CHAPTER 61

WAVE FORCES ON PILES IN RELATION TO WAVE ENERGY SPECTRA

by

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ABSTRACT

The determination of wave forces on piles is for an important part based upon data obtained with regular laboratory waves. Non-linearities in the mechanism that underlies these forces may lead to deviations when applying the data to predict forces exerted by irregular waves.

Experiments have been performed with irregular waves to investigate wave forces, more particularly to study the influence of the energy density spectrum of the waves.

Within the range of conditions in the experiments, the wave motion is sufficiently characterized by its energy and the frequency (or wave period) at which the energy density is maximum to determine the probability distribution of wave forces.

INTRODUCTION

Wave forces on piles are mainly determined at present either by computation or by performing experiments in regular laboratory waves. The latter also provided information on coefficients of drag (C_D) and intertia (C_M) to be applied in the methods of computation. A wide scatter in these coefficients was often found. This is partially due to the fact that the influence of the flow pattern around the piles on the forces exerted is not adequately characterized by coefficients C_D and C_M which are supposed to be a function of the geometrical shape of the pile only. Various authors have shown the influence of of the time history of flow on the forces exerted on submerged bodies [1], [2]. This implies that the phenomenon of forces in oscillating flow is in principle non-linear. even if inertia forces are predominant.

In engineering practice one is mainly interested in forces exerted by irregular waves. In consequence of the problems outlined above it is doubtful whether the results of regular-wave experiments can be used to predict forces by irregular waves. Moreover the description of irregular waves in a way that suits this procedure is rather complicated.

Experiments have been performed in irregular waves in order to study the effect of non-linearities and to establish a method for predicting the statistical distribution of wave forces suitable for design purposes. Some results of these experiments are presented in this paper. The aim of the study was to get some insight rather than establishing a set of data for design purposes. The range of variables was very limited, up till now. It is stressed that the conclusions which have been drawn are not necessarily valid beyond the range of conditions applied in the experiments.

REVIEW OF SOME METHODS

The starting point for the computation of wave forces is generally the description of the force as given by Morrison's equation:

$$F(t) = \tfrac{1}{2} \cdot C_D \cdot \rho D \int_{-d}^{\eta} u|u| dy + C_M \cdot \rho \frac{\pi}{4} \cdot D^2 \int_{-d}^{\eta} \frac{du}{dt}\, dy$$

For regular waves the integrals have been expressed in terms of wave height H, wave period T and water depth d. Various methods have been applied to derive the probability distribution of wave forces by irregular waves starting from Morrison's equation. A first method has been proposed by Borgman[3] for the case of a narrow-band spectrum. In this method the wave energy is supposed to be concentrated at a fixed frequency (hence the wave period is a constant). Consequently the wave height H is the only statistical parameter in the wave motion. By computing the forces for various values of H, the probability distribution of wave forces can be derived from the probability distribution of H. When considering actual wave energy spectra it is obvious that the assumption of constant period is a rather crude one, which has always been recognized by Borgman himself.

Pierson and Holmes [4] and Borgman [5] have derived the probability densities of velocities u and accelerations $\frac{du}{dt}$ and applied in Morrison's equation. In this way the probability distributions of drag forces and inertia forces can be determined from which the probability distribution of peak forces is derived. An objection to this method is that u and $\frac{du}{dt}$ are considered to be independent stochastic variables. In natural waves with a given energy spectrum the high waves most often have high values of both u and $\frac{du}{dt}$, especially if the spectrum is rather narrow. This has been overcome by Borgman [5] by computing the time functions of u and $\frac{du}{dt}$ from a sea surface simulation on a computer and applying these functions in an approximation of the Morrison formula. This method is probably the most advanced one in computations of irregular wave forces. It is obvious, however, that it is subject to the restrictions to be made when applying wave theories to irregular waves. Moreover the assumption of constant C_D and C_M has to be made.

The considerations given above show the need of having data from experiments in irregular waves. It will then be interesting to investigate the influence of the energy density spectrum of the waves on the forces and to consider the transfer functions converting wave spectra into force spectra. A simple expression for this transfer function is found in case of small amplitude waves and inertia forces being predominant. Some experiments have been performed by Jen [6] for this case which showed good agreement with theory.

EXPERIMENTS

ARRANGEMENT OF THE EXPERIMENTS

The experiments were performed with circular and square piles in vertical position and extending from the bottom of the flume up to a level above the highest crest level of the waves. The piles were attached to force transducers in such a way that the overall horizontal force was measured as well as the moment with respect to a fixed level in order to determine the point of application of the force. No information is presented in this paper, however. on the point of application.

The water depth was kept constant in the experiments at d = 0.45 m. The pile diameters D were:

circular piles: ⌀ 0.125, 0.063 m

square piles : ⌀ 0.12 , 0.06, 0.03 m

The experiments were carried out in a windwave flume, 55 m long and 4 m wide. In this flume irregular waves were generated by means of a wave board driven by servo-controlled hydraulic actuators. The input signals for the hydraulic servo mechanism were obtained by filtering a random noise signal such that the required wave energy spectra were obtained. See also ref.[7]. The wave energy spectra were determined on-line by means of a special analogue computer. Simultaneous wave and force records were set on punched tape for elaboration on a digital computer.

The wave motion was measured by making continuous records of the sea-surface elevation $\eta(t)$ with respect to still water level in a fixed point beside the piles. As a first characteristic of the wave motion the energy density spectrum $S_{\eta\eta}$ has been determined. In the computations of $S_{\eta\eta}$ the autocorrelation function was determined first. using samples of $\eta(t)$ at time intervals of 0.125 sec. From the autocorrelation function 60 points were used to compute $S_{\eta\eta}$, applying a triangular screen filter. The total energy of the wave motion is also used as a parameter. This energy is characterized by $M_{o\eta}$ which is equal to the area enclosed by $S_{\eta\eta}$ and the frequency axis. Some examples of spectra are given in figure 1. For an easy comparison of the shape of the various spectra the relative energy density $S_{\eta\eta}/M_{o\eta}$ has been plotted along the vertical axis. Hence the area enclosed by the curves and the f-axis is equal to unity in all cases.

The energy spectrum is furthermore characterized by the frequency or period at maximum energy density, $\hat{f}$ or $\hat{T}$ respectively, and a parameter for the width of the spectrum. For the latter the relative width as defined by Cartwright and Longuet-Higgins (see also [7]) has not been used in this case as it presents some problems in practical computations. As a simple parameter for denoting the relative width, the ratio of the maximum energy density $S_{\eta\eta}max$, to the total energy $M_{o\eta}$ has been chosen. It is recognized that $S_{\eta\eta}max/M_{o\eta}$ has not the advantage of being dimensionless. Besides the characteristics of the spectra also the probability distributions of wave heights have been determined. Examples are given in figure 3.

As has been stated above only the overall forces on the piles are considered in this paper. Continuous records of the forces versus time were made, F(t). From these records the spectra of the force S_{FF} were determined in the same way as described for the wave motion. Similarly M_{OF} was computed. When determining the probability distributions of wave forces it is useful to consider separately the distributions of maximum forces exerted by individual waves in the direction of wave propagation and the distributions of maximum forces in the direction opposite to the direction of wave propagation. Only the forces in the direction of wave propagation are considered here. Examples are given in figure 4.

PRESENTATION OF RESULTS

The examples shown in figure 1 give an indication about the variation of the width of the wave-energy spectra applied in the experiments. For reasons of comparison a Pierson-Moskowitz spectrum for fully developed sea has been plotted. For $\hat{f} = 0.8$ ($\hat{T} = 1.25$ sec) the value of $S_{\eta\eta\, max}/M_{o\eta}$ is equal to 1.95 sec. for the Pierson - Moskowitz spectrum. In the experiments values between 1.27 and 3.1 sec. were used for $S_{\eta\eta\, max}/M_{o\eta}$. For $\hat{f} = 0.6$ ($\hat{T} = 1.67$ sec.) the Pierson - Moskowitz value of $S_{\eta\eta\, max}/M_{o\eta}$ is 2.25 sec. whereas values in the experiments ranged from 1.14 to 4.1 sec. Consequently it may be expected that the experiments covered reasonably well the variation in spectrum width which occurs in nature.

The probability distributions of wave heights are close to the Rayleigh distribution in all cases. Examples are given in figure 3. Apparently the width of the energy spectrum did not affect the properties of the wave-height distribution.

As a starting point for the description of the wave forces, the wave-force spectra and the probability distributions of forces in the direction of wave propagation have been used. Examples are given in figure 2 and 4 respectively. For many design purposes this way of describing the forces to be expected under natural wave conditions is the most adequate one. From a set of probability distributions of forces corresponding to various storm-wave conditions a design load can be selected taking into account the probabilities of occurrence of the various storm conditions. The wave force spectra may be important when considering the response of structures to the dynamic loads.

When comparing the wave and force spectra of figure 1 and 2 it appears that there is a considerable shift of the energy to the higher frequencies in the force spectra. This is quite reasonable as in the situation of these examples the wave steepness is moderate, whereas the pile diameter is such that inertia forces are predominant. For the conditions of small amplitude waves and inertia forces being predominant there exists, theoretically, a transfer function converting wave spectra into force spectra which is a function of wave frequency only (Notice that the water depth was constant in the experiments). This holds only, however, if the coefficient C_M is a constant for the particular shape of the pile. A suitable way to define such a transfer function T(f) is:

$$S_{FF} = \left[\rho g \frac{\pi}{4} D^2\right]^2 . T(f). S_{\eta\eta}$$

in which:

ρ = density of water in kg/m^3

g = acceleration of gravity in m/sec^2

According to linear wave theory T(f) is equal to:

$$T(f) = \left[C_M \cdot \frac{\omega^2}{gk}\right]^2$$

in which:

$\omega = 2\pi f = \frac{2\pi}{T}$

$k = \frac{2}{L}$

T = wave period

L = wave length

The theoretical values of T(f) together with data obtained from the experimental spectra have been plotted in figure 5 and 6. A discussion will be given later on.

The examples of probability distributions of wave forces F, given in figure 4 appears to be rather close to the Rayleigh distribution. Only slight deviations occur for the higher forces. For the approximation of a narrow band spectrum by a single constant frequency as introduced by Borgman in ref. [3] and inertia forces being predominant, the probability distribution of wave forces is of the same nature as the probability distribution of wave heights, hence a Rayleigh distribution. Again this holds only in case C_M is a constant. In all other cases the probability distribution of forces is in principle affected by the wave-energy spectrum and the diameter of the pile compared to the magnitude of the wave heights and cannot be described by a standard distribution law. In order to get an idea about the variability of these distributions some simple parameters are introduced:
Let Fn be the wave force exceeded by n% of a series of peak forces in the direction of wave propagation. The parameters
$\frac{F_{50}}{F_{13.5}}$ and $\frac{F_1}{F_{13.5}}$ will be used to have some indication on the variability of the distributions.
Note: In a Rayleigh distribution $F_{13.5}$ is in its meaning comparable to the significant wave height $H_s = H_{13.5}$ in a wave height distribution. If the forces would satisfy the Rayleigh distribution, then:

$$\frac{F_{50}}{F_{13.5}} = 0.62 \qquad \text{and} \quad \frac{F_1}{F_{13.5}} = 1.5$$

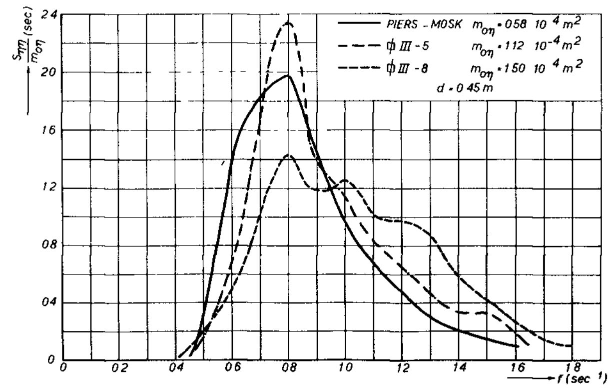

FIG. 1

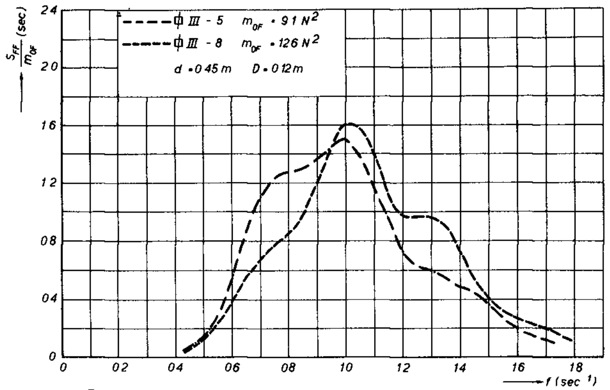

FIG. 2

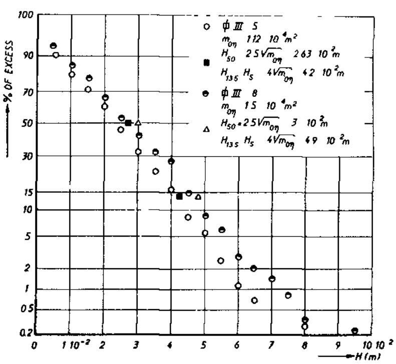

FIG. 3

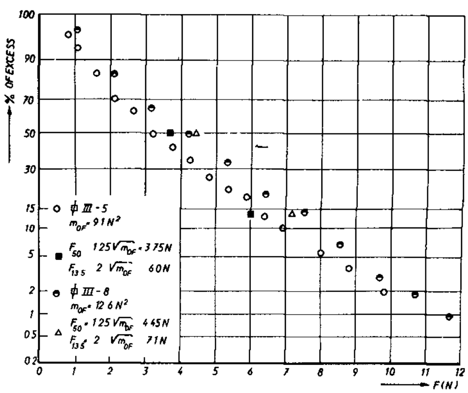

FIG. 4

Applying $F_{13.5}$ as a characteristic force in the probability distribution of forces one may try to relate this force to the parameters selected for the description of the wave conditions and the pile diameter, hence to:

$$\hat{T},\ M_{o\eta},\ \frac{S_{\eta\eta\,max}}{M_{o\eta}},\ D.$$

Instead of $F_{13.5}$ the parameter $\dfrac{F_{13.5}}{(\rho g \frac{\pi}{4} D^2)\sqrt{M_{o\eta}}}$ has been used.

The results of the experiments expressed in these parameters have been listed in table I.

DISCUSSION OF RESULTS

The values of T(f) obtained from a number of experiments with circular and square piles of 0.125 m and 0.12 m diameter respectively have been plotted in figure 5. In these cases the inertia forces are predominant. The curves give T(f) according to linear wave theory for the values of C_M indicated in the figure. The experimental results show some scatter, especially in the extreme high and low frequency ranges. The energy densities in these parts of the spectra are rather small, however, and consequently the values of T(f) obtained by dividing wave and force spectral densities are relatively inaccurate. It may be said that on the whole reasonably uniform transfer funotions were obtained. No distinct influence of $M_{o\eta}$ can be observed, hence no appreciable effect of non-linearity is apparent. It is remarkable that both for the circular and square piles T(f) has a maximum at $f \approx 1.1\ sec^{-1}$. The results of Jen [6] slightly indicate a similar effeot. There is also an appreciable difference between the experimental and theoretical values of T(f). The application of higher order wave theories does not lead in this case to a better agreement with the experimental data. An explanation for the deviations cannot be given. It might be that a more or less periodic development of eddies around the piles introduces the effect of a filter, but no further evidence is available so far to support this hypothesis. In evaluating the importance of the irregularities in T(f) and the scatter in results it should be kept in mind that T(f) as defined here is proportional to the square of the forces.

The results presented in figure 5 furthermore indioate that C_M is close to 2 and 3 for the circular and square pile respectively.

For a smaller circular pile the transfer function T(f) has been plotted in figure 6. In this case the contribution of drag forces is no longer negligible. Consequently there should be a non-linear effect resulting in higher values of T(f) with increasing $M_{o\eta}$. Moreover the values of T(f) should be higher for the smaller pile diameters. (This is a result of the definition of T(f)).

It is understood that if non-linearities occur the description of forces in terms of spectra is in fact not very meaningful.

The dotted line in figure 6 gives the mean values of T(f) for the bigger pile with D = 0.125 m. Examination of the results reveals

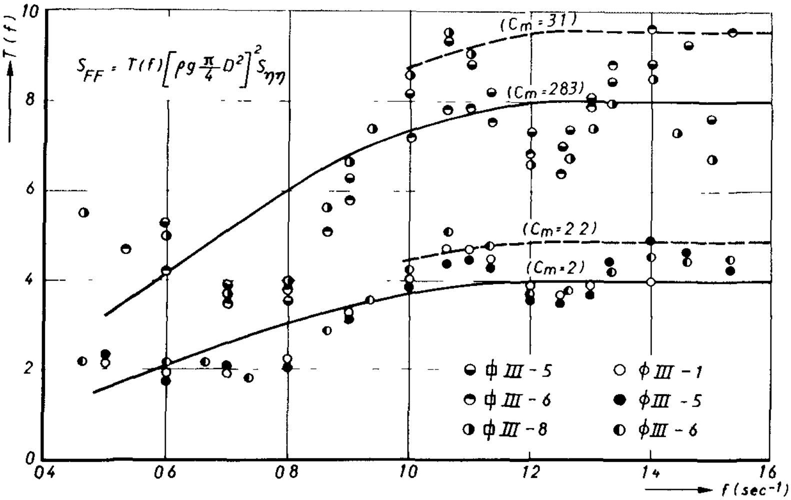
$S_{FF} = T(f)\left[\rho g \frac{\pi}{4} D^2\right]^2 S_{\eta\eta}$
(Cm = 3.1)
(Cm = 2.83)
(Cm = 2.2)
(Cm = 2)
ф III - 5
ф III - 6
ф III - 8
ф III - 1
ф III - 5
ф III - 6
T(f)
f (sec⁻¹)

FIG. 5

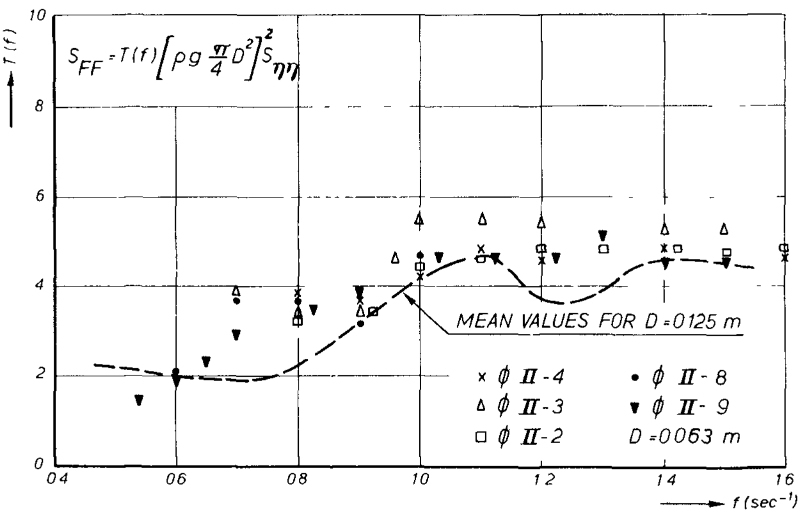
$S_{FF} = T(f)\left[\rho g \frac{\pi}{4} D^2\right]^2 S_{\eta\eta}$
MEAN VALUES FOR D = 0.125 m
ф II - 4
ф II - 3
ф II - 2
ф II - 8
ф II - 9
D = 0.063 m
T(f)
f (sec⁻¹)

FIG. 6

some typical aspeots. It is obvious that the mean values of $T(f)$ are higher for the smaller pile diameter as could be expected, the explanation being the increased contribution of drag forces. This should also lead, however, to increasing values of $T(f)$ with increasing wave heights, hence increasing $M_{o\eta}$. Suoh a tendency does not appear as oan be seen by comparing the various series using the information listed in table I. Furthermore a distinct maximum as observed in figure 5 does not occur in this oase.

Again no reasonable explanation can be given and it is obvious that further experiments are necessary to arrive at reliable conclusions. For the time being it can only be said that it is doubtful whether a simple superposition of drag and inertia forces can be applied in predicting irregular wave forces.

Still nowadays a method which is often applied to determine the design load on piles consists of selecting a design-wave height and period and subsequently computing the wave force using Morrison's equation and data obtained from experiments with regular waves. In this respect it may be interesting to consider a number of waves in an irregular wave train and to compare actual forces with those computed according to Morrison's equation. In the irregular wave train the wave period was defined as the time interval between two downward zero-crossings and the wave height as the vertical distance between the wave crest and the preceding wave trough. The wave foroe considered is the peak force in the direction of wave propagation. In the computations the integrated orbital velocities and accelerations were determined using the results of Reid and Bretschneider[8]. C_D and C_M values were selected in such a way that on the average the computed wave foroes correspond as good as possible with the forces actually recorded. Figures 7 and 8 show actual and computed forces for both a square and a oircular pile of such a diameter that the inertia forces are predominant. The C_M values applied are 3.1 and 2 respectively. Figure 9 shows the results for a smaller square pile ($D = 0.06$) in which case there is an appreciable contribution of drag forces for the higher waves. The coefficients applied in the computations were: $C_D = 2$ and $C_M = 2.83$. From these figures it appears that the actual forces may deviate considerably from the computed foroes. One of the reasons is of course that the wave profile in the irregular wave train is a superposition of components with different frequencies. It is felt, however, that also the time history of the flow around the piles has an influence. To illustrate this, an example of simultaneous wave and force records has been given. in figure 10. Apparently the wave following the maximum wave in this example produces a relatively small force. Such phenomena were observed several times in the records. It might be that the higher preoeding wave generates strong persistent eddies which hamper the development of the flow pattern. The considerable scatter of results from irregular waves compared to results of computations has been observed before. In the author's opinion this leads to the following recommendation:

If a design load has to be determined on the basis of an acceptable probability of occurrence, it shall preferably be derived from probability distributions of forces, rather than

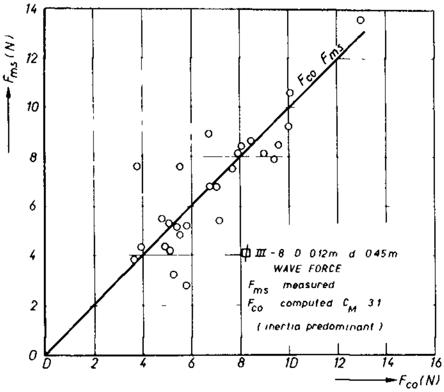

FIG. 7

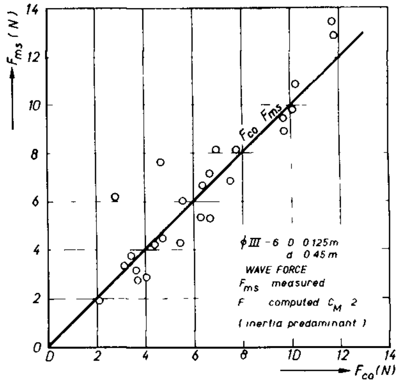

FIG. 8

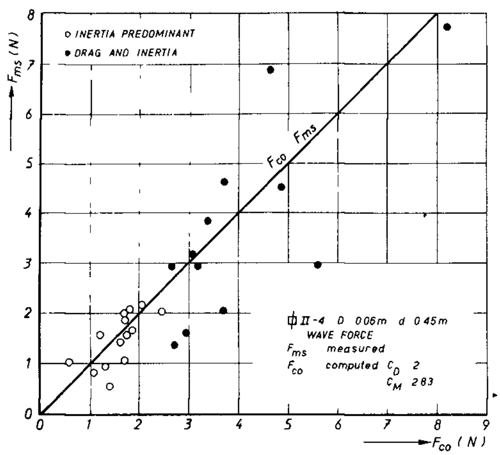

FIG. 9

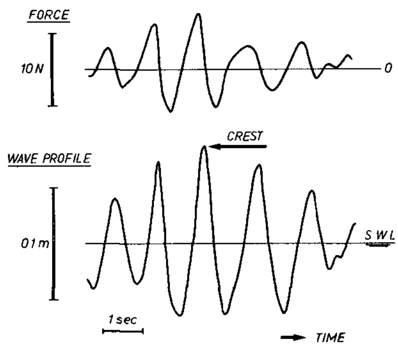

FIG. 10

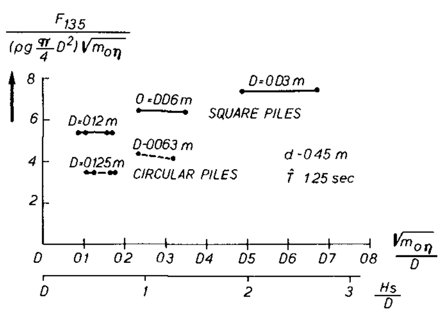

FIG. 11

selecting a design wave which has the accepted probability and computing the force which is expected to be exerted by this wave. For each wave condition the probability distribution of wave forces must then be known.

As has been outlined before the probability distributions will for the time being be characterized by

$$\frac{F_{13.5}}{(\rho g \frac{\pi}{4} D^2)\sqrt{M_{o\eta}}}, \quad \frac{F_{50}}{F_{13.5}} \quad \text{and} \quad \frac{F_1}{F_{13.5}}$$

The results of a number of experiments expressed in these parameters have been listed in table I, together with the data on wave conditions.

When considering the ratio $\frac{F_{13.5}}{(\rho g \frac{\pi}{4} D^2)\sqrt{M_{o\eta}}}$ it appears that there is first of all an influence of the pile diameter. The ratio increases with decreasing pile diameter. This is in agreement with the results obtained for T(f) and it would be reasonable to say that this is due to the increased contribution of drag forces. For the same reason there should be an increase with increasing M . However, as could be seen already from the results of T(f), such a tendency does not appear. It must be kept in mind that the range of variables was rather limited. Nevertheless there is a rather significant indication that the influences of pile diameter and wave height are different from those established in methods of computation.

From the transfer functions given in the figures 5 and 6 one might expect that there is an influence of the width of the spectrum, especially if the spectrum has the maximum energy density at $\hat{f} = 0.8$ sec^{-1}. From table 1, however, one may conclude that in practice the influence of the width, expressed in

$\frac{S_{\eta\eta \max}}{M_{o\eta}}$, is negligible. In this respect it must be stated oncemore that T(f) is proportional to the square of the wave forces. Hence there is not such a strong influence of the wave frequency on the forces as suggested by the path of T(f).

In order to illustrate the various problems mentioned above, the results obtained from the experiments with $\hat{T} = 1.25$ sec have been summarized in figure 11. According to theories there should be a gradual increase of

$\frac{F_{13.5}}{(\rho g \frac{\pi}{4} D^2)\sqrt{M_{o\eta}}}$ with increasing $\frac{\sqrt{M_{o\eta}}}{D}$. It is clear from figure 11 that there is an influence of the pile diameter which is not adequately incorporated in present theories.

The values of $\frac{F_{50}}{F_{13.5}}$ and $\frac{F_1}{F_{13.5}}$ show a variability in the probability distributions of forces. There is a scatter in the results

TABLE I

Results of experiments. Water depth d = 0.45 m

series D=0.12m	$M_{o\eta}.10^4$ (m^2)	$\hat{T}$ (sec)	$\frac{S_{\eta\eta\,max}}{M_{o\eta}}$	$\frac{F_{13.5}}{(\rho g \frac{\pi}{4} D^2)\sqrt{M_{o\eta}}}$	$\frac{F_{50}}{F_{13.5}}$	$\frac{F_1}{F_{13.5}}$	remarks
□ III-1	5.47	1.36	1.8	4.5	0.56	1.57	
III-2	2.39	1.36	2.12	4.6	0.54	1.6	
III-5	1.13	1.25	2.26	5.4	0.5	1.7	
III-6	4.31	1.25	2	5.4	0.57	1.57	
III-7	3.98	1.25	1.7	5.4	0.54	*	
III-8	1.50	1.25	1.38	5.4	0.58	1.54	
D=0.125m							
○ III-1	6.25	1.5	2.16	3.4	0.59	1.54	
III-2	3.5	1.5	2.08	3.3	0.57	1.54	
III-3	4.94	1.25	2.54	3.5	0.57	1.57	
III-4	2.43	1.25	2.34	3.5	0.55	*	
III-5	4.47	1.25	1.66	3.5	0.5	1.74	
III-6	1.76	1.25	1.42	3.5	0.56	1.6	
D=0.06m							
□ II-1	1.75	1.16	1.6	6.2	0.6	1.71	
II-2	4.45	(1.1)	*	6.4	0.54	1.73	1)
II-3	1.97	1.25	2.94	6.5	0.62	1.63	
II-4	4.35	1.25	2.7	6.4	0.54	1.92*	
II-5	2.38	(1.19)	*	6.0	0.6	1.56	2)
II-7	2.33	1.67	4	5.9	0.58	1.72	
II-8	4.7	1.67	3.6	6.3	0.57	1.74	
II-9	5.51	1.68	1.2	6.3	0.54	1.74	
D=0.063m							
○ II-1	2.33	(1.14)	*	3.8	0.59	1.6	3)
II-2	3.85	1.20	1.27	4.1	0.59	1.59	
II-3	2.14	1.25	3.1	4.4	0.63	1.60	
II-4	4.03	1.25	3.02	4.2	0.61	1.71	
II-7	1.98	1.65	4.1	3.5	0.61	1.5	
II-8	4.03	1.63	3.6	3.5	0.61	1.59	
II-9	3.66	1.66	1.14	3.9	0.62	1.62	
D=0.03m							
□ I-1	1.81	1.6	1.65	7.5	0.64	1.58	
I-2	3.89	(1.65)	1.65	7.3	0.57	1.73	4)
I-3	2.12	1.4	3	7.4	0.63	1.63	
I-4	3.92	1.3	3	7.5	0.57	2.0	

* not reliable

1) maxima in $S_{\eta\eta}$ at T = 1.1 and T = 1.38 sec
2) " " " " T = 1.19 " T = 1.74 "
3) " " " " T = 1.14 " T = 1.59 "
4) " " " " T = 1.4 " T = 1.65 "

which makes it somewhat difficult to draw reliable conclusions. Still the following tendencies can be observed:

- The distributions deviate from the Rayleigh distribution. For the smaller probabilities of occurrence the forces are higher than predicted by a Rayleigh distribution $\left(\frac{F_1}{F_{13.5}} > 1.5\right)$.
- The deviations from the Rayleigh distribution increase with decreasing pile diameters. A relation between $\frac{F_1}{F_{13.5}}$ and $\frac{\sqrt{M_{o\eta}}}{D}$ could not yet been established.
- The tendencies mentioned above are in agreement with the influence of non-linear drag forces.
- It may be expected that a set of standard distributions can be established which are sufficiently accurate for design purposes.

It is obvious that the information presented in this paper is by no means sufficient to get a reasonable insight into the problems related to irregular-wave forces on piles. Some typical features have been found which could not yet be explained. There exists non-linearities in the mechanism. that underlies the forces exerted by irregular waves which may lead to deviations between actual forces and the forces derived from data obtained with regular waves or predicted by computations. It is felt that further investigations are necessary to establish a reliable method for the prediction of wave forces which is based on the statistical properties of irregular wave motion.

CONCLUSIONS

The prediction of irregular-wave forces from theories and regular-wave force data reveals many problems. The phenomenon of wave forces is non-linear and the magnitude of the forces is affected by the time-history of flow. The information presented in this paper does not provide sufficient insight into these problems.

The results obtained so far with irregular waves indicate that with respect to forces on piles the wave motion is sufficiently characterized by $\hat{T}$ and $M_{o\eta}$, hence that for design purposes the width of the energy spectrum is of minor importance. The results moreover suggest that a set of standard probability distributions of forces can be established in which the water depth, the pile diameter, the period of maximum wave energy and the total energy of the wave motion appear as parameters.

Any tendency or conclusion derived in this paper may not be applied for the time being beyond the range of variables covered by the experiments.

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