CHAPTER 5

SPECTRAL COMPUTATIONS ON PRESSURE WAVE GAUGE RECORDS

by Manuel Mendes de CARVALHO*
Fernando Silveira RAMOS*
Carlos de Campos MORAES*

ABSTRACT

With a view to establish sea wave data processing procedures to be applied to records obtained at the Portuguese coast, a detailed study is made of different choices of certain parameters used in one-dimensional spectral analysis of a pressure wave gauge record Statistics computed by the selected spectral procedure are then compared with results of a Tucker-Draper analysis of the same record Finally a hindcast of sea conditions for the date and place of the record is made by different methods and comparisons with previous results are presented

1 - INTRODUCTION

The study of the coastal sea wave regime in Portugal needs development both in programming an adequate deployment of sea wave gauges and in what concerns establishing the best data processing procedures The authors form a research team charged at present with the study of the instrumentation and use of an irregular wave flume at LNEC It is known that for a perfect simulation of irregular sea waves a detailed knowledge is necessary of the wave regimes to be reproduced There are two phases to consider the first is a qualitative one in which methods for sea wave data analysis and computation procedures are discussed and developed, the second is quantitative, results from the first being extensively applied, to try and acquire a more exact knowledge of the configuration of sea waves in the zone of interest

The present paper relates to the above mentioned first phase It represents the study path followed by the authors Although most of the techniques presented are already known, it is thought that eventually some usefulness may be derived from reading it, particularly in pointing out some doubts and difficulties inherent in the methods used The work being still restricted to the first phase, only one record was used This record was made by a pressure wave gauge

2 - CONDITIONS OF WAVE MEASURING, RECORDING AND ANALYSIS

In the approaches to Leixões Harbour (in the northwestern coast of Portugal) a pressure wave gauge is installed on the bottom at a mean depth of 22 m (Fig 1) It is an autonomous St Chamond Granat pressure wave gauge, type LNH Its working schedule is as follows at both 0900 hrs and 2100 hrs a twenty-minute recording period starts, when the surface waves exceed 4 m it automatically produces a twenty-minute record every two hours Normally a 250 milibar manometer is in use The present study is based on a twenty-minute record made at 3 47 GMT on December 18th, 1968 For the record digitalization a Boscar LNF 630 projector by Benson, France, was used This projector provides a twelvefold magnification of the 35 mm film from the wave gauge and can also handle 16 and 70 mm films The computations were made in the LNEC's computing centre in a NCR-Elliott 4130 computer with 24 k 24-bit words and 3 magnetic tape handlers

3 - SPECTRAL ANALYSIS

Estimates of the energy spectrum, $P(f)$, were obtained by the indirect method through the autocovariance function $c(\tau)$

$$P(f) = \int_{-\infty}^{+\infty} c(\tau)\, e^{-i2\pi f\tau}\, d\tau = TF\left[c(\tau)\right] \qquad (1)$$

where TF means Fourier transform A measure of the mean energy of the waves

* - Trainee Research Officers, Laboratorio Nacional de Engenharia Civil (LNEC), Lisbon, Portugal

during the record duration T_R is given by

$$c(0) = 2 \int_0^{+\infty} P(f)\, df \qquad (2)$$

3 1 - Lag and spectral windows

If $Z(t)$ is the water surface elevation above an arbitrary level as a function of time, t, and at a certain place, then the autocovariance function is the mean value of $Z(t)\ Z(t+\tau)$ along time

$$c(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} Z(t)Z(t+\tau)\, dt \qquad (3)$$

$c(\tau)$ expresses covariance between $Z(t)$ and $Z(t+\tau)$, that is, between water surface elevations at any two instants separated by a time lag of τ secs In case there are no periodicities in $Z(t)$, then $c(\tau)$ tends to zero as τ tends to infinity If there is a periodicity in $Z(t)$, then an oscillation about zero, with the same period, appears in $c(\tau)$ In practice, the autocovariance function, $c(\tau)$, is computed as the mean value of $Z(t)Z(t+\tau)$ in the available recording interval, whose length or, rather, duration, is T_R $c(\tau)$ never really damps out to zero, not only because T_R is finite but also owing to periodicities which always occur in records* This leads to the necessity of truncating** the autocovariance function, that is, to consider instead of $c(\tau)$ the new function $c(\tau)\ D(\tau)$ where

$$D(\tau) = \begin{cases} 1 & , |\tau| \leq T_M \\ 0 & , |\tau| > T_M \end{cases}$$

$D(\tau)$ is the rectangular lag window and T_M is the maximum lag of the autocovariance function Instead of $D(\tau)$ and to get more stability in the spectrum estimates, other functions or lag windows, $D_i(\tau)$ are used, which are also identically zero outside the interval $(-T_M, T_M)$ and take the value 1 for $\tau = 0$ Using these lag windows, which is unavoidable in practice, leads to the fact that, for every frequency, one gets a wheighted average of neighbouring values of the record spectrum The wheighting function is $Q_i(f) = TF[D_i(\tau)]$, the so-called spectral window corresponding to the lag window $D_i(\tau)$ In this study the Parzen, Tukey, Hamming, Bartlett and rectangular lag windows were considered Their lag and spectral versions are presented in Fig 3

It is known that to each window there corresponds a statistical estimator for the spectrum To chose one among these, various criteria have been proposed, most of them based in the minimization of the mean square error of the estimator or some function or functional of it (See, for instance, Jenkins and Watts [4]) According to this kind of criteria the rectangular window distinguishes itself by being considerably worse than the others mentioned, which in turn are similar to one another in performance In consequence the rectangular window should be avoided and the choice among the Parzen, Tukey, Hamming and Bartlett windows becomes of secondary importance, as compared, for example, with the choice of the maximum lag for $c(\tau)$, to be considered in 3 6 However, differences do exist and a choice had to be made

The fact that the side lobes of the Parzen spectral window are much smaller and that its small variance originates narrower confidence intervals for the spectrum estimates led to its selection For this window, the number of degrees of freedom of the spectrum estimates is $3\ 71\ T_R/T_M$ The spectral window bandwidth is

* - As shown later, (see Fig 2) the autocovariance function relative to the record considered is far from showing any tendency to become zero

** - Of course, if $c(\tau)$ equalled zero only outside a very large interval a truncation would be in order

$$b = \frac{1}{\int_{-\infty}^{+\infty} D^2(\tau)\,d\tau} = \frac{1}{\int_{-\infty}^{+\infty} Q^2(f)\,df} = \frac{1.86}{T_M} \qquad (5)$$

and it is approximately the frequency interval between two practically independent spectrum estimates It is seen that the smaller is b, the greater is the resolution of the estimates, i e , the smaller is the influence exerted on an estimate by estimates in neighbouring frequencies Applying the rectangular lag window is the same as simply truncating the autocovariance function The spectrum obtained is termed the raw spectrum The raw spectrum would be exactly equal to the true record spectrum if the autocovariance function were zero for $|\tau| > T_M$, which never happens in sea wave analysis Using any other of the mentioned windows produces smoothed estimates, that is, more stable estimates For a given window, if T_M is varied then stability and resolution increase or decrease in opposite senses Fig 4 shows the raw spectrum computed for $T_M = 80$ s and its Parzen smoothed version

3 2 - Prewhitening and prefiltering

It is known that spectral windows delete accidental details of the spectrum, that is, they smooth it In the process of smoothing, the spectrum estimate in a given frequency is influenced by the estimates in neighbouring frequencies A sharp peak is "spread" over its neighbourhood in a way similar to the spectral window configuration 5 This leads to the alteration of the true form of the spectrum To avoid this difficulty as much as possible, Blackman and Tukey [7] suggested a prewhitening, that is a preliminary digital filtering of the record aiming to reduce the importance of its spectrum peaks This is bringing the spectrum closer to that of a white noise

The suggested digital filtering is in general

$$y_t = a\,x_t + b\,x_{t-1} \qquad (6)$$

$\{x_t\}$ being the input time series and $\{y_t\}$ the output time series The input and output spectra, $P_x(f)$ and $P_y(f)$ respectively, are then related by

$$P_y(f) = (a^2 + b^2 + 2\,ab\cos 2\pi f \Delta t)\,P_x(f) \qquad (7)$$

In this way the spectral window will be applied to a spectrum without sharp peaks and the obtained spectrum $P_y(f)$ may then be corrected for prewhitening by (7)

Computations showed that, in the present case, estimates were practically the same with and without prewhitening This led to giving up the use of prewhitening As for other kinds of prefiltering, it was not deemed necessary to eliminate the spurious energy of very low frequencies A first difference filter was nevertheless considered, as is suggested by Jenkins [4], results showing that it was not adequate, as its influence reaches regions of too high frequencies

3 3 - Confidence intervals

Let $\bar{P}(f)$ be the smoothed spectrum estimate at frequency f, P(f) the true value of the spectrum, χ^2_ν a random variable following the chi-square distribution with ν degrees of freedom and b the spectral window bandwidth It can be proved that $2\,T_R\,b\,\bar{P}(f)/P(f)$ is a random variable following the chi-square distribution with $2\,T_R\,b$ degrees of freedom

$$2\,T_R\,b\,\frac{\bar{P}(f)}{P(f)} = \chi^2_{2T_R b}$$

This permits confidence intervals to be constructed for the estimates Those intervals will be

$$\left(\frac{\nu}{\chi_\nu(1-\alpha/2)}\,\bar{P}(f),\ \frac{\nu}{\chi_\nu(\alpha/2)}\,\bar{P}(f)\right) \qquad (9)$$

where $1-\alpha$ is the confidence level and $x_\nu(k)$ is a number such that

$$P(\chi^2_\nu \leq x_\nu(k)) = k \qquad (10)$$

In Fig 5, curves of variation with of $A(\nu) = \frac{\nu}{x_\nu(1-\alpha/2)}$ and $B(\nu) = \frac{\nu}{x_\nu(\alpha/2)}$ for $1-\alpha = 80\%$ are presented

This was the confidence level used in the computations made Most spectrum figures in this paper show a 80% confidence zone obtained in the manner described

3 4 - Sampling interval

From a continuous record a time series can be extracted using a certain sampling interval Δt (digitalization) If F_D is the frequency above which spectrum values are negligible one must have

$$F_D \leq \frac{1}{2\Delta t} \qquad (11)$$

to avoid the so called aliasing $F_N = \frac{1}{2\Delta t}$ is the Nyquist frequency Aliasing is a consequence of the digitalization of the record In fact, the result of digital computations is not the true spectrum $P(f)$ but the aliased spectrum

$$P_a(f) = \sum_{q=-\infty}^{+\infty} P(f - \frac{q}{\Delta t}) \qquad (12)$$

If condition (11) is met, however, then (12) gives $P(f)$ values between 0 and F_D Having fixed $T_M = 80$ s (and so, b = 0 023 cps), which is equivalent to having fixed resolution and stability of the estimates, computations were carried out with $\Delta t = 0$ 5, 1 0, 1 5, 2 0, 2 5 and 3 0 s, corresponding to Nyquist frequencies of 1, 0 5, 0 33, 0 25, 0 20 and 0 16 cps As $F_D \approx 0$ 25 cps in the present case, aliasing should be negligible for $\Delta t = 0$ 5, 1 0, 1 5 and 2 0 s In fact, it was seen that estimates were practically the same For $\Delta t = 2$ 5 and 3 0 s, that is, $F_D > F_N$, aliasing is already strongly apparent (Fig 6)

The conclusion is that inasmuch as there is no aliasing, the sampling interval may be as high as wished In these computations $\Delta t = 2$ s could have been used

3 5 - Cutoff frequency

For reasons of economy, computation of spectrum values should not be carried much beyond F_D Frequency F_C up to which calculations are made will be called cut-off frequency One should then have $F_D < F_C < F_N$ The choice of F_C can have great influence in practical computation of the spectral moments

The spectral moment of order n is defined as

$$m_n = \int_{-\infty}^{+\infty} f^n P(f)\, df \qquad (13)$$

For even values of n and since $P(f)$ is an even function, we may write

$$m_n = 2 \int_0^{+\infty} f^n P(f)\, df \qquad (14)$$

In practice, estimates of m_n (even n) may be obtained from

$$m_n = 2\Delta f \sum_{r=1}^{k} (r\Delta f)^n P(r\Delta f) \qquad (15)$$

with $k\Delta f = F_C$ and f being the frequency interval between two adjacent estimates of $P(f)$

Theoretically we have $m_o = c(0)$ and so it is suggested that, in practice and with a view to calculating spectral moments, F_C be chosen such that

$$c(0) = 2\Delta f \sum_{r=1}^{k} P(r\Delta f) \qquad (16)$$

with $F_C = k\Delta f$

Fig 7 shows the variation with F_C of m_o values obtained from (15) It is seen that F_C should be chosen between 0 25 and 0 3 cps approximately

3 6 - Maximum lag for the autocovariance

The choice of T_M is very important, as on it are closely dependent the resolution and stability of estimates A great T_M produces high resolution estimates owing to the narrow bandwidth of the spectral window, smoothing is however small and in consequence estimates present greater instability If T_M is small then the inverse is true we get small resolution and great stability A compromise is therefore necessary between stability and resolution

Three criteria for the choice of T_M were considered

a) The window closing technique

This technique is suggested by Jenkins [4] and consists in considering successively increasing T_M values, or, equivalently, decreasing b values (window closing), which will have the effect that initially obscured spectrum details will become more defined There are no rules to decide when certain details, as, for instance, a peak which is beginning to show, are real or due to instability Jenkins suggests that three spectra should be presented, computed for T_M values from the range where the initial form of the spectrum starts to change, that is, where after the initial convergence of shape a divergence begins to appear Fig 8 shows that the general spectrum configuration is kept until T_M reaches about 70 s Afterwards, a swelling begins to appear around frequency 0 12 cps and it becomes quite distinct when $T_M \approx 90$ s A T_M value of about 80 s seems therefore indicated

b) Using the rectangular lag window

If the autocovariance function $c(\tau)$ is zero for $|\tau| > \tau_o$, using the rectangular lag window with $T_M > \tau_o$ produces the true record spectrum If one uses $T_M < \tau_o$, negative values may appear for the spectrum, which is an indication that T_M is not sufficiently high (Barber [6]) This could eventually serve as a criterion for the choice of T_M Yet, in practice, with sea wave records, the autocovariance function never really comes to zero however great we may make T_M This is due to periodicities which always turn up in natural records and the result is that negative values may always arise whatever the T_M Fig 9 shows that, though T_M was increased tenfold, negative estimates show no tendency to disappear On the other hand, when the autocovariance function looks like the one pictured in Fig 2, where there are regions of almost total damping, as in the neighbourhood of $\tau = 44$ s, a rectangular lag window truncation in that region should produce a spectrum equal to one from a record for which $c(\tau) = 0$ from that region on This spectrum should not exhibit negative values, that is really the case when, in the present computations, $T_M = 44$ 5 s was used, as is seen in Fig 9 The conclusion to draw is that using the rectangular lag window is not a satisfactory way to decide on T_M

c) Variation of ε with T_M

One parameter to which the name spectrum width is generally given is

$$\varepsilon = \sqrt{\frac{m_o m_4 - m_2^2}{m_o m_4}} \qquad (17)$$

where m_n is defined by (13) ε values computed by (15) from spectra in which T_M

varied from 22 s to 222 s and where F_C was constantly equal to 0 3 cps are presented in table 1 and graphically in Fig 11 It is seen that for T_M > 70 s the ε value varies only very slightly This may mean that a sufficiently high value for T_M has been reached

Table 1

VARIATION OF T_M								Δt = 0 5 s F_C = 0 3 cps	
T_M	22	44 5	50	60	70	80	89	100	222 5
ε	0 61	0 56	0 55	0 55	0 54	0 54	0 54	0 54	0 53
m_o	0 50	0 51	0 51	0 51	0 51	0 51	0 51	0 51	0 51
f_{peak}	0 085	0 080	0 080	0 080	0 080	0 080	0 080	0 08	2 peaks
$P(f_{peak})$	3 17	4 55	4 79	5 15	5 41	5 59	5 69	5 77	Low smo-othing

These 3 criteria considered, it was decided to use 80 s as an adequate value for T_M, which is about 6 to 7 times the period corresponding to the peak frequency of the spectrum

3 7 - Record partition

The record was partitioned in four 5-minute parts Table 2 and Fig 10 show results of computations made m_o values increase clearly from the 1st to the 2nd part and from the 2nd to the 3rd part The 3rd and 4th part values are about the same Spectrum variations may have different reasons They may result from the fact that we are now dealing with four different samples and variation may be merely statistical in nature These variations should, perhaps, be considered large, which might mean that a five-minute period is too short for getting a representative record Since m_o is a measure of the mean energy of sea waves, another reason for the variations may be that the record should not be considered completely stationary In future, when an extensive study is undertaken of records made at the same date as this one, the matter may be clari - fied further

Table 2

<table>
<tr><th></th><th>F VE MIN</th><th>F VE IN</th><th>VE M</th><th>F VE M</th></tr>
<tr><td>ε</td><td colspan="4">0 54</td></tr>
<tr><td>m_o</td><td colspan="4">0 51</td></tr>
<tr><td>f_{pe k}</td><td colspan="4">0 08</td></tr>
<tr><td>P(f_{pa k})</td><td colspan="4">5 59</td></tr>
<tr><td>ε</td><td>0 51</td><td>0 53</td><td>0 54</td><td>0 54</td></tr>
<tr><td>m_o</td><td>0 39</td><td>0 45</td><td>0 57</td><td>0 60</td></tr>
<tr><td>f_{peak}</td><td>0 09</td><td>0 08</td><td>0 08</td><td>0 08</td></tr>
<tr><td>P(f_{peak})</td><td>3 18</td><td>4 88</td><td>5 75</td><td>5 74</td></tr>
<tr><td>ε</td><td colspan="2">0 53</td><td></td><td></td></tr>
<tr><td>m_o</td><td colspan="2">0 42</td><td></td><td></td></tr>
<tr><td>f_{peek}</td><td colspan="2">0 085</td><td></td><td></td></tr>
<tr><td>P(f_{peak})</td><td colspan="2">3 96</td><td></td><td></td></tr>
<tr><td>ε</td><td></td><td colspan="2">0 55</td><td></td></tr>
<tr><td>m_o</td><td></td><td colspan="2">0 51</td><td></td></tr>
<tr><td>f_{peak}</td><td></td><td colspan="2">0 08</td><td></td></tr>
<tr><td>P(f_{paak})</td><td></td><td colspan="2">5 28</td><td></td></tr>
<tr><td>ε</td><td></td><td></td><td colspan="2">0 54</td></tr>
<tr><td>m_o</td><td></td><td></td><td colspan="2">0 59</td></tr>
<tr><td>f_{peak}</td><td></td><td></td><td colspan="2">0 08</td></tr>
<tr><td>P(f_{pe k})</td><td></td><td></td><td colspan="2">5 72</td></tr>
<tr><td>ε</td><td colspan="3">0 53</td><td></td></tr>
<tr><td>m_o</td><td colspan="3">0 47</td><td></td></tr>
<tr><td>f_{paek}</td><td colspan="3">0 08</td><td></td></tr>
<tr><td>P(f_{p ek})</td><td colspan="3">4 50</td><td></td></tr>
<tr><td>ε</td><td></td><td colspan="3">0 55</td></tr>
<tr><td>m_o</td><td></td><td colspan="3">0 54</td></tr>
<tr><td>f_{pe k}</td><td></td><td colspan="3">0 08</td></tr>
<tr><td>P(f_{pe k})</td><td></td><td colspan="3">5 44</td></tr>
</table>

3 8 - Variation of some statistics with T_M, Δt and F_C

Tables 1, 3 and 4 show the variation of ε, m_o, f_{peak} and $P(f_{peak})$ with T_M, Δt and F_C respectively Variation of ε with the same parameters is also shown in Fig 11

As said in 3 5 variation of Δt will not influence spectrum estimates, as long as $F_D \leq F_N$ This is illustrated in table 3

Convergence of ε values when T_M increases is apparent from table 1 and Fig 11, from T_M = 70s onwards f_{peak} is practically the same for all T_M but $P(f_{peak})$ increases with the closing of the spectral window since the influence of neighbouring low values steadily decreases This increase, however, is small for T_M > 80 s m_o values vary only slightly with T_M and F_C as seen in tables 1 and 4 It is clear that F_C should exert no influence on f_{peak} and $P(f_{peak})$ Choice of F_C is nevertheless very important for the calculation of ε as is seen in Table 4 and Fig 11, and should be made according to 3 5

Table 3

c(0)=0 506	Variation of Δt			T_M= 80s f_C=0 25
Δt	0 5	1 0	1 5	2 0
ε	0 52	0 53	0 53	0 53
m_o	0 506	0 506	0 506	0 506
f_{peak}	0 080	0 080	0 080	0 080
$P(f_{peak})$	5 59	5 58	5 65	5 56

Table 4

c(0)=0 506	Variation of F_C					Δt=0 5 T_M= 80s
f_C	0 150	0 200	0 250	0 300	0 650	1
ε	0 45	0 50	0 52	0 52	0 79	0 95
m_o	0 495	0 505	0 506	0 506	0 507	0 508
f_{peak}	0 08					
$P(f_{peak})$	5 59					

3 9 - Conclusions

The considerations made from 3 1 to 3 8 led to the adoption of the following characteristics for the final computation of the bottom spectrum

- No prefiltering
- Window Parzen
- Maximum lag 80 s
- Cutoff frequency 0 3 cps
- Sampling interval 0 5 s (although this could have been wider the corresponding computations were already available)
- 80% confidence intervals

The number of degrees of freedom was 55 and the bandwidth 0 023 cps The resulting values for ε, m_o, f_{peak} and $P(f_{peak})$ are presented on table 1 Value computed for c(0) was 0 5056

This final bottom spectrum is shown in Fig 12

Table 5

T_Z	10 8
ε	0 58
H_1	4 5
H_2	3 5
H'_1	7 1
H'_2	5 5
$(H_s)_1$	2 8
$(H_s)_2$	2 4
$(H'_s)_1$	4 4
$(H'_s)_2$	3 8
$(m_o)_1$	0 48
$(m_o)_2$	0 35

4 - WAVE STATISTICS AND WAVE HEIGHT DISTRIBUTION

4 1 - Computations of wave statistics by the Tucker-Draper method

The Tucker-Draper method [8], [9], was used to compute the statistics of Table 5 By this method, the fundamental values read from the record are T_Z (mean zero up crossing period) H_1 (sum of the highest crest with the lowest trough), H_2 (sum of the second highest crest with the second lowest trough) To obtain surface values H' from bottom values H the classical Hydrodynamics Formula was used

$$H' = H \operatorname{ch} \frac{2\pi d}{L} \qquad (18)$$

where d (depth) ≈ 23 3 m

$$L \text{ (wave length)} = \sqrt{gd}\left(1 - \frac{\pi d}{4\ 68\, T_Z^2}\right) T_Z{}^{*} \qquad (19)$$

g being the acceleration of gravity

Computations were based on the digitalized record using a sampling interval $\Delta t = 0\ 5$ s

4 2 - Wave height distribution compared with the Rayleigh distribution

Wave height in the present paper is the distance between the le-

* - This formula is derived from $c = \sqrt{\frac{gL}{2\pi} \operatorname{th} \frac{2\pi d}{L}}$ by a series expansion of the th and substituting L by its approximate value $L = \sqrt{gd}\, T_Z$

vels of a crest and the preceding trough For the record considered, Fig 13 shows that the wave distribution is different from the Rayleigh distribution, which was to be expected since $\varepsilon \simeq 0\ 51$, a value greater than the limit (0 4) below which, according to Cartwright and Longuet-Higgins [10], wave heights follow reasonably well that distribution

5 - COMPARISON BETWEEN WAVE STATISTICS OBTAINED FROM THE PRESSURE RECORD BY SPECTRAL ANALYSIS AND THE TUCKER--DRAPER METHOD

In Table 6 wave statistics obtained from the pressure record by spectral analysis and the Tucker-Draper method are compared

Both for m_o and H_s a pair of values is presented corresponding to calculations made from H_1 e H_2

Table 6

	S P	T-D
ε	0 54	0 58
m_o	0 51	0 48
		0 35
H_s	2 84	2 8
		2 4

6 - CORRECTION TO SURFACE OF PRESSURE RECORD STATISTICS

Though the aim of the work undertaken is mainly the processing of digital data extracted from wave records, which makes unimportant the nature (surface or bottom) of the record used, a comparison of results arising from a weather hindcast with those from spectral analysis and the Tucker-Draper method is possible only if a correction to surface is applied to the latter Formula (18) was used with and without the instrument factor 1 25 which is recommended by the makers of the LNH type pressure wave gauge as adequate for the correction to surface of the significant wave height The same formula, without the instrument factor, and the linearity hypothesis for spectrum decomposition as a sum of infinitesimal sinusoids leads to the following relation between surface and bottom spectra, $P_S(f)$ and $P_d(f)$ respectively

$$P_s(f) = P_d(f)\ \mathrm{ch}^2\ \frac{2\pi d}{L(f)} \qquad (20)$$

where $L(f)$ is the wave length corresponding to frequency f

It should be noted that (1) the instrument factor is very important and an investigation should be made to determine if the recommended value of 1 25 is adequate for sea wave regime at the Portuguese Coast, (2) formula (20) is based on an unverified hypothesis, namely, on linearity of the spectrum decomposition as a sum of infinitesimal sinusoids In Fig 14 results of the surface correction for the spectrum are shown Some pertinent remarks are the following

- The peak of the surface spectrum occurs at frequency 0 085 cps, which is very close to the one of the bottom spectrum
- The existence of a second peak at 0 11 cps may or not have a physical significance (as, for instance, a local storm superimposed on the pre-existent one) as pointed out in 3 6 a)
- Up from f = 0 13 cps, P(f) values increase without any physical significance Owing to the great depth (23 3 m) of the recording wave gauge, $\mathrm{ch}\ \frac{2\pi d}{L}$ values increase rapidly to absurd results from that frequency on

7 - WAVE HINDCAST FOR DECEMBER 18th, 1968

7 1 - Analysis of the synoptic charts

Examining the synoptic charts for some days prior to December 18th, 1968, it is apparent that the pressure field has taken a little varying shape in the last three days (Fig 15), the isobarics following approximately a northwesterly direction, and that Leixões is included in the generating area over which this three-day storm actuated As will be seen later the sea should be considered as fully arisen

7 2 - Approximate determination of the fetch to be considered

In Fig 16, the maximum fetch relating to the meteorological situations of the storm days (16^{th}, 17^{th} and 18^{th}) is delimited by a dashed line After a mean wind speed during the storm is fixed, it can be seen that any of the indicated fetches is greater than the minimum fetch corresponding to a fully arisen sea

7 3 - Wave refraction up to the wave gauge site

Recorded values at Leixões at a depth of 23 3 m cannot be transformed in off shore values without some criticism

As examination of Fig 17* reveals, waves following a course close to NW have a refraction coefficient given by He/Ho (where He is wave height at the harbour entrance and Ho the corresponding off-shore value) which is independent of period (the latter varying between 10 and 17 s) and has a nearly constant value of 0 7 approximately This is the adopted value for a correction of statistics for off-shore conditions, according to a rough, yet seemingly adequate, procedure for the present case Concerning the course followed by waves the following remarks should be noted. It was not possible to secure radar photographs made at Leixões and so there is no direct information on which course to consider However, the shape of the isobarics shows that that course should be close to NW On the other hand, for a better accuracy of the He/Ho coefficient, a refraction plan including the wave gauge site should be available, indeed, elements provided in [2] concern only the harbour entrance Therefore the adopted 0 7 value should not be accepted unreservedly

7 4 - Hindcast

7 4 1 - Observations from North Atlantic Weather Ships

For a wave hindcast for the day the pressure record was made, knowledge of wind, sea and swell data provided by the North Atlantic Weather Ships is necessary According to their position and keeping in view the NW direction of the storm, the most important ships are, by increasing order of distance to Leixões K ship (730 km), J ship (1500 km) and C ship (2400 km)

K ship is the only one included in the zone common to the three fetches indicated in Fig 16 and, even so, she is outside the minimum fetch corresponding to a fully arisen 30-knot wind sea, as will be seen later This is then the ship from which the most important information stems, as regards waves at Leixões (Fig 18)

We may roughly guess at the day and the period of time in which waves recorded at Leixões were passing by each one of the three ships (Table 7) Once more we are simplyfying things by assuming that the propagation speed is $V_T = g\ T/4\pi$ and considering the period range to be 8 to 16 s

Table 7

Ship	Dist to Leix	T (s)	V_T (m/s)	Time to reach Leix	Ship passed by on day	hr
K	730	8	6 2	33	16	19
		16	12 5	16	17	12
J	1500	8	6 2	67	15	9
		16	12 5	33	16	19
C	2400	8	6 2	108	13	16
		16	12 5	54	15	22

Table 8

Ship	Swell			Sea	
	Dir	H (m)	T (s)	H (m)	T(s)
K	W (const	6 (const)	12 (const)	9 5 (8-10)	10 (const)
J	276 (270-282)	7 5 (7-8)	12 (const)	6 5 (6-7)	8 5 (8-9)
C	–	–	–	5 (4-6)	8 (7-9)

Table 8 is a résumé of sea and swell conditions recorded by each one of the three ships in such a way that waves could have arrived at Leixões during the twenty-minute recording period at 3 a m on December 18^{th}, 1970

* - Reproduced from Fig 5 in [2]

Another important problem in the application of the two most common forecasting methods (SMB and PNJ) concerns the choice of a mean wind speed likely to have occurred during the storm Fig 16 shows that the mean values of wind speed recorded at K ship on the two days before Dec 18th are

Dec 16th U = 42 knots (a)

Dec 17th U = 32 knots (b)

Assuming that the sea is fully arisen (Leixões being included in the generating area) it will be interesting to know land-recorded values at the closest weather station to Leixões (Pedras Rubras)

Table 9

Day		17		18
Hour		6	18	6
Dir		W	NW	W
Speed (knots)	Pedras Rubras	22	14	18
	Leixões	29	20	26

Mean value at Leixões (sea) U=25 knots (c)

With U values indicated in (a), (b) and (c) a mean value should now be adopted If a Neumann spectrum is chosen corresponding to U=30 knots according approximately with the pressure spectrum (Fig 14) and assuming a fully arisen sea, the following minimum values are obtained for duration and fetch t_m = 23 h, F_m = 518 km Hence, in fixing U, only Leixões and K ship should be considered (K ship is even beyond F_m in case U=30 knots)

In short with mean U values of 25 knots (at Leixões) and 32 knots (during the previous 24 hours) and now reasoning backwards, the choice of U=30 knots seems to be adequate as the mean prevailing wind speed which brought about the fully arisen sea relative to the Dec 18th storm

7 4 2 - Forecasts of the S M N *

Based in lighthouse observations the S M N wave forecast for Dec 16th, 17th and 18th was "strong northwesterly waves" which corresponds to wave heights between 2 5 and 5 5 m

7 4 3 - The Sverdrup-Munk-Bretschneider Method (S M B)

For U=30 knots and assuming a 800 nautical mile fetch corresponding approximately to Dec 18th, we get t=50 h, H_s=6 09 m, T_s=14 s

There is small accuracy in the choice of fetch, but observing Fig 1-7 on page 19 in [3] one can see that wave characteristics vary only slightly between 800 and 1000 miles The mean wind speed of 30 knots can be assumed constant during two days which corresponds approximately to t=50 h

Wave forecast for the wave gauge zone, taking into account a He/Ho value of 0 7 is then H_s=4 3 m

7 4 4 - Pierson-Neumann-James method (P N J)

As mentioned above a 30-knot Neumann spectrum was chosen according approximately with the pressure spectrum (Fig 14)

The corresponding hindcast statistics have the following off-shore values

H_s = 6 6 m

$H_{1/10}$ = 8 3

H_{av} = 3 m

T_{av} = 8 5 s

f_{peak} = 0 08 cps

Taking into account the correction for the wave gauge zone we get

H_{av} = 2 1 m H_s = 4 4 m $H_{1/10}$ = 5 7 m

* - Serviço Meteorológico Nacional (Portuguese Weather Service)

8 - COMPARISON OF COMPUTED AND HINDCAST SURFACE SIGNIFICANT WAVE HEIGHT VALUES

Tucker-Draper method

H_s (from H_1) = 4 42 m

H_s (from H_2) = 3 78 m

Taking the arithmetic mean and correcting to surface with ch $\frac{2\pi d}{L}$ only, we get H_s=4 1 m If the instrument factor 1 25 recommended by Chatou is used, then H_s=5 1 m

P N J method H_s=4 4 m

S M B method H_s=4 3 m

S M N forecast for Dec 18^{th} Wave heights between 2 5 and 5 5 m

These values are close to one another and so it seems reasonable to conclude that at Leixões, on December 18^{th}, 1968, the surface significant wave height was 4 to 5 m

9 FINAL SUMMARY OF THE MORE NOTEWORTHY ASPECTS OF THIS PAPER

Partial detailed conclusions were drawn in preceding chapters of seemingly more salient aspects A summary of important points is

- The sampling interval may be as high as 2 s if spectral analysis only is intended
- If a smaller interval is used, which means a higher Nyquist frequency (F_N), then a convenient cutoff frequency (F_c) should be chosen, possibly according to coincidence of m_o and c (0)
- A good estimate of the spectrum width depends on a well balanced choice of some parameters, especially on the cutoff frequency
- A good choice of maximum lag for the autocovariance function is of great importance to spectral analysis
- Results from record partition seem to indicate that a more careful study must be made of the necessary record duration
- Some aspects of wave data acquisition which are well-known yet should be kept in mind concern
    - correction to surface of bottom spectra (lack of linearity) and of bottom statistics (adequate instrument factor)
    - great influence of depth on wave attenuation
    - faulty characteristics of pressure records for spectral analysis of sea waves
- For hindcast and related meteorological problems, the usual difficulties arise in fetch determination (to the resolution of this problem, directional spectra may provide a useful contribution) It is also important to make a realistic criticism to sea wave data secured from meteorological services, indeed, it is frequent to detect absurd correlations between wave heigths and periods and sometimes fallacious distinctions are made between sea and swell

As final conclusion we would like to stress the importance of a close collaboration between different techniques used in Maritime Hydraulics This collaboration is essential for an intimate connexion in the development of both the physical (meteorology, oceanography, fluid mechanics) and the mathematical approaches

ACKNOWLEDGEMENTS

The authors are grateful to the Instituto Hidrografico for the facilities granted concerning reading and digitizing wave records They are also in debt to Dr Anthímio de Azevedo from the Serviço Meteorologico Nacional for the meteorological elements provided and their interpretation

REFERENCES

[1] - Bonnefille, Germain, Lepetit - Statistiques des houles naturelles mesures par le houlographe autonome type LNH, Houille Blanche nº 8, 1967

[2] - Reis de Carvalho, J J e D Vera-Cruz - Regime de ondulação no Porto de Leixões, Estudo realizado para a Sacor, LNEC, Julho de 1964

[3] - Shore Protection, Planning and Design, Technical Memorandum nº 4, CERC (formerly Beach Erosion Board), 3rd Edition, June 1966

[4] - Jenkins, G M and D G Watts - Spectral Analysis and its applications, Holden-Day, San Francisco, 1969

[5] - Barber, N F - Experimental Correlograms and Fourier Transforms, Pergamon Press, 1961

[6] - Barber, N F - A plea for the rectangular lag window, Ocean Wave Spectra, Prentice-Hall, Englewood Cliffs, 1963

[7] - Blackman, R B and J W Tukey - The Measurement of Power Spectra from the Point of View of Communications Engineering, The Bell System Technical Journal, January and March, 1958

[8] - Tucker, M J - Analysis of Records of Sea Waves, Proceedings of the Institution of Civil Engineers, October 1963

[9] - Draper, L M - The Analysis and Presentation of Wave Data - A plea for Uniformity, Proceedings of the Xth Conference on Coastal Engineering, Tokyo, 1966

[10] - Cartwright, D E and M S Longuet-Higgins - The Statistical Distribution of the Maxima of a Random Function, Proceedings of the Royal Society, A, 237, 1956

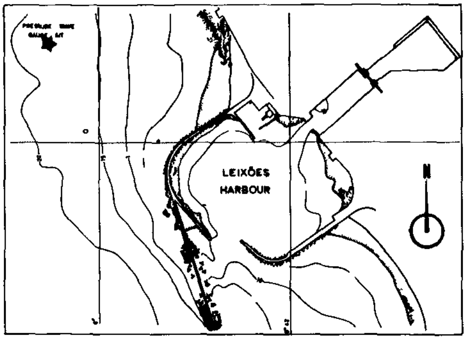


Fig 1 - Wave gauge site

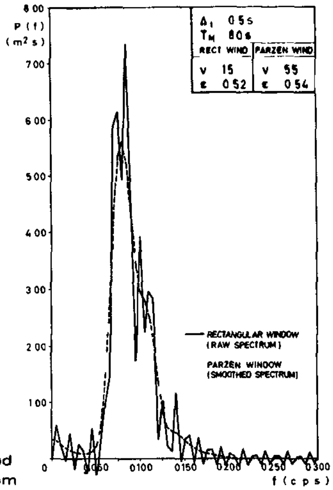


Fig 4 - Raw and smoothed spectrum

Fig 2 - Autocovariance function

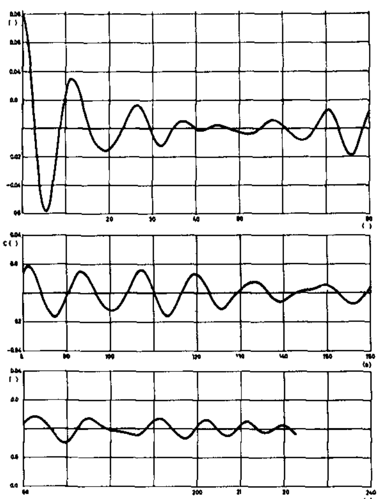
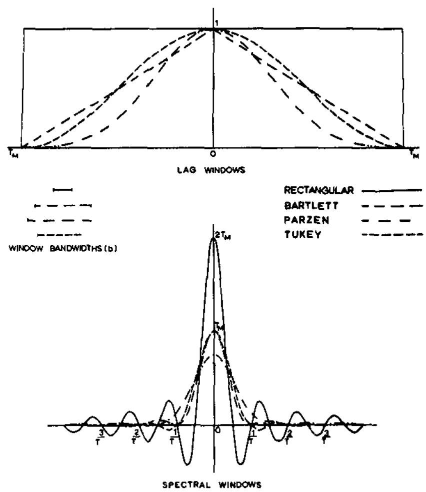


Fig 3 - Windows considered



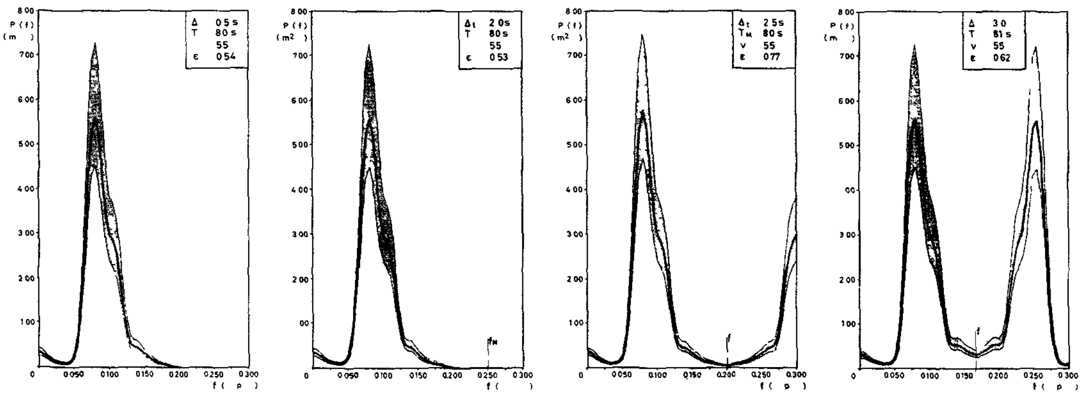
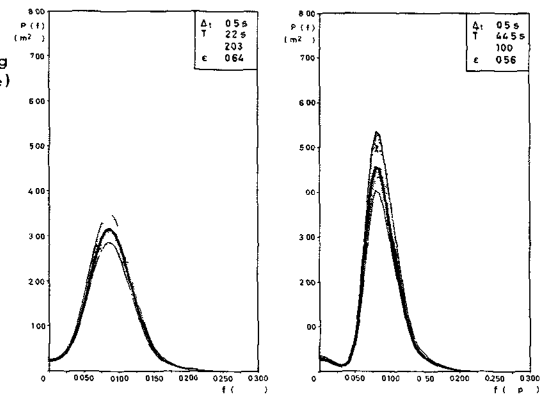


Fig 6 - Effect of Δt on aliasing

Fig 8 - The window closing technique (cont next page)



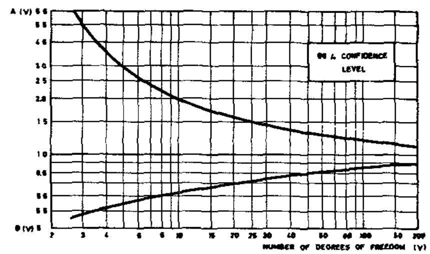
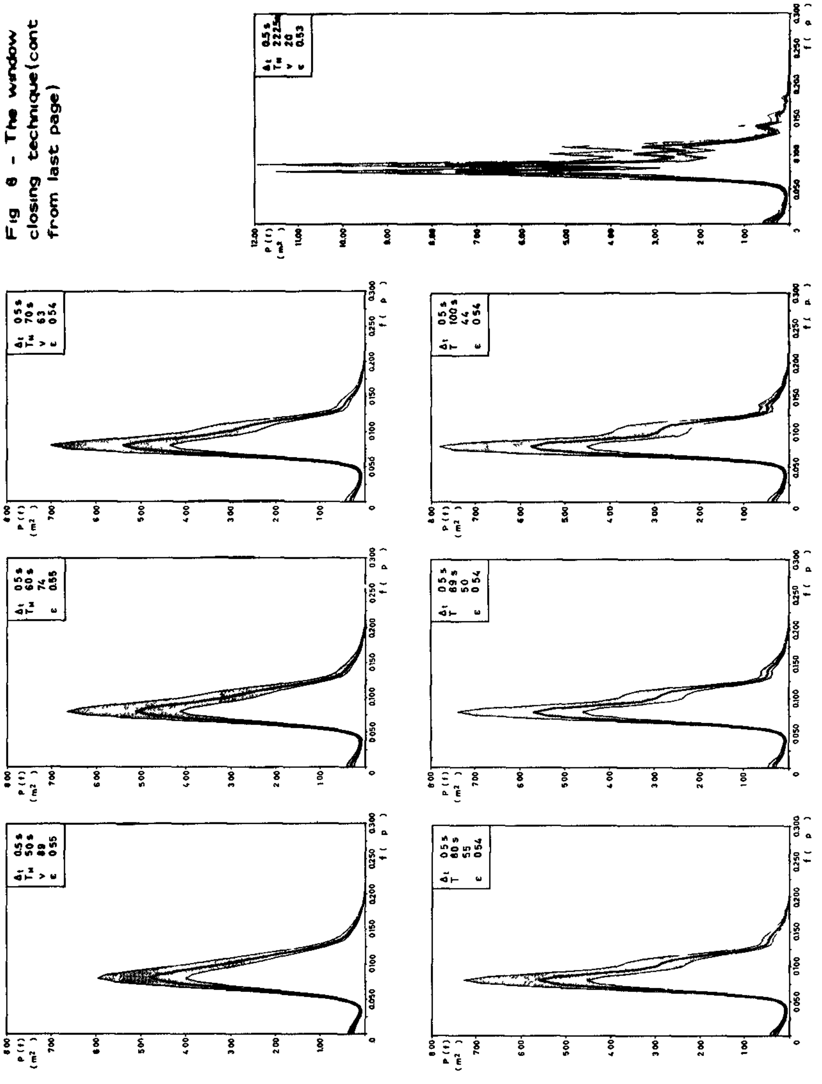


Fig 5 - Variation of the confidence interval with v

Fig 8 - The window closing technique (cont from last page)



45

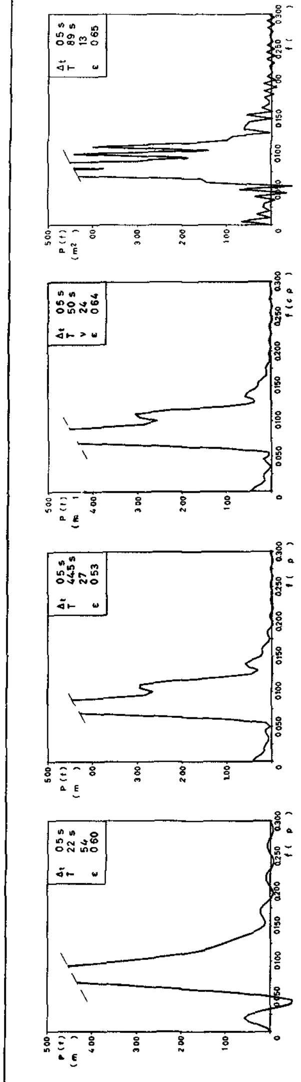


Fig 9 - The rectangular lag window and the choice of T_M

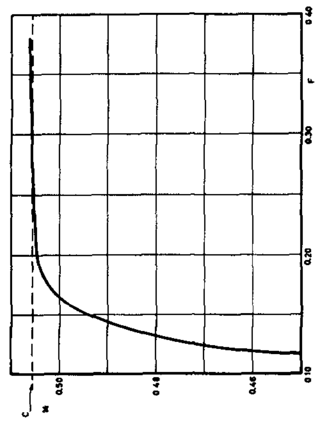


Fig 7 - Variation of m_o with f_c

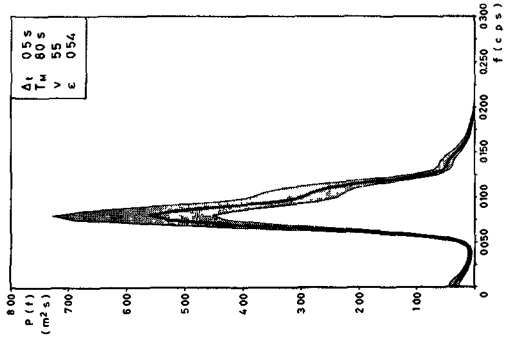
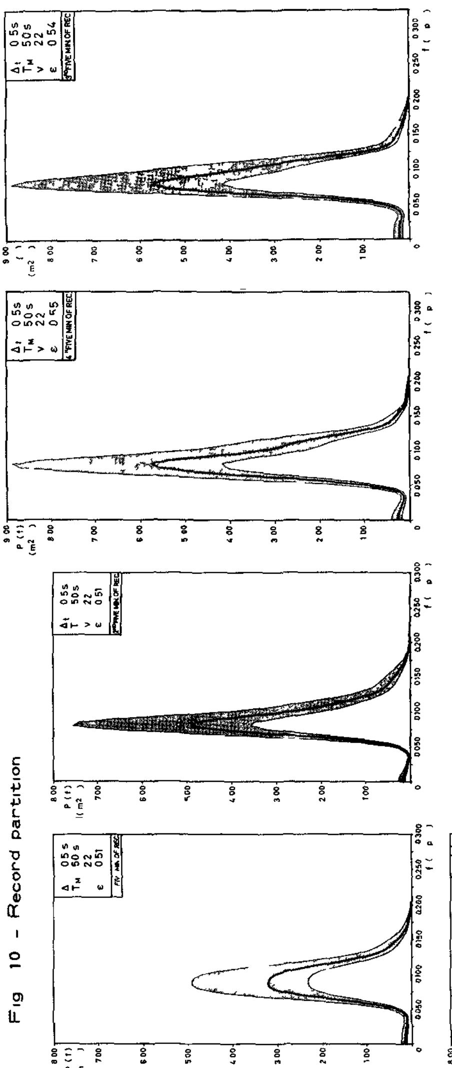


Fig 12 - Selected bottom spectrum



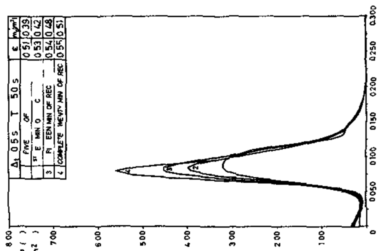


Fig 10 - Record partition

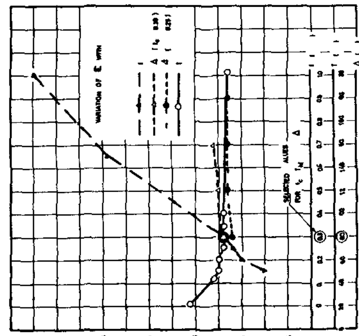


Fig 11 - Variation of ε with Δt T_M and f_c

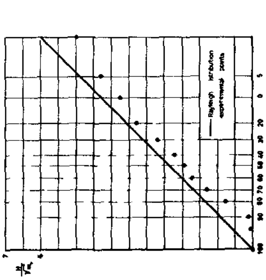


Fig 13 - Exceedance probabilities of bottom wave heights compared with Rayleigh distribution

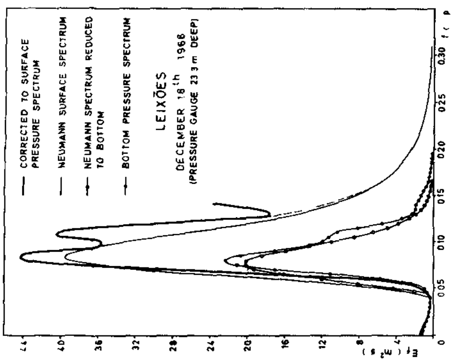
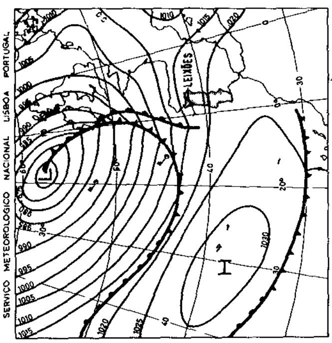


Fig 14 - Surface version of bottom pressure spectrum compared with Neumann spectrum



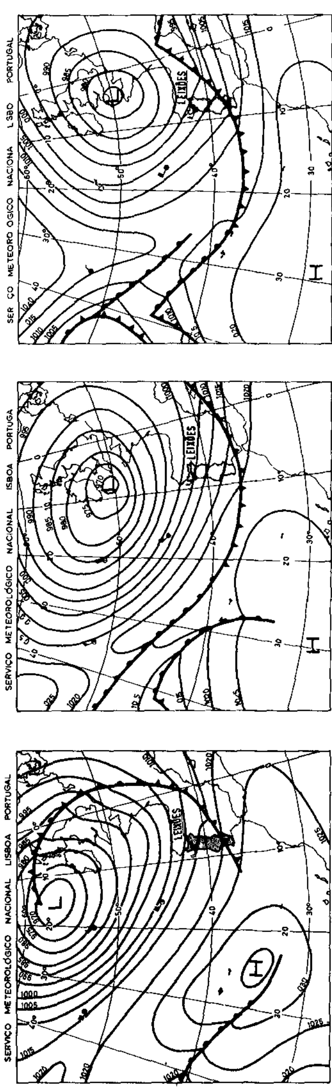
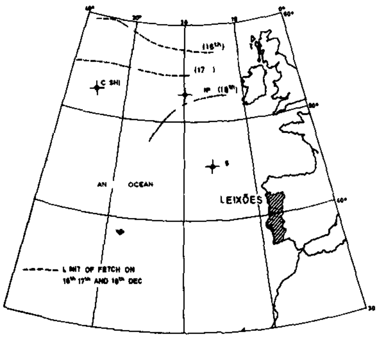


Fig 15 - Synoptic charts

Fig 16 - Fetch delimitation



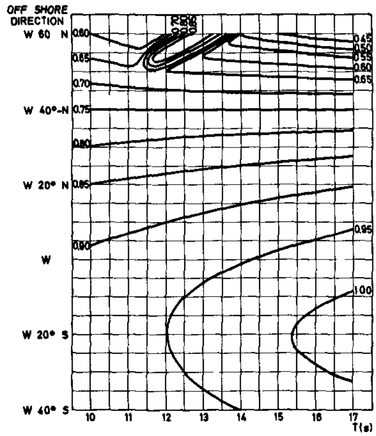


Fig 17 - Refraction coefficients at Leixões harbour entrance

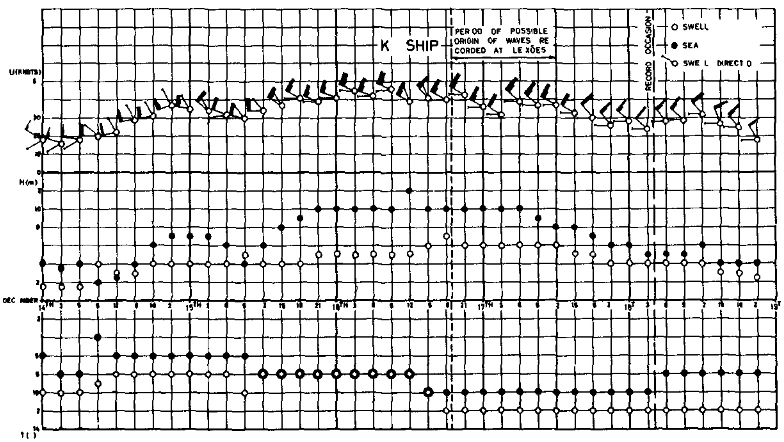


Fig 18 - Observations at North Atlantic K weather ship