CHAPTER 57

THE EFFECT OF WAVE ENERGY SPECTRA ON WAVE RUN-UP

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1. ABSTRACT

Previous investigations carried out by the Delft Hydraulics laboratory have shown the necessity of applying irregular waves in studies on wave run-up. The installation of a wave generator driven by hydraulic actuators has created the possibility of producing irregular waves with arbitrary wave spectra.

Investigations performed with this type of wave generator show the influence of the shape of the energy spectrum on the wave run-up on smooth straight slopes of 1:4 and 1:6. The results are compared with run-up figures derived from experiments with wind generated waves and with monochromatic waves.

2. INTRODUCTION

A seawall is generally designed to withstand certain wave conditions. Depending upon the circumstances the conditions may be a storm surge, attack by waves or a combination thereof. The present study is restricted to one aspect of the attack by waves only, i.e. the determination of the crest height above the still water level. An accurate assessment of the required crest level is important as the cross-sectional area of the dyke increases approximately proportional to the square of the crest height.

To determine the crest level on the basis of experiments suitable criteria have to be established with respect to run-up or overtopping. The criteria shall define either the crest level relative to a level of wave run-up under design condictions or an acceptable amount of overtopping. Little is known at present about the mechanism that underlies the damage caused by wave attack on dykes. The assumption seems justified however, that overtopping is one of the major factors causing damage to the inner (grass grown) slopes. It is therefore generally accepted nowadays in the Netherlands that no overtopping is allowed under design conditions. This means that the crest level should be at least at the level of wave run-up.

The prevailing windwaves are unfortunately of a statistical nature with respect to their height and period. Consequently the wave run-up is of a statistical nature too and a distinct maximum cannot be defined. The design level for wave run-up is now adopted in the Netherlands as the level which is exceeded by 2% of the uprushing waves under design conditions. If it is expected that the inner slope of a particular dyke is more or less vulnerable to overtopping this percentage can be adapted accordingly. Paape(Ref. 1) has shown already in 1960 that the amount of overtopping water is a parameter which is in principle more relevant to damage at the inner slope than the run-up. However, it is still a point of investigation and discussion which amount of overtopping will initiate damage to the inner slopes. As long as the proper relationship between damage (onset and extent) and overtopping or run-up is not known neither of these parameters are preferable to each other.

In view of the simplicity of measuring technique, in the present study the run-up is selected as a parameter.

3. PREVIOUS INVESTIGATIONS

In the past extensive model investigations were carried out in several Institutes to determine the wave run-up on smooth and rough, straight and composite slopes. A majority of these tests have been carried out applying monochromatic waves. Though the applicability of these investigations was limited, their great value lies in the insight that was obtained regarding the influence of wave steepness and waterdepth on the relative run-up. (Ref. 2 and 3).

On the basis of the abovementioned data Saville calculated a statistical distribution of run-up assuming a joint distribution of wave heights and periods as described by Bretschneider for the case of fully developed sea.

Apart from the investigations with regular waves, tests have been carried out applying wind waves, generated in a wind flume. (Ref. 1). Due to the limited fetch available, the wind speeds had to be exaggerated in order to generate waves of appreciable height. This resulted in waves with a great steepness and consequently a low relative run-up.

Recently the wind flume of the Delft Hydraulics Laboratory has been equipped with new wave generating facilities and it was decided to extend the investigations of wave run-up applying irregular waves with variable wave spectra, in order to study the influence of the spectral form and the applicability of previous investigations and calculations.

4. RECENT INVESTIGATIONS

4.1 Arrangemen of the model

The experiments have been performed in the wind-wave flume of the Laboratory at Delft. The flume is 4 m wide and 1 m high, and has an effective length of 55 m. The models consisting of plywood, had straight impermeable slopes of 1:4 and 1:6 respectively, with sufficient height to prevent overtopping. The waterdepth in front of the model was 0,40 m. The models had a width of 1 m each and were placed in the flume as shown on Figure 1.

During the tests the wave motion was recorded in between the models. A continuous sampling of these records was fitted into an analogue spectrum analyser for an on-line calculation of the energy spectrum of the waves. Also the frequency distribution of wave heights was measured directly. Apart from the on-line process, all records have been digitized and analysed afterwards on a digital computer, in order to obtain the wave energy spectrum and from it the spectral width parameter.

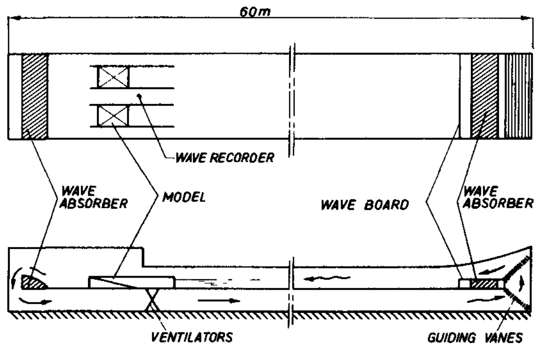


FIG 1 SITUATION OF MODELS

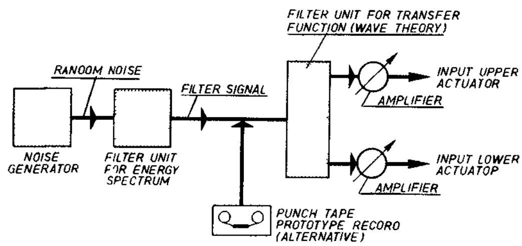


FIG 2 FORMING INPUT SIGNAL FOR THE WAVE GENERATOR

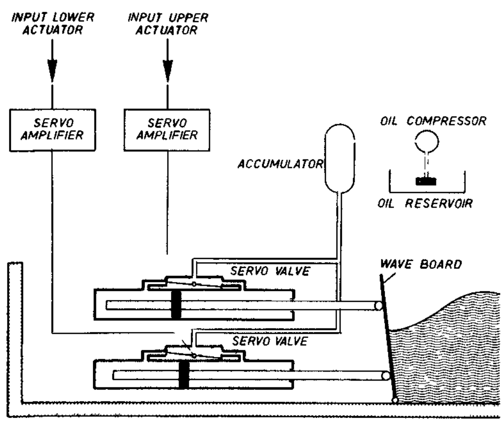


FIG 3 PRINCIPLE OF WAVE GENERATOR

The statistical distribution of the run-up was measured by means of an electronic multi-point gauge, which was fitted just above the slope.

4.2. Generation of waves

Waves were generated by a wave board. The board is driven by two servo-controlled hydraulic actuators to produce waves which have the same statistical properties as the waves in nature.

Via a set of variable analogue filters the signal of a random-noise generator is shaped conform to the required energy spectrum. This signal is filtered again in such a way that the horizontal movements of upper and lower actuator are obtained. This transfer process is based on second order wave theory.

In addition to the artificially generated wave profiles two punchtape records of prototype conditions in the North Sea, made available by the Hydraulics Division of Rijkswaterstaat, were used as an input signal for the wave board. A schematic sketch of the wave generator and a block diagram of the control procedure is given in Fig. 2 and 3.

The adaption of the details of the shape of the waves is obtained by an air stream running over the full length of the flume.

5. WAVE CHARACTERISTICS

The present investigation was directed mainly on the determination of the influence of the spectral form. Therefore a variation of the spectral form was required with other variables kept constant. However, to extend the range of applicability of the experiments, the wave steepness and relative waterdepth, expressed in the dimensionless terms $Hs/g\hat{T}^2$ and $D/g\hat{T}^2$ have been varied too. The variation in wave heights and periods was limited by the required accuracy of the measurements and by the capacity of the wave generating facilities.

TABLE I

Test	$\hat{T}$* (sec)	Hs (cm)	$\varepsilon_{5\%}$	$Hs/g\hat{T}^2$ $\cdot 10^{-3}$	$D/g\hat{T}^2$ $\times 10^{-3}$	Wave generation by:
T 1	0,97	3,7	0,38	4,0	43,4	random noise
T 2	1,04	4,2	0,44	4,0	38,1	"
T 3	0,95	3,7	0,50	4,2	45,1	"
T 4	0,97	5,6	0,38	6,1	43,4	"
T 5	0,95	5,6	0,50	6,3	45,1	"
T 6	1,54	10,2	0,34	4,3	17,0	"
T 7	1,54	10,0	0,45	4,3	17,2	"
T 8	1,47	10,2	0,57	4,8	19,0	"
T 6 repeated	1,54	9,3	0,34	4,0	17,0	"
T 8 repeated	1,47	9,7	0,57	4,6	19,0	"
T 9	1,64	13,6	0,38	5,2	15,3	"
T 10	1,55	13,5	0,55	5,7	16,9	"
T 11	1,58	13,5	0,59	5,5	16,3	
T 12	1,44	8,7	0,42	4,3	19,8	punch tape
T 13	1,40	7,4	0,52	3,9	21,1	"
T 14	0,71	6,0	0,22	12,2	81,4	wind only

* $\hat{T}$ is the period in the spectrum with maximum energy density

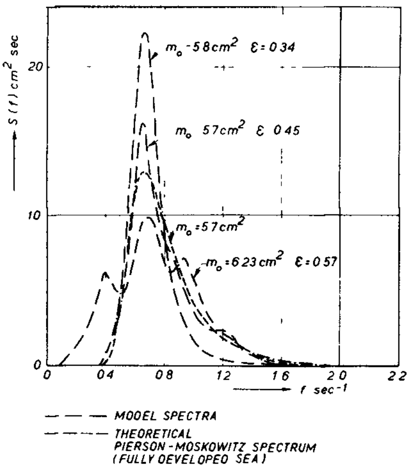


FIG 4 EXAMPLE OF APPLIED WAVE ENERGY SPECTRA

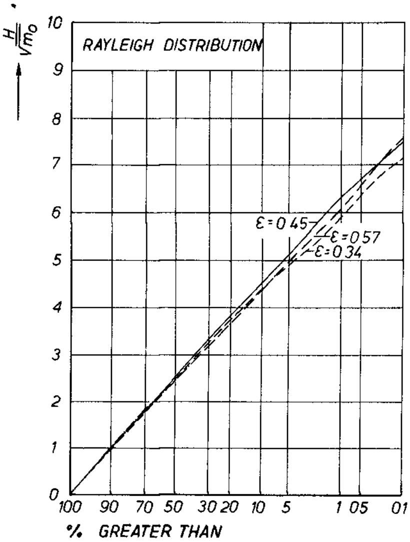


FIG 5 FREQUENCY DISTRIBUTION OF WAVE HEIGHTS

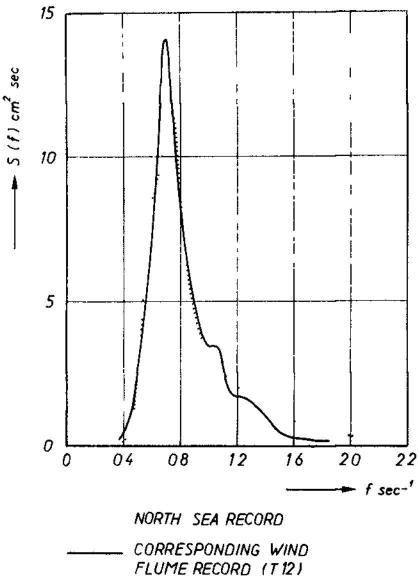


FIG 6 WAVE ENERGY SPECTRUM USING PUNCH TAPE PROTOTYPE RECORD

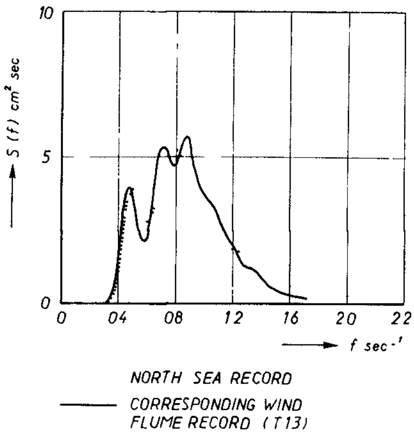


FIG 7 WAVE ENERGY SPECTRUM USING PUNCH TAPE PROTOTYPE RECORD

The relationship between wind speed and energy spectrum of the waves as described by Pierson and Moskowitz for a fully developed sea (Ref. 5) has been used as a reference for the actually applied spectral forms and wave steepnesses. Figure 4 shows such a spectrum type for a particular wave period (wind speed). As the requirements for a state of full development however are restricted to a few cases only, wave spectra and wave steepnesses with appreciable deviations may occur.

By varying the relative waterdepth $D/g\hat{T}^2$ from 15.3×10^{-3} to 45.1×10^{-3}, conditions were accomplished for almost shallow water waves to almost deep water waves.

The spectral form is expressed in the relative width parameter ε. defined as follows (Ref. 6):

$$\varepsilon^2 = \frac{m_0\, m_4 - m_2{}^2}{m_0\, m_4} \quad \text{in which } m_n = \int_0^\infty S_{(\omega)} \cdot \omega^n \, d\omega$$

The calculation of the spectrum and ε has been carried out on a digital computer using the digitized wave records. The Nyquist frequency was 3.2 cps. The frequency interval between adjacent estimates of the spectral density 0.0533 cps. The correlation function was passed through a triangular screen filter. Inherent to the definition of ε, the higher frequencies have an unproportional large influence on the calculated value of ε, whereas both the accuracy and the practical interest of this frequency range is small. Therefore the calculation of m_n is carried out for that part of the spectrum for which the energy density $S(\omega)$ at $\omega > \hat{\omega}$ is greater than 5% of the maximum energy density $s(\hat{\omega})$. The consequences of this cut-off procedure for one special case (T13) are shown in Table II.

TABLE II

limitation at	0%	1%	2%	3%	4%	5%
ε	0.648	0.599	0.550	0.533	0.523	0.518

From the two test runs in which the input signal consisted of a prototype wave record from the North Sea a comparison has been made between the wave spectra and wave height distributions from the prototype and those from the wave flume. Figures 6 and 7 show the wave spectra, Figures 8 and 9 the wave height distributions

6. RESULTS

The frequency distribution curves of the run-up of all tests have been plotted on Gaussian distribution paper, like Fig. 10. Comparing sets of distribution curves, each set for wave conditions with the same significant wave height (Hs) and period ($\bar{T}$) but different ε values, shows that there is an influence of ε in that way that the "steepness" of the run-up distribution is greater for higher ε values (Fig. 10).

Though only a few tests have been performed in which the waterdepth and the wave steepness have been varied independently it appeared that neither a variation in waterdepth nor a variation in wave steepness has a significant influence on the "steepness" of the run-up distribution curve, so all curves can adequately be characterized by the run-up at

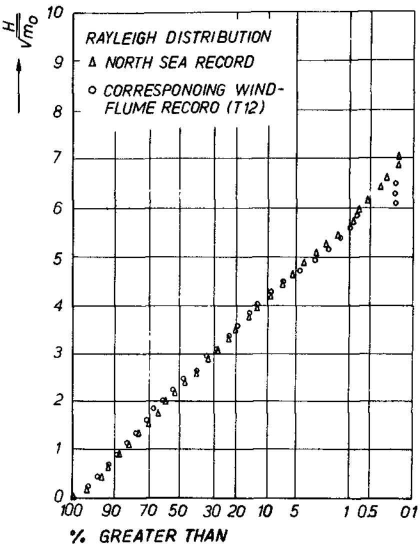


FIG 8 FREQUENCY DISTRIBUTION OF WAVE HEIGHTS USING PUNCH TAPE PROTOTYPE RECORD

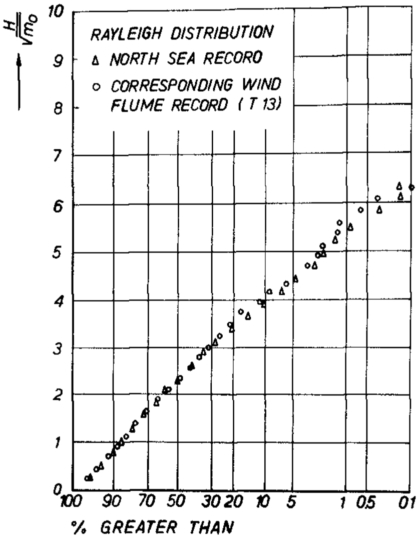
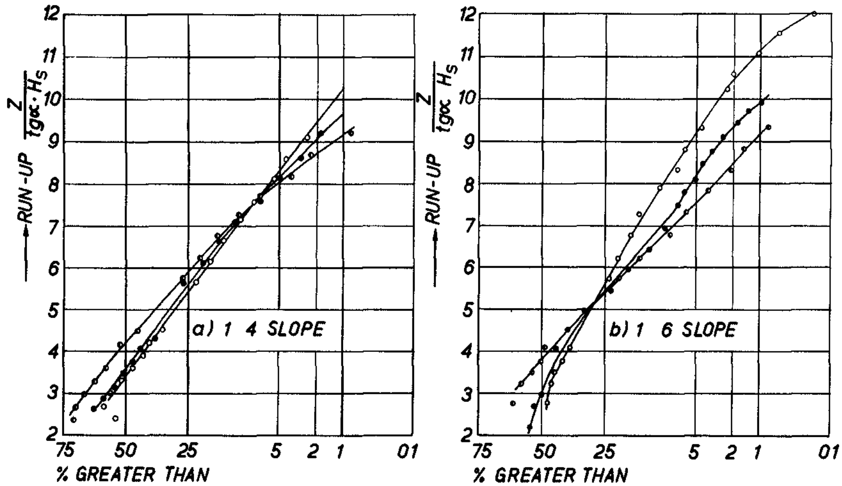


FIG 9 FREQUENCY DISTRIBUTION OF WAVE HEIGHTS USING PUNCH TAPE PROTOTYPE RECORD



○ $\bar{H}_S$ = 10 2cm $\hat{T}$ = 1 47 sec WIDE ε=0 57

● $\bar{H}_S$ = 10 0cm $\hat{T}$ = 1 54sec MEDIUM ε= 0 45

◦ $\bar{H}_S$ = 10 2cm $\hat{T}$ = 1 54sec NARROW ε= 0 34

FIG 10 FREQUENCY OF RUN-UP

one specific exceedance percentage.

The run-up distribution curve is characterized by the level of run-up surpassed by 2% of the waves (Z_2) for reasons explained in the Introduction.

In the case of monochromatic waves it is known that for $D < 3H$ the influence of the waterdepth is perceptible. One may expect therefore that the same holds for irregular waves, with a comparatively small D/H ratio. Such a situation has not been investigated so far.

Hunt (Ref. 3) developed an empirical formula for the wave run-up on smooth impermeable slopes, based on tests with monochromatic waves:

$$\frac{Z}{H} = \frac{C \ \mathrm{tg}\ \alpha}{\sqrt{H/L_o}} \tag{1}$$

The experiments indicate that also in the case of irregular waves the influence of the slope is well expressed by this formula as far as the differences between the slopes 1:6 and 1:4 are concerned.

Substituting the irregular wave characteristics in (1) the following expression is obtained:

$$Z_n = \frac{C_n(\varepsilon).Hs.\mathrm{tg}\ \alpha}{\sqrt{Hs/L}} \tag{2}$$

or

$$Z_n = \frac{C_n(\varepsilon).Hs.\mathrm{tg}\ \alpha}{\sqrt{Hs/g\hat{T}^2}} \tag{3}$$

The subscript n denotes the frequency of exceedence. The unknown factor C_n which is directly proportional to the wave run-up is not a constant as for monochromatic waves but a function of ε. In Fig. 11 the C_2 values of all tests have been collected and expressed as a function of ε. Notwithstanding the scatter in the C_2 values, it is evident that ε is an important parameter in the run-up phenomenon in that way that a wave motion with a wider spectrum produces considerable higher run-up for the smaller exceedance frequencies than a wave motion with a narrow spectrum. The C values are separated for both slopes because the influence of ε in the case of 1:6 slope is slightly larger than for the 1:4 slope. The scatter is fairly small if one considers that the calculation of C_2 is based on an empirical formula derived from tests with monochromatic waves. Consequently it seems justified to assume that the influence of the wave characteristics H_s and $\hat{T}$ is well expressed in formula (3). However it should be stressed that this is valid only within the range of conditions tested.

As mentioned before two tests have been conducted with waves generated by making use of wave records from the North Sea (T 12 and T 13) The wave run-up obtained in this way, expressed in the C_2 values of formula (3), is in good agreement with the run-up values obtained by making use of filtered noise (fig. 2) as an input signal for the wave generator. See Fig. 11.

Wind blowing over a water surface in a wind flume has been for many years the only possibility of simulating wind generated ocean waves in the Delft Hydraulics Laboratory, and this method is still used in many other laboratories. At the D.H.L. many tests have been performed with

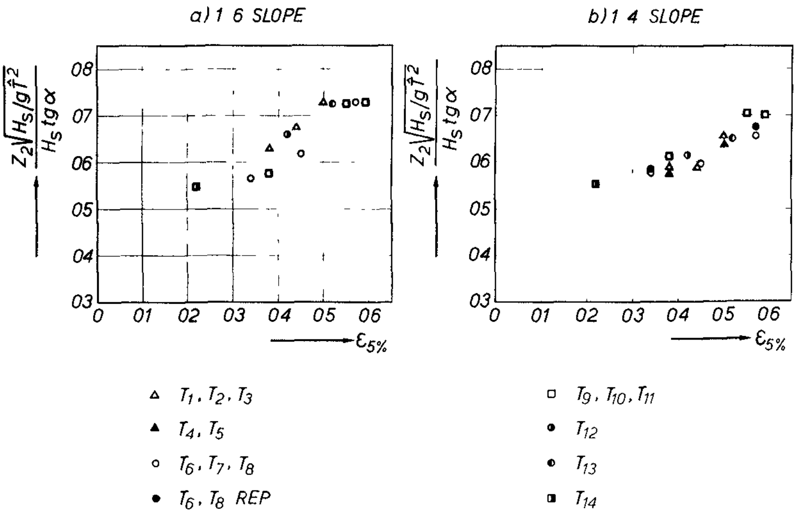


FIG 11 RELATIVE WAVE RUN-UP AS A FUNCTION OF THE SPECTRAL WIDTH $\varepsilon_{5\%}$

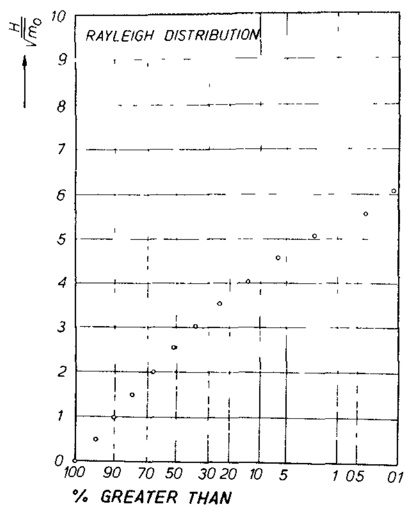


FIG 12 FREQUENCY DISTRIBUTION OF WAVE HEIGHTS FOR WIND GENERATED WAVES

this type of wave generation (Ref. 1). However, since the fetches are mostly relatively short, the wind speed has to be exaggerated to obtain waves of sufficient height and consequently also the wave steepnesses are exaggerated. In view of this, one test has been carried out using wind generated waves instead of mechanically generated waves. (See T 14) of Table I).

The wind speed was such that a comparative wave height was obtained in relation to former experiments. The resulting large wave steepness ($Hs/g\hat{T}^2 = 12.2 \times 10^{-3}$) was primarily the reason for a relatively low run-up on the two slopes, as the run-up decreases at increasing wave steepness. Moreover the wave spectrum was extremely narrow, as may appear from the ε-value of only 0.22, and the wave height distribution was quite different from the Rayleigh distribution as opposed to all other tests. Compare Figure 5 and 12. In spite of the deviating wave characteristics however the location of the corresponding C_2-values (T 14) in Figure 11 indicates that formula (3) is still applicable.

7. ANALYSIS OF RESULTS

To permit an estimation of the variability in irregular wave run-up without the use of facilities for irregular wave generation, Saville has proposed the following calculation of a statistical distribution of wave run-up (Ref. 4), using standard (monochromatic) wave run-up curves.

The probability of a combination of a particular wave height and wave period may be given as:

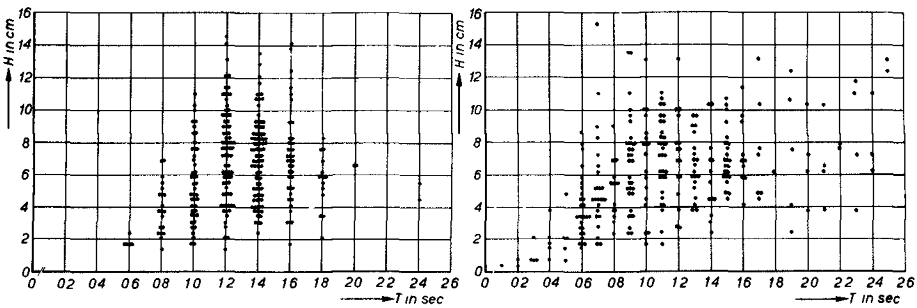
$$p(H,T) = p(H).p(T), \qquad (4)$$

consequently assuming a zero-correlation between wave height and wave period. The probability distributions used were those proposed by Bretschneider (Ref. 7) for fully developed sea.

With the expression (4) one can obtain a joint distribution of wave steepness $p(H/g\hat{T}^2)$ and wave height, and subsequently

using standard run-up curves giving the relation between relative run-up and wave steepness (Ref. 2), a distribution of individual relative run-up $p(\frac{Z}{H})$. Multiplication of a particular run-up $\frac{Z}{H}$ with the associated wave height from the assumed joint distribution of wave heights and periods for a particular wave condition, results in a distribution of run-up $p(z)$.

To check the applicability of this calculation and to get more insight in the influence of the spectral shape on the wave run-up a calculation as described is performed for two wave conditions (a wide and narrow spectrum) for which run-up distributions have been measured (T 6 and T 8 of Table I). Instead of assuming a zero-correlation between wave heights and periods however actual wave records of the two wave conditions have been used for the determination of the joint distribution $p(H,T)$. The records were analysed by hand, taking 300 successive waves into consideration. The result is shown on Figure 13 in which every point represents a wave with both height and period within a certain class. Following the method of Saville a distribution of run-up of 300 individual waves is obtained for two different spectra and 2 different corresponding H/T correlations. The solid lines on Figure 14 show the result of the calculation. Although this calculation is rather simplified it appears that,



a) NARROW SPECTRUM (m_o = 5 8 cm^2 ε = 0 34) b) WIDE SPECTRUM (m_o = 6 23 cm^2 ε = 0 57)

FIG 13 JOINT DISTRIBUTION OF WAVE HEIGHT AND PERIOD

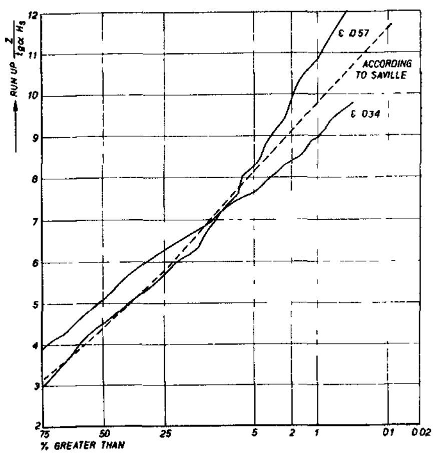


FIG 14 FREQUENCY OF RUN-UP CALCULATED

thanks to the fact that a large amount of waves is considered, the obtained result is in remarkable good agreement with the recorded wave run-up for the same wave condition. Compare Figure 10 and Figure 14.

The calculation shows in addition that the difference in the higher part of the run-up curve for the two wave conditions considered was caused by the few long, low waves present in the case of a wide spectrum only (Fig. 13). This was also confirmed by visual observation. These waves with extreme low steepness hardly break on the slope and consequently cause an unproportional high run-up. The assumption of a zero-correlation between wave height and period according to Saville's method results in a run-up distribution (the dotted line on Figure 14) which corresponds with the measured run-up distribution for the middle one of the three spectra in Figure 4 (the Pierson Moskowitz spectrum for fully developed sea).

Calculated and measured distribution curves diverge at higher exceedance percentages as a result of the applied measuring technique.

8. CONCLUSIONS

- The statistical run-up distribution curves diverge at the smaller exceedance percentages for different spectral forms. Wider spectra are accompanied by higher run-up.
- Increase in wave run-up for wider spectra is caused by differences in the joint distribution of wave heights and periods. The distributions of wave heights only are the same for the different spectra.
- The relationship $Z_n = \frac{C_n(\varepsilon).Hs.tg\ \alpha}{\sqrt{Hs/g\hat{T}^2}}$ gives a good approximation of the influence of wave height, wave period and slope for the conditions investigated.
- Wind waves generated in the laboratory applying exaggerated wind speed, produce wave run-up which is generally to low due to the extreme narrow wave spectrum.
- The calculation of a statistical wave run-up distribution curve assuming a zero-correlation between wave height and period as described by Saville agrees fairly well with the measured one for a state of fully developed sea. Taking into consideration variations in the joint distribution of wave heights and periods for spectra different from the spectrum of fully developed sea, a higher run-up is obtained for wider spectra; conversely lower run-up goes with narrower spectra.

9. REFERENCES

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