# CHAPTER 99

## THE MOTIONS OF SMALL BOATS IN STANDING WAVES

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## ABSTRACT

Some dynamical aspects of the surge motions of small boats moored asymmetrically with elastic non-linear restraints are discussed herein. In connection with the theoretical analysis of mooring dynamics, experiments were conducted to determine the periods of the free oscillations of a 26-ft boat moored in various ways in a floating slip. These results are in reasonable agreement with those predicted theoretically. An analytical study of the mooring dynamics of seven small boats indicated that the periods of free oscillation were less than about 10 sec., hence, for these boats the important wave periods for forced oscillations would be in the range of those of storm waves.

## INTRODUCTION

An important question which is raised in the design of harbors for small boats of the size usually used for pleasure is the range of wave periods which could be expected to cause damage to these moored boats If these periods can be predicted then the problem of minimizing wave-induced oscillations in a small-boat harbor may be simplified, because it would be sufficient to investigate the response of a harbor to waves only over the limited range of wave periods which affects small craft. The objective of the study reported herein was to develop a method of predicting this range of wave periods for surge motions of a small boat given the dimensions and characteristics of the boat and its mooring system.

It is first important to consider some of the major differences between small and large boats which relate to their dynamics. The most obvious difference between the two cases is size, in the case of small pleasure boats the vessels which are of interest are less than approximately 60 ft long and the displaced weights are less than approximately 10 tons as compared to large vessels whose length may be 300 ft to 700 ft with displaced weights ranging from 9,000 to 50,000 tons. However, it is not the displaced weight alone which is of importance in defining the response of the vessel to waves, but the important parameter is the ratio of the restoring force associated with the mooring system to the inertial force (which depends on the displaced weight of the vessel).

With respect to this ratio, the nature of the restoring force for large ships can be quite different from that for small boats. Wilson (1967a) has described the mooring systems used by a number of large vessels. In general those systems consisted of a large number of lines extending from the bow, the stern, and the midship of the vessel to the dock which restrict motion both in the fore and aft direction as well as in the direction perpendicular to the dock. Even though the elastic characteristics of the individual mooring lines may be quite different from one another, on the average the restoring force for motion in the fore and aft direction for similar displacements would be approximately the same. (It should be noted, however, that for motions of a large vessel in sway, i.e., in a direction perpendicular to the dock, the restoring forces would be highly asymmetrical.) In addition to these features, usually the mooring lines for large ships have slack to allow for motions due to changes in the tide.

In contrast, generally small boats are moored with only a few lines, e.g., in mooring in a "U-shaped" slip usually only two bow lines and two stern lines are used. Under these conditions, it is evident that the restoring force for motion in one direction may be quite different from the restoring force for similar displacements in the opposite direction. This asymmetry can have a significant effect upon the motion of the boat and potential boat damage. For instance, if a boat is moored in the slip with little clearance between the bow of the boat and the front of the slip, impact damage to the bow may be possible due to the asymmetrical restoring forces. This type of damage possibly qould be eliminated by using a stiffer more symmetrical mooring system.

Although only two examples of the differences have been presented, through these it is evident that attention must be given to the details of mooring for small boats which could perhaps be neglected for large vessels.

## THEORETICAL CONSIDERATIONS

An analysis is presented in this section which describes the motions of a moored body in surge when exposed to a standing wave system. The mooring system used in the analytical model can result in non-linear asymmetrical restoring forces which resist the wave-induced motions. Only surge motions (boat displacements either toward the bow or toward the stern) are considered, and the boat is treated as a block body with no attempt being made to fully describe in detail the shape of the vessel. The innovation in this development concerns incorporating a non-linear asymmetrical restoring force in the equation of motion of the analytical model. For a more complete development of the basic equation of motion the interested reader is referred to Wilson (1958), Kilner (1960), and Raichlen (1965).

The block body is a rectangular parallelepiped of length 2L, beam B, and draft D moored in a way such that the only allowable motions are in surge. The standing wave is formed in water of a constant depth d by a progressive wave which is reflected from a perfectly reflecting surface located a distance b from the center of the moored body. The x-coordinate is measured from the center of the body in the at-rest-position and denotes the movement of the center in surge motion. (A schematic diagram of this body moored in a standing wave system is shown in Fig. 1.)

The equation of motion in surge of the moored body is

$$M\ddot{x} = F_p + F_i + F_d + F_r \tag{1}$$

where $\ddot{x}$ is the acceleration of the body, $(d^2x/dt^2)$, M is the mass of the body and $F_p$ = net pressure force acting on the ends of the body, $F_i$ = inertial force due to the unsteady motion of the body and the fluid, $F_d$ = force due to viscous effects, $F_r$ = restoring force due to the mooring system.

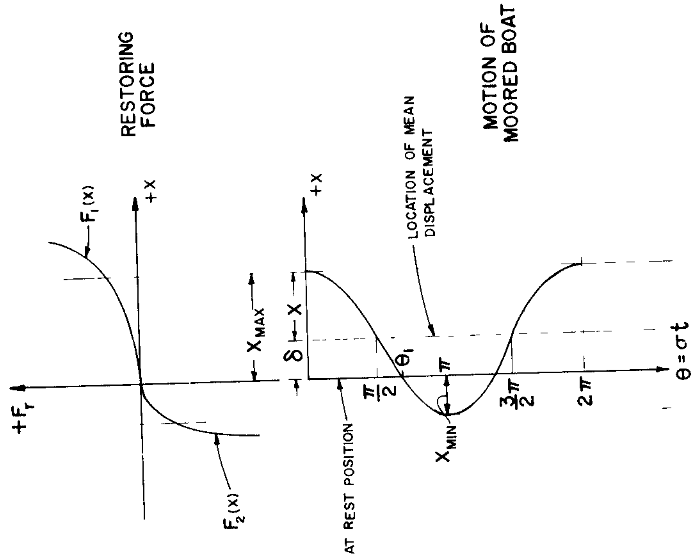


Fig 2 Schematic Diagram of Non-Linear Asymmetrical Restoring Force and Assumed Resultant Boat Motion

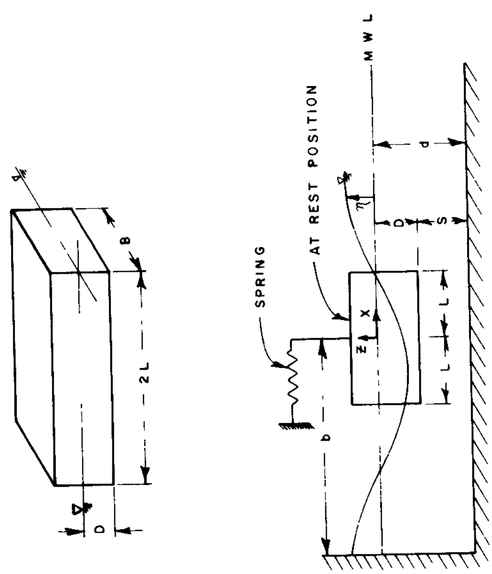


Fig 1 Schematic Diagram of Block Body

Considering small amplitude standing waves and certain assumptions relating to the water particle accelerations it can be shown (see Wilson (1958)) that, neglecting viscous effects, Eq. 1 becomes

$$\ddot{x} + (F_r/C_M M) = \dot{U} \tag{2}$$

where $C_M M = M + M_x'$ ($M_x'$ is the added hydrodynamic mass) and

$$\dot{U} = \zeta\,\sigma \cos \sigma t \tag{3a}$$

$$\zeta = Ag\,[\sinh kd - \sinh ks]\,\sin k L \sin kb/(LD\sigma\, k \cosh kd) \tag{3b}$$

The forcing function $\dot{U}$ is the average over the displaced volume of the block body of the water particle accelerations in the standing wave. It is introduced as a simplification in describing the pressure force and the inertial force in Eq. 1 (see Wilson (1958) and Raichlen (1965)).

The problems in solving Eq. 2 arise primarily due to the form of the restoring force, $F_r$. Although the variation of the virtual mass coefficient, $C_M$, with boat shape and other conditions is not well known, the effect of this quantity on the dynamic response of a small moored boat in most cases is not as important as the effect of the restoring force if a reasonable estimate of $C_M$ is made initially.

Consider the following general expression for the restoring force

$$F_r(x) = F_1(x) \text{ for } x > 0 \tag{4a}$$

$$F_r(x) = F_2(x) \text{ for } x < 0 \tag{4b}$$

where x is defined as positive for motion toward the bow and negative for motion toward the stern. (The functions of Eqs. 4 may take any form linear or non-linear, symmetrical or asymmetrical, free travel or no free travel.)

For simplicity, the equation of motion (Eq. 2) is rewritten so that the phase relation between the forcing function and the body response is incorporated in the forcing function

$$\ddot{x} + (F_r/C_M M) = \zeta\,\sigma \cos(\theta - \varphi) \tag{5}$$

where $\theta = \sigma t$ and $\varphi$ is the phase angle and Eq. 4 describes the variation of $F_r$.

If the restoring forces $F_1(x)$ and $F_2(x)$ are different then it is reasonable to assume that the mean position of motion of the vessel will be different from the at-rest-position of the vessel. Therefore, the general solution to Eq. 5 is taken as

$$x = \delta + X \cos \theta \tag{6}$$

Eq. 6 is an approximate solution which neglects the effect of higher harmonics; however, it has been shown in most mooring problems (see Kilner (1960)) that this is justified. The motion described by Eq. 6 is shown in a schematic manner in Fig. 2 along with assumed restoring forces described by Eq. 4.

By setting $x = 0$, Eq. 6 describes the degree of asymmetry of the restoring force, that is

$$\delta = -X \cos \theta_1 \tag{7}$$

and for $\theta_1 = \pi/2$ the displacement of the mean position from the at-rest-position, $\delta$, becomes equal to zero. It is noted from Fig. 2 that $\theta_1 = \pi/2$ implies that $F_1(x) = -F_2(x)$, hence, the restoring forces and the resultant motion for that case are symmetric about the at-rest-position of the boat. If, on the other hand, $\theta_1 = \pi$ the boat motion is highly asymmetrical and in accordance with Eqs. 6 and 7 there would be no motion of the vessel in the minus x-direction, i.e., for a given displacement $F_2(x) >> F_1(x)$.

Klotter (1951) has described the solution of an equation such as Eq. 5 with non-linear restoring forces based on the averaging method of W. Ritz. Rewriting Eq. 5 as

$$E(x) = \ddot{x} - \zeta\, \sigma \cos (\theta - \varphi) + (F_r/C_M M) \tag{8}$$

the averaging method furnishes the following two conditions

$$\int_0^{2\pi} E(x) \cos \theta \, d\theta = 0 \tag{9a}$$

$$\int_0^{2\pi} E(x) \sin \theta \, d\theta = 0 \tag{9b}$$

Assuming the form of the displacement described by Eq. 7, the first two terms of Eq. 8 can be integrated directly in accordance with Eq. 9a (or Eq. 9b). The integral in Eq. 9a which is comprised in part of the third term in Eq. 8, referring to Fig. 2, can be integrated over three intervals, hence·

$$\int_0^{2\pi} \frac{F_r}{C_M M} \cos \theta \, d\theta = \int_0^{\theta_1} \frac{F_1(x)}{C_M M} \cos \theta \, d\theta + \int_{\theta_1}^{2\pi-\theta_1} \frac{F_2(x)}{C_M M} \cos \theta \, d\theta$$
$$+ \int_{2\pi-\theta_1}^{2\pi} \frac{F_1(x)}{C_M M} \cos \theta \, d\theta \tag{10}$$

In order to solve Eq. 10 so that the solution to Eq. 9a can be obtained it is necessary to evaluate the restoring forces $F_1(x)$ and $F_2(x)$ for a particular mooring system. To carry out the indicated integrations it is assumed that the functions $F_1(x)$ and $F_2(x)$ can be represented reasonably well over the range of displacements which are of interest by polynomials consisting of odd powers of x, that is to say

$$F_1(x) = a_1 x + a_2 x^3 + a_3 x^5 \tag{11a}$$

$$F_2(x) = r_1 x + r_2 x^3 + r_3 x^5 \tag{11b}$$

Eq. 6 is substituted into Eqs. 11a and 11b and the resulting expressions are then substituted into Eq. 10. Although the integrations indicated in Eq. 10 are straightforward, the result consists of numerous terms in powers of sines and cosines of $\theta_1$ and other system parameters and for the sake of brevity will not be given explicitly in this discussion (see Raichlen (1968)). In order to evaluate $\theta_1$ the equation of motion (Eq. 8) is averaged over one wave period. When this is done only one term remains

$$\int_o^{2\pi} \frac{F_r}{C_M M} d\,\theta = 0 \tag{12}$$

Eq. 12 can be integrated over the same intervals as used in Eq. 10 and a solution can be obtained for the variation of $\theta_1$ with X (see Raichlen (1968) for details of this analysis).

Solving Eq. 9b in a similar way it can be shown that $\varphi = 0, \pi$, which is as it should be for the forced oscillations of an undamped dynamic system.

Using this approach the following general equation is obtained which describes the response in surge of the arbitrarily moored body in terms of its maximum displacement in the positive x-direction from the at-rest-position

$$\sigma^2 + \frac{\zeta(1 - \cos\theta_1)}{X_{max}}\sigma - \frac{(1 - \cos\theta_1)}{\pi X_{max} C_M M}\int_o^{2\pi} F_r \cos\theta\, d\,\theta = 0 \tag{13}$$

where $X_{max} = \delta + X = X\,(1 - \cos\theta_1)$

The integral in Eq. 13 is evaluated from Eq. 10 with the variation of $\theta_1$ with X determined from Eq. 12. It should be noted, the most direct solution to Eq. 13 is to solve for $\sigma$ given values of $\zeta$, X, and $\theta_1$. Since the maximum displacement in the negative x-direction, $X_{min}$, can be obtained from the relation $X_{min} = (2\delta - X_{max})$, the response curve is fully defined.

To demonstrate some of the general features of the solution the problem is reduced to one with linear restoring forces by letting $a_2 = a_3 = r_2 = r_3 = 0$. It can be shown that Eq. 13 reduces to:

$$\sigma^2 + \frac{\zeta(1 - \cos\theta_1)}{X_{max}}\sigma - \frac{S}{C_M M} = 0 \tag{14a}$$

where $S = (a_1 - r_1)\left[\frac{\theta_1}{\pi} - 1 - \frac{\sin 2\theta_1}{2\pi}\right] + a_1$ (14b)

and $\theta_1$ is given by $\tan\theta_1 = \left[r_1\,\pi + (a_1 - r_1)\,\theta_1\right]/(a_1 - r_1)$ (14c)

For $a_1 \neq r_1$ this case corresponds to a bi-linear asymmetrical restoring force and for $r_1 >> a_1$, $\theta_1$ tends to $\pi$ which indicates highly asymmetrical motions. When $a_1 = r_1$, Eq. 14a becomes the equation of motion for a linear spring system and Eq. 14c shows that $\theta_1$ equals $\pi/2$, the condition of symmetry as seen in Fig. 2.

In an actual mooring problem neither of these two cases is likely to occur and the problem must be treated as indicated by Eqs. 10 and 13 after fitting Eqs. 11a and 11b to the predicted (or measured) restoring forces. In order to develop a method of predicting these restoring forces, consider first the schematic diagram of Fig. 3 which shows a block body (representing the small boat) moored with four lines to a dock. Initially the lines are slack by some arbitrary amount $\Delta\ell$. Upon movement of the body in the positive x-direction this slack becomes zero after the boat moves through a distance $\Delta_f$ denoted as the free travel. Until the vessel has moved this distance the restoring force is considered to be zero. This is because the lines usually have a small unit weight and the restoring force is assumed to develop only due to the elastic characteristics of the lines Therefore, when $x = \Delta_f$ the line tension is zero ($T^* = 0$), and $T^*$ becomes greater than zero for $x > \Delta_f$.

The restoring force $F_r$ shown in the plan view in Fig. 3 is equal to the sum of the x-components of the line tensions, $T^*$, for all lines acting to restrain the boat's motion in a given direction. Hence

$$F_r = \sum_{n=1}^{N} T^*_{x_n} \tag{15}$$

where from the geometry of Fig. 3

$$T^*_{x_n} = T^*_n \cos \beta_n \cos \alpha_n \tag{16}$$

The elastic characteristics of the mooring line can be represented as

$$T^*/T^*_{Brk.} = R\,\epsilon^m \tag{17}$$

where $T^*_{Brk.}$ is the average breaking strength of the particular line, $\epsilon$ is the strain (total elongation divided by length), and R and m are constants.

Consider the definition sketch of a typical line shown in Fig 4 where the line goes from a cleat on the dock to a cleat on the boat passing through a guide at its point of first contact with the boat. Therefore, the line is divided into two sections the first section going from dock to boat with a length $\ell' = \ell + \Delta\ell$, and the second section on the boat with a length $\ell_1$.

For small elongations and considering the geometry shown in Fig. 4 the x-component of the line tension can be expressed as

$$T^*_{x_n} \cong K_1 \left[\frac{f + \Delta_f}{\ell'(\ell'+\ell_1)}\right]^m \frac{\left[1 - \left(\frac{y_o}{\ell'}\right)^2\right]^{1/2}}{\left[1 - \left(\frac{z_o}{\ell'}\right)^2\right]^{(m - 1/2)}} [x - \Delta_f]^m \tag{18}$$

where $K_1 = RT^*_{Brk.}$ (see Raichlen (1968)).

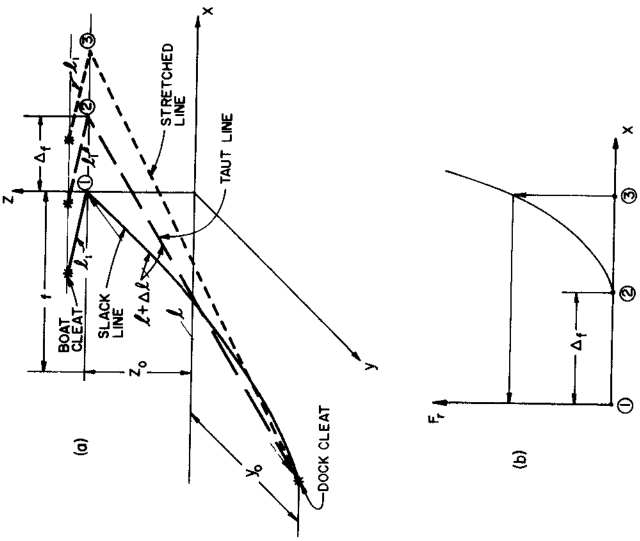


Fig 4 a) Geometry of Typical Mooring Line
b) Schematic Diagram of Non-Linear Restoring Force with Free Travel

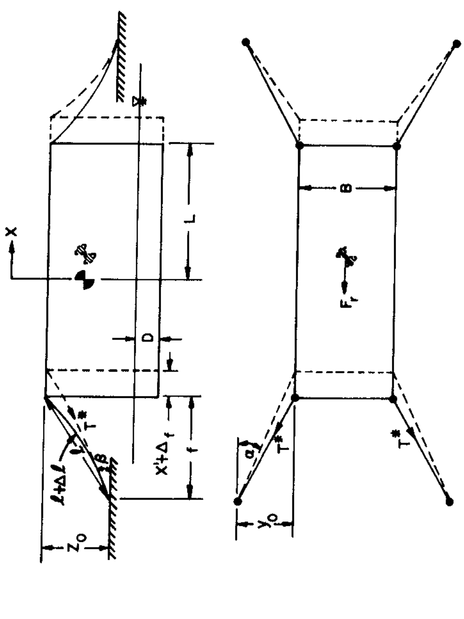

Fig 3 Schematic Diagram of Four-Point Mooring System

The method of approach used in determining the variation of the restoring force with the displacement of the boat in the direction of either the bow or the stern follows directly from Eqs. 17 and 18. For a given displacement the component tensions, $T^*_{x_n}$, computed from Eq. 18 and the corresponding elastic characteristics $K_1$ and m, are summed in accordance with Eq. 15, repeating this procedure until the restoring force vs. displacement curve is constructed.

In order to determine the coefficients R and m in Eq. 17 the elastic characteristics of various materials used for mooring have been obtained from various manufacturers. In addition to these data, tensile tests were conducted in the laboratory using a short section of 5/8-inch diameter manila rope. Based on the available information the values used in this study are: R = 18, m = 1.48 for manila and R = 2.54, m = 1.65 for nylon. The breaking strength of ropes composed of various materials, $T^*_{Brk.}$, were obtained from the manufacturer's data or from Wilson (1967b).

## RESULTS AND DISCUSSION OF RESULTS

A series of tests were conducted at a small craft harbor near Los Angeles, California, (Marina del Rey) to determine the periods of free oscillation of a 26 ft boat (referred to herein as Harbor Boat No. 3) moored with a system of four 5/8 in. diameter manila lines (two bow and two stern lines) to a "U" shaped floating slip. The boat had a length of 26 ft, a beam of 9 ft 2 in., a maximum draft of 2 ft 4 in., and a loaded displacement of approximately 7,000 lbs. The hull in cross section was a modified Vee shape and for this reason the block body approximation used for the shape is perhaps not as good as when applying this to larger vessels, such as tankers. The dimensions of the mooring system are presented in Table 1 using the nomenclature of Fig. 4.

Table 1. Dimensions of Mooring Lines of Harbor Boat No. 3

| | Bow Lines | | Stern Lines | |
|---|---|---|---|---|
| | Port | Starboard | Port | Starboard |
| $z_o$ | 27.5* | 29. | 18. | 16. |
| $y_o$ | 27. | 32. | 33. | 36. |
| f | 60. | 57. | 76. | 80. |
| $\ell$ | 71. | 72. | 85. | 89. |
| $\ell_1$ | 36. | 36. | 0. | 0. |

*Note all dimensions in inches

These dimensions show that although an attempt was made to moor the boat in the center of the slip, this objective was not achieved exactly. The length of the mooring line, $\ell$, shown in Table 1 is for the case of all lines taut, in the test two other cases were examined· 4 in. and 8 in. of slack in all lines.

In these tests the boat was displaced in either the direction of the bow or the stern with known applied forces and the displacement relative to the dock was recorded photographically by means of a time-lapse movie camera. A photograph of the boat and its mooring system is presented in Fig. 5, which also shows the experimental arrangement for the tests. A scale is fixed to the boat along with an electric stop clock to facilitate measurements of the displacement and of the period of the free oscillation. The vessel was displaced using rigging that went either from the bow or the stern of the boat to piling located nearby. After the force vs. displacement characteristics of the mooring system were determined the boat was displaced either in the positive or the negative x-direction with an applied force of 200 lbs or 500 lbs and then suddenly released. Although it would have been desirable to use an applied force greater than 500 lbs this was not considered possible due to the condition of the pilings. The motions of the boat after it was released were recorded using a time-lapse camera and the displacement-time history of the boat's motion as well as the period of free oscillation were determined from a frame-by-frame analysis of the film. Such experiments were conducted for the condition of taut lines, 4 in. slack in all lines, and 8 in. slack in all lines, with the initial force applied first to the bow and then to the stern.

The variation of the restoring force with displacement for Harbor Boat No. 3 moored with taut 5/8-in. manila lines is presented in Fig. 6. In addition to these data, predicted restoring force curves are presented in Fig. 6 which were obtained from Eqs. 15 and 18 using the elastic characteristics presented along with the breaking strength of this rope (4400 lbs). The agreement between measurements and theory in Fig. 6 is relatively good, one of the main reasons for disagreement is the fact that the point of application on the boat of the applied load was above the center of gravity of the boat. Therefore, some pitching of the boat was introduced, contrary to the conditions of the analysis.

Predicted force displacement curves are presented in Fig. 7 for the three conditions which were tested. Curves are shown for the bow lines (resisting motion in the negative x-direction) and the stern lines (resisting motion in the positive x-direction) for the four-point mooring system which is shown in the inset in Fig. 7. The effect of the addition of even a small amount of slack between approximately 5% and 10% of the line length) is seen in Fig. 7, for these cases the force displacement curves approach a constant displacement for small applied forces corresponding to the free displacement of the vessel. The measured data for the two cases of slack lines are not included in this figure, the agreement between the experiment and the theory was good with respect to the slope of these curves but it was poor with respect to the prediction of the free displacement of the vessel. The primary reason for this is attributed to difficulties in measuring the at-rest-position of the vessel for the case of slack lines.

Typical examples of the free oscillations of the small boat are presented in Figs. 8a and 8b for the case of all lines taut and 8 in. slack in all lines. The periods of free oscillation were determined from such displacement-time histories for all cases tested.

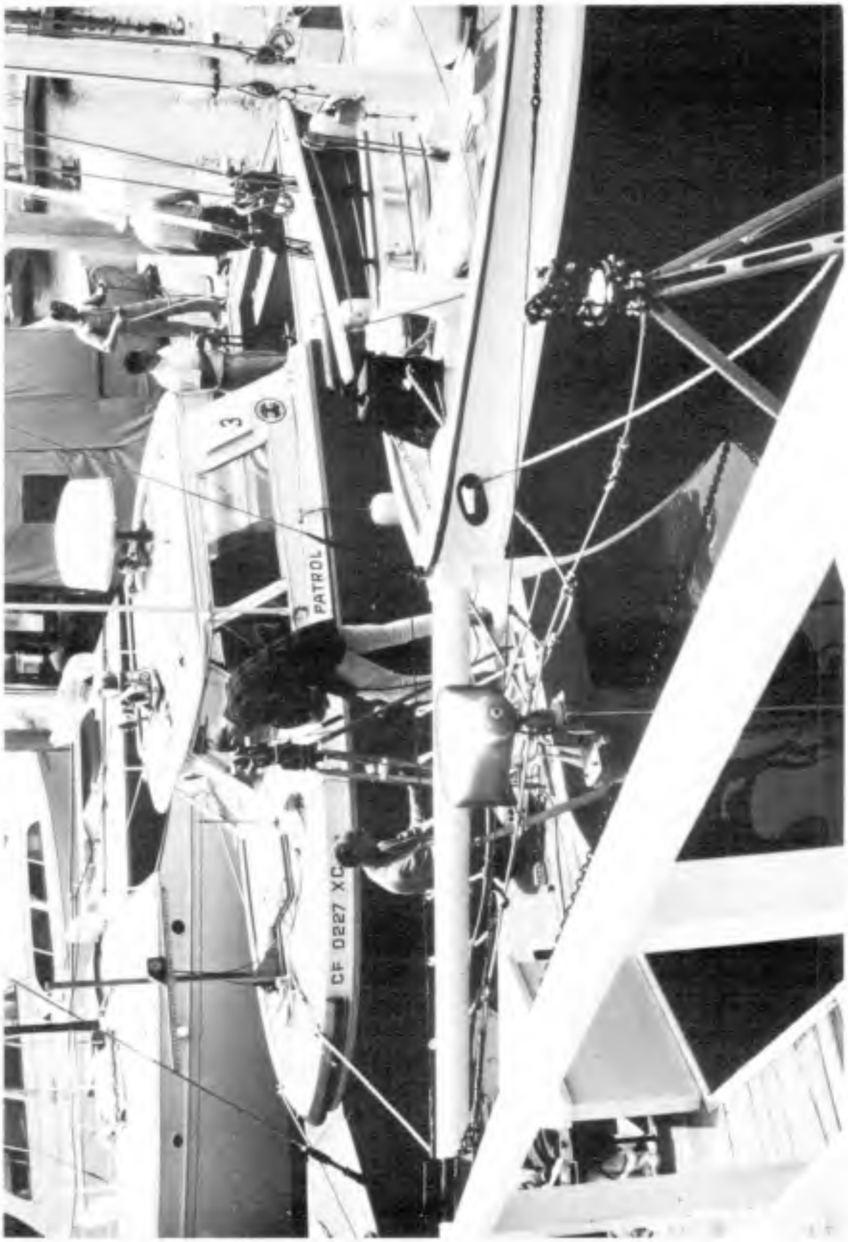


Fig. 5 Photograph of Harbor Boat No. 3 and Experimental Equipment

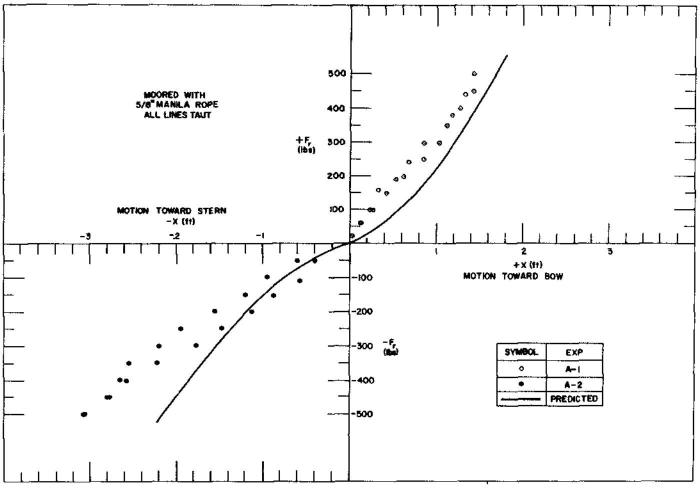


Fig 6 Measured and Predicted Restoring Force vs Displacement Harbor Boat No 3, All Lines Taut

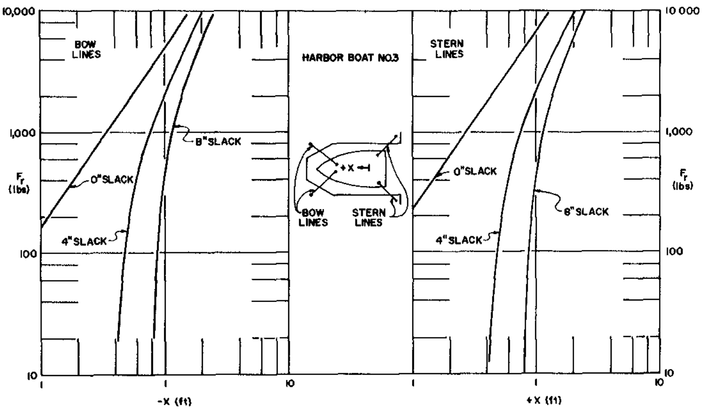


Fig 7 Predicted Restoring Force vs Displacement Harbor Boat No 3, All Lines. Taut, 4 inches Slack, 8 inches Slack

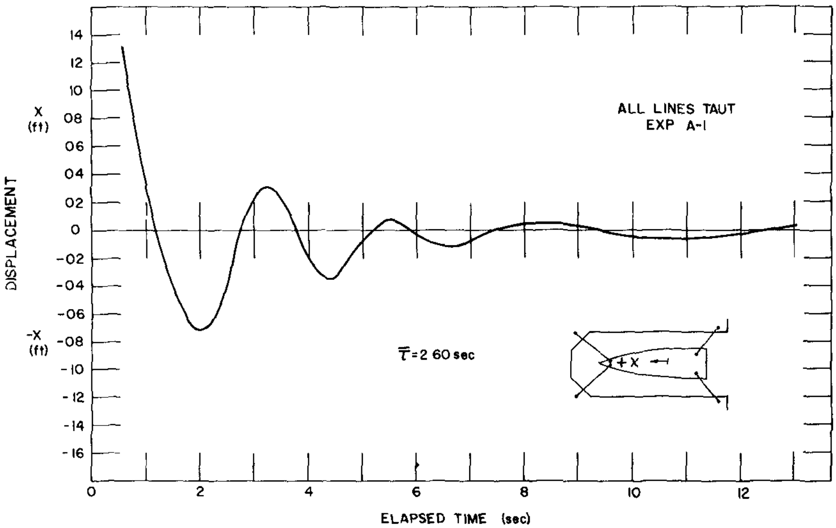


Fig 8a Free Oscillations, Harbor Boat No 3. Taut Lines, 500 lbs Force Applied to Bow

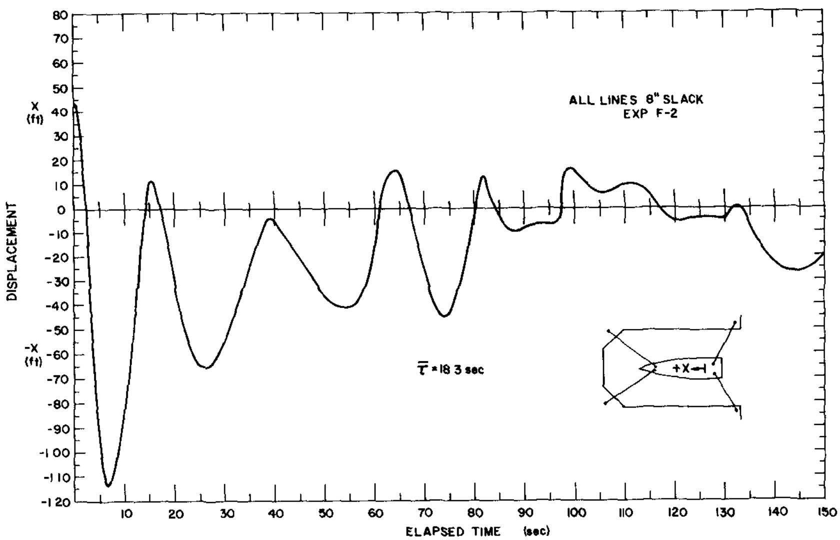


Fig 8b Free Oscillations, Harbor Boat No 3. 8 inches Slack, 500 lbs Force Applied to Bow

The results of these experiments are compared to the results of the theoretical analysis in Fig. 9. The equation of motion for the case of free oscillations is given by Eq. 5 with the right-hand side set equal to zero. A value of the virtual mass coefficient of $C_M = 1.2$ has been used in Eq. 5 for Harbor Boat No. 3 in accordance with data from Wilson (1958) for a floating body with a similar beam-to-length ratio. The curves which result are presented in Fig. 9 for the three cases investigated taut lines, 4 in. slack in all lines, and 8 in. slack in all lines. Measured data, similar to that obtained from figures such as 8a and 8b are included in Fig. 9 for two different definitions of the initial displacement. One set of data is based on initial displacements which were determined experimentally and the second set of data is for initial displacements which were theoretically determined for the given applied force, the former is labeled Meas. Init Displ and the latter is labeled Pred. Init. Displ. in Fig. 9. It is seen that both sets of experimental data show reasonable agreement with the theory except for the case of 4 in. slack and the case for taut lines with a force of 500 lbs applied in the negative x-direction. For small values of displacement the theoretical curve of free oscillations for the case of taut lines tends to a constant value of the period of oscillation because of the approximation to the force which was used (Eqs. 11) and not due to the nature of the mooring system. For displacements less than the free travel, for the cases with slack, motion is undefined since the restoring force is zero.

The undamped non-linear response curves for Harbor Boat No. 3 moored under the three different line conditions are presented in Figs. 10, 11 and 12. These response curves, obtained from Eq. 13, are plotted as the variation of the maximum displacement of the boat in surge from the at-rest-position ($X_{max}$ or $X_{min}$) with the wave period, T, for various values of the forcing function $\zeta$. It has been seen from Eq. 3b that the parameter $\zeta$ is the maximum with respect to time of the water particle velocity averaged over the displaced volume of the moored body. Since this is a function of the standing wave amplitude as well as certain parameters of the system, in general the interpretation of $\zeta$ is not simple and its value must be determined for given system parameters and wave amplitudes as a function of the wave period. The "backbone" curves, $\zeta = 0$, correspond to the case of free oscillation and are the same as those shown in Fig. 9.

These examples demonstrate that simple changes in the mooring system can have a profound effect upon the dynamics of the moored boat. For instance, the motions induced by storm waves (8 sec. to 12 sec. periods) for the boats moored with 8 in. of slack in the lines are approximately five to six times greater than those for the boat moored with taut lines. Where one mooring arrangement may be considered safe, the other may be potentially dangerous. In addition, damage to the boat moored with taut lines may be due to the failure of the lines or fittings, however, for the case with slack lines impact of the boat with the dock due to excessive motion may be the probable source of damage. It is interesting that for the three cases which are considered resonance occurs well below the periods associated with storm waves, viscous effects would not tend to alter this significantly.

Consider the variation of the forcing function $\zeta$, with the wave period. A specific case is presented in Fig. 13 where the variation of the ratio of the forcing function to the wave amplitude, $\zeta/A$, with wave period is shown for a block body whose center is located 120 ft from a reflecting surface

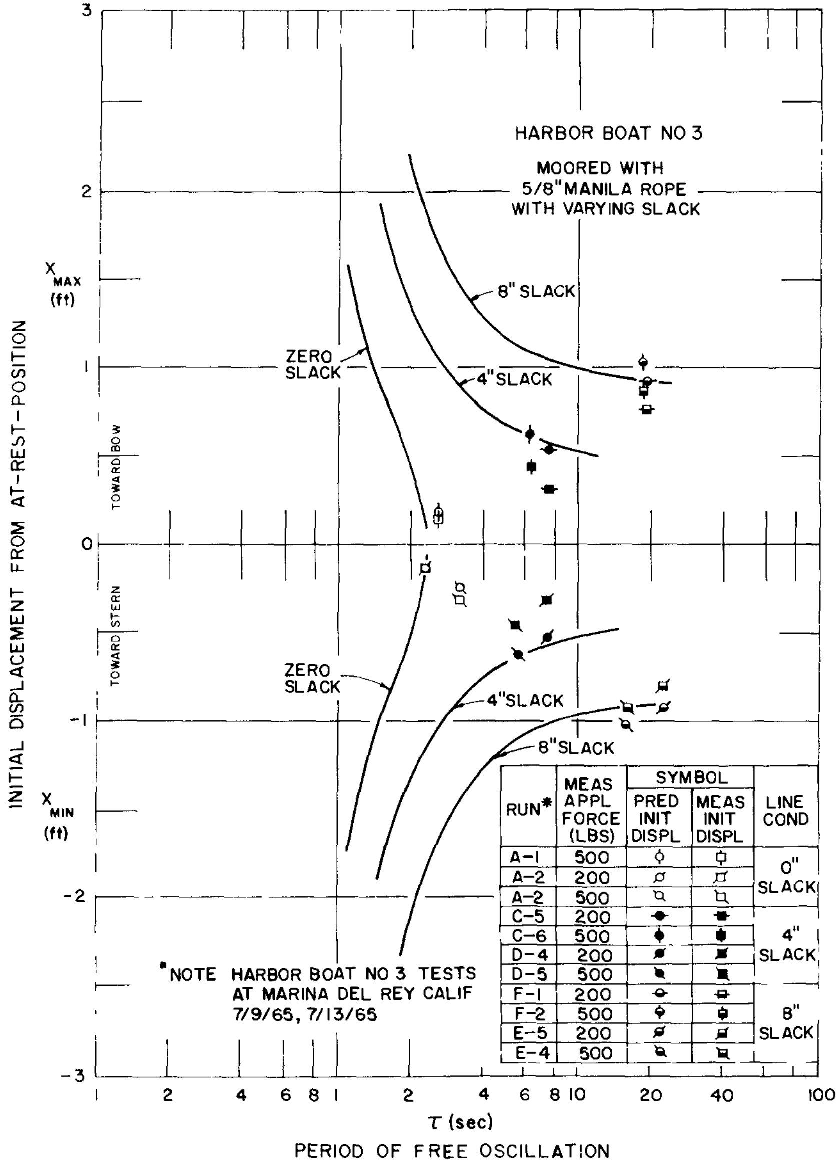


Fig 9 Measured and Predicted Periods of Free Oscillation Harbor Boat No 3

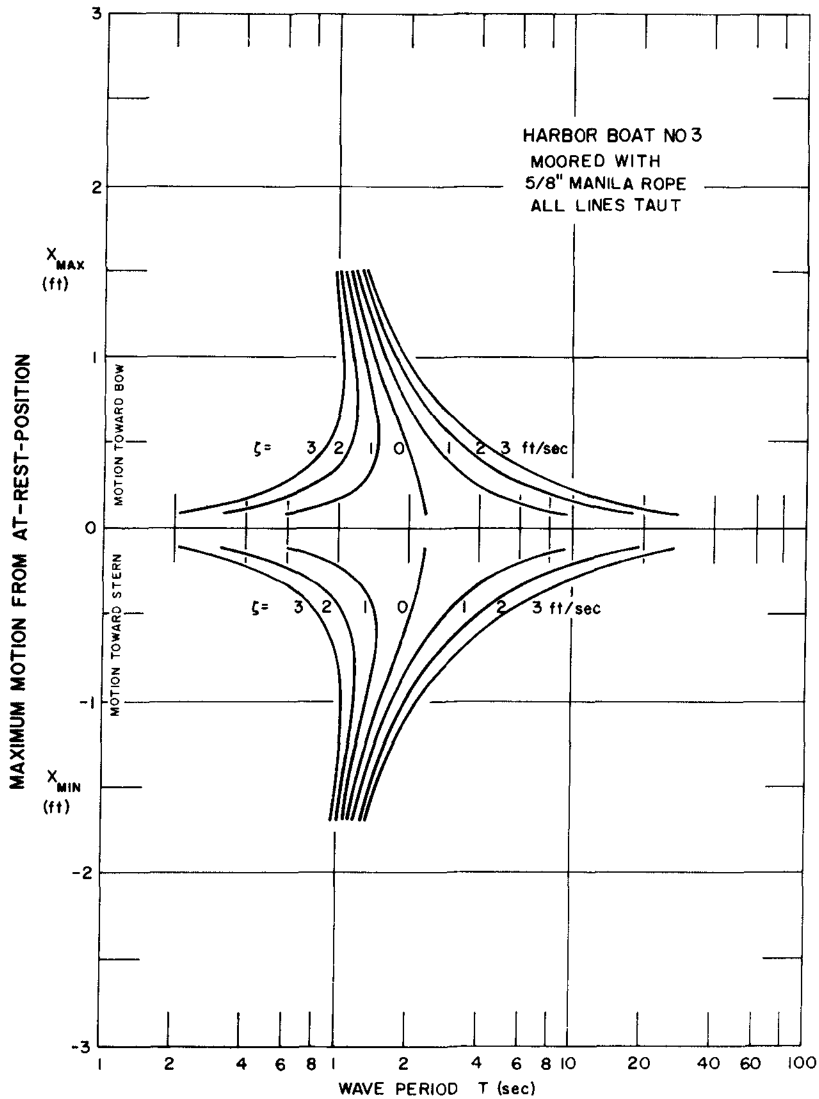


Fig 10 Response Curve Harbor Boat No 3, Taut Lines

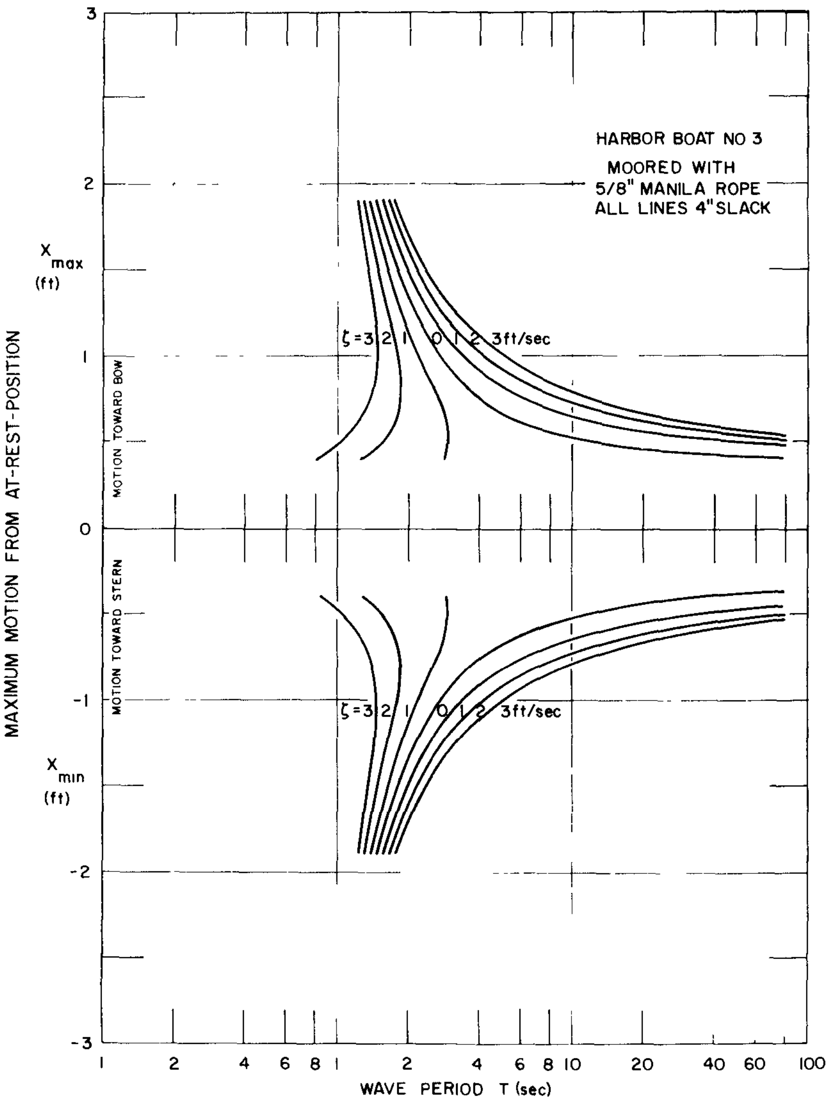


Fig 11 Response Curve Harbor Boat No 3
4 inches Slack in All Lines

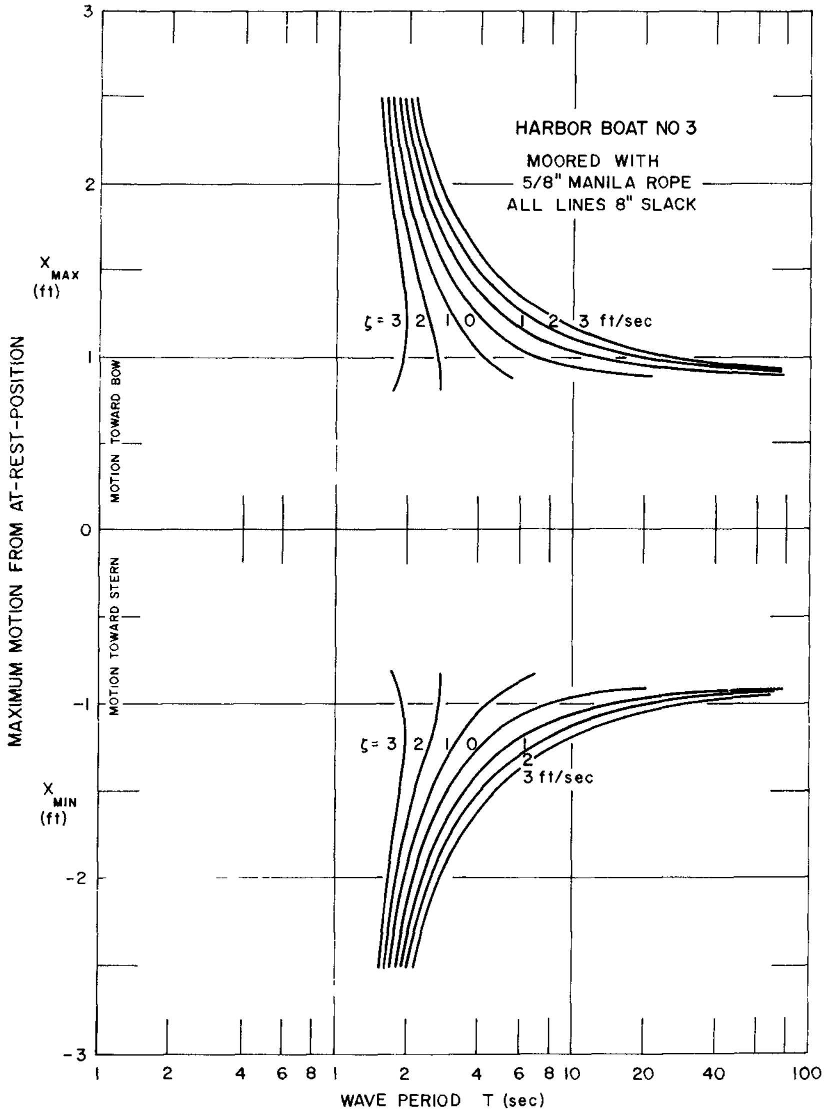


Fig 12 Response Curve Harbor Boat No 3
8 inches Slack in All Lines

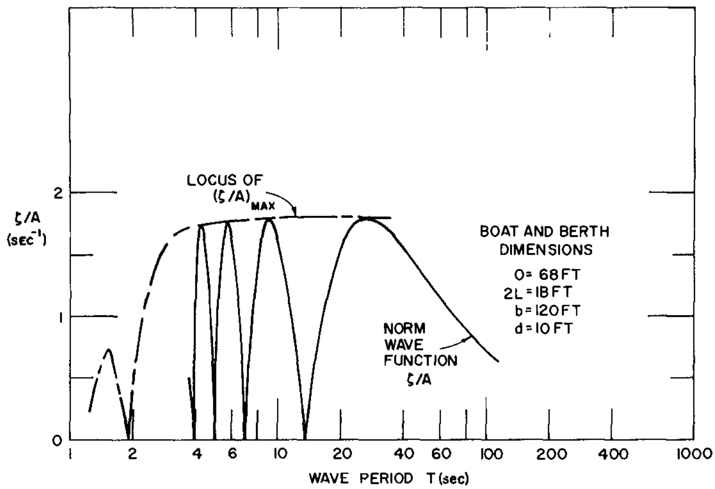


Fig 13 Variation of Normalized Forcing Function, ζ/A as a Function of Wave Period, T

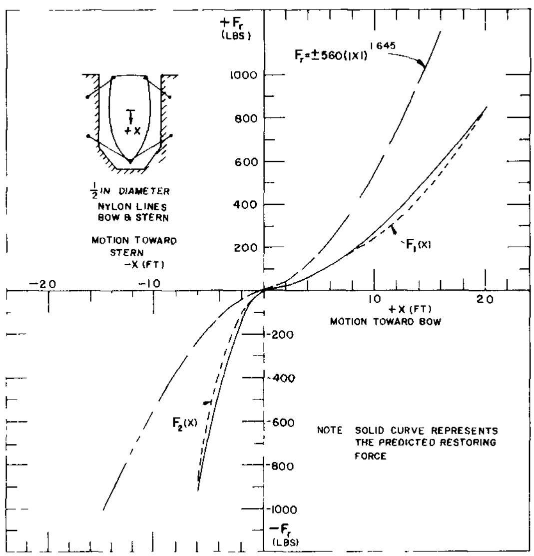


Fig 14 Predicted Restoring Forces vs Displacement, 25 ft moored Boat, Calif Reg No , CF 0394 CV

(b = 120 ft), the depth of water at the mooring site chosen for this example is 10 ft (d = 10 ft). These dimensions were chosen to demonstrate some of the features of the forcing function and only in a general way do they represent conditions at the berth at Marina del Rey. However, Fig. 13 demonstrates the periodic nature of the variation of the quantity $\zeta/A$ with wave period, at certain intervals this function and the locus of the maxima of this function become zero. This occurs when the crest of the standing wave is located at the center of the body and there is no net force acting, conversely at maxima nodes occur at the center of the body. Therefore, in general it is not sufficient to view the response curves for a particular boat simply as curves of constant $\zeta$ since the variation of the forcing function $\zeta$ with wave period is also important in completely describing the dynamic motion of the boat. However, to investigate potential damage it is possible to use a relation such as that shown in Fig. 13 to determine the maximum value of the forcing function, $\zeta_{max}$, which would be realized for a particular wave amplitude and then to view the response curves (such as Figs. 10, 11 and 12) which correspond to that maximum value.

A consequence of neglecting the asymmetric nature of the mooring systems for small boats is illustrated by the response curve of another boat whose in situ mooring dimensions were obtained at Marina del Rey. The boat is approximately the same size as Harbor Boat No. 3 a length of 25 ft., a maximum beam of 9 ft. and a maximum draft of approximately 2.5 ft. The estimated displaced weight of this boat is 5200 lbs. The variation of the restoring force with displacement predicted for this boat using Eqs. 15 and 18 is presented in Fig. 14. The solid curve corresponds to the predicted force-displacement relation and the curve with short-dashed lines corresponds to Eqs. 11a and 11b fitted to this relation. It is seen that due to the dimensions of the mooring system the restoring force for motion in the negative x-direction is greater than the force which results from similar displacements in the positive x-direction.

One set of response curves for $\zeta$ = 2 ft/sec. is presented in Fig. 15a based on the restoring forces labeled as $F_1(x)$ and $F_2(x)$ in Fig. 14 showing the asymmetrical nature of the response of this boat to a periodic forcing function. Included in Fig. 15a is the variation of $\delta$ (the distance from the average position to the at-rest-position) with wave period which also shows that the motion is not symmetrical about the at-rest-position but it is symmetrical about a mean position a distance $\delta$ away.

Since large vessels are usually treated as symmetrically moored systems (see Wilson, 1967a) it is of interest to treat the mooring system of this boat in a similar fashion to investigate the resulting response curves. A symmetrical restoring force was obtained by first plotting the predicted curves shown in Fig. 14 in the positive $F_r$ vs. x-quadrant and then computing the average displacement for a given applied force. The expression which best describes the resulting average curve is shown in Fig. 14 with long-dashed lines. The response curves for $\zeta$ = 2 ft/sec. based on the "averaged" restoring force are presented in Fig. 15b for motions only in the positive x-direction, the curves which correspond to motions in the negative x-direction would be mirror images. A comparison of Fig. 15a and Fig. 15b shows the problems which may arise in assuming symmetry where the mooring system is highly asymmetrical. For example, if the clearance between the bow of the boat and the slip had been small it would be possible

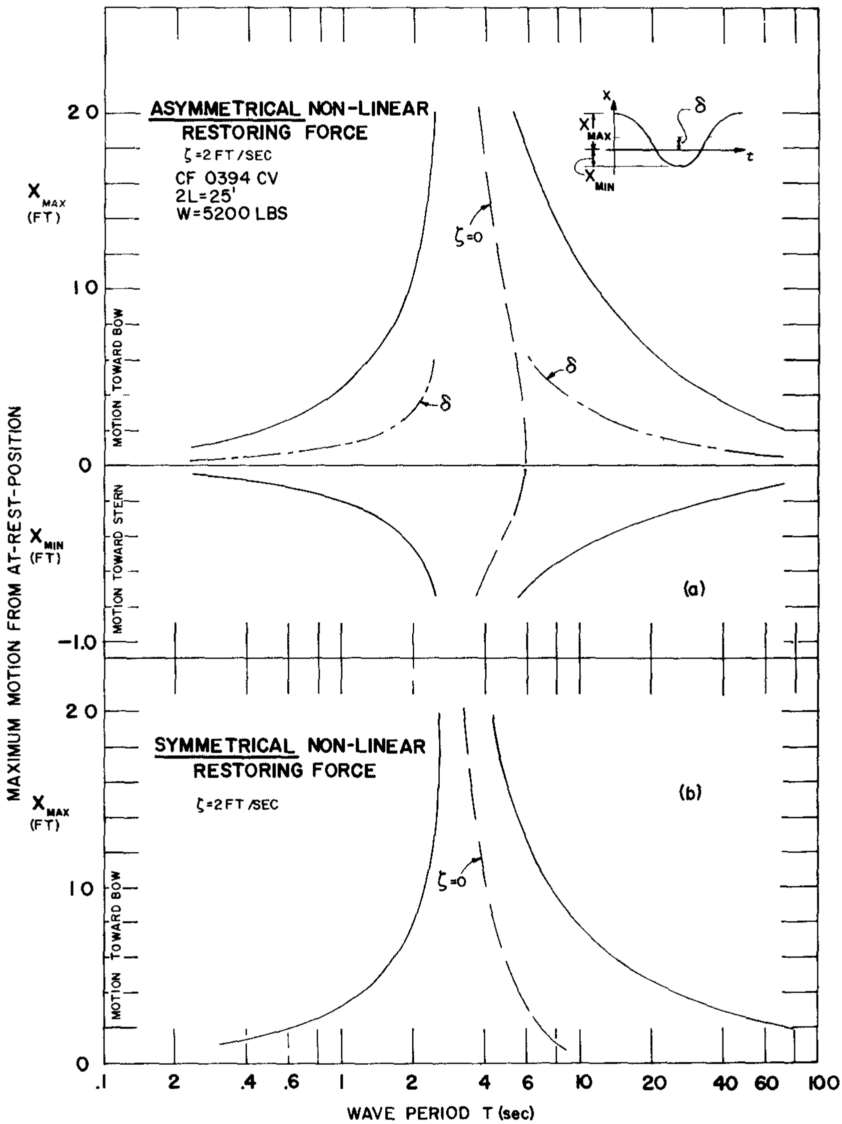


Fig 15 (a) Response Curve, 25 ft Moored Boat with Predicted Restoring Force

(b) Response Curve, 25 ft Moored Boat with Averaged Restoring Force to conclude, using an average restoring force system, that no damage would be expected due to waves of certain heights and periods whereas using the predicted restoring forces a distinct possibility could exist that the boat would actually strike the dock.

In a similar manner the response curves were obtained for six other small boats moored at Marina del Rey. The force-displacement relations were determined from the elastic characteristics of manila and nylon described previously and in situ measurements of the mooring dimensions. The lengths and estimated displaced weights of the boats are presented in Table 2, for more details of the mooring systems the interested reader is referred to Raichlen (1968).

Table 2. Measured Lengths and Estimated Weights of a Sample of Small Boats

| State of California Registration Number | Length (Ft-In.) | Estimated Weight (Lbs) |
|---|---|---|
| CF 6141 CB | 22'-5" | 3,700 |
| CF 0675 CJ | 24'-8" | 5,200 |
| CF 0394 CV | 25' | 5,200 |
| CF 0310 CV | 25' | 5,200 |
| CF 7651 CL | 29'-3" | 8,200 |
| CF 7198 CJ | 33' | 11,500 |
| CF 1756 CW | 38' | 17,000 |

The important range of wave periods for these small boats is shown in Fig 16 where the variation of the period of free oscillation is plotted as a function of the ratio of the displaced weight of the boat to the restoring force which corresponds to a given boat displacement in either the positive or the negative x-direction. Fig. 16 shows that for the range of restoring forces (or displacements) considered the periods of free oscillation of the boats varied from approximately 2 sec to 10 sec. In the region of large line stresses, $10 > W/F_r > 1$, the periods generally were less than 6 sec. It is interesting to note that the smallest boat had associated periods which were about twice as large as the periods associated with the largest boat investigated. This was due to the characteristics of the mooring systems, demonstrating that it is possible without mooring restrictions in a small boat harbor for the range of the important periods of oscillation of small pleasure boats to be close to the periods of storm waves. However, Fig. 16 also indicates the incidehce of small boat damage due to waves in a harbor may be reduced by adopting restrictions on the elastic characteristics of lincs and the mooring geometry which are permitted for certain size boats.

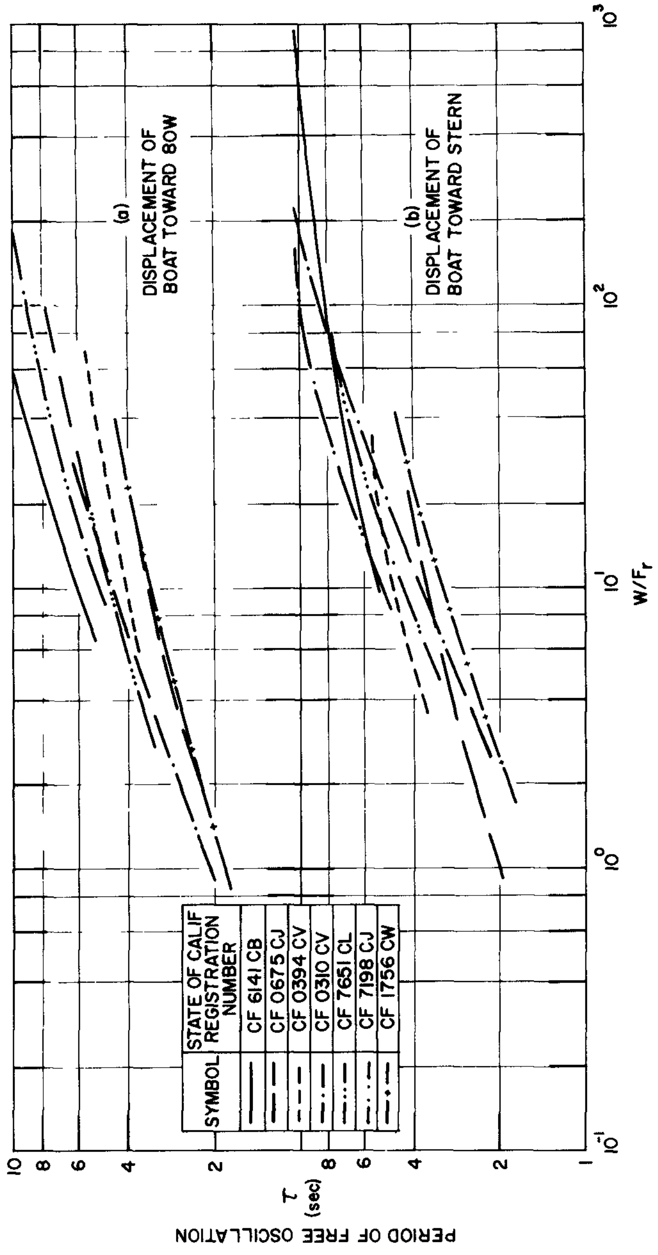


Fig 16 Variation of Period of Free Oscillation with $W/F_r$
a) Initial Displacement of Boat Toward Bow
b) Initial Displacement of Boat Toward Stern

## CONCLUSIONS

The following major conclusions may be drawn from this study

1. The theory which is developed to treat the surge motions of small boats which are moored asymmetrically with elastic non-linear restraints appears adequate when the results are compared to experimental results of the free oscillations of a small moored boat.

2. A major difference between the mooring systems of small boats and large ships is the restoring forces which oppose surge motions, for small boats these forces will probably be asymmetrical whereas relatively symmetrical restoring forces are expected for large moored ships.

3. An analytical study of the mooring dynamics of seven small boats has indicated that the nature of the restoring force may be more important in defining the periods of oscillation of these boats than the displaced weight of the boat.

4. For the seven small boats studied the periods of free oscillation were less than about 10 sec. For forced oscillations of a small boat in surge the important wave periods would be increased by an amount dependent upon the magnitude of the forcing function and the permitted motion of the boat, but they would still probably remain in the range of those of storm waves.

## ACKNOWLEDGMENT

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