Tagonoura Harbor

Part 1
THEORETICAL AND OBSERVED WAVE CHARACTERISTICS

Tone River Mouth, Choshi

CHAPTER 1

THE ANALYSIS AND PRESENTATION OF WAVE DATA - A PLEA FOR UNIFORMITY

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ABSTRACT

This paper describes a method of analysis of sea-wave records and ways of presentation of the results. The author hopes that standard techniques can be agreed amongst users of wave data, and puts forward this paper as a possible model.

INTRODUCTION

The results of the analysis of sea-wave records are occasionally published in the scientific press. Various methods of analysis have been used, but it is rarely possible to compare measurements made at different places, because the presentation varies according to the needs and imagination of the primary user. There are, of necessity, many different ways of measuring waves, but if the techniques of analysis and presentation can be agreed between theoreticians, analysts and users, the usefulness of the data will be considerably increased, and its publication would make it available to others.

As a result of theoretical studies of the statistical properties of sea waves, the collection and analysis of many thousands of wave records, and discussions with engineers on the uses to which wave data is put, the N.I.O. has evolved techniques of analysis which are simple and relatively quick to use, yet which yield parameters which are theoretically meaningful. The results of such analyses can be presented in ways which are simple in format yet which contain a large amount of information, and which seem to be of value to the user. An example of the ways in which wave data is now presented by the N.I.O. can be found in the associated paper "Waves at Sekondi, Ghana". Although it is the fourth paper (see References) in which data is presented in this form, the N.I.O. does not claim that it is the optimum method for all or any users of such data, and suggestions for improvement would be welcomed.

FACTORS AFFECTING ANALYSIS OF WAVE RECORDS

This paper is confined to the ways in which pen records of the variation of wave height with time at one place may be analysed, although the principles involved can easily be applied to any automatic system of recording and analysis. The ways in which this has been done are probably as numerous as the number of organizations undertaking the work. Many methods are based on the crest-to-trough height of individual waves and on the apparent period of the highest wave in the record, or the average period of some arbitrarily selected group of large well-defined waves.

The majority of wave recorders make use of some form of filtering which removes the shorter-period waves; very often this is the hydrodynamic filter inherent in the use of pressure recorders (the so-called Attenuation of Waves with Depth). This is of great apparent help to the analyst, to whom the short-period "embroidery" is a nuisance; when the shorter waves are removed, the longer-period and more powerful waves are shown clearly. The difficulty is that such filtering is arbitrary, depending strongly on the depth at which the transducer happened to be placed, and on the wave periods present at any given time. The absence of the filtered-out short-period waves is probably the reason why so many analysts have been able to adopt methods based on the height of the highest individual wave as it appears on a record. Such a simple and convenient method cannot be operated satisfactorily on unmodified true-surface records. The increase in the number of true-surface wave recorders in regular use, such as the Waverider accelerometer buoy, adds urgency to the task of solving the problems which will have to be faced in the analysis of these records. Figure I explains the problem. This is a typical wave record of wide-spectrum sea waves, with some loss of shorter waves due to attenuation with depth. If one decides to ignore the shorter waves remaining on the record, how short and how small does each one have to be before it is ignored? Clearly, such a method of analysis cannot be defended. No technique of analysis is free from errors, but at least if all records could be analysed in the same way, any errors resulting from the comparison of results would be minimized.

Tucker[1] proposed a simple method of analysis of sea waves, based on theoretical studies of their statistical properties by Cartwright and Longuet-Higgins[2,3,4] and others. Checks on the reliability of these statistics have been made and have been in good agreement with the theory, but the results have not been published. The methods now suggested for general use are a modification of Tucker's method, which is reproduced in the Appendix.

PARAMETERS EXTRACTED FROM EACH RECORD

Wave height parameters are calculated from the distances of the highest crest (A) and lowest trough (C) from the mean water level, irrespective of whether or not they are part of the same wave; such distances have a mathematical significance, whereas the height of an individual wave, if such a thing could be defined, has no such significance. A and C are added to give H_1 , as described in the Appendix. The wave-period parameter which can be interpreted mathematically and yet has physical significance is the zero-crossing period T_z; the apparent period of one large wave is difficult to measure accurately, and cannot be interpreted mathematically. H_1 must be corrected for the response of the recording instrument and for attenuation of waves with depth[5,7] (if appropriate), to yield H_1' . It is suggested that the appropriate period to use for these corrections is T_z.

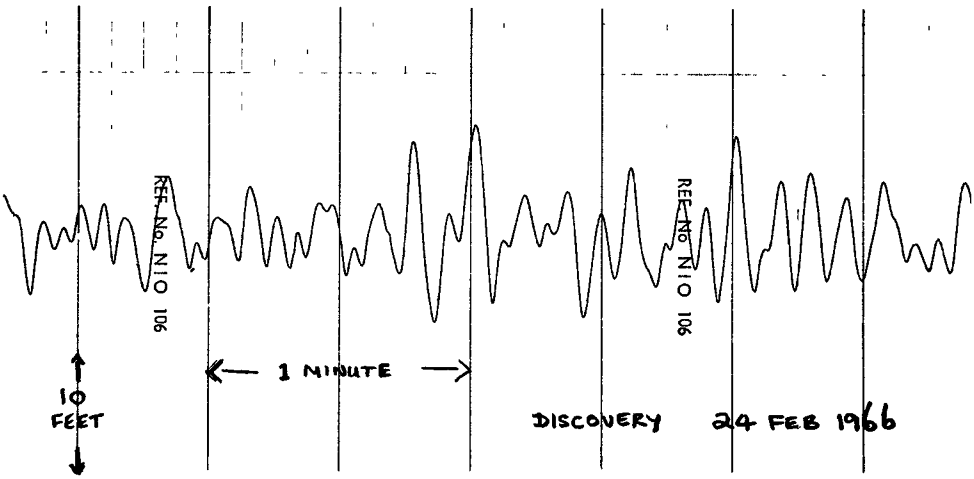

Fig. 1. A typical wave record of wide-spectrum sea waves, with some loss of shorter waves due to attenuation with depth.

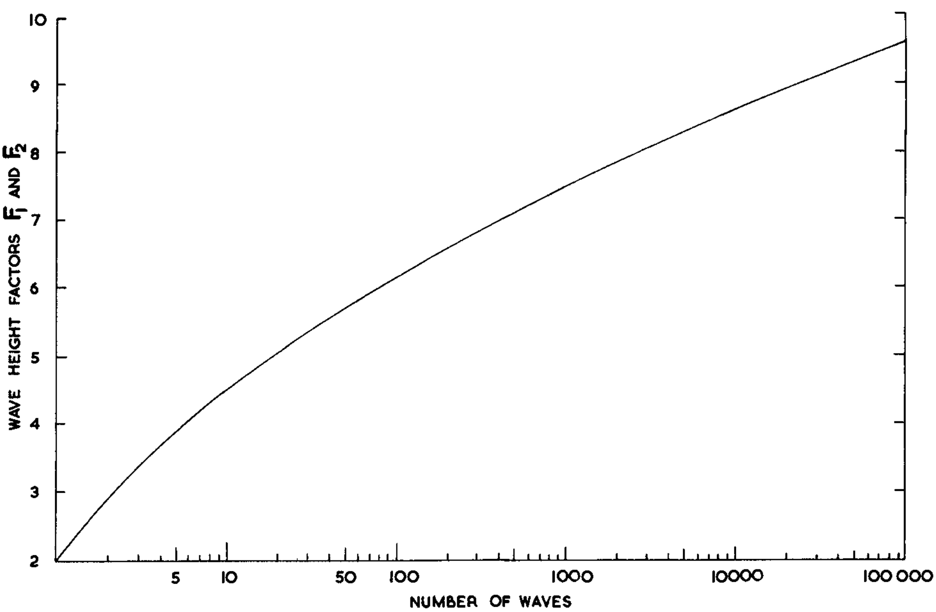

Fig. 2. The relationship between wave-height factors F_1, F_2 and the number of waves.

The zero-crossing period T_z is derived as follows:- From a known length of record, of say ten or fifteen minutes' duration, count the number of times the record crosses the mean water level in both the upwards and downwards directions; occasions when a crest or trough just touches the mean line are counted as one crossing. This number $N_{z(total)}$ is divided by two to give the number of zero crossings in the upwards direction, N_z. Hence $T_z = \frac{\text{Duration of the record in seconds}}{N_z}$

Another easily-measured parameter is the number of wave crests, N_c from which the mean crest-period T_c can be calculated. From these two wave periods (T_z and T_c) can be calculated the spectral-width parameter ϵ, which is a number giving a simple but useful measure of the width of the wave spectrum; once again this parameter has theoretical significance. These are the only parameters taken from the wave records (A, C, N_z, N_c). The above procedure is followed on each of the records, which usually amount to, say, eight per day or nearly 3,000 in a year.

Tucker's paper[1] also explains how the r.m.s. wave displacement $D_{r.m.s.}$ can be derived; (this was originally termed r.m.s. wave height $H_{r.m.s.}$ but to avoid confusion the new notation has been adopted). This parameter is of use in more fundamental studies of waves, but it is not calculated or used in the method of presentation of wave data suggested in this paper.

PROCESSING AND PRESENTATION OF THE DATA

To some extent the accompanying paper is self-explanatory. We have found that wave height is best presented as percentage exceedance, rather than percentage occurrence per unit wave-height interval, as is sometimes done. The exceedance graphs present the information in a form which allows the user to decide at a glance the percentage of time in which wave conditions exceeded any particular height. We have chosen to give (a) significant height, and (b) the most probable value of the height of the highest wave in some specific interval of time, usually three hours or six hours. Examples of uses of these are, for the former, the calculation of useable construction time in that area, and the latter in estimating the probable utilization of high-speed craft such as hydrofoils or hovercraft, where even one unusually large wave encountered on a journey can impede progress.

The calculation of H_s from H_1' is a simple process, and depends only on the number of waves (i.e., number of upwards zero crossings). Table I gives the relationship between $\frac{H_s}{H_1'}$ and N_z. (This is adapted from Fig. 2 of Tucker, 1963[6]).

TABLE I

$H_s = H_1' \times$ Factor

No. of zero-crossings, N_z, in a record	Factor	No. of zero-crossings, N_z, in a record	Factor
20-21	0.77	64-73	0.65
22-23	0.76	74-85	0.64
24-25	0.75	86-100	0.63
26-27	0.74	101-118	0.62
28-29	0.73	119-139	0.61
30-32	0.72	140-166	0.60
33-35	0.71	167-202	0.59
36-39	0.70	203-253	0.58
40-44	0.69	254-315	0.57
45-49	0.68	316-390	0.56
50-55	0.67	391-488	0.55
56-63	0.66	489-615	0.54

The factors by which H_1' must be multiplied to obtain H_s.
For example: N_z = 54 waves; H_1' = 18 feet.
From the Table, the factor = 0.67 $\therefore$ H_s = 12 feet.

The calculation of the most probable value of the height of the highest wave in some specific interval of time such as three hours, again depends on the number of waves, and is calculated directly from H_1' using Fig. II. The factor F_1 appropriate to the number of waves in the record (N_z) is obtained, and also the factor F_2 appropriate to the number of waves in the longer interval (3 or 6 hours, perhaps). The height required is then $H_1' \times \frac{F_2}{F_1}$. The absolute value of these factors is unimportant in this application. (The diagram is taken directly from Draper (1963)[7] and relates the ratio of the most probable value of H_{max} to $D_{r.m.s.}$ for various numbers of waves.)

In the presentation of the occurrence of period throughout a year, it is often the percentage of occurrence of waves of some specific period which needs to be known, and the data is given in this form. Various measures of period have been used; we have chosen to use zero-crossing period because, as explained earlier, it is a parameter which is susceptible to mathematical treatment and can be determined unambiguously. It can be related to the significant period; the relationship between the two has been given[8] from investigations made by M. Darbyshire (1962)[9].

The inter-relationship between wave height and period is often important and is well presented by a scatter diagram. We have given significant height and zero-crossing period as being the most likely parameters to be used. The inclusion of lines of constant wave steepness helps to give an indication of severity of conditions, and contouring gives a quick indication of the frequency of occurrence of specific conditions. We have given occurrences in parts per thousand to avoid the use of the decimal point.

METHOD OF PREPARING THE SCATTER DIAGRAM

A large sheet of paper is laid out in the form of Fig. 4 of "Waves at Sekondi, Ghana". The wave height and wave period range should each be divided into not less than about ten equal intervals; it is better to err on the numerous side because adjacent sections can later be combined by adding two or more groups together, whereas one section cannot later be split up. Each analyzed record yields a pair of values of H_s and T_z and a mark is made in the section of the diagram appropriate to these values. This is repeated for all the results of the records, when a picture of the distribution of the wave parameters will emerge. One diagram should be prepared for each season, or, if the distribution of seasons is not known, for each month separately; the monthly diagrams can then be compounded into seasons having similar characteristics. For the scatter diagram for publication, up to the present time we have found that one figure containing the data for a whole year is adequate.

PREPARATION OF THE WAVE HEIGHT DIAGRAMS

Use can be made of the scatter diagrams in the preparation of the exceedance graphs. For each season the numbers of occurrences of waves in each height range are added irrespective of period. These totals are then added, starting at the greatest height, to give the numbers of occasions when wave height exceeded a given value. For instance, the number of occasions in the highest range is the number of times in which waves exceeded the lowest value of that height range. The number of occasions exceeding the lowest value of the second highest range is the number of waves in that range plus the number in the highest range, and so on. After this successive summation the figures are expressed as percentages and a graph can be plotted showing the information at a glance; this is then directly in the form of the percentage of the season during which waves exceeded any given height.

PREPARATION OF THE PERIOD DIAGRAMS

The same scatter diagrams can then be analysed to give the numbers of times in which wave period lay between any two values. Again we have found it convenient to express the data on a seasonal basis, but period is more usefully expressed as percentage occurrence rather than percentage exceedance.

PREPARATION OF THE SPECTRAL-WIDTH PARAMETER DIAGRAM

Information on the width of the spectrum is sometimes valuable, and therefore the distribution of the spectral width parameter ϵ is included. In the areas investigated so far its distribution has been found not to vary significantly from season to season, and it has therefore been expressed as a percentage occurrence over the whole year.

WAVE PERSISTENCE

For constructional and operational purposes it is often important to know how long a given condition will last. We have found the Cumulative Persistence Diagram to be useful; from this it is possible to decide at a glance how often and for how long wave conditions of certain specific heights and above are likely to persist in one year (or season). This is of importance in planning the probable utilization of hovercraft and other vessels on a projected route, and in many operations from moored vessels. The persistence diagram is prepared by plotting a graph of the significant wave height throughout the year. For each height level, the duration of every occasion when conditions are at or above that level is listed; the information is again presented as an exceedance.

"LIFETIME WAVE"

A prediction of the most severe wave conditions which might occur over a long period of time such as fifty years is often required. This can be made by plotting on probability paper the most probable value of the height of the highest wave in, say, 3 or 6 hours; its derivation has been described in detail[7]. Although more work needs to be done in studying the distribution of extreme wave heights, useful estimates can sometimes be made and an example is given in the accompanying paper.

CONCLUSION

The methods described in this paper have evolved as a result of theoretical studies of sea waves, the existence of various methods of recording wave data, and especially as a result of trying to make optimum use of wave data in ways which will be of benefit to the engineer. The object of this exercise has been to make the easiest and most useful presentation of data required by engineers, whilst not losing useful information. There may be other requirements which have not come to our notice and which could be usefully filled from the available data. Suggestions would be welcomed on either the improvement of these methods or their replacement by better systems. The whole object of this paper is to ask that criticisms, constructive and otherwise, be made so that some standards of data presentation can be accepted as widely as possible. The ideal which is worth striving for is a universally adopted system enabling data collected by every instrument throughout the world to be compared easily and reliably.

PROPOSAL

It is the author's proposal that if there should be no major objections, the associated paper be accepted as a model until such time as it becomes inadequate for the purposes of the engineer.

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LIST OF SYMBOLS

A	= Height of highest crest above mean water level.
C	= Depth of lowest trough below mean water level.
H_1	= $A + C$.
H_1'	= True height (= H_1 corrected for attenuation of waves with depth, and response of instrument, as appropriate).
H_s	= Significant wave height.
$H_{max}(\ \ )$	= Maximum true wave height (defined in the same way as H_1') for the duration stated in the subscript brackets.
N_z	= Number of upwards zero-crossings in a record (same as the number of waves in the record).
N_c	= Number of crests.
T_z	= Mean zero-crossing period.
T_c	= Mean crest period.
ϵ	= Spectral width parameter.

APPENDIX

The following is the method suggested by Tucker for the practical measurement of wave records.

Measure off a ten-minute length of record, as shown in Fig. 3, and consider only waves in this interval.

Draw in a mean water-level line by eye (zero line).

Count the number of crests N_c.

A crest is defined as a point where the water level is momentarily constant, falling to either side. Some crests may be below mean water level.

Count the number of times N_z that the record crosses the zero line moving in an upward direction.

Measure the height A of the highest crest and the height B of the second highest crest, measuring from the zero line.

Measure the depth C of the lowest trough and the depth D of the second lowest trough, measuring from the zero line and taking both quantities as positive.

From these measurements: $H_1 = A + C$

$$H_2 = B + D$$

$$T_c = \frac{600}{N_c} = \text{period of crests}$$

$$T_z = \frac{600}{N_z} = \text{period of zero crossings}$$

The theoretical basis for this system of measurement is given by Cartwright and Longuet-Higgins and by Cartwright (see also Putz), and is briefly as follows:

The statistical distribution of wave heights is governed by the r.m.s. wave-height $D_{r.m.s.}$ and by a spectral-width parameter ϵ.

From the measurements, the best estimate of ϵ is

$$\epsilon^2 = 1 - (T_c/T_z)^2$$

One can think of the significance of this parameter as follows:

If the wave components cover a wide range of frequencies, the long waves will carry short waves on top of them and there will be many more crests than zero crossings, so that T_c will be much smaller than T_z and ϵ will be nearly one. If, on the other hand, there is a simple swell which contains only a narrow range of frequencies, each crest will be associated with a zero crossing, so that T_c will approximately equal T_z and ϵ will be nearly zero.

Using the measured value of ϵ, the values of the other wave-height parameters can be estimated from H_1 and H_2. Thus $D_{r.m.s.}$ is estimated as follows:

From H_1:

$$D_{r.m.s.} = \tfrac{1}{2}H_1(2\theta)^{-1/2}\,(1 + 0.289\theta^{-1} - 0.247\theta^{-2})^{-1}$$

From H_2:

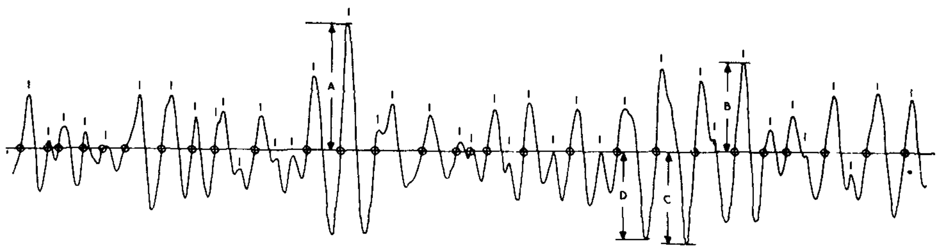
$$D_{r.m.s.} = \tfrac{1}{2}H_2(2\theta)^{-1/2}\,(1 - 0.211\theta^{-1} - 0.103\theta^{-2})^{-1}$$

where $\theta = \log_e N_z$

These are the best estimates to a good degree of approximation.

If these conversions were to be used a great deal, the ratio of $D_{r.m.s.}$ to H_1 could easily be tabulated as a function of N_z. The statistical errors in these estimates are less than might be expected and are not much worse than that of the mean of the highest $\frac{1}{3}$rd waves in the records. The formulae for them are complicated, but in a typical case where $N_c = 100$, $\epsilon = 0.8$, the proportional standard error in the estimate of $D_{r.m.s.}$ from H_1 is approximately 13% and from H_2 about 10%. In practice, for many civil engineering purposes, the relevant wave height is H_1', and the relevant period T_z.

A point which must not be overlooked is that most wave records have to be corrected for the frequency response of the recording instrument and also in the many cases where the instrument records pressure change, for the attenuation of waves with depth. This cannot be done exactly, but in practice the best answer can be obtained by using the correction factor appropriate to the zero-crossing period, T_z. Fig. 4 relates the attenuation of waves with depth and wave period, for instruments located on the sea bed. For instruments located in mid-water, some calculations have been made and are available in graphic form.



An illustration of the simple measurement of a wave record (only 5 minutes of the record is shown here). The points marked with a dash are wave crests, and with a circle are zero crossings in an upward direction.

Fig. 3. Section of a wave record illustrating some of the definitions of wave height and period.

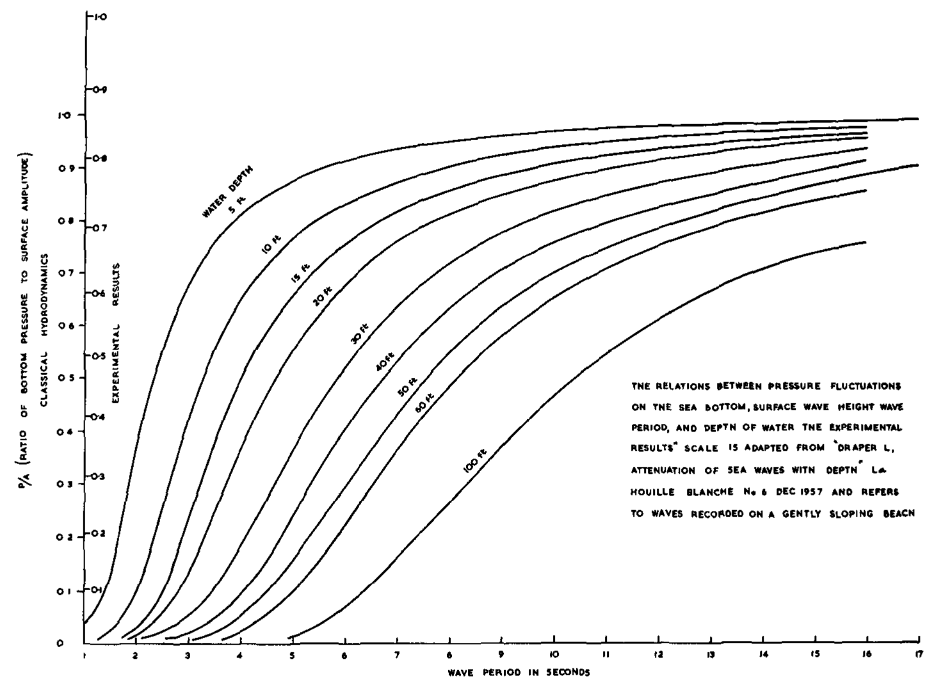


Fig. 4. Relationship between pressure fluctuations on the sea bottom, surface wave height, wave period and depth of water.