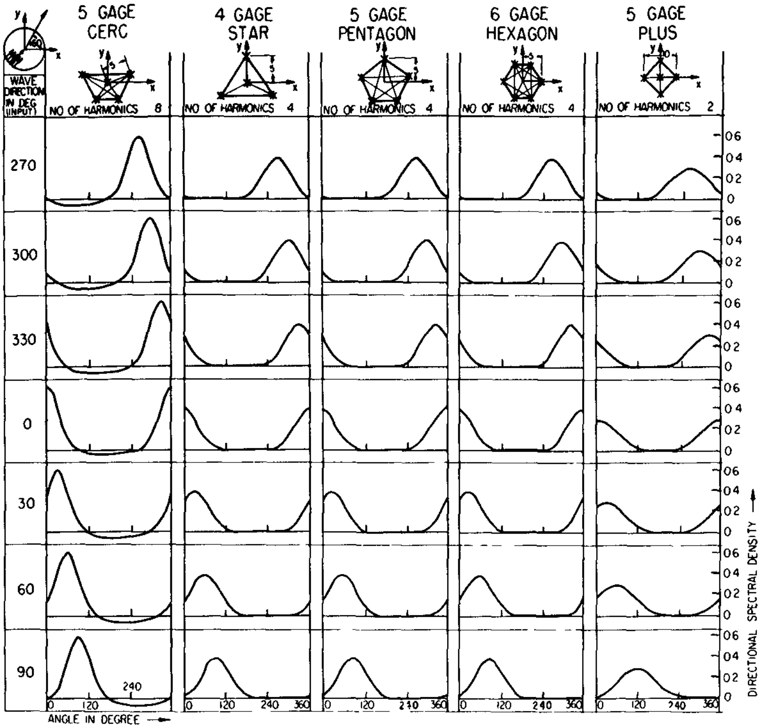


FIG 3 DIRECTIONAL RESOLVING POWER OF GAGE ARRAYS
DIRECTIONAL SPECTRA OBTAINED ON DIFFERENT ARRAYS WITH W_2 SMOOTHING FOR FREQUENCY 0 5 Hz FOR A SINGLE WAVE TRAIN OF PERIOD 2 SEC AND AMPLITUDE 0 1' COMING FROM DIFFERENT DIRECTIONS

Analysis of Data From the Pacific Ocean

Figure 4 gives the location map and Fig 5 the gage orientation for a 5-gage array put up by the Coastal Engineering Research Center in the Pacific Ocean off Point Mugu, California This array was specifically designed for the determination of directional spectrum of ocean waves of periods between 7 sec and 25 sec , see Borgman and Panicker (13) for the design The gages were subsurface pressure transducers placed 3 feet from the bottom at a water depth of 30 feet and 1600 feet away from the shore The wave data were analyzed for directional spectrum using the method developed Some examples of results obtained are given below

The choice of a frequency band for averaging the spectra was made by studying Fig 6 which shows the effect of the use of different frequency bands for averaging When the band width is large, the confidence interval of the spectral estimate is close, but the spectrum obtained is rather too smooth, see, for example, the spectrum obtained when averaging is made in blocks of 64 FFT coefficients, i e , in band width of 64/1024 Hz The spectrum is so smooth that it does not show the bimodality indicated by most other cases with block averaging in narrower frequency band width But when the block averaging is done in too narrow a frequency width, as in block averaging of 4 Fourier coefficients, the spectrum shows much erratic natuie Tentatively, therefore, it was decided to average in blocks of 32 FFT coefficients and 16 FFT coefficients Figure 7 shows a comparison of the spectra obtained at the different gages They compare well but there is some discrepancy at peak frequencies Figures 8 and 9 show typical directional spectra obtained for the locality using the random phase method of analysis described The spectral densities shown are based on pressure in ft of water and not adjusted for surface elevations The dominant directions would not be affected, anyway Figure 8 shows a comparison of the directional spectra obtained at two different times Notice the arrival of the prominent swell in the morning of March 28 Figure 9 shows a contour plot of directional spectrum obtained with a block averaging of 16 FFT coefficients, or a frequency band width of 16/1024 Hz The typical bimodal spectrum of the Pacific coast can be seen with the ridges showing the strong sea and swell, both being shown to come generally from the West South West

CONCLUSIONS

The following conclusions may be drawn from the above discussions

1 Use of wave gage arrays seems to be well suited for the determination of directional spectra in a routine manner

2 The analytical procedure and the scheme of computation developed seem to work well for the different situations tested, viz numerically simulated data, laboratory data and ocean data

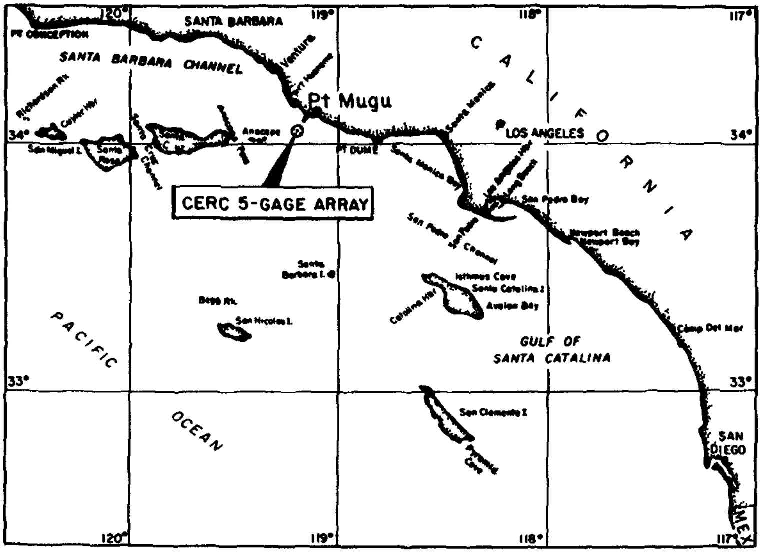


FIG 4 LOCATION MAP OF CERC 5-GAGE ARRAY OFF POINT MUGU, CALIFORNIA

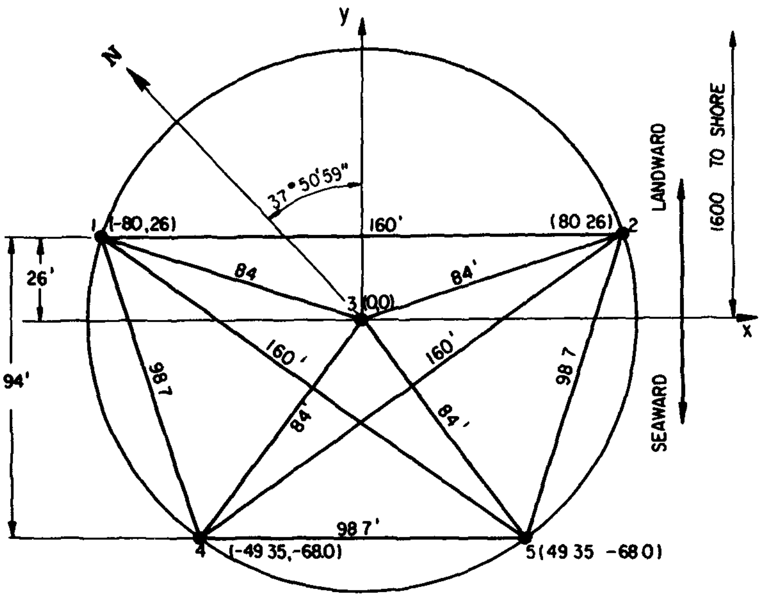


FIG 5 CERC ARRAY OF WAVE GAGES OFF POINT MUGU, CALIFORNIA

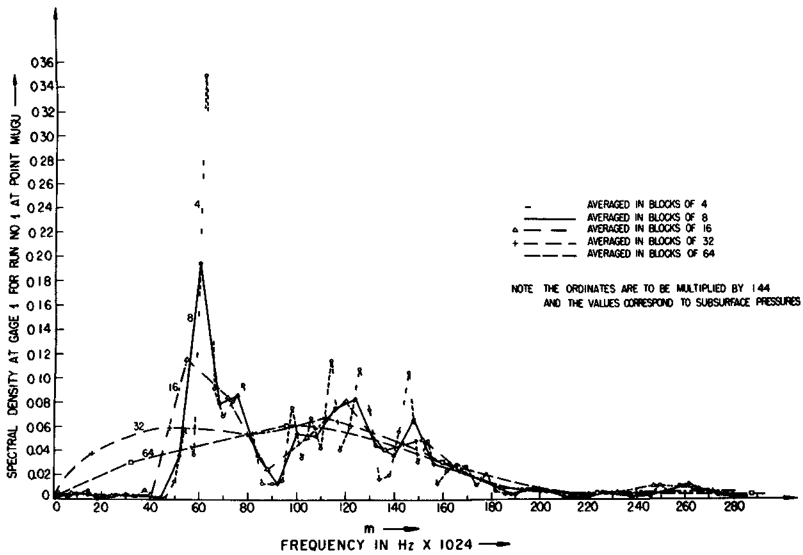


FIG 6 SPECTRA AT GAGE 1 OFF POINT MUGU FOR RUN NO 1, AVERAGE IN BLOCKS 4, 6, 16, 32 and 64

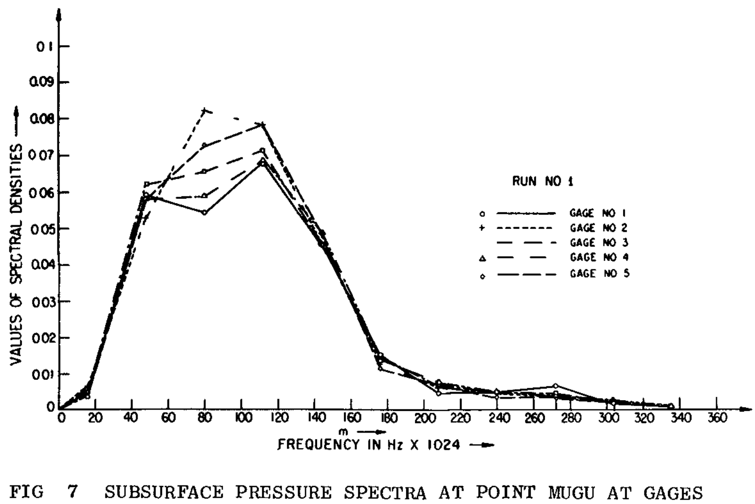
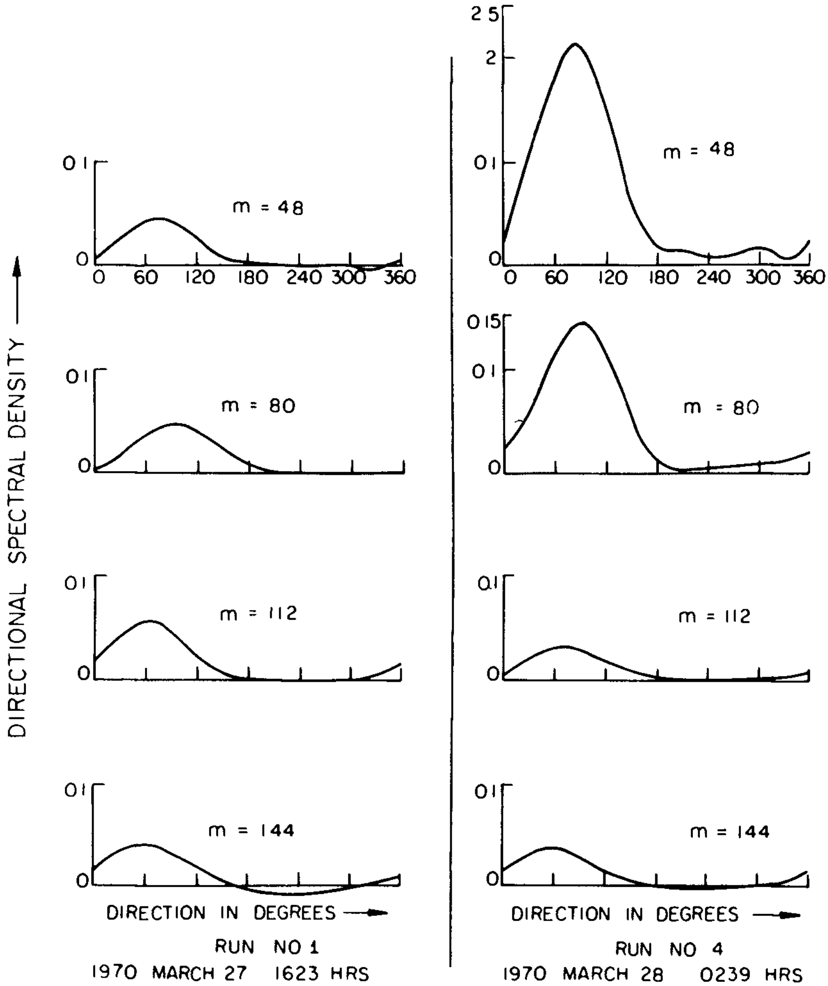


FIG 7 SUBSURFACE PRESSURE SPECTRA AT POINT MUGU AT GAGES 1, 2, 3, 4 AND 5 AVERAGED IN BLOCKS OF 32



NOTE FREQUENCY $f = \frac{m}{1024}$ LENGTH OF BLOCKS IN AVERAGING SPECTRA = 32 NO OF HARMONICS TO WHICH DIRECTIONAL SPECTRAL DENSITY FUNCTION IS REPRESENTED = 4

FIG 8 DIRECTIONAL SPECTRA OBTAINED FROM CERC 5-GAGE ARRAY OFF PT MUGU, CALIFORNIA

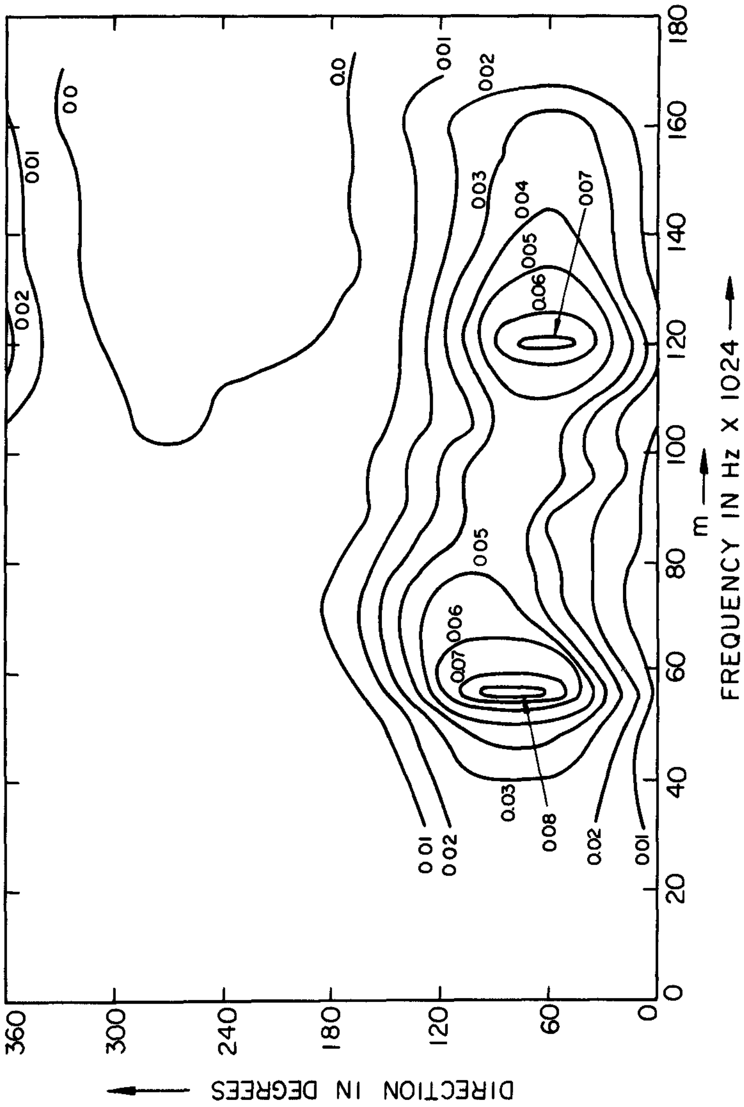


FIG 9 SPECTRA OBTAINED AT POINT MUGU, CALIFORNIA FROM CERC 5-GAGE ARRAY FROM RUN 1, MARCH 27, 1970 AT 1623 HOURS

3 Much more research work is needed in almost all aspects of the problem

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