CHAPTER 15

A STUDY ON WAVE TRANSFORMATION INSIDE SURF ZONE

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ABSTRACT

The wave transformation inside surf zone is treated analytically in this paper under the several appropriate assumptions. The theoretical curves computed numerically have a consistant agreement with the experimental data in the case of wave transformation on a horizontal bottom. On the other hand, in the case of wave transformation on a uniformly sloping beach, the analytical treatment seems to be inadequate to clarify the actual phenomena. Besides them the numerous data on wave height attenuation and others are presented in the graphical forms.

INTRODUCTION

The phenomena of wave transformation in the surf zone has been a matter of great interest to the coastal engineers, therefore the numerous investigators have treated the same problem on the basis of the appropriate assumptions.[1,2,5,6] The assumptions are such that the wave has its critical height as a progressive wave at each depth of water, or that the wave height decreases exponentially with the distance of wave propagation from the breaking point. These foregoing treatments seem to be inadequate to clarify the phenomena of wave transformation inside the surf zone, thus more reasonable method is required to be applied. The aim of this paper is to present an approach to the stated problem on the basis of the analytical and experimental treatments.

THEORETICAL ANALYSIS

In the analysis on the attenuation of wave height in the surf zone, the following assumptions are introduced as the basis of the analytical treatment:

a) The 2nd order approximation of solitary wave theory introduced by Laiton[4] is adopted to express the features of the broken waves progressing in the surf zone. That is, the wave profile, wave celerity, and horizontal component of water particle velocity are given by the following equations respectively:

$$\eta = H \operatorname{sech}^2 \sqrt{\frac{3H}{4h^2(H+h)}}\,(x - ct) \tag{1}$$

$$C = \sqrt{gh\,(1 + H/h)} \tag{2}$$

$$u = \sqrt{gh}\left[\frac{\eta}{h}\left\{1 - \frac{5H}{4h} - \frac{3H}{2h}\left(\frac{2z}{h} + \frac{z^2}{h^2}\right)\right\} + \left(\frac{\eta}{h}\right)^2\left\{\frac{5}{4} + \frac{9}{4}\left(\frac{2z}{h} + \frac{z^2}{h^2}\right)\right\}\right] \tag{3}$$

where η is surface elevation measured from the still water level, H wave height, h water depth, C wave celerity, u horizontal component of water particle velocity, z vertical axis taking upward from the still water level, and x horizontal axis taking along the still water level.

b) The wave is attenuated by the effects of turbulence and bottom friction. The effect of percolation on the attenuation of waves is negligibly small.

c) The friction coefficient has the same value in the entire region of surf zone.

d) The turbulence is isotropic and decreases exponentially according as the increase of showeward distance measured from the breaking point.

ENERGY DISSIPATION DUE TO BOTTOM FRICTION

In an oscillatory flow the shearing stress at bottom may be expressed approximately by the following equation[3]:

$$\tau_b = C_f\left(\frac{8}{3\pi}\right)\hat{u}^2 \tag{4}$$

where τ_b is shearing stress at bottom, C_f friction coefficient and $\hat{u}$ amplitude of the average velocity in depth, $\bar{u}$.

The energy dissipation by bottom friction per unit width, per unit time, dE_b/dt , is given by

$$\frac{dE_b}{dt} = \int_{-\infty}^{\infty} \rho\bar{u}\,\tau_b\,dx = \frac{16\rho C_f}{3\pi\alpha}(gh)^{3/2}\frac{H}{h}\left(1 - \frac{13H}{12h} + \frac{101H^2}{80h^2} - \frac{37H^3}{80h^3} + \frac{3H^4}{20h^4}\right) \tag{5}$$

where

$$\alpha = \sqrt{(3H/4h^2)(h+H)} \tag{6}$$

ENERGY DISSIPATION DUE TO TURBULENCE

When the wave breaks at a certain point, a great amount of air bubble is entrained into the water, causing a large scale disturbance in flow. Such kind of disturbance seems to take the main role of energy dissipation at least at the initial stage of wave transformation in the surf zone. By the assumption that the turbulence is statistically isotropic, the energy dissipation due to turbulence per unit volume, per unit time, is given by

$$\overline{W} = 15\mu \frac{\overline{u'^2}}{\lambda^2} \tag{7}$$

where $\overline{W}$ is the rate of energy dissipation due to turbulence, μ coefficient of fluid viscosity, u' fluctuation of horizontal velocity component, and λ microscale of turbulence or dissipation length.

The kinetic energy of turbulence seems to be inversely proportional to the distance from the breaking point. Therefore it may be possible to express the decay of turbulence as follows:

$$\overline{u'^2} \propto \exp(-\beta x/L) \tag{8}$$

where β indicates a damping coefficient of turbulence, x distance measured from the breaking point and L wave length. Thus the dissipation length may be expressed by the following relation:

$$\lambda^2 = \frac{-10\nu\,\overline{u'^2}}{\frac{d\overline{u'^2}}{dt}} = \frac{-10\nu\,\overline{u'^2}}{c\,\frac{d\overline{u'^2}}{dx}} = 10\,\frac{\nu T}{\beta} \tag{9}$$

Here we assume that the mixing length, ℓ , in the Prandtle's hypothesis is proportional to the height above the bottom,

$$u' = \ell\,\frac{du}{dz} = \kappa\,(z+h)\frac{du}{dz} \tag{10}$$

where κ is the Kármán's universal constant, h water depth, u horizontal component of particle velocity and z axis taking upward from the still water level. Therefore we obtain the following expressions on the rate of energy dissipation due to turbulence, $\overline{W}$, and the loss of energy due to turbulence per unit width, dE_t/dt :

$$\overline{W} = 15\,\rho\,\frac{\kappa^2\beta}{T}(z+h)^2\left(\frac{du}{dz}\right)^2 \tag{11}$$

$$\frac{dE_t}{dt} = \int_{-\infty}^{\infty}\int_{-h}^{\eta} \overline{W}\, dz\, dx = \frac{0.825 g \beta h^2}{T\alpha}\left(\frac{H}{h}\right)^4 \left[1 + 3.99\left(\frac{H}{h}\right)\right.$$
$$\left. + 7.29\left(\frac{H}{h}\right)^2 + 7.65\left(\frac{H}{h}\right)^3 + 8.60\left(\frac{H}{h}\right)^4 + 2.08\left(\frac{H}{h}\right)^5\right] \qquad (12)$$

On the other hand, the total wave energy of a solitary wave per unit width, E_s , is given by

$$E_s \doteqdot 2E_p = \frac{8}{3\sqrt{3}}\rho g h^3\left(\frac{H}{h}\right)^{3/2}\left(1 + \frac{H}{h}\right)^{1/2} \qquad (13)$$

The time rate of energy transport, dE_s/dt , is as follows:

$$\frac{dE_s}{dt} = \frac{4.4}{\sqrt{3}}\rho g h\left(\frac{H}{h}\right)^{1/2}\left(1 + \frac{H}{h}\right)^{1/2}\frac{d(H/h)}{dx} \qquad (14)$$

WAVE TRANSFORMATION ON A HORIZONTAL BED

The rule of energy conservation is expressed in the next equation:

$$\frac{dE_s}{dt} = -\left(\frac{dE_b}{dt} + \frac{dE_t}{dt}\right) \qquad (15)$$

Substituting Eqs. (5), (12) and (14) into Eq. (15), we find the following differential equation:

$$\frac{dx}{T\sqrt{gh}} = \frac{d\left(\frac{H}{h}\right)}{0.0374\beta\left(\frac{H}{h}\right)^3\left(1+\frac{H}{h}\right)^{-1/2}F\left(\frac{H}{h}\right) + 0.772 C_f T\sqrt{g/h}\left(\frac{H}{h}\right)^2\left(1+\frac{H}{h}\right)^{-1/2}\varphi\left(\frac{H}{h}\right)} \qquad (16)$$

where

$$\left.\begin{aligned} F\left(\frac{H}{h}\right) &= 1 + 3.99\left(\frac{H}{h}\right) + 7.27\left(\frac{H}{h}\right)^2 + 7.65\left(\frac{H}{h}\right)^3 + 8.60\left(\frac{H}{h}\right)^4 + 2.08\left(\frac{H}{h}\right)^5 \\ \varphi\left(\frac{H}{h}\right) &= 1 - 1.08\left(\frac{H}{h}\right) + 1.26\left(\frac{H}{h}\right)^2 - 0.463\left(\frac{H}{h}\right)^3 \end{aligned}\right\} \qquad (17)$$

The integration of the above equation can not be done analytically, but be done numerically as shown in Fig. 1. Here β is selected to be equal to 5, and the effect of bottom friction is included in a factor of $C_f T/\sqrt{g/h}$ selected as a parameter of family of curves.

WAVE TRANSFORMATION ON A UNIFORMLY SLOPING BED

The slope of the bottom is defined by $S = -dh/dx$, and the time rate of energy transport per unit width can be

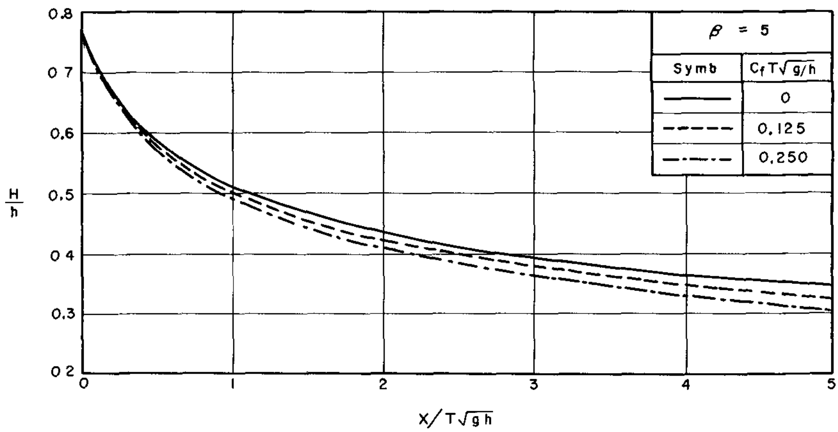


Fig. 1. Numerical integration curves of Eq. (16). (Horizontal bottom)

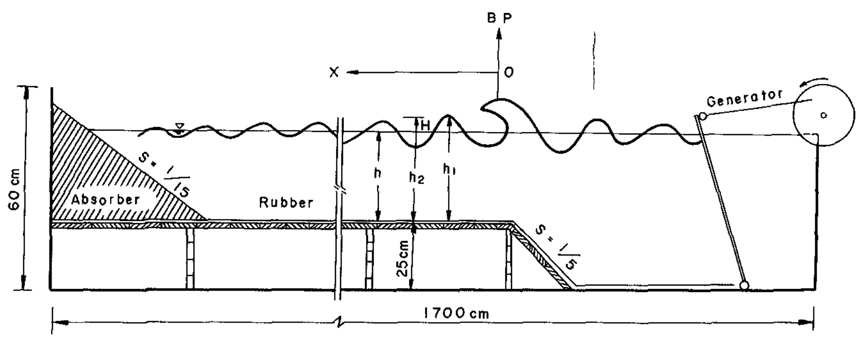


Fig. 2. Laboratory installation. (First set of experiments)

expressed by the following:

$$\frac{dE_s}{dt} = c\left(-S\frac{\partial E_s}{\partial h} + \frac{\partial E_s}{\partial H}\frac{\partial H}{\partial x}\right) \tag{18}$$

Taking into consideration the next relationships,

$$\left.\begin{aligned} \frac{\partial E_s}{\partial h} &\doteqdot \frac{3.56}{\sqrt{3}}\rho g H h\left(\frac{H}{h}\right)^{1/2}\left(1+\frac{H}{h}\right)^{1/2} \\ \frac{\partial E_s}{\partial H} &= \frac{4.4}{\sqrt{3}}\rho g h^2\left(\frac{H}{h}\right)^{1/2}\left(1+\frac{H}{h}\right)^{1/2} \\ L &= 2\pi h\left(1+\frac{H}{h}\right)^{1/2}\left(\frac{3H}{h}\right)^{1/2} \end{aligned}\right\} \tag{19}$$

we may obtain the following differential equation:

$$\frac{dh}{h} = \frac{S\, d\left(\frac{H}{h}\right)}{0.0103\beta\left(\frac{H}{h}\right)^{9/2}\left(1+\frac{H}{h}\right)^{-1/2}F\left(\frac{H}{h}\right) + 0.772C_f\left(1+\frac{H}{h}\right)^{-1/2}\left(\frac{H}{h}\right)^{3}\varphi\left(\frac{H}{h}\right) - 1.81S\left(\frac{H}{h}\right)} \tag{20}$$

where the functions of $F(H/h)$ and $\varphi(H/h)$ are the same as given in Eq. (17). The integration of the above equation can be done by the method of numerical computation.

EXPERIMENTAL ANALYSIS

HORIZONTAL BED

The experimental studies by using a horizontal bed were conducted for the purpose of determining the damping coefficient of turbulence, β, which was defined in Eq. (8). It seems to be quite reasonable to assume that the turbulence of flow induced by breaking of waves takes the most important role on the wave attenuation in the surf zone on a smooth horizontal bed comparing to the bottom friction and others.

The wave channel used for the present studies is 17 m long, 0.7 m wide and 0.6 m high. At one end of the channel, an elevated wooden horizontal bottom was installed and connected to the channel bottom with a slope of 1/5 as shown in Fig. 2. The surface of the horizontal bottom mentioned above was covered with a smooth rubber plate. Waves were generated at the other end of the channel by a flap type wave generator. The incident waves were forced to break themselves on the sloping bottom and then propagated to the elevated horizontal region. Among the various kinds of waves generated, we selected only the particular ones which broke just at the corner between the elevated horizontal bottom and the sloping bed. The characteristics of the selected waves are given in Table 1.

A sample plotting of the experimental data is presented in Fig. 3. Here the tested waves have the same period of 1 sec, but the water depth above the elevated bottom has a defferent value for each run. Figure 4 gives a comparison of the laboratory data with the analytical curve determined by taking $\beta = 5$. From this figure it may be recognized that the wave steepness in deep water of incoming waves seems to have very little effect on the wave transformation in the surf zone.

Summarizing the results of our laboratory experiments, we conclude that the damping coefficient of turbulence, β , can be taken a certain value between 4 and 5 in the present experiments.

In order to investigate the applicability of the above treatment to the practical phenomena in field, we took the field data of wave height in the surf zone which were obtained by Ijima by means of stereo photography on the Niigata West Coast. The bottom slope of beach at the questioned site is so gentle, therefore the beach slope is assumed to be horizontal. Figure 5 shows the comparison of various curves such as (1) laboratory curve, (2) analytical curve calculated under the assumption of $\beta = 1$, and $C_f = 0.05$, (3) curve proposed by Ijima empirically, and (4) mean curve of field data. The agreement between the analytical curve and the curve of field data seems to be quite consistant. But here it is necessary to remark that the value of β in laboratory is 4 $\sim$ 5, while the value in field is 1. The above fact suggests us the existance of scale effect of turbulence in the present problem. From this point of view more field works are certainly necessary to be done.

UNIFORMLY SLOPING BED

Another series of experiments were carried out to reveal the influence of bottom slope on the wave transformation inside the surf zone. The first set of experiments in which we tested the bottom slopes of 1/20 and 1/30 was conducted by using the same channel as in the previous experiments. The second set of experiments in which we tested the bottom slope of 1/65 and 1/80 was done by using another channel at Cheng Kung University, the size of which was 75 m long, 1.0 m wide and 1.2 m deep. The slope in the latter two cases was made of concrete. The conditions of both sets of experiment are given in Table 2.

The dimensional analysis introduces the following relationship among the wave characteristics, water depth, and bottom slope condition in a non-dimensional form.

Table 1. Experimental conditions on horizontal bottom.

Period T(sec)	Water depth d(cm)	Wave height H_0(cm)	Steepness H_0/L_0
2.6	15.0 18.18	8.54 - 12.0	0.007 - 0.010
2.0	6.5, 10.5, 15.0 8.5, 12.5, 18.2	5.73 - 15.3	0.009 - 0.025
1.5	6.0, 10.0, 15.0 7.5, 12.7	3.43 - 14.4	0.010 - 0.044
1.0	6.1, 10.0, 15.0 7.5, 12.5, 10.1	3.65 - 15.1	0.025 - 0.100

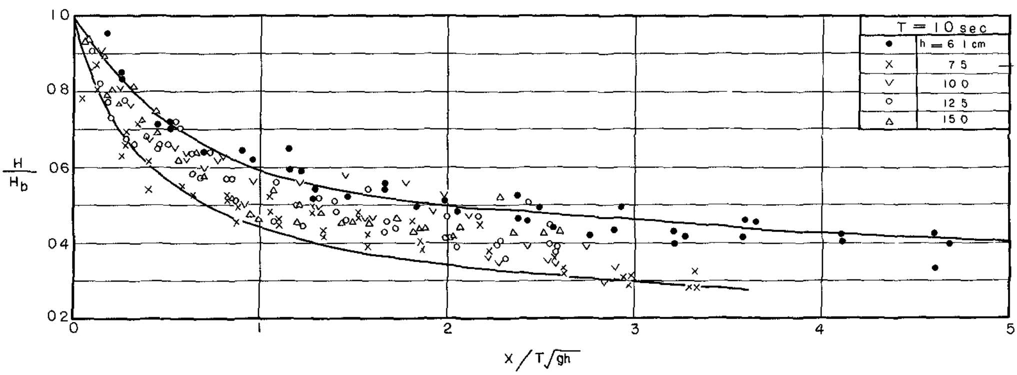


Fig. 3. A sample of experimental results. (Horizontal bottom)

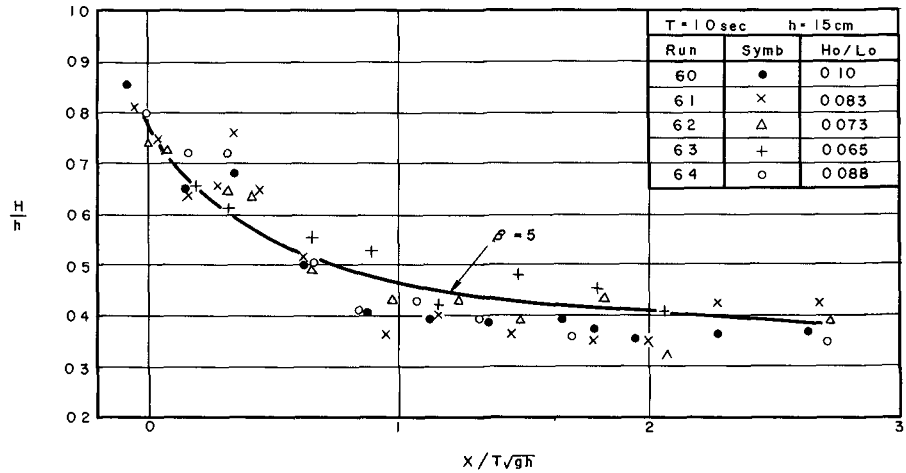


Fig. 4. Comparison of the experimental results with the theoretical curve. (Horizontal bottom)

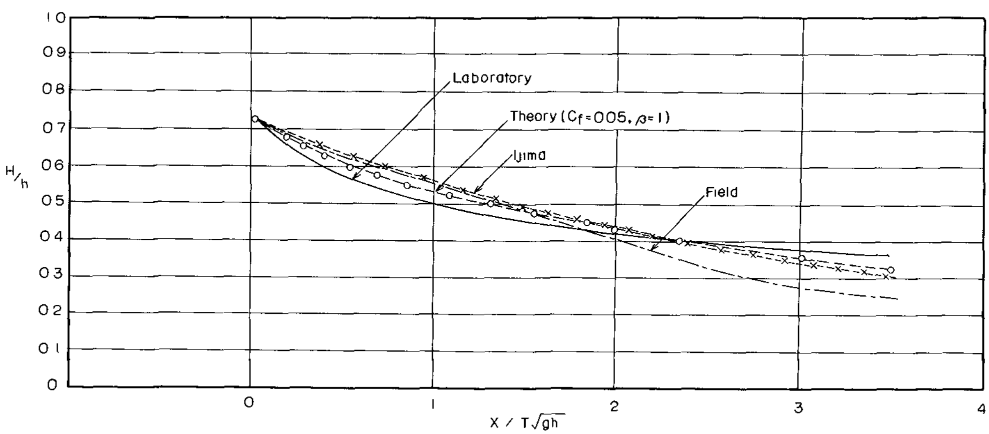


Fig. 5. Relationship between H/h and $x/T\sqrt{gh}$ obtained from various sources. (Horizontal bottom)

Table 2. Experimental conditions on sloping bottom

Bed condition		Wave characteristics			Depth d(cm)
Surface	slope	T(sec)	H_0(cm)	H_0/L_0	
Rubber surface	1/20	2.0, 14.1	5.6 - 16.9	0.008 - 0.053	43.3
	1/30	2.2, 14.1	4.7 - 17.2	0.007 - 0.052	39.0
Concrete bed	1/65	1.56, 2.0 2.0, 1.8	5.9 - 24.5	0.009 - 0.065	78.0
	1/80	1.6, 1.4 1.2	5.8 - 16.7	0.011 - 0.072	75.0

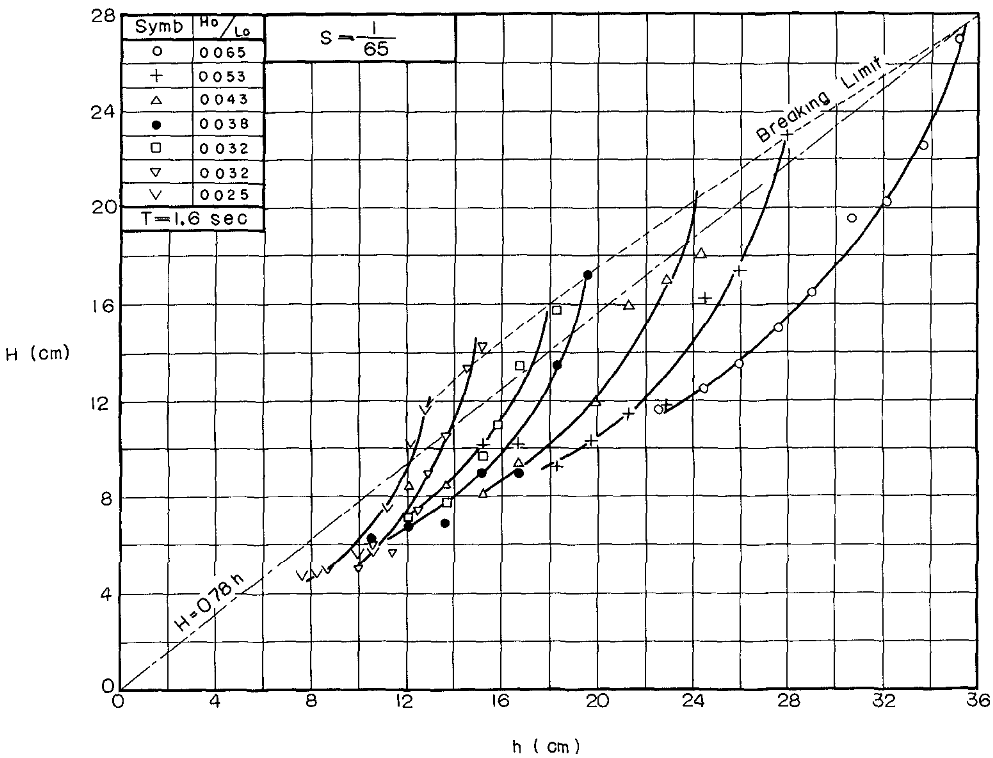


Fig. 6. Transformation of wave heights inside surf zone with 1/65 bottom slope.

$$\frac{H}{H_b} = \phi\left(\frac{H_o}{L_o}, \frac{h}{h_b}, S\right) \tag{21}$$

or

$$\frac{H}{h} = \psi\left(\frac{H_o}{L_o}, \frac{h}{h_b}, S\right) \tag{22}$$

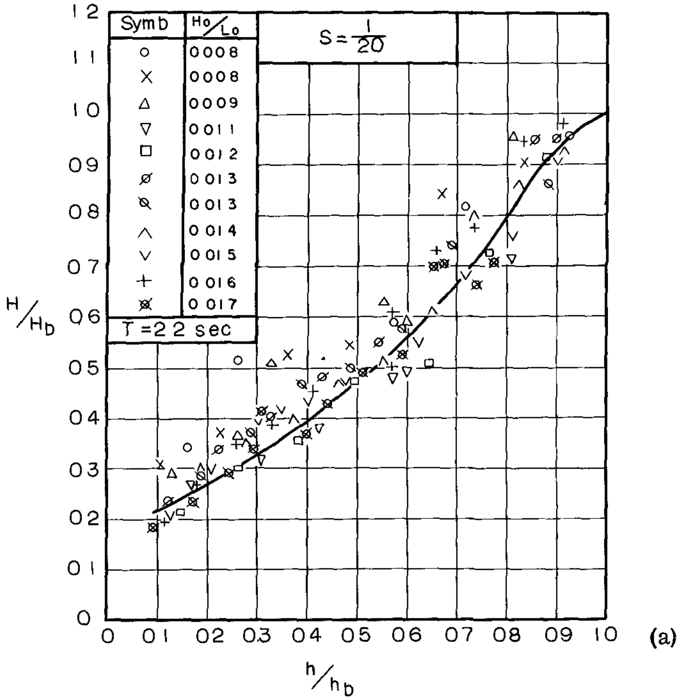
where subscript o and b denote the values in deep water and at breaking point respectively.

According to the result of dimensional analysis the experimental data were plotted as shown in the following figures. Figure 6 gives several examples of wave transformation on the sloping bottom of 1/65, and indicates that the limiting condition of solitary wave, $H=0.78h$, is not suitable to express the wave height inside the surf zone. In Fig. 7 is shown the correlation between the relative wave height, H/H_b, and the relative water depth, h/h_b, for each bottom slope. Scatter of data in these figures seems to be caused mainly by the instability of waves inside surf zone, but the steepness of incident waves in deep water seems to have small influence on the stated relationship. Therefore the effect of bottom slope on the wave attenuation in the surf zone is summarized in Fig. 8, from which it may be recognized that the gentler the bottom slope becomes, the smaller the relative wave height, H/H_b, becomes at the same relative water depth, h/h_b. The above fact is due to that the decay distance from the breaking point on a gentle slope is larger than on a steep slope.

In the same way we plotted the data on the following graphs as shown in Fig. 9 in order to find out the relationship between the relative wave height with respect to water depth, H/h, and the relative water depth, h/h_b, for each particular bottom slope. There is a large scatter of data, but it is possible to draw mean curve through the plotted data. A family of curves thus determined is given in Fig. 10 with the parameter of bottom slope. The figure shows that the relative height, H/h, has its minimum on the condition of $h/h_b \doteq 0.6$. The analytical results obtained by the integration of Eq. (20) under the conditions of $\beta=4$, and $C_f = 0.02$ are also plotted in the same figure by dots and dashes. The agreement between the computed and experimental results is not fully satisfactory, therefore it is quite necessary to treat the present problem by more regorous approach.

Lastly, it will be mentioned here that the Boussinesq's expression for wave celerity has the better agreement with the experimental results as shown in Fig. 11. The equation is as follows:

$$C = \sqrt{gh\{1 + (a_1/h)\}\{1 + (a_1/2h)\}} \tag{23}$$



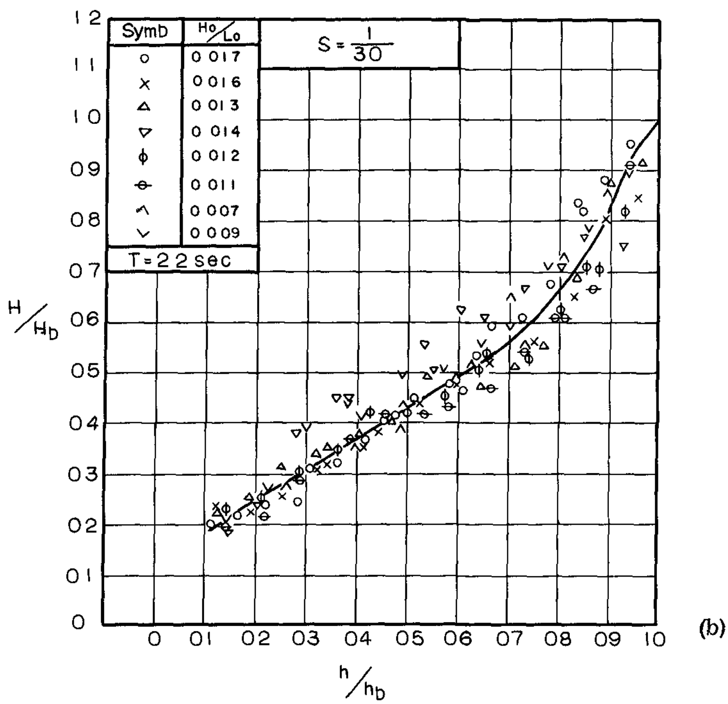
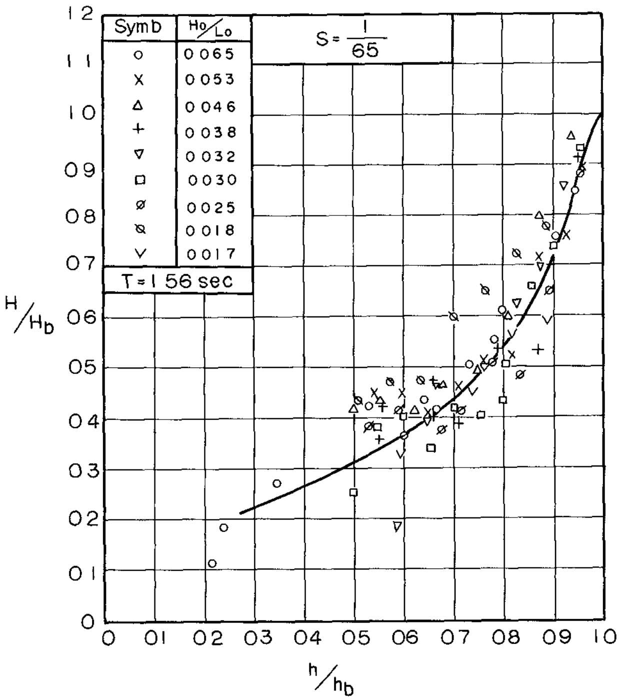


Fig. 7
Correlation between H/H_b and h/h_b with 1/20, 1/30, and 1/65 bottom slope respectively.



(c)

Fig. 7
Correlation between H/Hb and h/hb with 1/20, 1/30, and 1/65 bottom slope respectively.

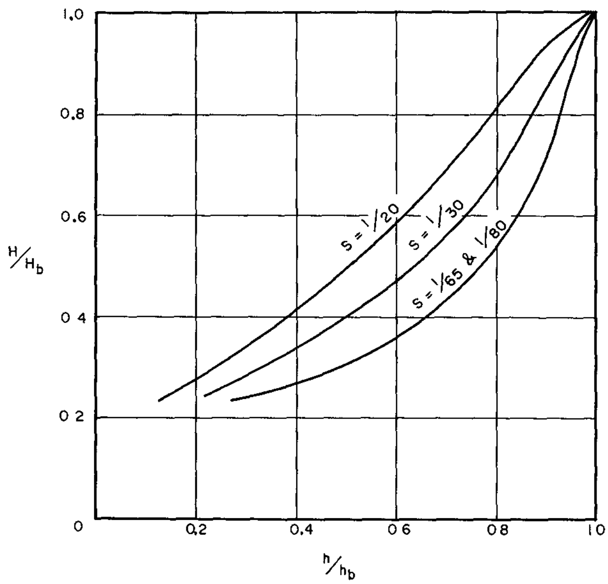


Fig. 8. Effect of the bottom slope on the wave attenuation inside surf zone.

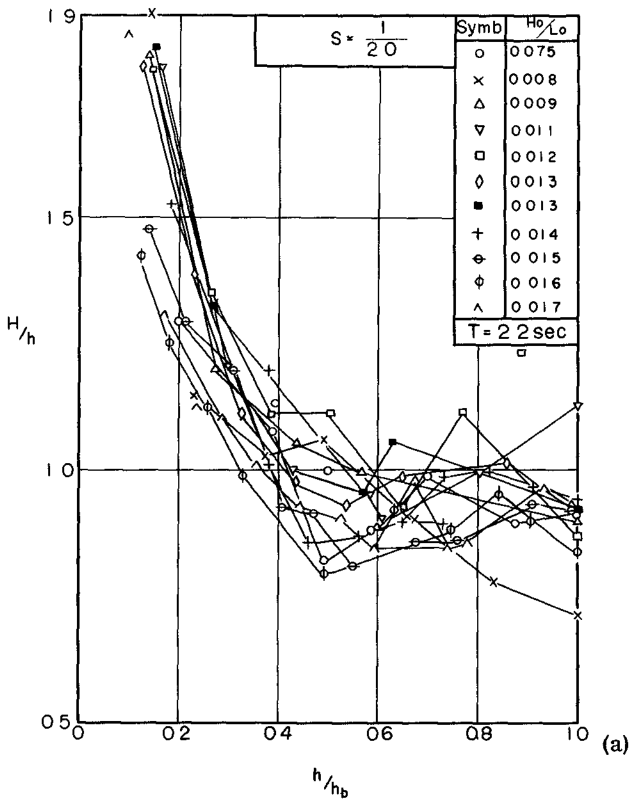
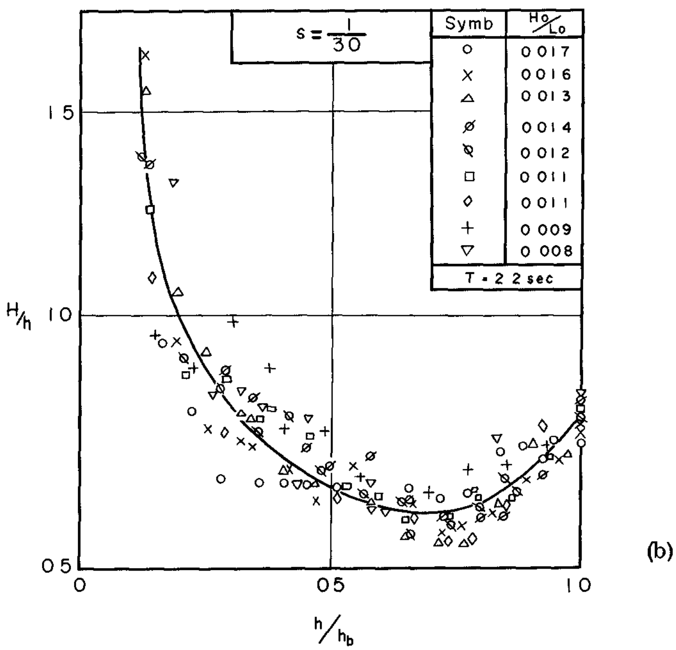
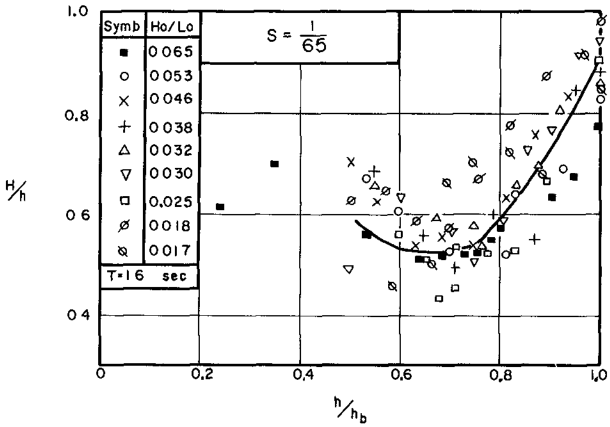


Fig. 9

Correlation between H/h and h/h_b with 1/20, 1/30, 1/65, and 1/80 bottom slope respectively

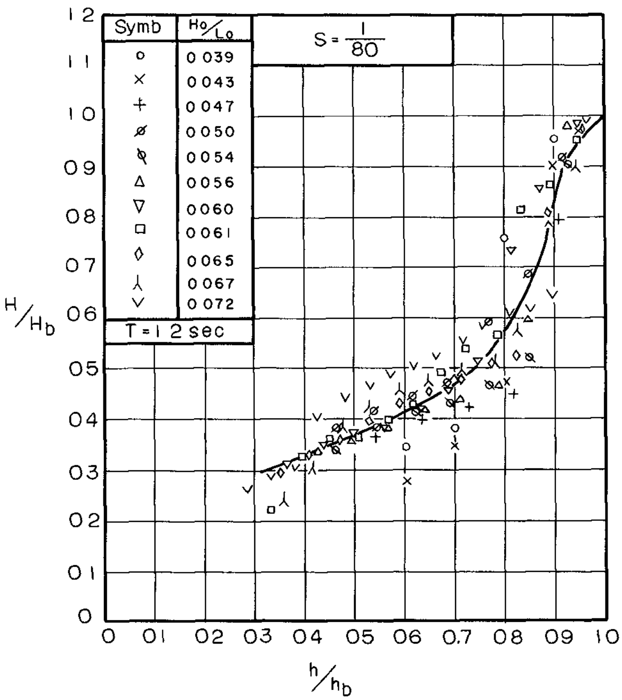




(c)

Fig. 9

Correlation between H/h and h/h_b with 1/20, 1/30, 1/65, and 1/80 bottom slope respectively.



(d)

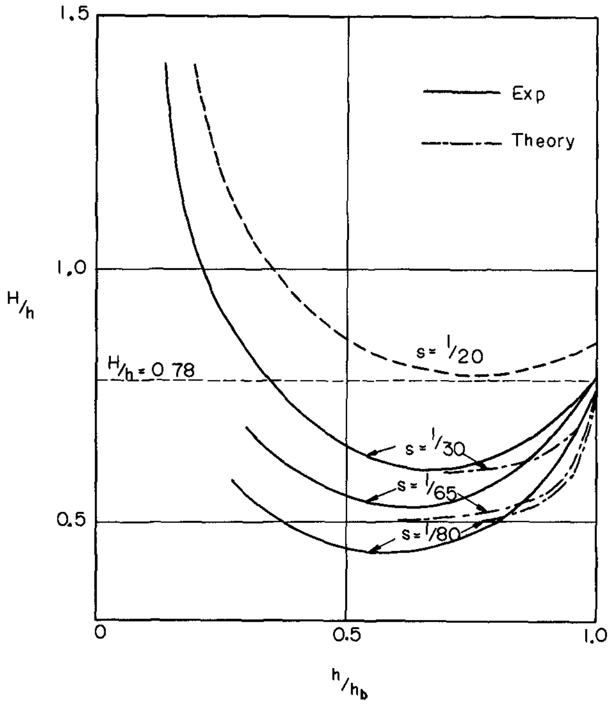


Fig. 10. Comparison of the theoretical and experimental results.

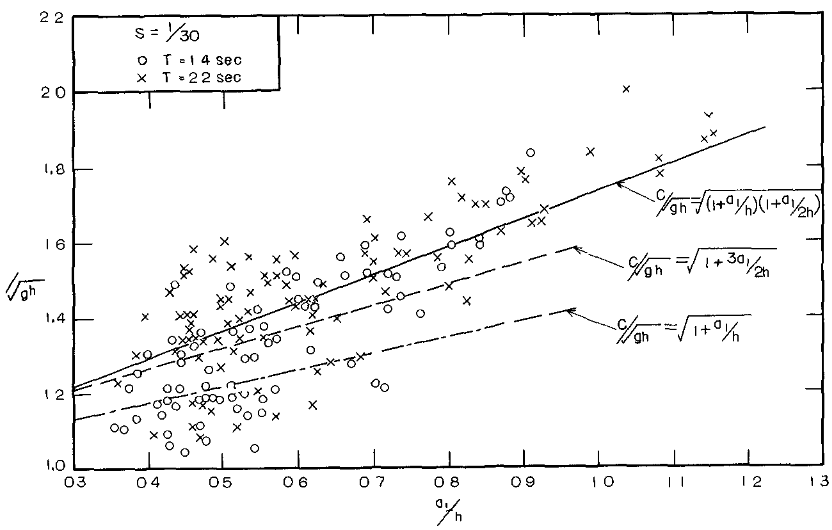


Fig. 11. Comparison of the Boussinesq's expression for wave celerity with the experimental results.

where a_1 is the crest height above the still water level.

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