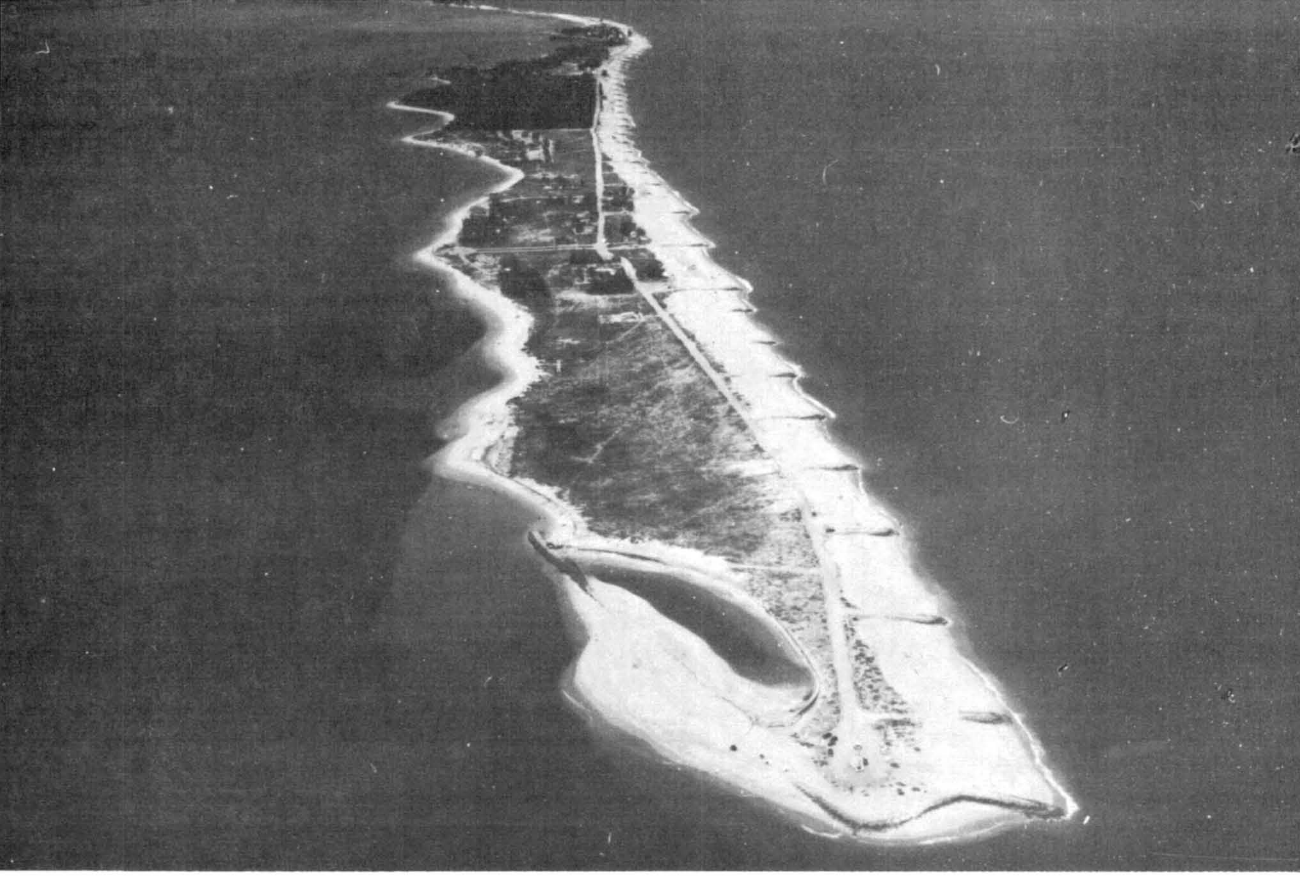

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Part 2
COASTAL SEDIMENT PROBLEMS

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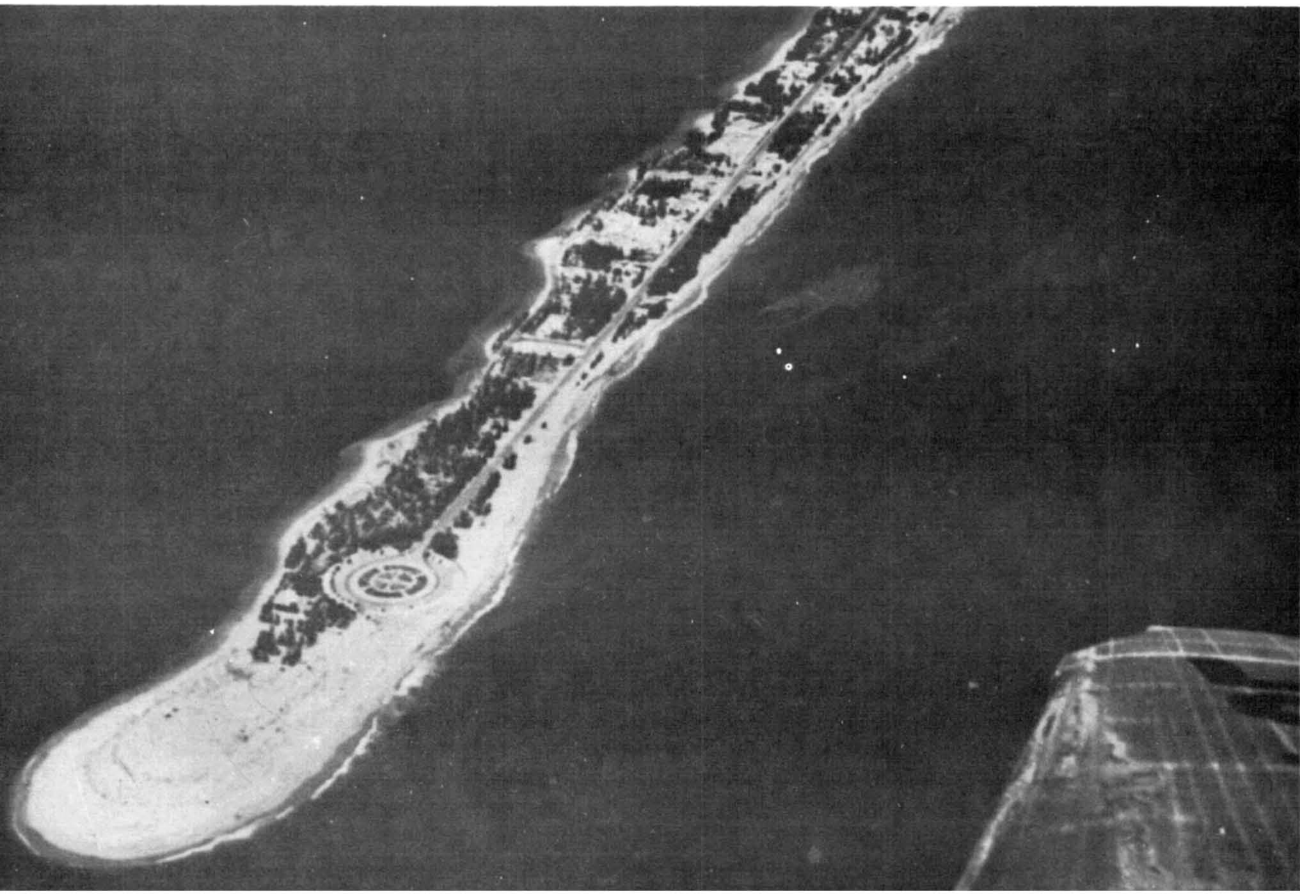

Chapter 13

THE COASTLINE OF RIVER-DELTAS

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INTRODUCTION.

The purpose of this paper is to investigate the shape of the coastline of a river delta on a coast, along which sediment is transported by waves only. In order to make the problem suitable to a mathematical treatment it is necessary to simplify to a large extend the processes occurring in nature. We assumed:

1e. There are no tides and no tidal currents.
2e. The influence of currents on the sedimenttransport will be neglected.
3e. In the beginning the coastline is a straight line (x -axis).
4e. The fore shore has a constant slope until a depth D; the influence of the waves reaches until this depth D; further seawards the waterdepth has a constant value D.
5e. The mouth of the river lies in the origin of the system of coördinates, and stays there.
6e. The river continually brings a constant quantity of sediment into the sea.
7e. Waves with a constant height and a constant wave lenght are approaching the coast continually under a angle β with the x - axis.
8e. Refraction, diffraction and reflexion of the waves are neglected.
9e. The relation between the quantity of sediment Q, transported by the waves along the coast, and the angle α between the wave crests and the coastline, will be simplified in such a way, that an analytical solution of the partial differential equation will be possible. The next paragraph deals with this problem.

THE TRANSPORT FUNCTION

The sediment transport along shore, caused by oblique waves, is often represented by the formula:

$$Q = Q_m \sin 2\alpha$$

in which Q_m is the maximum value of Q, occuring when $\alpha = 45^\circ$. EDELMAN (1963) showed, that this formula is a fair approximation of the transport in the breaker zone. GRIJM (1960) showed, that an application of this formula to the delta problem leads to a very complicated partial differential equation, the solution of which seems to be possible only by means of a computer or a graphical method (GRIJM 1964).

LARRAS (1957) produced experimental data on the relation between Q/Q_m and α (Table I), that are not in agreement with the formula: $Q = Q_m \sin 2\alpha$

TABLE I

α =	0	10	20	30	40	50	60	70	80	90 degrees
$\frac{Q}{Q_m}$ =	0	0,30	0,57	0,70	0,83	0,98	0,70	0,28	?	?

Considering the problem from a theoretical point of view, however, it seems to be unprobably, that any single formula or any single series of data on this topic could be correct. Since refraction of waves depends on wave-height and wave-lenght, it seems to be impossible to set up a simple formula, covering the transport by all types of waves occuring.

The formulae and data, proposed by investigators in this field, until now, have, however, some things in common. Transport is increasing from zero at $\alpha = 0$ towards a maximum at a value of α lying somewhere between 40 and 65 degrees. At greater values of α the transport decreases and perhaps becomes zero at $\alpha = 90^\circ$.

Since we want a solution of the delta problem only in a general way, in order to obtain a general insight into the behaviour of the coastline in the neighbourhood of a river mouth, it seems to be admissible to make use of a new transport equation, provided, that this equation covers the above mention characteristics in a general way. Moreover, the new equation must lead to a delta-equation of which the solution can be obtained along a analytical way.

The authors believe to have succeeded in finding an equation which sufficiently satisfies these conditions. For that purpose the range of α was split up into two areas. The transpo within the first area, in which $0 < \tan\alpha < 1.23$ is described by the formula:

$$Q = Q_m K_1 \tan\alpha$$

In the second area, in which $1.23 < \tan\alpha < \infty$ the formula

$$Q = Q_m \frac{K_2}{\tan\alpha}$$

will sufficiently cover the few data known about wave-transport in this area.

$$\frac{1}{K_1} = K_2 = 1.23.$$

The maximum wave transport according to this formula would occur at $\tan\alpha = 1.23$ or $\alpha \approx 51^\circ$.

The data and formulae given in this paragraph are plotted in figure 1.

THE DELTA EQUATION

From the continuity equation it follows that:

$$\frac{\delta q}{\delta x} + \frac{\partial y}{\partial T} . D = 0$$

If ρ is the angle between the coastline and the x - axis, and β is the angle between the wave crests and the x - axis, figure 2 shows that $\alpha = \beta - \rho$

Therefore:

$$\tan\alpha = \tan(\beta - \rho) = \frac{\tan\beta - \tan\rho}{1 + \tan\beta . \tan\rho} = \frac{\tan\beta - \frac{\partial y}{\partial x}}{1 + \tan\beta . \frac{\partial y}{\partial x}}$$

Area I

$$q = K_1 q_m \tan\alpha$$

$$q = K_1 q_m \frac{\tan\beta - \frac{\partial y}{\partial x}}{1 + \tan\beta \frac{\partial y}{\partial x}}$$

$$\frac{\partial q}{\partial x} = - K_1 q_m (1 + \tan^2\beta) \frac{\frac{\partial^2 y}{\partial x^2}}{\left(1 + \tan\beta \frac{\partial y}{\partial x}\right)^2}$$

thus

$$K_1 q_m (1 + \tan^2\beta) \frac{\frac{\partial^2 y^2}{\partial x^2}}{\left(1 + \tan\beta \frac{\partial y}{\partial x}\right)^2} = \frac{\partial y}{\partial T} . D$$

Area II

$$q = K_2 . \frac{q_m}{\tan\alpha}$$

$$q = K_2 q_m \frac{1 + \tan\beta \frac{\partial y}{\partial x}}{\tan\beta - \frac{\partial y}{\partial x}}$$

$$\frac{\partial q}{\partial x} = K_2 q_m (1 + \tan^2\beta) \frac{\frac{\partial^2 y}{\partial x^2}}{\left(\tan\beta - \frac{\partial y}{\partial x}\right)^2}$$

thus

$$- K_2 q_m (1 + \tan^2\beta) \frac{\frac{\partial^2 y}{\partial x^2}}{\left(\tan\beta - \frac{\partial y}{\partial x}\right)^2} = \frac{\partial y}{\partial T} . D$$

Putting $t = \frac{Q_m}{D} (1 + \tan^2\beta) T$, we obtain

Area I

$$K_1 \frac{\frac{\partial^2 y}{\partial x^2}}{\left(1 + \tan\beta . \frac{\partial y}{\partial x}\right)^2} = \frac{\partial y}{\partial t}$$

Area II

$$- K_2 . \frac{\frac{\partial^2 y}{\partial x^2}}{\left(\tan\beta - \frac{\partial y}{\partial x}\right)^2} = \frac{\partial y}{\partial t}$$

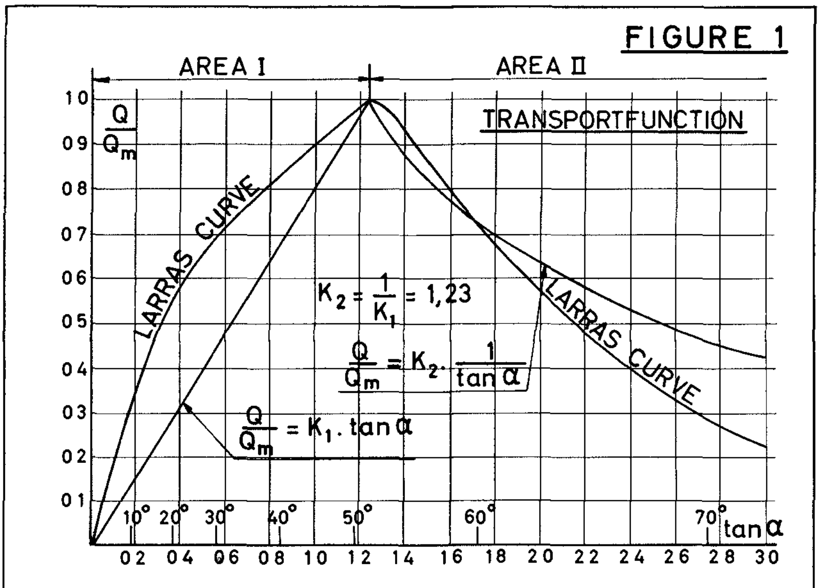


FIGURE 1
AREA I
AREA II
TRANSPORTFUNCTION
$\frac{Q}{Q_m}$
LARRAS CURVE
LARRAS CURVE
$K_2 = \frac{1}{K_1} = 1,23$
$\frac{Q}{Q_m} = K_2 \cdot \frac{1}{\tan\alpha}$
$\frac{Q}{Q_m} = K_1 \cdot \tan\alpha$
10
09
08
07
06
05
04
03
02
01
10° 20° 30° 40° 50° 60° 70°
$\tan\alpha$
02 04 06 08 10 12 14 16 18 20 22 24 26 28 30

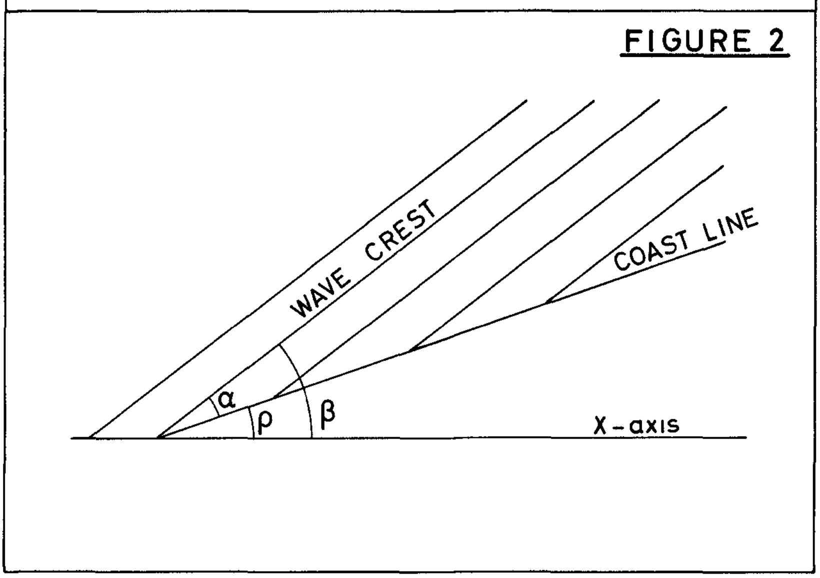


FIGURE 2
WAVE CREST
COAST LINE
α
ρ
β
X-axis

If Q_m is expressed as a volume per time-unit, t possesses the dimension of a surface.

SOLUTION OF THE DELTA EQUATION

If we put: $y = t^n f(u)$ and $x = ut^m$ we obtain:

Area I

$$K_1 \frac{t^{n-2m}\,\frac{d^2f}{du^2}}{\left(1+\tan\beta \cdot t^{n-m}\frac{df}{du}\right)^2} = t^{n-1}\left(nf - mu\frac{df}{du}\right)$$

Area II

$$-K_1 \frac{t^{n-2m}\,\frac{d^2f}{du^2}}{\left(\tan\beta - t^{n-m}\frac{df}{du}\right)^2} = t^{n-1}\left(nf - mu\frac{df}{du}\right)$$

t disappears, if we put: $n = m = \frac{1}{2}$ and we obtain:

$$y = f\sqrt{t}\,,\quad x = u\sqrt{t}\,,\quad \frac{\partial y}{\partial x} = \frac{df}{du} \quad \text{and} \quad \frac{\partial y}{\partial t} = \frac{1}{2\sqrt{t}}\left(f - u\frac{df}{du}\right)$$

The delta equations become

$$2K_1 \frac{\frac{d^2f}{du^2}}{\left(1+\tan\beta\frac{df}{du}\right)^2} = f - u\frac{df}{du}$$

$$-2K_2 \frac{\frac{d^2f}{du^2}}{\left(\tan\beta - \frac{df}{du}\right)^2} = f - u\frac{df}{du}$$

The solutions of these non-partial differential equations are:

Area I:

$$u = -A\left[e^{-v^2} + v\sqrt{\pi}\left\{E(v)+a\right\}\right]$$

$$\frac{df}{du} = -\frac{1}{\tan\beta}\left[\frac{2\sqrt{K_1}}{A\sqrt{\pi}\left\{E(v)+a\right\}} + 1\right]$$

in which

$$v = \frac{u + f\tan\beta}{2\sqrt{K_1}}$$

$$E(v) = \frac{2}{\sqrt{\pi}}\int_0^v e^{-v^2}\,dv$$

A and a are integration constants.

Area II:

$$u = -B\left[e^{w^2} - 2w\left\{\varphi(w)+b\right\}\right]$$

$$\frac{df}{du} = \tan\beta - \frac{2\sqrt{K_2}}{2B\left\{\varphi(w)+b\right\}}$$

in which

$$w = \frac{u\tan\beta - f}{2\sqrt{K_2}}$$

$$\varphi(w) = \int_0^w e^{w^2}\,dw$$

B and b are integration constants.

The curve f(u) is the basic curve of the deltaproblem; a function $y = f(x)$ at any time T can be obtained from the basic curve by central projection from the origin of the system of coordinates, multiplying with the factor $\sqrt{t}$.

The basic curve of area I has a hyperbolic shape; it possesses two asymptots running through the origin.

The basic curve of area II has a parabolic shape; it does not possess asymptots.

INVESTIGATION ABOUT THE STABILITY

Introducing the new coördinates $\bar{x}$ and $\bar{y}$ of which the $\bar{x}$ - axis is taken parallel to the wave crests, we obtain

Area I

$$K_1 \frac{\partial^2 \bar{y}}{\partial \bar{x}^2} = (1+\tan^2\beta)\frac{\partial \bar{y}}{\partial t}$$

Area II

$$-K_2 \frac{\frac{\partial^2 \bar{y}}{\partial \bar{x}^2}}{\left(\frac{\partial \bar{y}}{\partial \bar{x}}\right)^2} = (1+\tan^2\beta)\frac{\partial \bar{y}}{\partial t}$$

or, changing the variables:

$$-K_2 \frac{\partial^2 \bar{x}}{\partial \bar{y}^2} = (1+\tan^2\beta)\frac{\partial \bar{x}}{\partial t}$$

The solution with separated variables is:

$$\bar{y} = e^{-(K_1 a^2 \cos^2\beta)t}\left\{C_1 \sin a\bar{x} + C_2 \cos a\bar{x}\right\}$$

$$\bar{x} = e^{(K_2 a^2 \cos^2\beta)t}\left\{C_3 \sin a\bar{y} + C_4 \cos a\bar{y}\right\}.$$

In area I the amplitude decreases with t. In area II, however, the amplitude increases with t.

If we understand these solutions to be disturbances, superimposed upon the solutions of the foregoing paragraph, we see, that in area I such a disturbance will decrease with time and, at last, will disappear. In area II, however, such a disturbance will grow larger and larger with time (fig. 3). The original curve grows with $\sqrt{t}$, but the disturbance grows with e^t. Therefore, in area II the disturbance will supersede the original curve after some time.

Obviously, the original solution (the basic curve) is stable in area I, but unstable in area II.

This is an important conclusion. It may be seen from figure 1, that two different values of tan α are associated with every value of $\frac{q}{q_m}$. That means that in every point of a

FIGURE 3

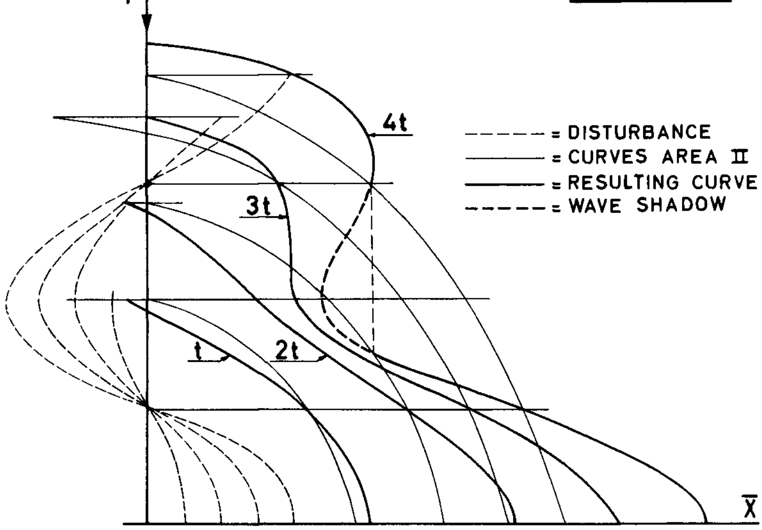
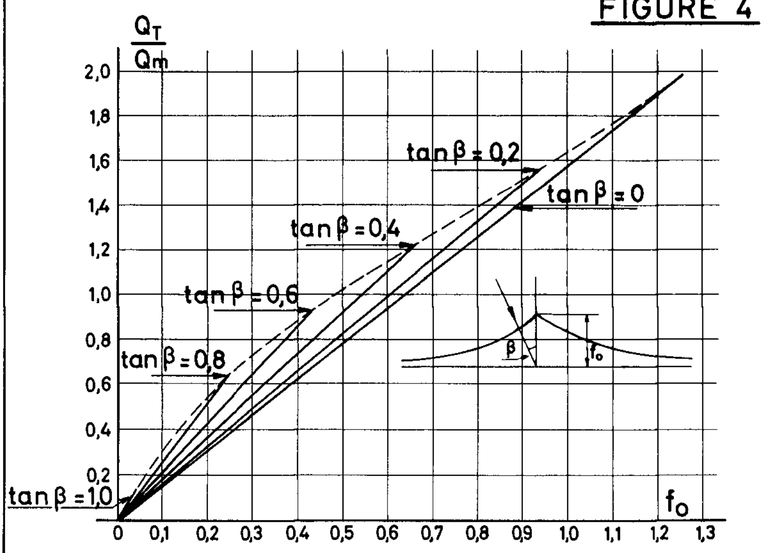


FIGURE 4



coastline two different directions can be found, along which an equal quantity of sediment can be transported by the waves. Since we have found that one of these directions is unstable, this ambiguity is eliminated from the problem.

Secondly, the problem of the delta shape can now be restricted to the cases in which $\tan\beta$ has values between zero and K_2, because a straight coastline is already unstable in itself if $\tan\beta > K_2$.

If $\tan\beta < K_2$, stable solutions can be obtained only if the coastal curves are in accordance with the formula of area I. This solution is an asymptotic one. In a stable delta, therefore, the influence of the river will be always perceptible, at both sides, towards infinite distance.

SHAPE OF THE POINTED DELTA

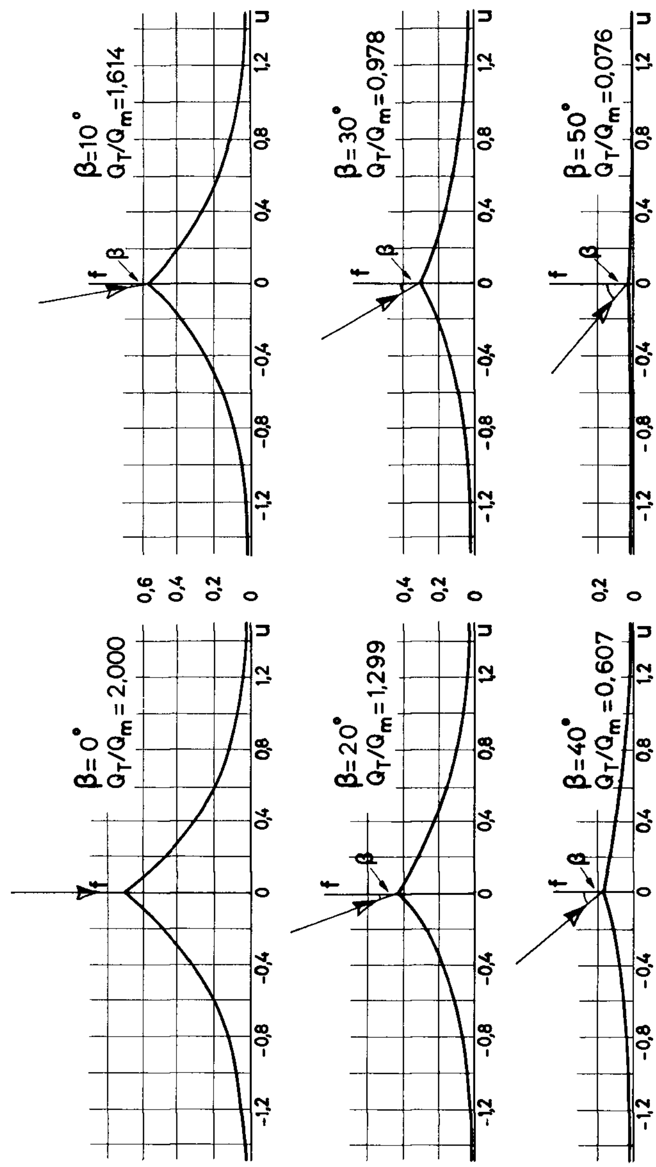
Using the foregoing formulae, we calculated f_o as a function of the quotient $\frac{Qt}{Qm}$ ($f = f_o$ when $u = o$; Q_t is the quantity of sediment, transported by the river) the result of which may be seen from figure 4. In figure 5 we show some basic curves of stable deltas if $\frac{Qt}{Qm}$ has the maximum value associated with distinct values of β. If $\frac{Qt}{Qm}$ exceeds this maximum value, the right hand side of the delta becomes unstable. (Area II).

Presumably, this type of instability means, that in nature spits will occur at the lee side of the delta.

The quantity Qt is related to Qm by the angle between the two tangents in the point of the delta. It can easily be seen, that the ratio $\xi = \frac{Qt}{Qm}$ can never exceed the value 2.

If $\xi > 2$, no solution at all can be obtained from our formula. It seems therefore, that the method in which the problem has been approximated until now, is not quite satisfactory; because we want to know the shape of a delta for the whole range of ξ between zero and infinity. Moreover, our pointed deltas possess the pecularity, that always a distance from the origin to the sea exists, which is shorter than the distance between the origin and the point of the delta. A better approximation of nature may be obtained if we alter some of our basic assumptions.

FIGURE 5



Shape of the pointed delta at maximum values of Q_T/Q_m

THE BEHAVIOUR OF THE RIVER WITHIN THE DELTA

In nature, a river will always try to take the shortest way to the sea. In the long run, and as an average, this will result in a delta with a more or less circular central part. In order to put this behaviour of a river into a mathematically usable shape, we made the following basic assumptions.

1e. The central part of the delta coastline is a circle with its centre in the origin.
2e. The river provides every point of the circle with a quantity q, necessary to maintain the circular coastline.
3e. At both sides the circular coastline is joined by coastal curves of the "pointed"-delta-type, belonging to area I.

From figure 6 it may be seen, that during a time-interval ΔT the circular part of the delta has grown with a quantity:

$$D.R.(\varphi_R + \varphi_L).\Delta R$$

At the right hand side the waves have removed during this time-interval a quantity $K_1 Q_m \tan\varphi_R \Delta T$ and at the left hand side a quantity $K_1 Q_m \tan\varphi_L . \Delta T$.

During the time interval ΔT the river has supplied a quantity : $Q_t . \Delta T$

Obviously: $\Delta T . Q_t = D.R(\varphi_R + \varphi_L)\Delta R + K_1 Q_m(\tan\varphi_R + \tan\varphi_L)\Delta T$

or : $\dfrac{Q_t}{Q_m} = z = \dfrac{D}{Q_m} R.\dfrac{\partial R}{\partial T}(\varphi_R + \varphi_L) + K_1(\tan\varphi_R + \tan\varphi_L)$.

If we put: $R = r\sqrt{t}$, we find $\dfrac{\partial R}{\partial t} = \dfrac{1}{2}\dfrac{r}{\sqrt{t}}$ and $R\dfrac{dR}{\partial T} = \dfrac{1}{2} r^2$.

Thus: $$z = \frac{1}{2} r^2(1+\tan^2\beta)(\varphi_R + \varphi_L) + K_1(\tan\varphi_R + \tan\varphi_L)$$

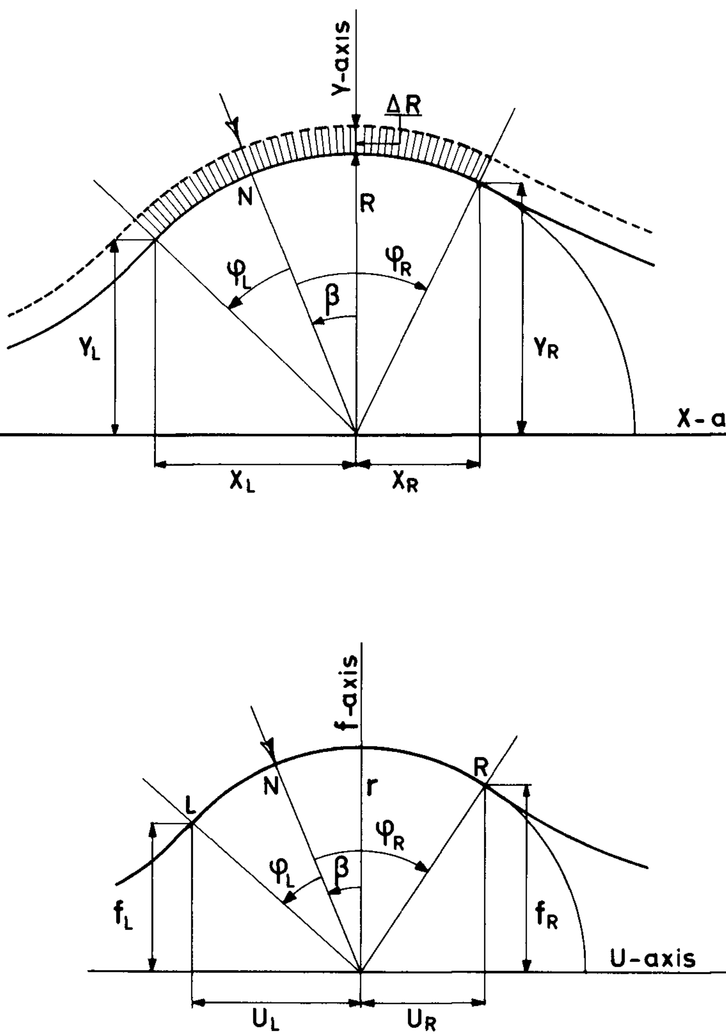
The circular part of the delta is stable if $\tan\varphi_R \leqq K_2$ and $\tan\varphi_L \leqq K_2$. Further conditions are

$$\tan\beta < \tan\varphi_R < K_2 \;\; ; \qquad -\tan\beta < \tan\varphi_L < \frac{1}{\tan\beta}$$

Admitting instability in the circular part of the delta, we see that a coastal curve of area I cannot join the instable circular part tangentially. The condition of joining is

$$K_1 \tan\alpha_L = \frac{K_2}{\tan\varphi_L}.$$

FIGURE 6


Y-axis
ΔR
N
R
φ_L
φ_R
β
Y_L
Y_R
X-axis
X_L
X_R
f-axis
L
N
r
R
φ_R
φ_L
β
f_L
f_R
U-axis
U_L
U_R

or $K_1 \tan \alpha_R = \frac{K_2}{\tan \varphi_R}$.

we find in these cases

$$\Sigma = \frac{1}{2} r^2 (1 + \tan^2 \beta)(\varphi_R + \varphi_L) + K_2 \left(\frac{1}{\tan \varphi_L} + \frac{1}{\tan \varphi_R} \right)$$

At the right hand-side another interesting case exists, in which the unstable delta causes erosion of the coast. In this case (figure 7):

$$\Sigma = \frac{1}{2} r^2 (1 + \tan^2 \beta)(\varphi_L + \pi - \varphi_E) + \frac{K_2}{\tan \varphi_L}$$

RADIAL DISTRIBUTION OF Q_t

If q = the quantity of river sediment per angle unit, continuity considerations lead to:

$$q = \frac{\partial Q}{\partial \varphi} + D.R. \frac{\partial R}{\partial T}$$

in which

$$D.R. \frac{\partial R}{\partial T} = \frac{1}{2} r^2 Q_m (1 + \tan^2 \beta).$$

In area I $(\tan \varphi \leqq K_2)$; $Q = K_1 Q_m \tan \varphi$ and thus:

$$\frac{\partial Q}{\partial \varphi} = K_1 Q_m \frac{1}{\cos^2 \varphi}.$$

In area II $(\tan \varphi > K_2)$; $Q = K_2 Q_m \frac{1}{\tan \varphi}$ and therefore

$$\frac{\partial Q}{\partial \varphi} = -K_2 Q_m \frac{1}{\sin^2 \varphi}.$$

Thus: in area I : $q = K_1 Q_m (1 + \tan^2 \varphi) + \frac{1}{2} r^2 Q_m (1 + \tan^2 \beta)$

in area II: $q = -K_2 Q_m \frac{1 + \tan^2 \varphi}{\tan^2 \varphi} + \frac{1}{2} r^2 Q_m (1 + \tan^2 \beta)$

In the direction OB, with $\tan \varphi = K_2$, we see that

out of area I: $q_b = K_1 Q_m (1 + K_2^2) + \frac{1}{2} r^2 Q_m (1 + \tan^2 \beta)$.

and out of area II: $q_b = -K_2 Q_m \frac{1 + K_2^2}{K_2^2} + \frac{1}{2} r^2 Q_m (1 + \tan^2 \beta)$.

We see that in the direction OB a jump occurs : $2 Q_m \frac{1 + K_2^2}{K_2}$.

(figure 8). This jump has been caused by the discontinuity we introduced in the transport equation. If we choose a continual transport equation no jump occurs. For instance, if we choose

$Q = Q_m \sin 2\varphi$, then

FIGURE 7

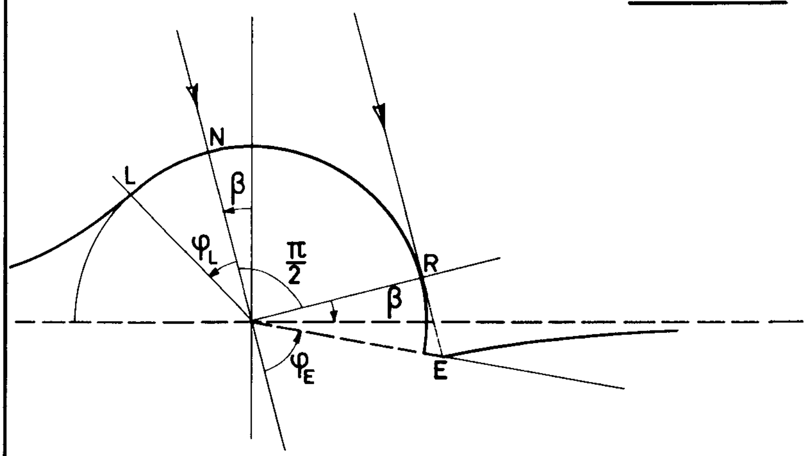
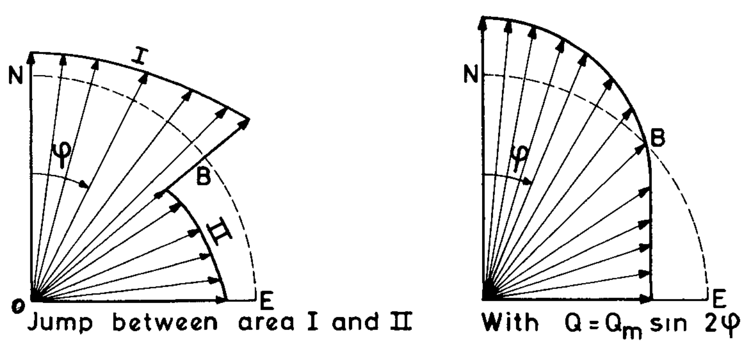


FIGURE 8



RADIAL DISTRIBUTION OF Q_T

$$\frac{\partial q}{\partial \varphi} = 2Q_m \cos 2\varphi \quad \text{and} \quad q = \tfrac{1}{2} r^2 Q_m (1+\tan^2\beta) + 2Q_m \cos 2\varphi \, .$$

being a curve without a jump (figure 8).

With our discontinual transport equation q may grow negative in area II. This occurs at first in the direction OB, where $\tan\varphi = K_2$ and

$$q_b = \tfrac{1}{2} r^2 Q_m (1+\tan^2\beta) - \frac{1+K_2^2}{K_2} \cdot Q_m$$

A negative value of q does not make sense. Thus the condition has to be fullfilled:

$$r^2 > \frac{2(1+K_2^2)}{K_2} \cos^2\beta \, .$$

or

$$r > 2.02 \cos\beta \, .$$

If $q = 0$ at $\tan\varphi = \infty$ (or $r \leqq 1.57 \cos\beta$) q is negative over the whole range of area II.

With the transportfunction: $Q = Q_m \sin 2\varphi$, the minimum value of q lies at $\varphi = \frac{\pi}{2}$ $\quad q_{min} = \tfrac{1}{2} r^2 Q_m (1+\tan^2\beta) - 2Q_m$

from which follows the same condition: $r > 2 \cos\beta$.

Obviously, a minimum value of r exists in these problems. Roughly we may put the condition:

$$r > 2 \cos\beta \, .$$

DELTAS WITH PARTLY CIRCULAR COASTLINE

a) Left hand side

There are two cases:

- a1: $\tan\varphi_L \leqq K_2$; stable solution
- a2: $\tan\varphi_L > K_2$. ; the circular part of the delta is partly unstable.

Case a1.

The infinity condition provides: $2\sqrt{K_1} = -A\sqrt{\pi}\,(a-1)$ - - - (1)

Point L provides:

$$2\sqrt{K_1}\, v_L = u_L + f_L \tan\beta \quad \text{- - - - - (2)}$$

$$u_L = -A\left[e^{-v_L^2} + v_L\sqrt{\pi}\,\{E(v_L)+a\}\right] \quad \text{- - - (3)}$$

$$\tan(\beta+\varphi_L) = -\frac{1}{\tan\beta}\left[\frac{2\sqrt{K_1}}{A\sqrt{\pi}\{E(v_L)+a\}} + 1\right] \quad \text{- - - - (4)}$$

$$\tan(\beta+\varphi_L) = -\frac{u_L}{f_L} \quad \text{- - - - - - - (5)}$$

$$u_L^2 + f_L^2 = r^2 \quad \text{- - - - - - (6)}$$

Eliminating A, a, u_L and f_L from these equations we obtain:

$$\frac{e^{-(-v_L)^2}}{(-v_L)\sqrt{\pi}\{1-E(-v_L)\}} = \frac{1+\tan^2\varphi_L}{\tan\varphi_L(\tan\varphi_L+\tan\beta)} \text{ ------ I}$$

and $$r^2(1+\tan^2\beta) = 4K_1\frac{1+\tan^2\varphi_L}{\tan^2\varphi_L}v_L^2 \text{ ----- II}$$

We find r as a function of φ_L by eliminating v_L along a graphical way from I and II. The result is shown in figure 9.

Case a2.

In the same way we find

$$\frac{e^{-(-v_L)^2}}{(-v_L)\sqrt{\pi}\{1-E(-v_L)\}} = \frac{K_2^2+1}{K_2^2+\tan\varphi_L\cdot\tan\beta} \text{ ------ I}$$

and $$r^2(1+\tan^2\beta) = 4K_1\frac{1+\tan^2\varphi_L}{\tan^2\varphi_L}, v_L^2 \text{ ----- II}$$

from which v_L has to be eliminated in order to find r as a function of φ_L. The result is shown in figure 9.

b) Right hand side.

There are three cases:

b1:	$\tan\varphi_R \leq K_2$	; stable delta
b2:	$\tan\varphi_R > K_2$	; the circular part of the delta is partly unstable;
b3:	$\tan\varphi_R = \infty$	; erosion at the lee side (fig. 7); the circular part of the delta is partly unstable.

Case b1.

In the same way as before we find the two equations

$$\frac{e^{-v_R^2}}{v_R\sqrt{\pi}\{1-E(v_R)\}} = \frac{1+\tan^2\varphi_R}{\tan\varphi_R(\tan\varphi_R-\tan\beta)} \text{ ---- I}$$

and $$r^2(1+\tan^2\beta) = 4K_1\frac{1+\tan^2\varphi_R}{\tan^2\varphi_R}v_R^2 \text{ ---- II}$$

from which V_R has to be eliminated in order to find r as a function of φ_R. The resulting curves are shown in figure 9.

Case b2.

We find:

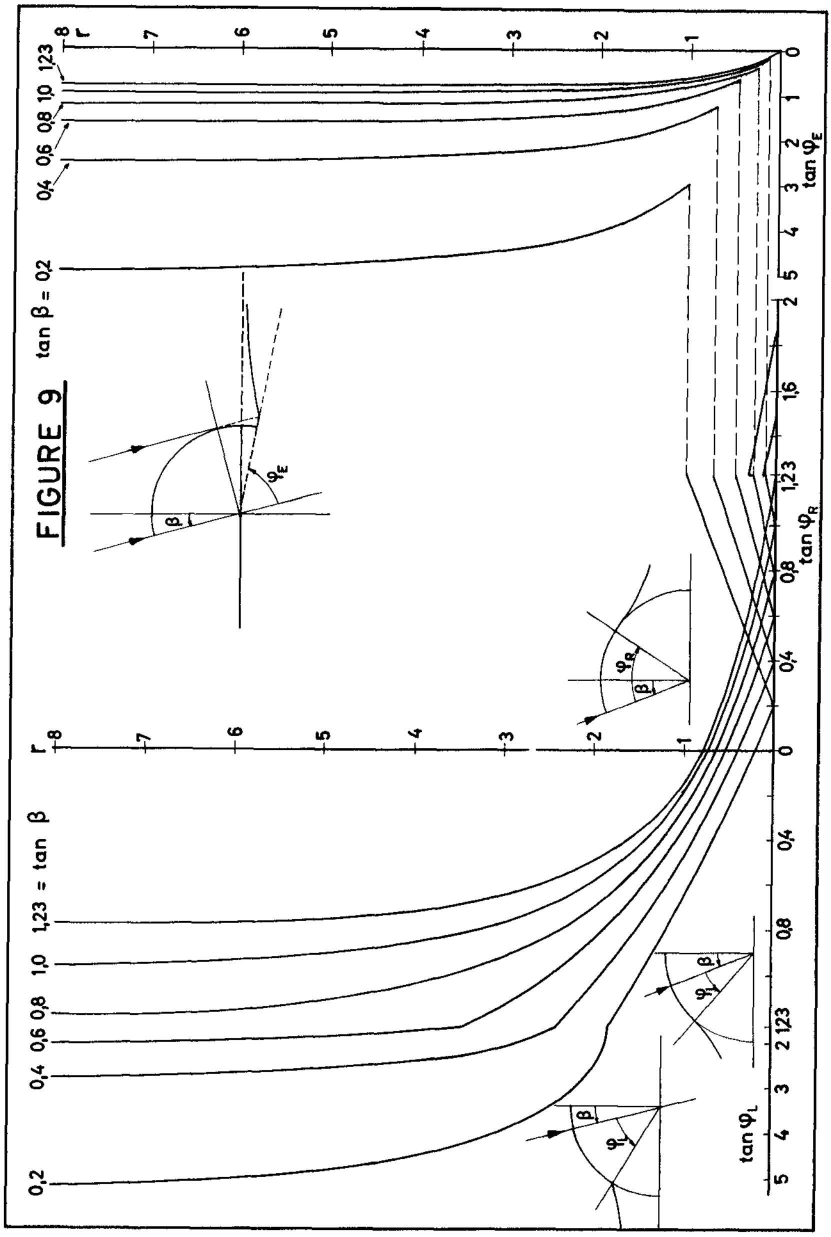


FIGURE 9
tan β = 0,2
0,4 0,6 0,8 1,0 1,23
1,23 = tan β
0,2 0,4 0,6 0,8 1,0
r
tan φ_E
tan φ_R
tan φ_L

$$\frac{e^{-v_K^2}}{v_K\sqrt{\pi}\{1-E(v_K)\}} = \frac{K_2^2+1}{K_2^2-\tan\varphi_K\cdot\tan\beta} \quad \text{-----} \quad \mathrm{I}$$

$$r^2(1+\tan^2\beta) = 4K_1\,\frac{1+\tan^2\varphi_K}{\tan^2\varphi_K}\cdot v_K^2 \quad \text{----} \quad \mathrm{II}$$

If we eliminate V_R from I and II, we find in this case, that always $r < 2\cos\beta$. Moreover, we find, that r decreases with increasing values of φ_K. This case, therefore, has to be rejected.

Case b3.

In a similar way we find here:

$$\frac{e^{-v_p^2}}{v_p\sqrt{\pi}\{1-E(v_p)\}} = \frac{1}{\tan\varphi_E\cdot\tan\beta} \quad \text{---} \quad \mathrm{I}$$

$$r = 2\sqrt{K_1}\cdot v_p\cdot\cos\beta \quad \text{------} \quad \mathrm{II}$$

Eliminating graphically V_p from both equations, we find r as a function of $\tan \varphi_E$. The result also is shown in figure 9.

The minimum values of r ($r_{min} = 2\cos\beta$) in this case b3 are higher than the maximum values of r, in the case b1. Therefore, if we maintain our condition: $r \geqq 2\cos\beta$, a gap will occur between b1 and b3. However, if we accept a certain disorder in the internal distribution of the river output, we can avoid this gap by calculating in case b3 the values of r between $r = 2\cos\beta$ and the maximum values of case b1.

OUTPUT OF THE RIVER

If we divide the total output of the river: Q_T into two parts, Q_L and Q_R, we are able to calculate r as a function of Σ from the curves of figure 9 by means of the following formulae:

$$\Sigma = \frac{q_T}{q_m} = \frac{q_L}{q_m} + \frac{q_R}{q_m}.$$

case a1: $\frac{q_L}{q_m} = \frac{1}{2} r^2 (1+\tan^2\beta) \cdot \varphi_L + K_1 \tan \varphi_L$.

case a2: $\frac{q_L}{q_m} = \frac{1}{2} r^2 (1+\tan^2\beta) \varphi_L + \frac{K_2}{\tan \varphi_L}$.

case b1: $\frac{q_R}{q_m} = \frac{1}{2} r^2 (1+\tan^2\beta) \varphi_R + K_1 \tan \varphi_R$.

case b3: $\frac{q_R}{q_m} = \frac{1}{2} r^2 (1+\tan^2\beta)(\pi - \varphi_E)$.

Our figure 10 shows r as a function of Σ for some values of $\tan\beta$.

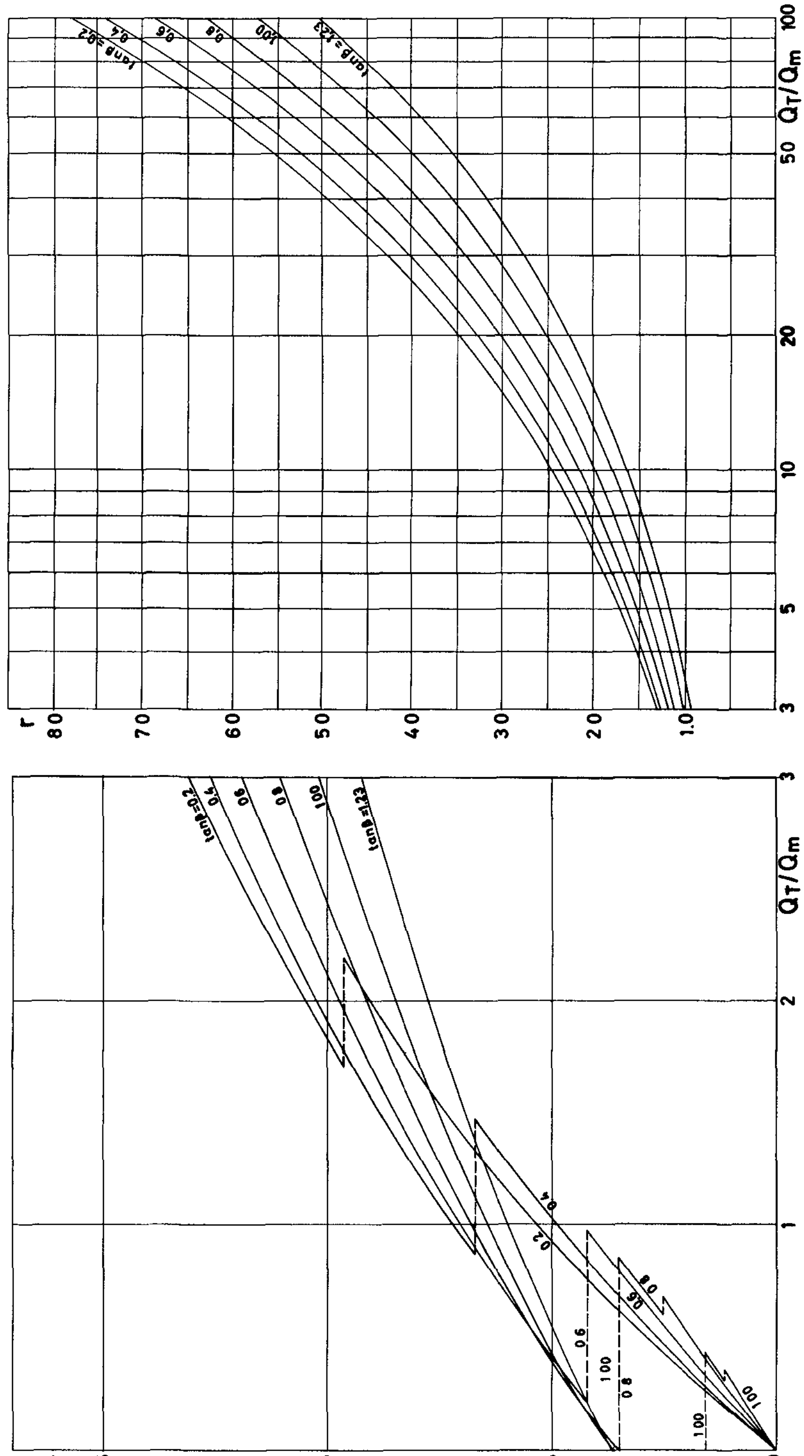
We can use the graphs of the figures 10 and 9 in the following way.

Supposing, that the values of Σ and β are given, we find the value of r from figure 10. From figure 9 we find, which values of φ_L and φ_R (resp φ_E) belong to this value of r. Also we can find out from figure 9, whether, at the left hand side, case a1 or case a2 occurs and, whether, case b1 or case b3 exists on the right hand side.

CONCLUSIONS

1. We considered the theoretical shape of a partly circular river delta with $0 < \tan\beta \leqq 1.23$. If $\tan\beta > 1.23$ a straight coastline is unstable already in itself and a stable solution for the delta shape cannot be found.

FIGURE 10



2. With $0 < \tan\beta \leqq 1.23$ we found, that a stable delta is possible only if $\xi \leqq \xi_1$, in which ξ_1 depends on β . To ξ_1 a distinct maximum value r_1 is associated.
3. If $\xi > \xi_1$, and $r > r_1$, the circular part of the delta becomes partly unstable and the internal distribution of river sediment becomes improbable. Erosion on the right hand coast occurs.
4. If $\xi > \xi_2 > \xi_1$ the improbability of the internal sediment distribution comes to an end, but the circular part of the delta remains partly unstable and the erosion of the coast at the right hand side of the delta goes on.
5. Instability along the circular part of the delta (all other parts are always stable) probably means, that "spits" occur. The shape of spits has not been dealt with in this paper.
6. Speaking in common language, we deduced that if the quantity of river sediment exceeds a certain amount, the delta becomes partly unstable and erosion on the lee side of the delta begins; the delta grows so large, that it is acting as a partly circular "groin" with erosion on its lee side and accretion on the weather side.

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