

CHAPTER 24

TRANSFORMATION OF SURGES

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ABSTRACT

This paper deals with the transient deformation of surges (bores) both theoretically and experimentally. Ideal surges in open channels would normally disperse into periodic waves and be transformed to the undular bore. A train of these dispersive waves may finally reach some stable form e.g. solitary or cnoidal waves.

In the transient process, with the development of the undulations, the height of the initial wave would not be constant as has been suggested by Keulegan-Patterson(1940), but would fluctuate in a complicated manner. The author indicates, theoretically, regions in which wave crests or troughs can exist and gives experimental criteria by which modes of breaking (spilling, surging or plunging) can be determined. He also believes that the curvature of the wave surface plays a leading role in the mechanism of dispersion and may act as a convective agent in the development of the undulation.

BACKGROUND

Favre(1935) had shown by his experiments that when the ratio h/H (h : the mean height of surge, H : still water depth) is less than 0.28, some stable undular bore may be expected and the height of initial wave η_1 is about $3/2$ times as large as the mean height h of original surges. Initial waves are expected to break when $0.28 < h/H < 0.75$, and for its greater ratio there will be no undulations. (Binnie and Orkney, 1955) Keulegan and Patterson (1940) introduced theoretically that $\eta_1/h = 3/2$ and each wave of the undular bore will reach the cnoidal wave in their forms. Lemoine(1948) calculated the energy loss due to radiation in the process of dispersion under the assumption of small amplitude and sinusoidal wave train and these restrictions in analyses were excluded by Benjamin and Lighthill(1954). The effect of bottom friction rather than the turbulence in wave front of bore was emphasized by Sturtvant(1965) or Chirriot-Bednarzyk(1964) because of consideration for the bottom friction to be important factors in deformation of undular bores. Under the assumption of laminar velocity-distribution in bores, Chester(1965) suggested the possibility of oscillatory surface on bores in criteria $F < 1.6$, where $F = V/\sqrt{gh}$, and h is a representative depth.

Very recently, Peregrine(1966) discussed the same problem with ours and he calculated numerically the wave patterns in the dispersion process of bores using the equation of Korteweg-de Vries. There are some discrepancies in understanding for the dispersion mechanism between him and the author, and discussions

on this point may be done in this paper.

THEORIES

MECHANISM OF THE DISPERSION OF IDEAL SURGES

It may be reasonable to assume each dispersive wave, in particular the initial wave, has properties of the shallow water waves with finite amplitudes and considerably large curvatures at the wave crest. The celerity of these dispersive waves C will be given by the following equation:

$$C = C_0 \left[1 + \frac{3}{2} \frac{\eta}{H} + \frac{H^2}{3\gamma} \frac{\partial^2 \eta}{\partial X^2} \right]^{\frac{1}{2}}, \quad C_0 = \sqrt{gH} \quad (1)$$

On the other hand, the celerity of ideal surges C_s is expressed in the equation

$$C_s = C_0 \left[1 + \frac{3}{2} \frac{h}{H} + \frac{1}{2} \left(\frac{h}{H} \right)^2 \right]^{\frac{1}{2}}. \quad (2)$$

According to the author's experimental verifications, as shown in Fig.2, it may be a reasonable assumption to put $C \cong C_s$, aside from the case just before breaking. Therefore, the following relation will be obtained by equating Eq.(1) with (2):

$$\frac{3}{2} \frac{\eta}{H} + \frac{H^2}{3\gamma} \frac{\partial^2 \eta}{\partial X^2} = \frac{3}{2} \frac{h}{H} + \frac{1}{2} \left(\frac{h}{H} \right)^2 \quad \text{const} \quad (3)$$

That is, the sum of relative height of dispersive wave η/H and curvature term must be conservative for a given surge. At the shoulder of actual surges in the early stage of development processes, the negative and larger curvatures must be expected to appear, and by the conservative function, the surface deviation η must increase. The junction point 2 in Fig.3 of this small superimposed hump with the back-layer surface of surge, will show the profile with positive curvature and by the above relation, some negative deviation (small depression) from the horizontal surface will be resulted. At next junction point 3 in Fig.4, $\partial^2 \eta / \partial X^2 < 0$ and then η will again increase, and so on.

According to these procedures, the undulation of horizontal back-layer seems to be developed, and so it is the author's opinion that convective agents in the transformation process may be the surface curvature.

In his interesting paper, Peregrine expresses the physical description concerning the matter, that in Fig.4, the vertical acceleration of the water is upward between E and C and there, the pressure gradient beneath the surface must be greater than hydrostatic. (Being contrary about the matter between C and A). The extra horizontal pressure gradients due to vertical acceleration of the water (or the effect of curvilinear flow) may be resulted as shown in Fig.4. By these additional pressure variation, extra elevation at B will be caused, he says.

In spite of careful reinspections of our observed wave

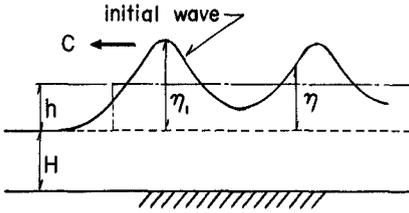


Fig. 1. Definition sketch for the undular bore.

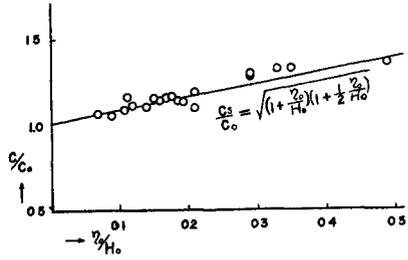


Fig. 2. The relation between the celerity of ideal surges, C_s , and the celerity of dispersive waves, C .

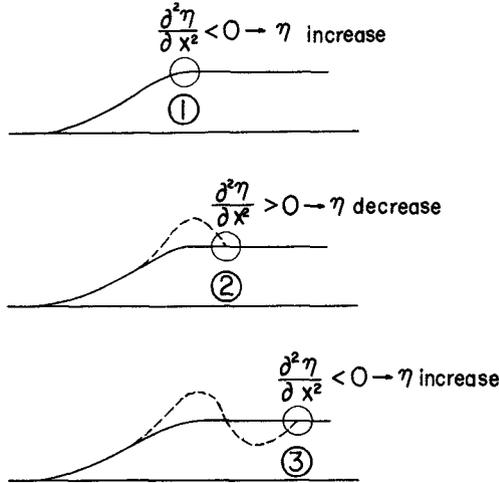


Fig. 3. The physical description of the mechanism of dispersion.

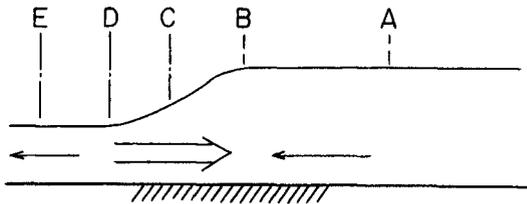


Fig. 4. The schematic explanation of Peregrine's physical descriptions.

patterns, any depressions at D which have to occur in same magnitude as the elevation at B by Peregrine's theory, is never noticed. Because the behaviour of wave forerunner near D is supposed to be essentially important, some analytical approaches will be shown later in this paper.

THE HEIGHT OF DISPERSIVE WAVES

As one example of fluctuations of dispersive wave heights in transient process, variations of initial wave-heights are shown in Fig.5. In cases including breaking phenomena in those processes, fluctuations of wave heights are much complicated and the usual prospect $\eta_1/h = 3/2$ is quite rare.

From Eq.(3),

$$\frac{\partial^2 \eta}{\partial x^2} = \frac{9}{2} \frac{\eta h}{H^3} \left(1 - \frac{\eta}{H} + \frac{h}{3H} \right) \tag{4}$$

For the crest of waves, putting the condition $\partial^2 \eta / \partial x^2 < 0$ into Eq.(4), it follows that

$$\frac{\eta}{h} > 1 + \frac{h}{3H} \tag{5}$$

Similarly, for the trough of waves, with the condition $\partial^2 \eta / \partial x^2 > 0$,

$$\frac{\eta}{h} < 1 + \frac{h}{3H} \tag{6}$$

Then, it may be concluded that the ordinary value $\eta_1/H = 3/2$ is only for the stable dispersed waves with small amplitude, while the expression of inequalities in Eqs.(5) and (6) is for more generalized cases.

SOME CONSIDERATIONS OF BREAKING OF THE INITIAL WAVE

It may be reasonable to suppose that there is a particular condition corresponding to each mode of breaking. Especially, surging or plunging breakers of the initial wave are caused by rapidly rising-up of the wave front and so analysis for the instability of wave front may give some informations about the growth of surging or plunging breakers. The suggestive institution of Stoker(1957) will be extended here to the case of breaking of undular surges.

The equation of motion and of continuity are

$$2C \cdot C_x + V_t + V V_x - g(S - S_f) = 0, \tag{7}$$

$$C V_x + 2V V_x + 2C_t = 0, \tag{8}$$

where $C = \sqrt{gH}$ $S_f = n^2 V |V| / R^{4/3}$

Let V_0 : velocity component in x-direction of the undisturbed flow, $\xi \equiv x$, $\tau \equiv (V_0 + C_0)t - x$, $C_0 = \sqrt{gH_0}$. then, Eqs.(7) and (8) will be rewritten as

$$2C(C_\xi - C_\tau) + V(V_\xi - V_\tau) + (V_0 + C_0)V_\tau - g(S - S_f) = 0 \tag{9}$$

$$2V(C_\xi - C_\tau) + C(V_\xi - V_\tau) + 2(V_0 + C_0)C_\tau = 0 \tag{10}$$

Putting

$$V \equiv V_0 + V_1(\xi)\tau + V_2(\xi)\tau^2 + \tag{11}$$

$$C \equiv C_0 + C_1(\xi)\tau + C_2(\xi)\tau^2 + \tag{12}$$

and substituting expressions (11) and (12) into Eqs.(9) and (10), we get

$$C_0(-2C_1 + V_1) + \left\{ 2C_0\left(\frac{dC_1}{d\xi} - 2C_2\right) - 2C_1^2 + V_0\frac{dV_1}{d\xi} - V_1^2 + 2C_0V_2 \right\} \tau - g(S - S_f) + \left[\quad \right] \tau^2 + \quad = 0 \tag{13}$$

$$C_0(-V_1 + 2C_1) + \left\{ 2V_0\frac{dC_1}{d\xi} - 3V_1C_1 + C_0\left(\frac{dV_1}{d\xi} - 2V_2\right) + 4C_0C_2 \right\} \tau + \left[\quad \right] \tau^2 + \quad = 0 \tag{14}$$

For prismatic channels with the rectangular section,

$$S_f = n^2g^{1/3}V|V| \left\{ \frac{1}{C^2} + \frac{2}{gB} \right\}^{3/2}, \quad S = n^2g^{1/3}V_0^2 \left\{ \frac{1}{C_0^2} + \frac{2}{gB} \right\}^{3/2} \tag{15}$$

From Eqs.(11), (12) and Eq.(15),

$$g(S - S_f) = -gS \left\{ \left(A + 2\frac{V_1}{V_0} \right) \tau + \left(\quad \right) \tau^2 + \quad \right\}, \tag{16}$$

where

$$A = -\frac{8}{3} \frac{C_1/C_0}{1 + (C_0^2/gB)}$$

Substituting Eq.(16) into (13) and (14), and considering for Eqs. (13) and (14) to be valid independently from τ , we get next relations.

$$2(V_0 + C_0)\frac{dC_1}{d\xi} - 6C_1^2 + 2C_0V_2 - 4C_0C_2 + 4gSC_1\left(\frac{1}{V_0} - \frac{2}{3C_0k}\right) = 0, \tag{17}$$

$$2(V_0 + C_0)\frac{dC_0}{d\xi} - 6C_0^2 - 2C_0V_2 + 4C_0C_2 = 0, \quad k = 1 + \frac{2C_0^2}{gB} \tag{18}$$

Adding Eqs.(17) to (18),

$$(V_0 + C_0)\frac{dC_0}{d\xi} - 3C_0^2 + C_0gS\left(\frac{1}{V_0} - \frac{2}{3C_0k}\right) = 0 \tag{19}$$

Under the initial condition:

$$\left(\frac{\partial C}{\partial \tau}\right)_{\tau=0} = \frac{g}{2\sqrt{gH(\xi,0)}} \left(\frac{\partial H}{\partial \tau}\right)_{\tau=0} = C_1(\xi), \tag{20}$$

the solution of Eq.(19) is as follows.

$$C_0(\xi) = 1/\left(\frac{\alpha}{\beta} + K \cdot e^{\beta\xi}\right), \tag{21}$$

where

$$\frac{\alpha}{\beta} = 3/gS(\frac{1}{V_0} - \frac{2}{3Ck})$$

$$K = \left\{ \frac{2C_0(V_0 + C_0)}{g} / \left(\frac{\partial H}{\partial \tau} \right)_{\xi=0, \tau=0} \right\} - \frac{\alpha}{\beta}$$

The angle between the forerunner and an undisturbed surface (see Fig.6), will be given by the equation:

$$\tan \theta = \left\{ \frac{\partial H(\xi, \tau)}{\partial \tau} \right\}_{\tau=0} = \frac{2C_0}{g} C_1(\xi) \tag{22}$$

Then,

$$\frac{d\theta}{d\xi} = -\frac{2C_0}{g} \frac{1}{(1 + \frac{2C_0 C_1}{g})^2} \frac{K\beta}{(\frac{\alpha}{\beta} + K \cdot e^{\beta\xi})^2} e^{\beta\xi} \tag{23}$$

where

$$K\beta = \frac{1}{V_0 + C_0} \left\{ \frac{2C_0(V_0 + C_0)S}{\left(\frac{\partial H}{\partial \tau} \right)_{\xi=0, \tau=0}} \left(\frac{1}{V_0} - \frac{2}{3Ck} \right) - 3 \right\}$$

According to Eq.(23),

when $\left(\frac{\partial H}{\partial \tau} \right)_0 > \frac{2}{3} C_0(V_0 + C_0) \left(\frac{1}{V_0} - \frac{2}{3kC_0} \right) S$, $\frac{d\theta}{d\xi} > 0$ and

the forerunner will gradually rise up and in the contrary case, the forerunner may damp out.

For simpler cases which are $S = 0$ (horizontal bed) and $V_0 = 0$ (intrusion on the still water region), we have Eq.(19) as follows.

$$C_0 \frac{dC_1}{dx} - 3C_1^2 = 0 \tag{24}$$

or $C_1(\xi) = -1 / \left\{ \frac{3}{C_0} \xi + K' \right\}$ $C_1(0) = -\frac{1}{K'}$ (25)

From Eqs.(20) and (25), K' will be

$$K' = -\frac{2C_0}{g}(V_0 + C_0) / \left(\frac{\partial H}{\partial \tau} \right)_0 \tag{26}$$

and

$$\tan \theta = -1 / \left\{ \frac{3}{2} \frac{\xi}{H_0} - \frac{C_0}{\left(\frac{\partial H}{\partial \tau} \right)_0} \right\} \tag{27}$$

$$\frac{d\theta}{d\xi} = \frac{3}{2H_0} \cos^2 \theta / \left\{ \frac{3}{2} \frac{\xi}{H_0} - \frac{C_0}{\left(\frac{\partial H}{\partial \tau} \right)_0} \right\}^2 > 0 \tag{28}$$

If we can roughly assume that the plunging of forerunner will be defined by the condition $\tan \theta \rightarrow \infty$, the site of breaking X_b

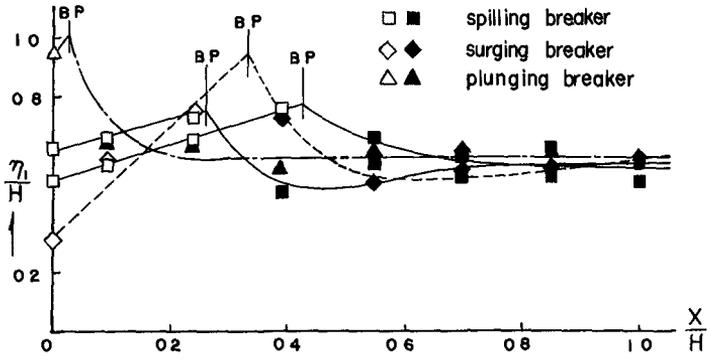


Fig. 5. Variations of the height of initial waves.

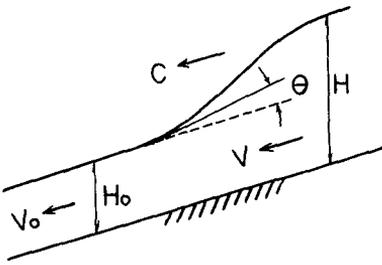


Fig. 6. Definition sketch for the forerunner.

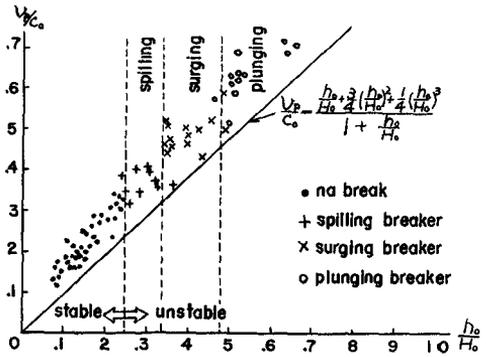


Fig. 8. The piston velocity, V_p of the wave generator.

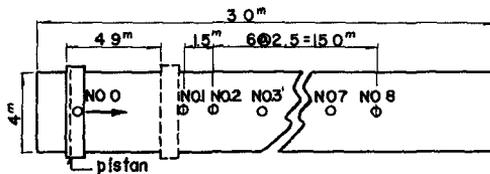


Fig. 7. Plan view of the wave tank and locations of wave meters.

may be estimated by the relation (putting the denominator of Eq. (27) to be zero)

$$\frac{3}{2} \frac{X_b}{H_0} = C_0 / \left(\frac{\partial H}{\partial \tau} \right)_{\tau=0} \quad (29)$$

EXPERIMENTS

METHODS AND PROCEDURES

The wave tank of horizontal bottom being 30m in length, 4m in width and 0.5m in depth, is shown in Fig.7, in which locations of 9 electric resistance-type wave-meters are also indicated. The No.0 wave meter is attached to and moves with the piston-type wave-generator and it can record the surface variation just in front of the piston plate.

It must be preferable to drive the piston with the large acceleration at its start for the purpose of generations of surges having the abruptly-rised front closely similar to the ideal surges. By our wave generator, $V_p/\tau = 2.2 \sim 3.5$ cm/sec, where V_p : the constant speed of piston, τ : the time required to reach constant-speed state from the start.

The relative height of surges h/H is determined theoretically by the piston speed as follows.

$$\frac{V_p}{C_0} = \frac{\left(\frac{h}{H} \right) \left[1 + \frac{3}{4} \frac{h}{H} + \frac{1}{4} \left(\frac{h}{H} \right)^2 \right]}{1 + \frac{h}{H}} \quad (30)$$

In Fig.8, the relation between V_p/C_0 and h/H are shown both in a theoretical curve and experimental plotting. To generate a expected surge, the slightly larger speed of piston-drive than the values given by the theory will be required. The reason of this discrepancy is supposed that the given energy by the piston motion can not perfectly transfer to the fluid.

Relative heights of surges in our experiments are in the range $0.08 < h/H < 0.64$ and corresponding generator speed are 18.3 cm/sec $< V_p < 85.5$ cm/sec.

Breaking phenomena are observed by 16 mm cine-camera loaded on the particularly designed wave tracer which is driven in remote controls to run at the same speed as the celerity of dispersive waves.

RECORDED WAVE PATTERNS

Each typical record of wave patterns in cases of the stable dispersion process and the unstable process including breaker phenomena, is shown in Fig.9a and 9b, respectively.

Because the dispersive waves in stable process are expected to reach finally to the conservative wave, it may be convenient

Fig.9a.

No breaking, stable

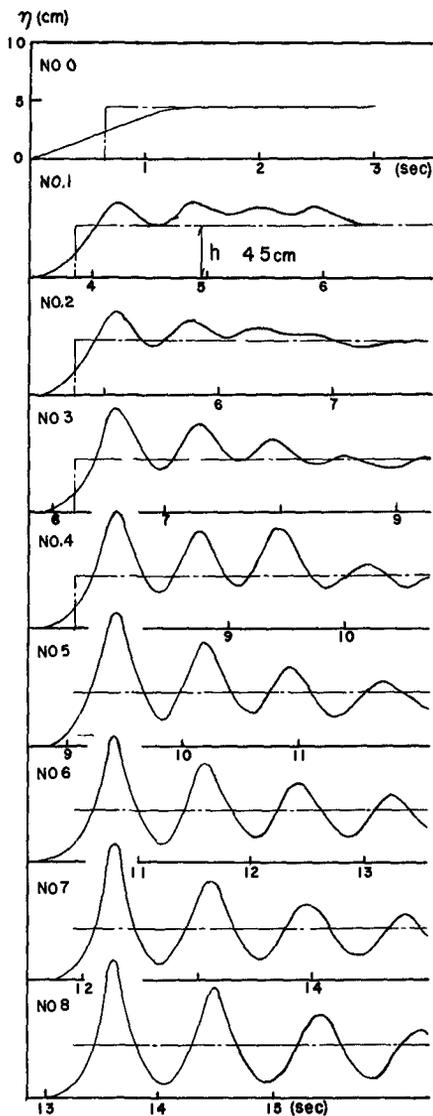


Fig.9b.

Surging breaker

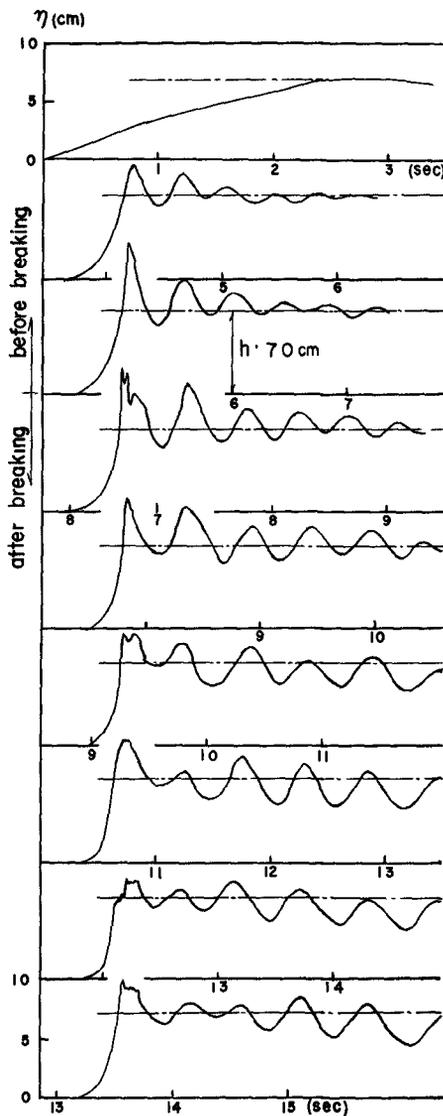


Fig. 9. Examples of recorded patterns of the dispersive wave.

for quantitative expression of the rate of approaching to a final state, to take the shaded area in Fig.10 as the amount of residual instability. Denoting this area by m and the initial value at the position of No.1 wave-meter by m_0 , we can see the tendency that the ratio m/m_0 in Fig.11 approach rapidly to zero and after running over the reach of 80 times as large as the still water depth H , the dispersive waves in the stable region will reach to the form of the conservative (solitary) wave.

Transformations of dispersed waves in the unstable process will be mentioned at the later article.

DISTRIBUTION OF THE HEIGHT OF DISPERSIVE WAVES AND RANGES OF THE STABLE OR UNSTABLE PROCESS

The distribution of surface deviation η from the undisturbed level are shown in Fig.12. The theoretical prospect is much satisfactory, that is, all of the wave crest and all of the trough exist over or under the theoretical limit-curve $\eta/h = 1 + h/3H$, respectively.

By the author's experiment, the upper critical value for the stable process including no breakers is $h/H = 0.25$ which is slightly smaller than Favre's value 0.28, but both Favre's and the author's value are the experimental and theoretically unfounded.

BREAKING

The initial waves always break and radiate their excess energy when the relative height of surge h/H exceeds the critical value 0.25. As we can see in Fig.5, the initial wave height η increase rapidly until the breaking occurs. The increasing rate of wave heights $d\eta/dx$ before the breaking seems to be proportional to h/H by our experiments and then the rise-up of initial waves before roller breaking are much dominant than that of spilling breaker.

According to the usual classification, breakings may be assorted into 3 types, i.e. spilling, surging and (rolling or) plunging breaker of the initial wave.

(a) Spilling breaker

Keeping the symmetry of wave forms by the crest, the white caps are observed at the cusped crest and the cusp angle are about $120^\circ \sim 130^\circ$ just in breaking. Entrained air bubbles are left over the breaking crest.

(b) Surging breaker

The dominant rise-up of wave front and the weak rolling at the crest are distinctive features of this type. Generated white water rush forward from the broken body. After breaking of the initial wave, sequential waves in the wave train are scarcely affected by the former breaking. Because surging breakers have

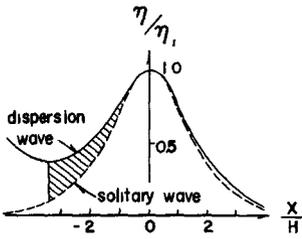


Fig. 10. The definition of the amount of residual instability, m .

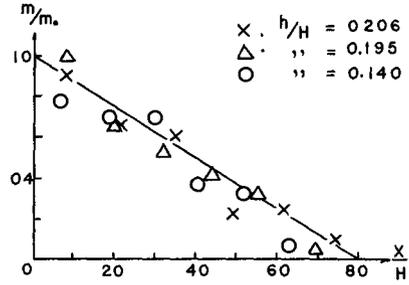


Fig. 11. $m/m_0 \sim X/H$.

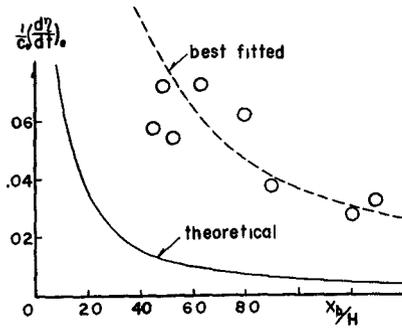


Fig. 13. The theoretical curve and experimental plotting of the breaking point, X_b .

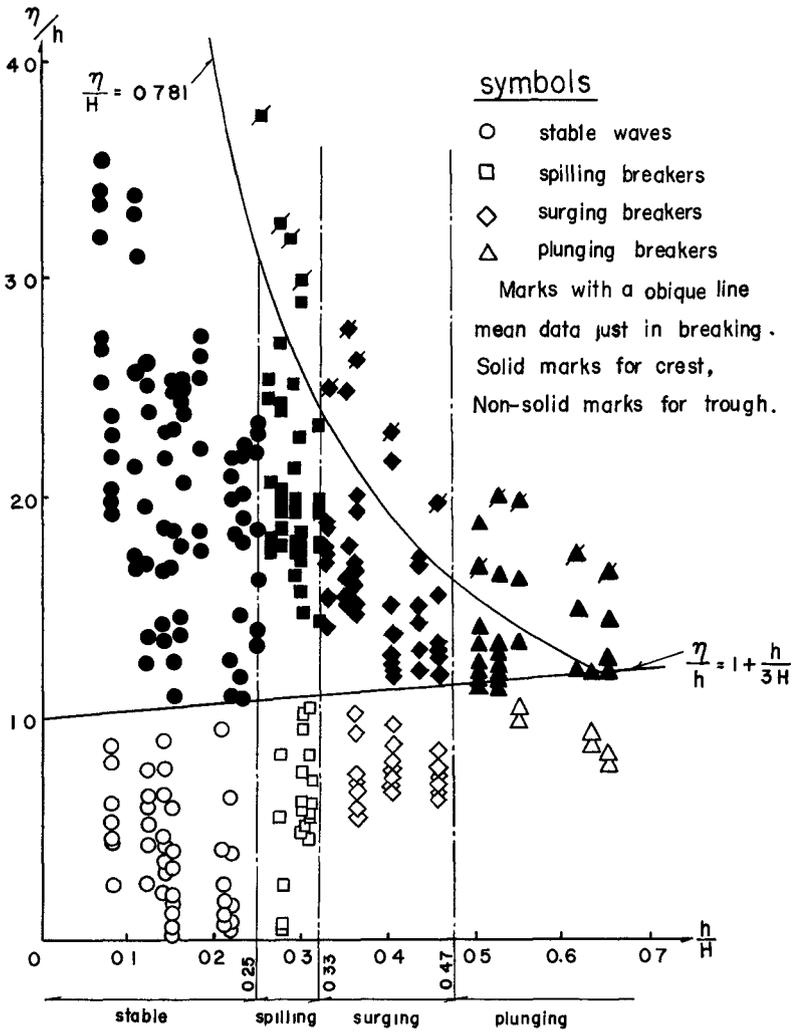


Fig. 12. The distribution of the height of dispersive waves and criteria of various modes of the breaking.

some intermediate characteristics between spilling and plunging, it may be difficult to strictly distinguish the surging from the plunging.

(c) Plunging breaker

Strong rollers will be observed at the breaking crest of this type. The white water rush forward and sequential waves are much affected by the initial breaking.

We observed the instantaneous feature of breaking by taking high-speed photographs (64 frames/sec) loaded on the wave-tracer and decided the type of breakings by those pictures.

Results are shown in Fig.12 and it may be concluded that the type of breakers seems to be determined only by the relative height of surges h/H as follows.

- | | | |
|-----|-----------------------|--------------------|
| For | $h/H < 0.25$, | stable. |
| For | $0.25 < h/H < 0.33$, | spilling breakers. |
| For | $0.33 < h/H < 0.47$, | surging breakers. |
| For | $0.47 < h/H$, | plunging breakers. |

Instantaneous wave heights at the breaking are also shown in Fig.12 by marks with an oblique segment. These are still something not to be clarified by the author that all of breakings of the initial wave occur beyond the Munk's criteria $\eta_{1b}/H = 0.78$ by our experiments, but it was reported conversely that $\eta_{1b}/H = 0.6$ by Sandover and Taylor's (1962) experiment.

Lastly, discussions on the position of occurrence of breakings will be mentioned. Experimental data are treated in Fig.13 using the parameter $\frac{1}{C_0} \left(\frac{d\eta}{dt} \right)_0$, which are designated by Eq.(29).

The theoretical curve in Fig.13 considerably deviates from experimental points. Because the reduction of Eq.(29) was based on the unreasonable assumption: $\tan\theta \rightarrow \infty$ for the breaking condition, the theoretical prospect will be not in the least exact quantitatively, but it may be asserted that the curve shows fairly well the general tendency of breaking position and $\frac{1}{C_0} \left(\frac{d\eta}{dt} \right)_0$ is supposed to be an important parameter in the analysis of breaking phenomena.

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