Chapter 6

WATER WAVE EQUIVALENT OF MACH-REFLECTION

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SUMMARY

Periodic (shallow water and transitional water) and solitary water gravity waves do not reflect from a wall in the manner commonly supposed, when the angle between the direction of wave advance and the wall is less than about 35 to 45 degrees. The wave front bends near the wall, becoming normal to the wall, with a small reflected wave. For angles less than about 20 degrees the reflected wave becomes almost negligible. The portion of the wave near the wall (called the Mach-stem in air blast waves) increases in height as the wave continues to move along the wall. Once the Mach-stem is formed, it will continue to grow even when the wall is bent around through almost 90 degrees; for periodic waves a Mach-reflected wave also develops. The Mach-stem is insensitive to undulation of the wall. Results of studies of this phenomenon in the laboratory are presented, together with some observation of its occurrence in the ocean. The importance of this phenomenon to the study of tsunami action at Hilo, Hawaii, is presented.

INTRODUCTION

On occasions, tsunamis in certain coastal regions have exhibited characteristics which do not seem to be accounted for when they are studied by means of the commonly used types of refraction drawings, diffraction calculations and run-up theory and measurements. It was suggested to the author by Professor John D. Isaacs in 1956 that this might be due to something analogous to the Mach-reflection phenomenon in acoustics. A number of aspects of this phenomenon have been investigated experimentally by several graduate students under the author's direction (Perroud, 1957; Chen, 1961; Sigurdsson and Wiegel, 1962; Nielsen, 1962). The first tests were made using a solitary wave, as it was believed that this was most nearly the water wave equivalent of a shock wave in compressible flow.

The first experiments were performed with a solitary wave incident to an oblique, vertical, impervious, smooth barrier; it was found that the phenomenon did exist. Later, tests were performed with periodic waves, and the Mach-reflection also occurred for these for values of $L/d >$ about 2 (where L is the wave length and d is the water depth). Several experiments were performed with types of models which were of importance to coastal engineering problems. One of these was an undulating vertical wall, which was found to affect only the small details of the wave motion. A second model consisted of a curved, vertical impervious barrier (similar to curved vertical wall jetties or breakwaters). Experiments in the laboratory showed that once the Mach-stem formed, it became so strong that it ultimately became independent of the incident wave.

Finally, experiments were made with an undistorted model of the bay at Hilo, Hawaii, and a phenomenon was found to exist which had the appearance of a Mach-reflection.

The only theory available on the Mach-reflection, to the author's knowledge, is due to Lighthill (1949), and it is not useful in its present form in the solution of water gravity wave problems.

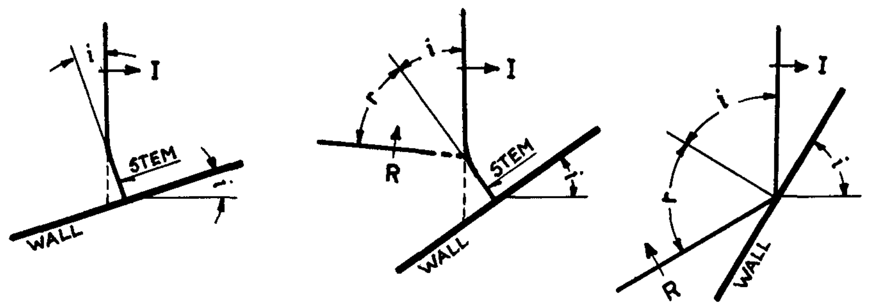
MACH-REFLECTION, SOLITARY WAVE

It is usually assumed that a wave which encounters an obstacle will either reflect from it, be dissipated, or both. In certain cases, however, a third possibility exists: the crest of the wave near the wall may bend, becoming normal to it, with the energy density adjacent to the wall increasing as the wave moves along. This phenomenon is known as a Mach-reflection.

Perroud (1957) made a series of tests in the laboratory of the reflection of a solitary wave from a straight, vertical, impervious wall. He found that three types of patterns occurred, with one of the patterns being a special case of one of the others. The critical angle of incidence, i, separating these two types of patterns appeared to be 45 degrees (Chen, 1961, found the critical angle to be between 35 and 40 degrees).

For incident angles (the angle between the direction of wave advance and the wall) greater than 35 to 45 degrees the reflection pattern is "normal" (Fig. 1). The incident and reflected waves are slightly disturbed near the wall, but the angle of reflection is equal to the angle of incidence, and the reflected wave height is only slightly less than the incident wave height. However, the reflected wave is followed by a trough, except for the case in which the angle of incidence is 90 degrees. It is interesting to note that in the latter case the wave height at the wall was 20 percent higher than twice the incident wave height.

For angles of incidence less than about 35 to 45 degrees the reflection appears to be of the type called a Mach-reflection in acoustics. When the angles of incidence are less than about 20 degrees, the wave crest bends, becoming normal to the wall, and no reflected wave appears. When the angle of incidence is greater than 20 degrees but less than 40 to 45 degrees, three waves are present, the incident wave I, a Mach-reflected wave R, and the Mach-stem wave M, the width of which grows as the wave moves along the wall. The angle δ ($\delta = r - i$) depends upon the angle of incidence in the manner shown in Fig. 2. The reflected wave height is smaller than the incident wave height, and the angle of reflection (r) is greater than the angle of incidence. The height of the Mach-stem portion is greater than the incident wave height and is at its maximum at the wall. The Mach-reflected and Mach-stem waves are followed by a trough.



(A) i < 20° Mach-stem without reflection

(B) 20°< i < 45° Mach-stem with reflection

(C) i > 45° Regular reflection

FIG 1. REFLECTION OF A SOLITARY WAVE WITH VERTICAL BREAKWATER.

I : Incident wave
R : Reflected wave

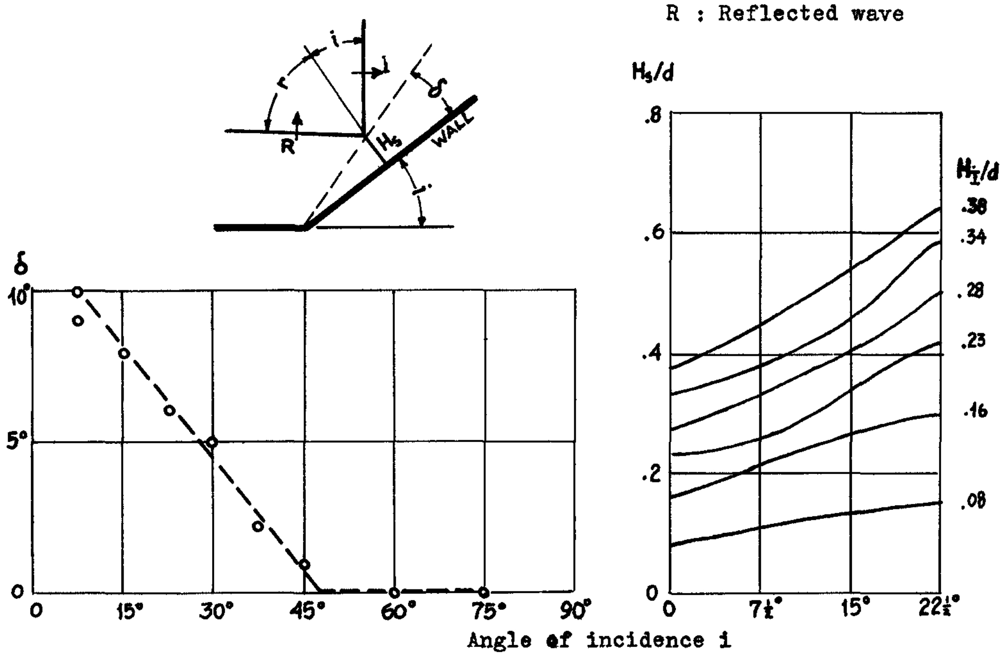


FIG 2a. STEM-ANGLE δ FOR SOLITARY WAVE. EACH PLOT IS AN AVERAGE VALUE FOR 6 DIFFERENT WAVE HEIGHTS. (H_i/d: .05-.43). Water depth d = 0.132 ft.

FIG 2b. HEIGHT OF STEM-WAVE FOR SOLITARY WAVE. Water depth d = 0.132 ft

(From Perroud, 1957)

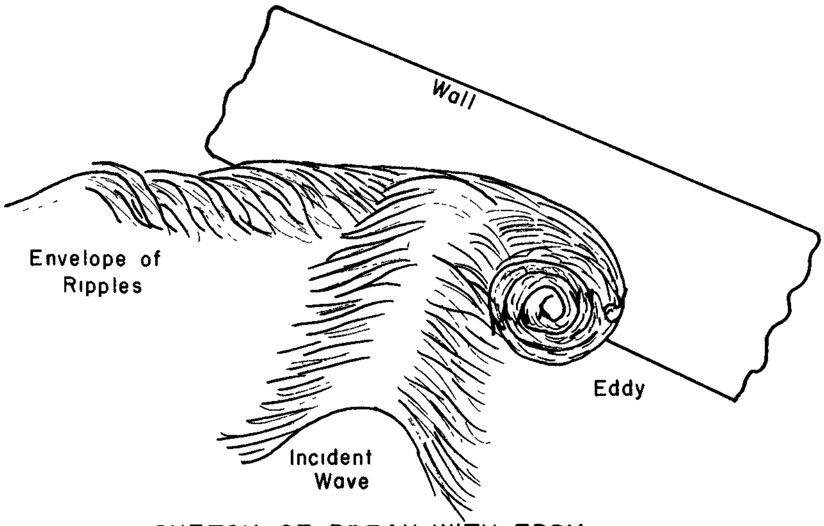
In practice, most structures and coastal areas have slopes. Experiments made with smooth impermeable slopes (Chen, 1961) showed that for nearly vertical walls the phenomenon looked the same as for the case of a vertical wall (β = 90 degrees). As the wall slope (β) was decreased a large horizontal eddy formed over the slope, and as the slope was decreased further, the wave broke along the slope (Fig. 3); the slope angle at which this occurred was found to be dependent upon the angle of incidence. The values of i and β which determine whether or not the wave breaks over the slope are shown in Fig. 4, for a specific value of H/d. For small values of H/d (where H is the incident wave height and d is the water depth) no break occurred when $\beta \approx$ 90 degrees. For intermediate values of H/d it was not possible to determine whether or not a wave would break because of the small size of the tank. In regard to this, Friedlander (1946) has shown theoretically for a sound pulse incident to a wedge (less than 90 degree wedge) that the pulse has to travel a considerable distance along the wedge before the pressure at the wedge builds up to its maximum value of twice the pressure of the incidence pulse, and that the smaller the wedge angle (hence, the smaller i) the greater this distance will be. For example, for a 30-degree wedge the pulse must travel nearly 60 pulse lengths to build up to about 90 percent of its final value.

Waves incident to an overhanging wall (β > 90 degrees) behaved in a manner similar to a vertical wall except that the stem did not grow with distance from the start as was found by Perroud for the vertical wall. At least, the width of the Mach-stem did not appear to grow within the limits of the experimental facilities.

The region of i and β for which various types of reflections occur are shown in Fig. 5.

The author (Wiegel, 1963) has observed ocean waves with the appearance of a Mach-stem occurring along a curved structure, and this led to another series of laboratory studies. A vertical wall breakwater was made of a piece of sheet metal. A 2 ft long straight section, starting at one wall of the tank, was placed at a 12 degree angle with the wall (Sigurdsson and Wiegel, 1962). This straight section was connected tangentially to a 66 degree segment of a circle of 3.33 ft radius, to make a total change in direction of 78 degrees (Fig. 6). Tests made with a barrier with a smooth wavy surface (corrugated aluminum, with corrugations 0.021 ft deep by 0.135 ft long, about 1/2 and 3 times the incident wave height, respectively) and with a rough barrier showed similar results (Fig. 7). It was observed that the amplitudes at the barrier were greater in the hollows than in the ridges of the corrugations.

The essential feature that was determined from the series of tests just described is that once the Mach-reflection starts, with an angle of incidence less than the required value, the non-linear Mach-stem becomes so strong that it continues to move around the curved barrier, normal to the barrier, and near the end of the barrier becomes independent of the incident wave. In this region there were essentially two separate waves, the "incident wave" and the Mach-stem advancing at an appreciable angle (up to 90 degrees) to the "incident wave." These two waves were connected



SKETCH OF BREAK WITH EDDY
FIGURE 3a

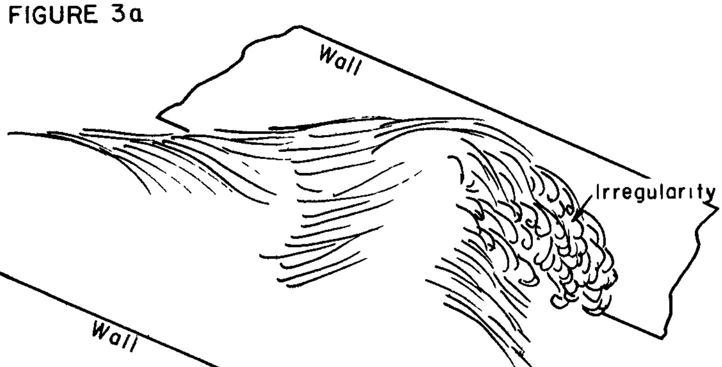
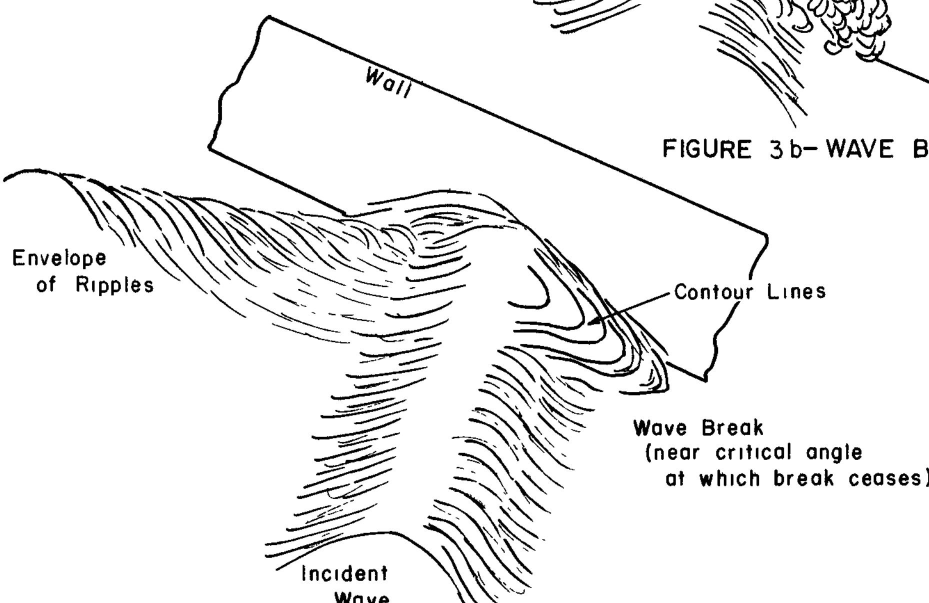


FIGURE 3b– WAVE BREAK



SKETCH OF BREAK WITHOUT EDDY
FIGURE 3c

(After Chen, 1961)

FIGURE 3 a,b,c

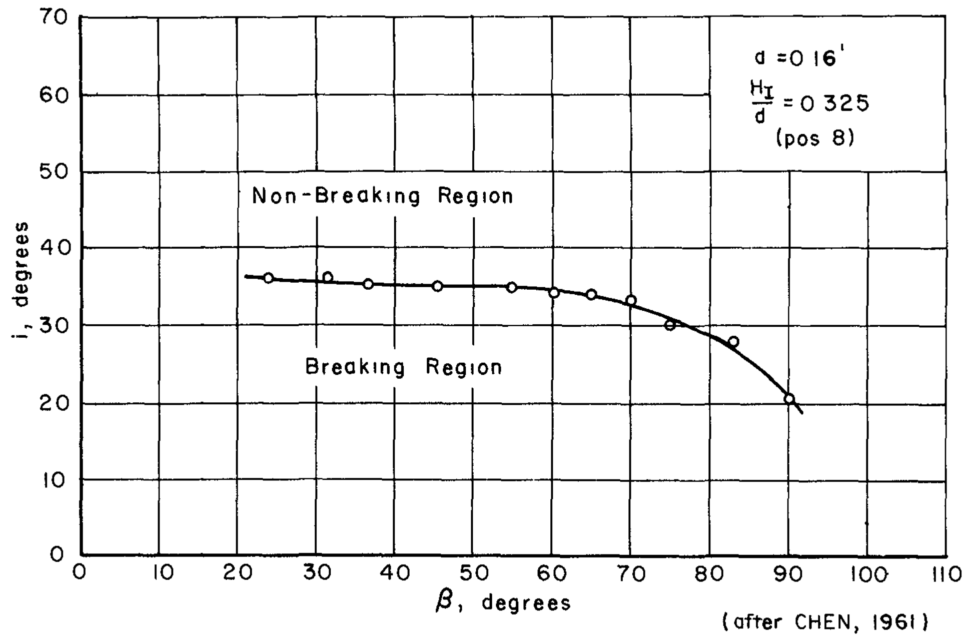


FIGURE 4 ANGLE OF INCIDENCE SEPARATING BREAKING AND NON-BREAKING REGIONS

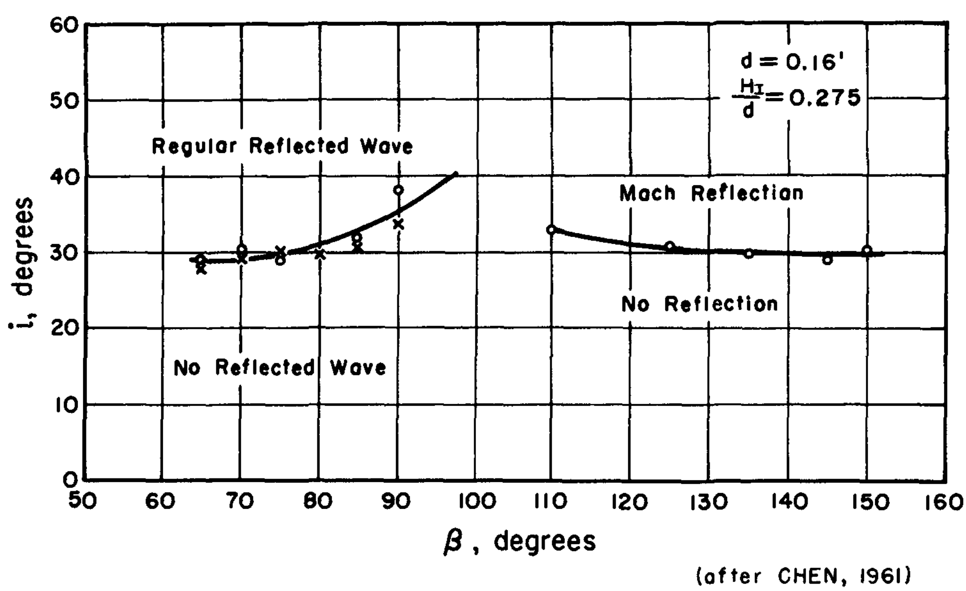


FIGURE 5 INCIDENT ANGLE i BEYOND WHICH REGULAR REFLECTED WAVE OCCURS FOR DIFFERENT SLOPES β, AND i AT WHICH MACH REFLECTION STARTS FOR EACH NEGATIVE SLOPE ($\beta > 90°$).

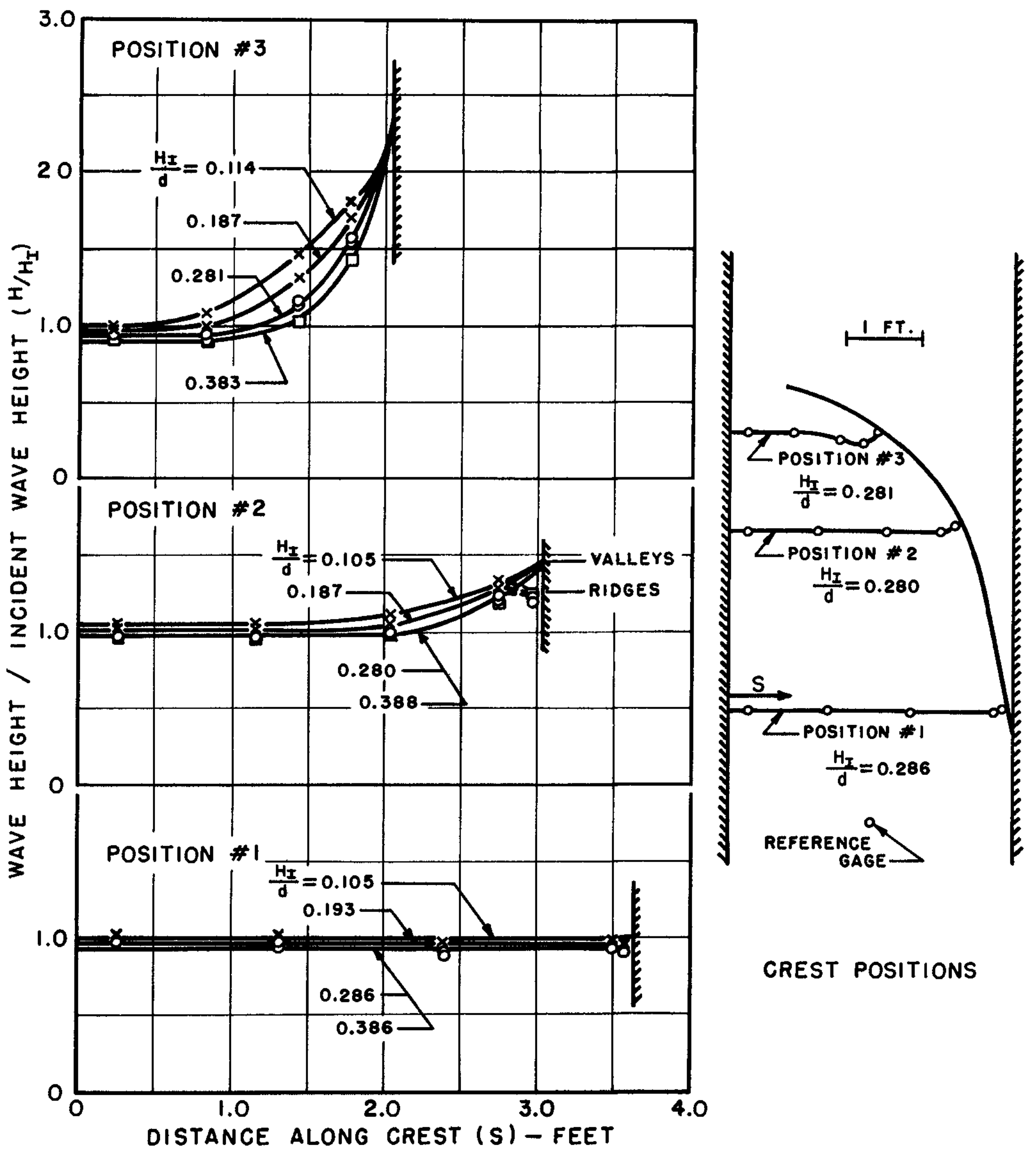
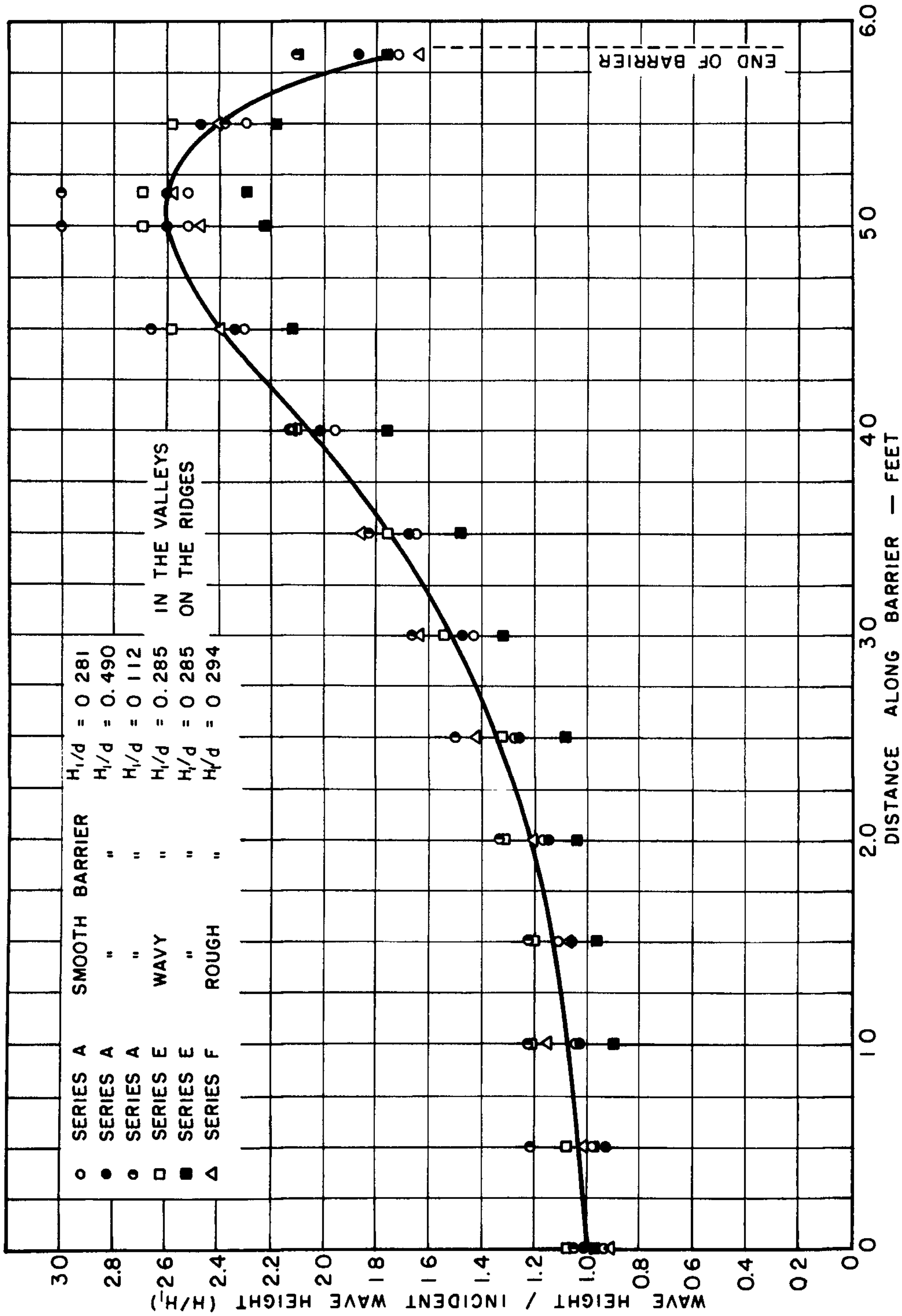


FIGURE 6. CREST POSITION AND CREST ELEVATION SERIES E

(From Sigurdsson and Wiegel, 1962)



MAXIMUM WAVE ELEVATION AT VERTICAL BARRIERS

(from Sigurdsson and Wiegel, 1962)

FIGURE 7

by a transition zone, and in this zone the incident and stem waves were superimposed, and there was a considerable variation in the profile.

When a barrier was made of 3/8 inch gravel to the same plan as the impervious barrier, but with a 36 degree side slope, there was no indication of either a build up of wave amplitude at the barrier, or of a reflected wave.

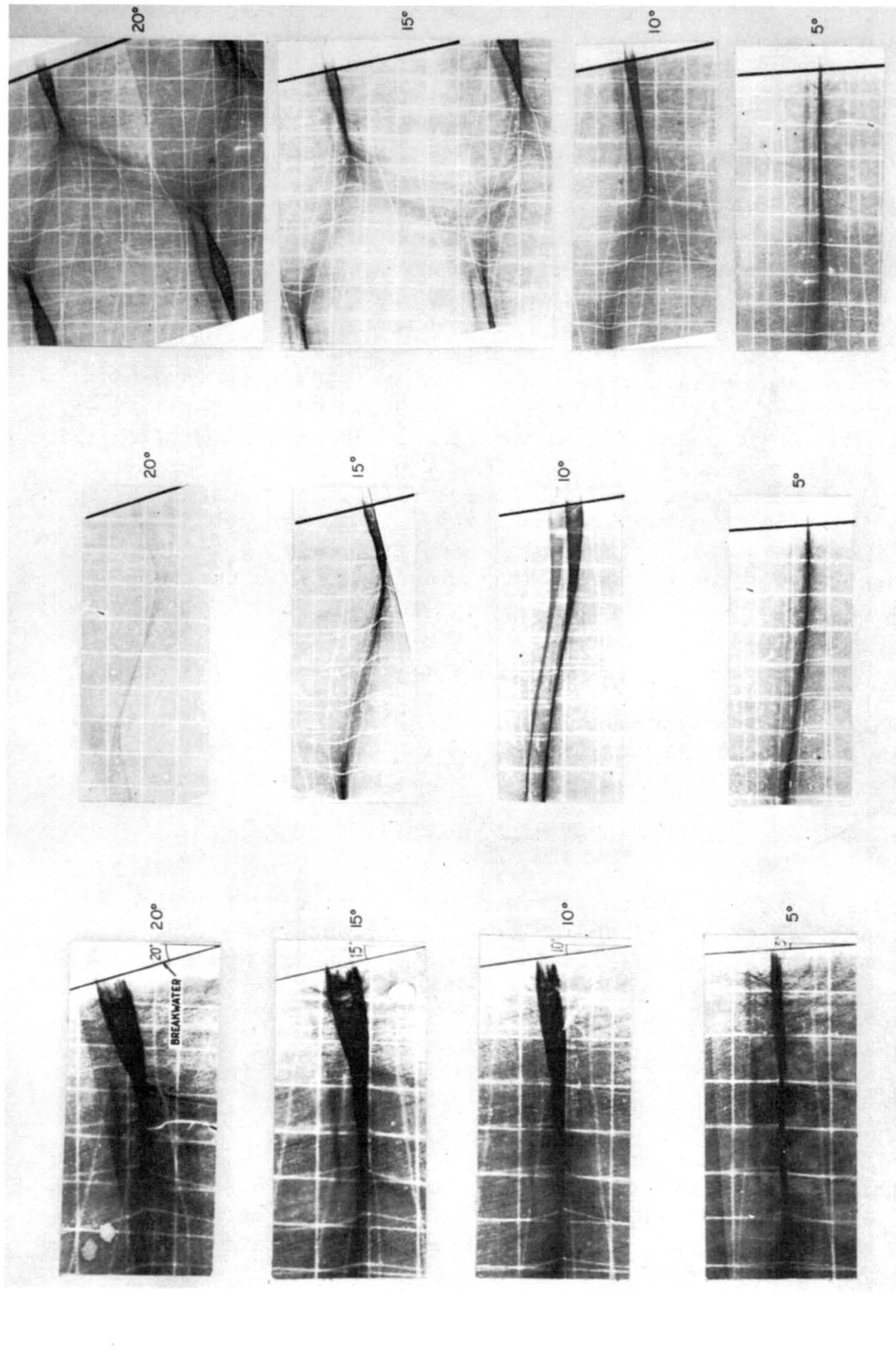
MACH-REFLECTION, PERIODIC WAVES

The experiments described so far were for solitary waves, as the solitary wave is analogous to the single pressure impulse in acoustic waves in air. The phenomenon had been observed for periodic waves in some coastal areas, so it was known that it did exist in actual conditions for nearly periodic waves. In order to see whether or not it existed for deep water as well as for shallow water waves, a program of laboratory studies was undertaken (Nielsen, 1962).

The first series of tests were made in the same tank as the one used in the various tests described in the previous section. Most of the tests were made using the same water depth, 0.160 ft, but some tests were made in deeper water, 0.240 ft. It was found that for shallow water waves a Mach-stem formed which was similar to the one formed by the solitary wave. This was true even for waves in the transitional region, as can be seen in Fig. 8a in which the phenomenon is shown for an L/d of 4.9 for angles of incidence of 5, 10, 15, and 20 degrees. In these shadow photographs the dark lines are the wave crest, and the wider the "line" the higher the wave. In deep water a type of Mach-stem forms, but it occurs to the rear of the incident wave and is connected to the incident wave by a crest with a compound curve, as can be seen in Fig. 8b. In addition, a curved Mach-reflected type of wave also occurs which is similar in appearance to the Mach-reflected wave of a solitary wave when 20 degrees $< i <$ 35 to 45 degrees. It is different in that it locks in with the intersection of the Mach-stem and incident wave of the following wave. For deeper water waves, $L/d < 2$, it appears that the reflected wave gradually develops into something similar to a normal type of reflection pattern, except that there appears to be a small Mach-stem adjacent to the barrier. For example, as can be seen in Fig. 8c for $i >$ 5 to 10 degrees, the more or less regular reflection pattern seems to be emerging, while for $i = 5$ degrees only the Mach-stem occurs.

It was found that the width of the Mach-stem increased as the wave moved along the barrier, as was the case for the solitary wave. It was found that the longer the wave length the greater the width for a given distance along the barrier; however, it was found that the solitary wave had the minimum width for a given distance, which is quite surprising as one normally thinks of a solitary wave having a long "length."

The ratio of wave height of the stem at the barrier to the incident wave height, H/H_I, was a complicated function of distance along the barrier, water depth and wave length. In general, for a given angle of incidence it increased up to a certain distance along the barrier, and



a. T = 0.43 SEC, L = 0.78 FT., d = 0.16 FT. L/d = 4.9

b. T = 0.33 SEC, L = 0.55 FT., d = 0.24 FT. L/d = 2.3

c. T = 0.30 SEC, L = 0.48 FT., d = 0.24 FT. L/d = 2.0

FIGURE 8. REFLECTION FROM VERTICAL STRAIGHT IMPERVIOUS WALL (after NIELSEN, 1963)

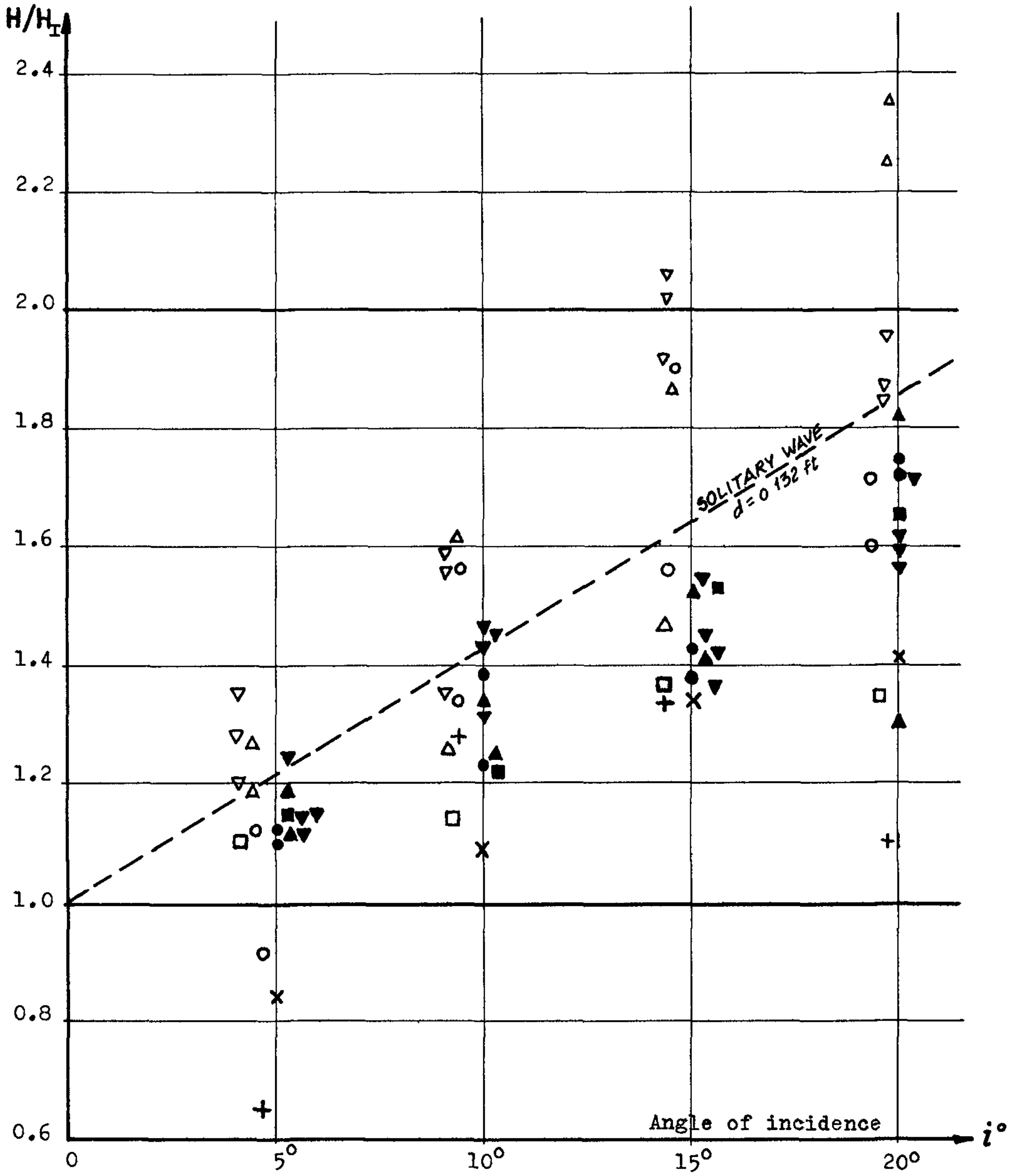
then remained nearly constant, and in some instances started to decrease. The effect of the angle of incidence upon H/H_I was similar to the case for the solitary wave, as can be seen in Fig. 9.

In order to study the phenomenon near the end of a curved breakwater, it was necessary to use a larger facility so that there would be no reflections from the wall opposite the barrier until a large number of waves could be measured. These experiments were made in a 150 ft long by 64 ft wide by 2 1/2 ft deep model basin. A vertical impervious barrier was constructed similar to the one described in a previous section, but about five times as large, as can be seen in Fig. 10. A photograph of the waves that were observed is given in Fig. 11.

The results were similar to the results obtained through the use of the small model basin. Color motion pictures were taken (16 mm), and they were studied thoroughly. Use of these motion pictures, together with the other types of results, permitted Nielsen to determine the details of the motion of a system of periodic waves moving along a curved barrier. His conclusions, in graphical form, are given in Fig. 12. The Mach-stem forms on the straight portion of the barrier (A), dropping behind the wave crest (which is different from the case of the solitary wave). As it moves along the curved section, a Mach-reflected wave also forms (B). As the wave moves still farther along the curve, the Mach-stem and Mach-reflected waves seem to coalesce (C), and then separate from the barrier as a free wave (D) moving nearly normally to the direction of the incident wave. At this stage the free wave is independent of the incident wave. The effect of this transition from one type of wave to another on the wave height along the barrier can be seen in Fig. 13. The amplitude increases in height gradually as it moves along the barrier, and then increases rather rapidly as the Mach-stem and Mach-reflected waves coalesce. Then, as the wave "separates" from the barrier the height at the barrier decreases rapidly, although the height a short distance from the barrier is still high.

TSUNAMI AT HILO, HAWAII

Hilo, Hawaii, is subject to severe damage from tsunamis which originate in the vicinity of Alaska and Chile. The orientation, topography and hydrography of the region (Fig. 14) are such that it appeared likely that a Mach-stem might have been associated with the 1 April 1946 tsunami which originated in the Aleutian Islands of Alaska. The height of the tsunami should increase by a factor of 4 to 5 as it moves onto the shallow portion of the reef off Hilo, due to shoaling effects alone, and it was believed that this, together with the Mach-stem effect, could account for the characteristics of several of the waves of the tsunami as they were observed. It was observed (M. L. Child of Hilo, Hawaii) that the two waves that did the most damage came in as a bore in a southerly direction along the cliff that forms the west border of the bay, swinging easterly and running up through the streets of the town and into the lee area of the breakwater, and waves also came over the top of the breakwater. A photograph taken at the time shows a wave that looks remarkably



LEGEND:

Wave length L ft	Period T sec	Water depth d ft	Distance X along breakwater 2½ ft	5½ ft
0.45	.30	.160	×	+
0.68	.38	.160	●	○
1.20	.58	.160	▲	△
1.46	.58	.240	▼	▽
1.50	.71	.160	■	□

FIGURE 9 INCREASE OF WAVE HEIGHT WITH ANGLE OF INCIDENCE.

H_I/d varies from 0.09 to 0.19

(from Nielsen, 1962)

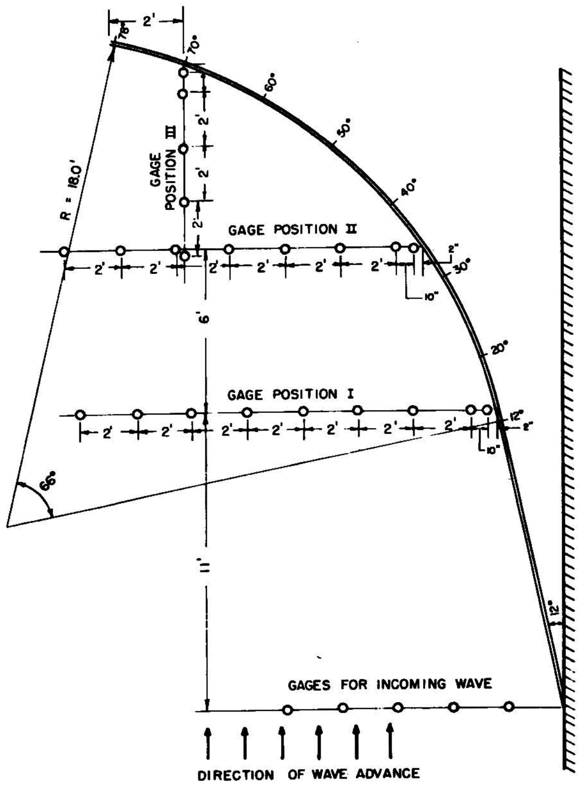


FIGURE 10 EXPERIMENTAL SET-UP IN THE MODEL BASIN IN RICHMOND

FIGURE 11 WAVE PATTERNS AT CURVED SECTION OF BREAKWATER. $L/d = 6.3$, $H_I/d = 0.25$, $d = 1.0$ ft.

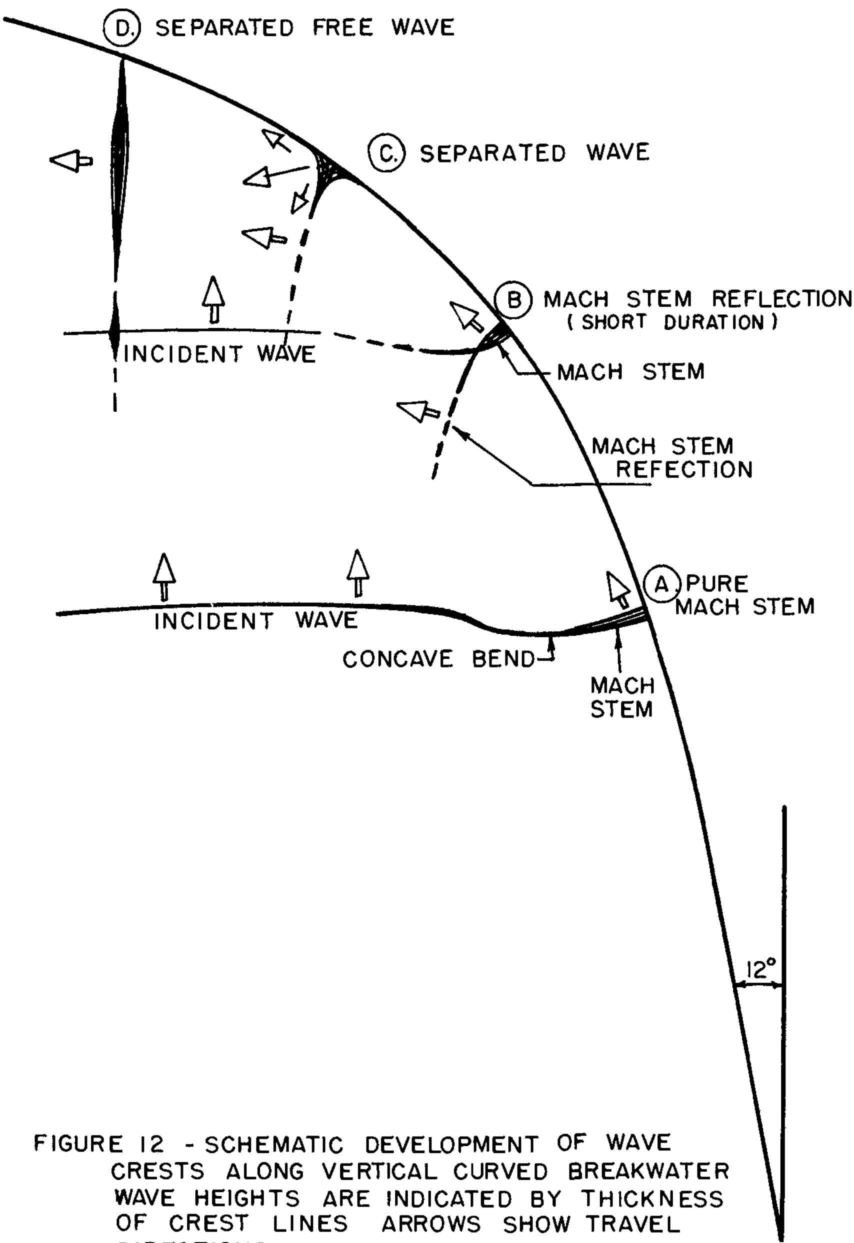


FIGURE 12 - SCHEMATIC DEVELOPMENT OF WAVE CRESTS ALONG VERTICAL CURVED BREAKWATER WAVE HEIGHTS ARE INDICATED BY THICKNESS OF CREST LINES ARROWS SHOW TRAVEL DIRECTIONS (from Nielsen, 1962)

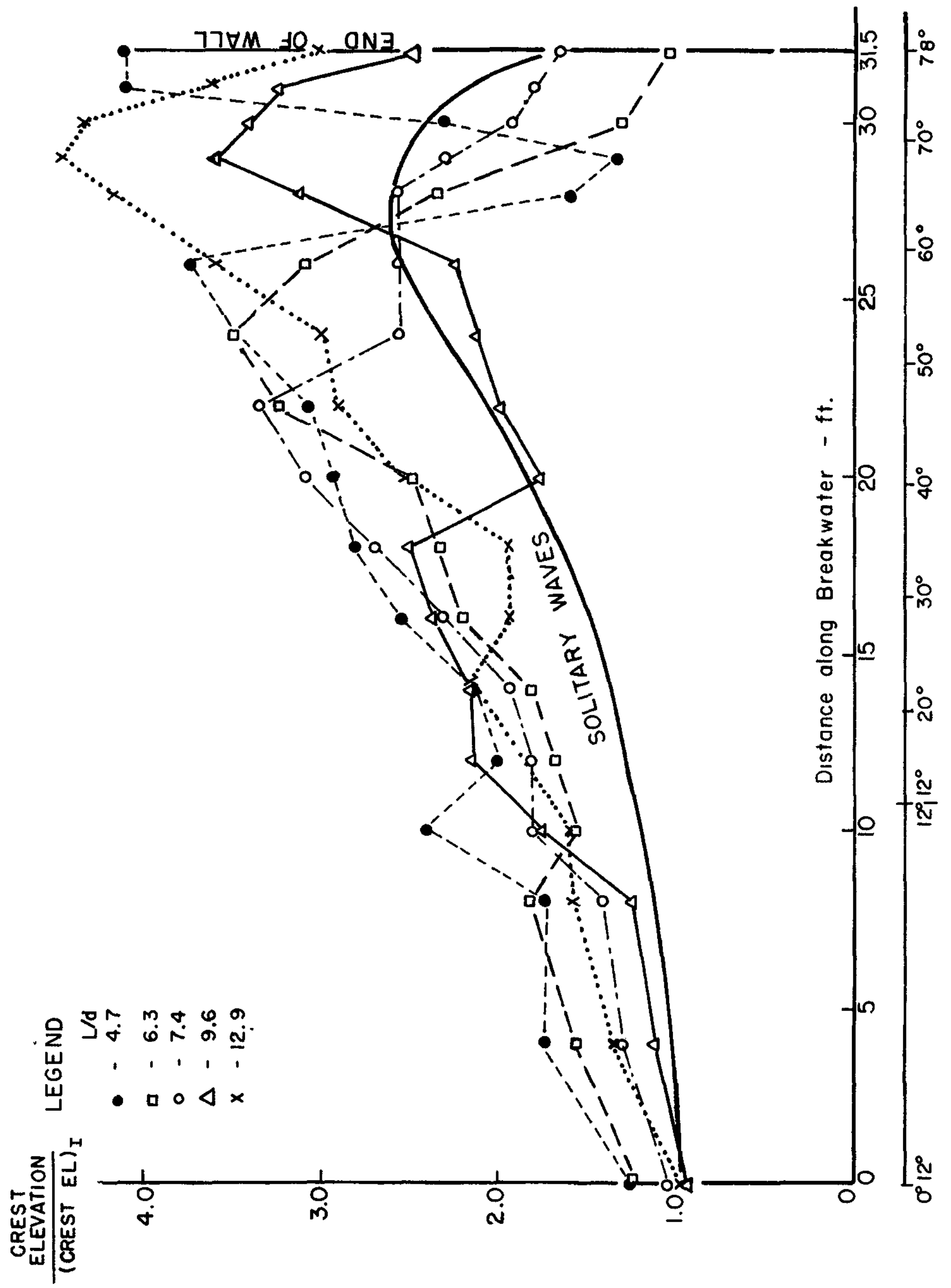


FIGURE 13 - MAXIMUM WAVE ELEVATION ALONG CURVED VERTICAL BREAKWATER FOR DIFFERENT WAVE LENGTHS (RICHMOND MODEL) $H_I/d \simeq 0.14$, d = 1.0 ft

(from Nielsen, 1962)

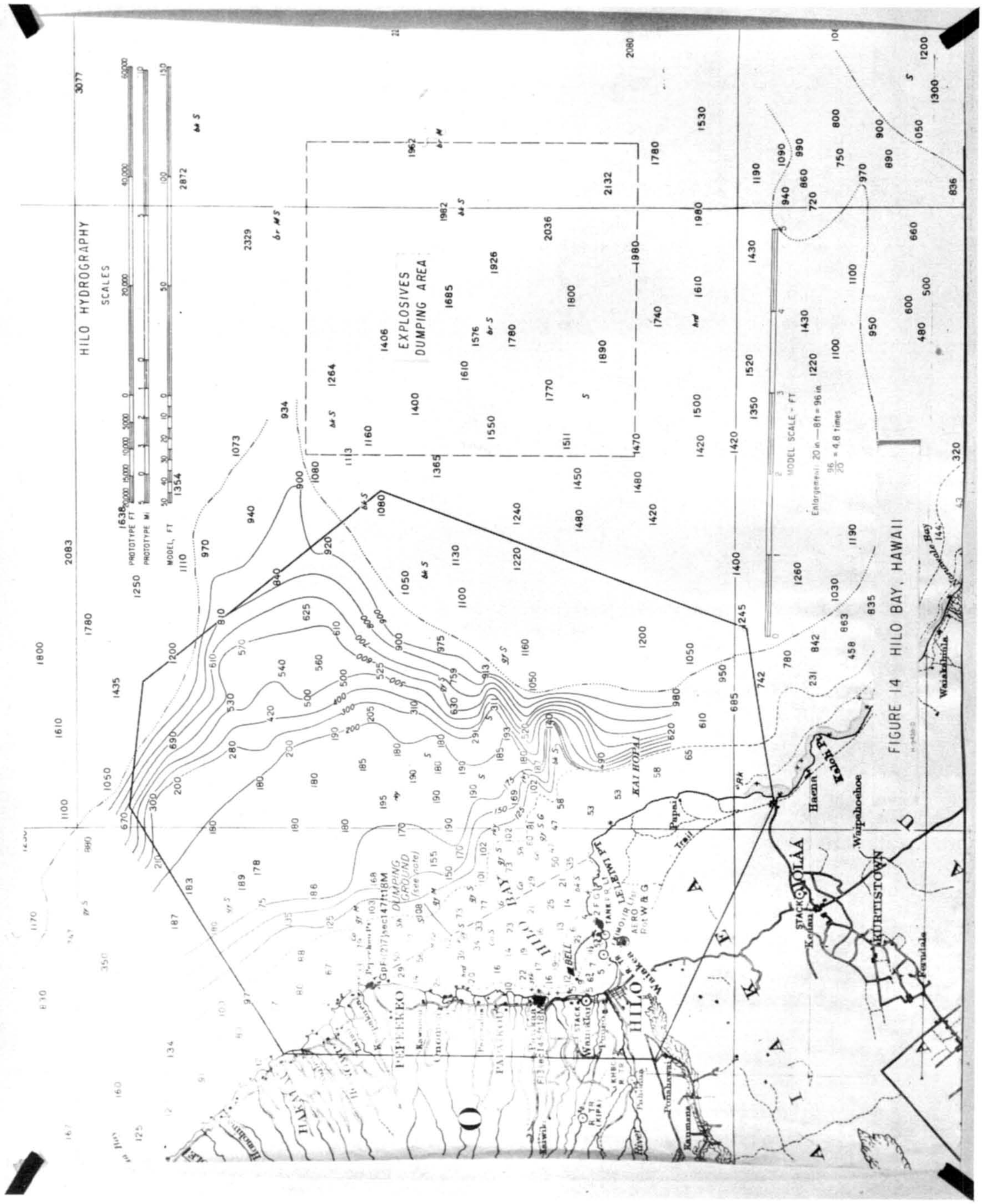


FIGURE 14 HILO BAY, HAWAII

like a Mach-stem (Fig. 15). Another photograph, showing the wave rolling into the town, is shown in Fig. 16.

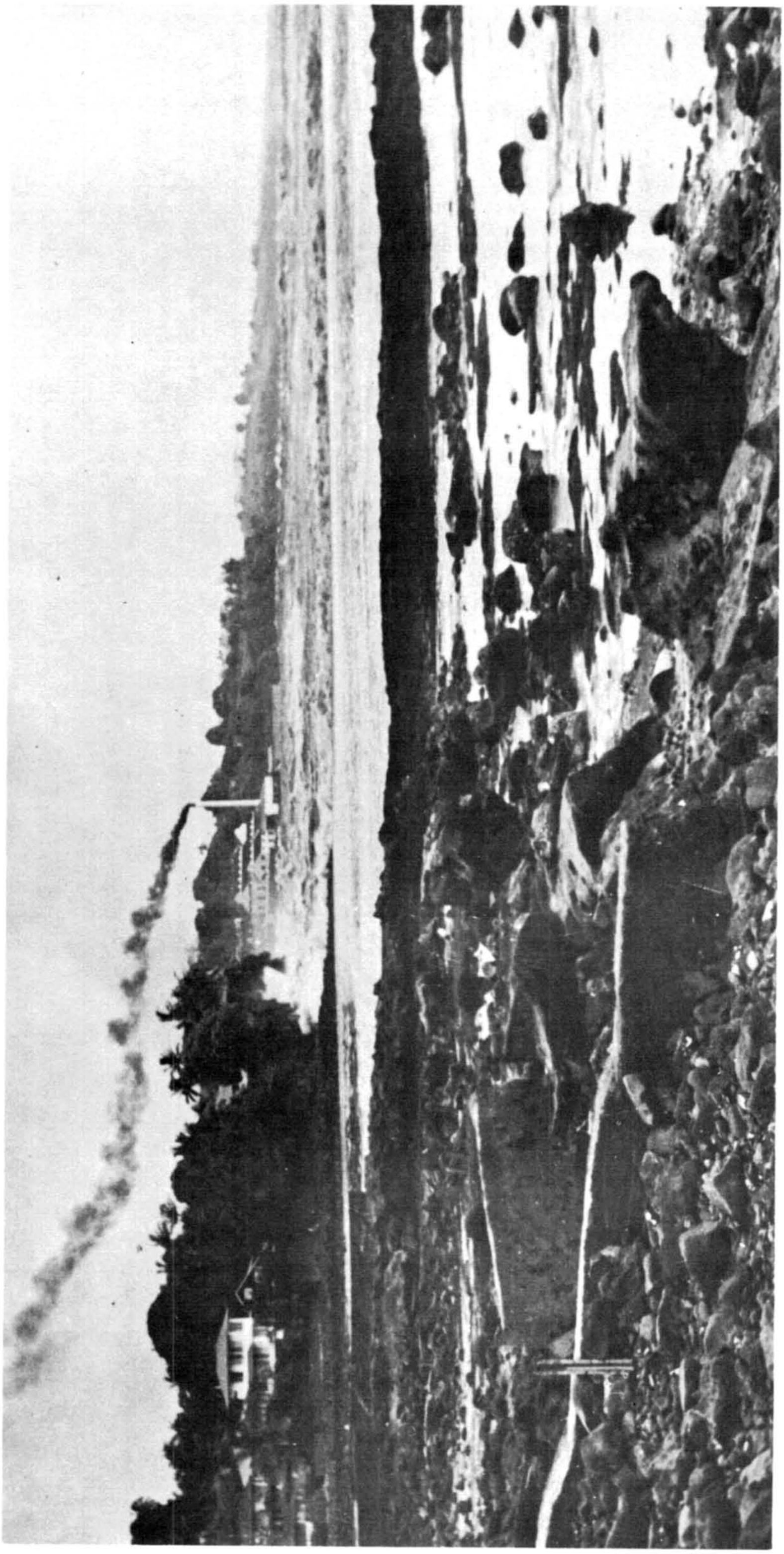
From some of the observations of the tsunami of 22 May 1960, originating in Chile, it appeared as if something similar must have happened, but the author could not visualize how it could have occurred.

In order to study the gross characteristics of tsunamis at Hilo, a 1:15,000 (1: $\sqrt{15,000}$ = 1:122 time and velocity scale) undistorted model was constructed of fiberglass. The model was approximately 8 ft on a side, so that the entire bay could be included, as well as the reef. A portion of the ocean was included, to a depth of 6,000 to 7,000 ft, prototype. The model was placed at one end of an 8-ft wide by 6-ft deep by 200-ft long tank so that a number of waves could be measured before reflections could be of importance. A series of runs were made with periods ranging from 6 to 24 minutes, prototype, and with waves from the N, E, and SE directions.

It was found, for waves from the north, that a wave which had the appearance of a Mach-stem was generated along the west cliff and rolled into the town of Hilo in a manner that was similar to observations. It was also found that the shoaling effect was about as theory predicted; that is, the wave height increased by a factor of about 4 over the reef, with respect to the wave height in the deep water portion of the tank.

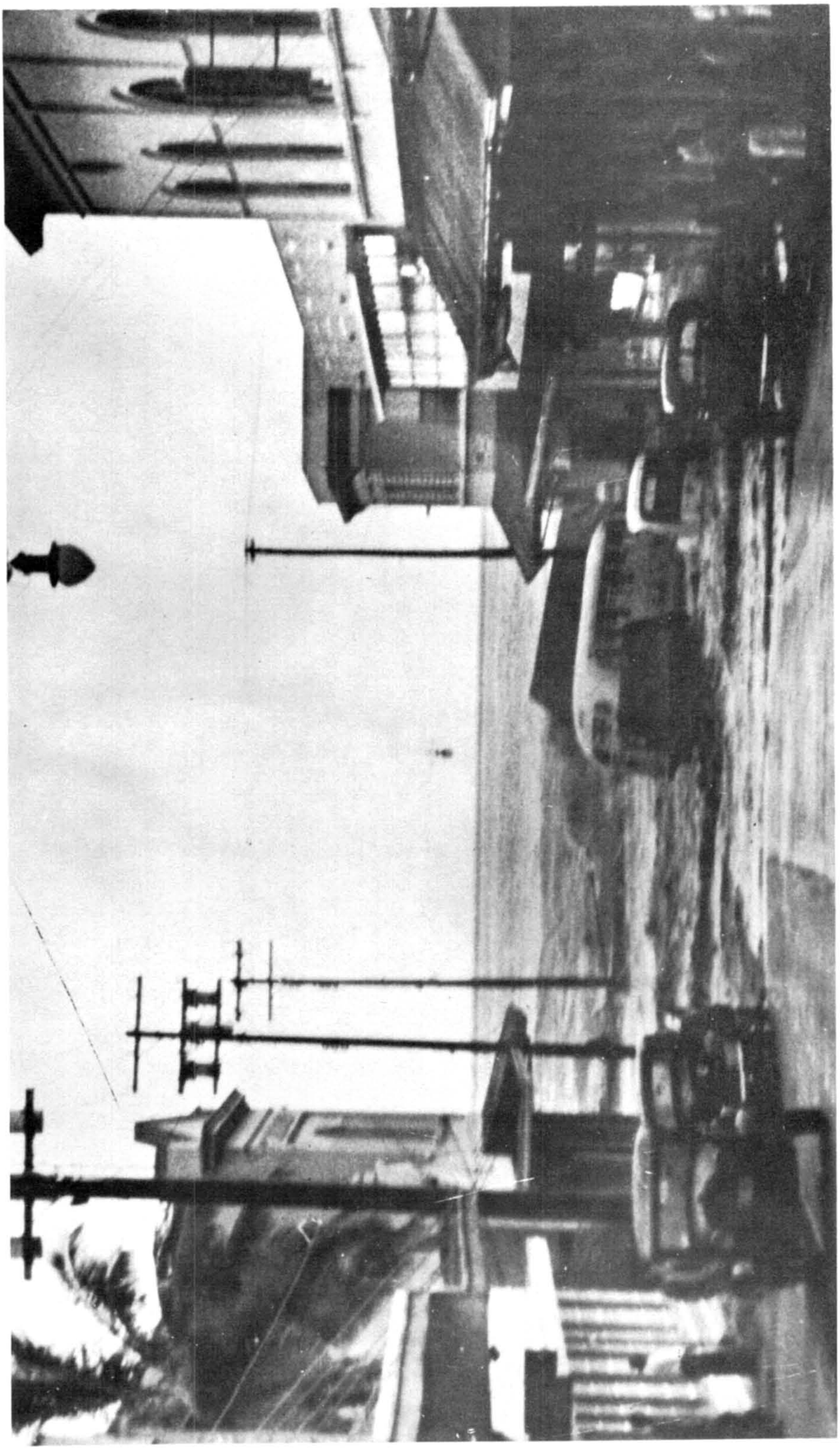
After the tests had been run, it was brought to the investigator's attention that due to refraction in the ocean, the tsunamis generated off Chile would most likely approach Hilo Bay from an easterly direction, rather than from a southeasterly direction as was originally supposed. Because of this, the results of the model tests for the waves from the east will be described herein. A remarkable phenomenon was observed in the 12 to 20 minute (prototype) period range. Referring to Fig. 17a, the initial wave refracted to about the position shown as (1)-(1). The northerly portion started to reflect from the coast while the southerly portion continued to move towards shore. This resulted in the pattern (2A)-(2A) as the reflected portion and (2)-(2) as the continuing portion. As the reflected portion (2A)-(2A) moved down the coast, it became independent of (2)-(2). At the same time the southerly tip of (2)-(2) diffracted into the harbor, raising the water level. About at the same time (2A)-(2A) progressed to position (3)-(3) with the portion near the coast being considerably higher than the portion offshore. The portion near the coast ran right along the coast, reaching positions (4)-(4) and (5)-(5) as a high wave running on top of the water which had diffracted into the harbor from (2)-(2). It then ran into the town of Hilo. The author believes that something similar to this must have happened during the actual tsunami.

The transformation of (2A)-(2A) to (3)-(3) was probably caused by a combination of refraction and reflection, together with some non-linear effects because of the relatively large wave height to water depth ratio along the coast, and the height of the tsunami at Hilo was probably due to this combined with the diffracted wave. At some place between



1946 TSUNAMI - HILO

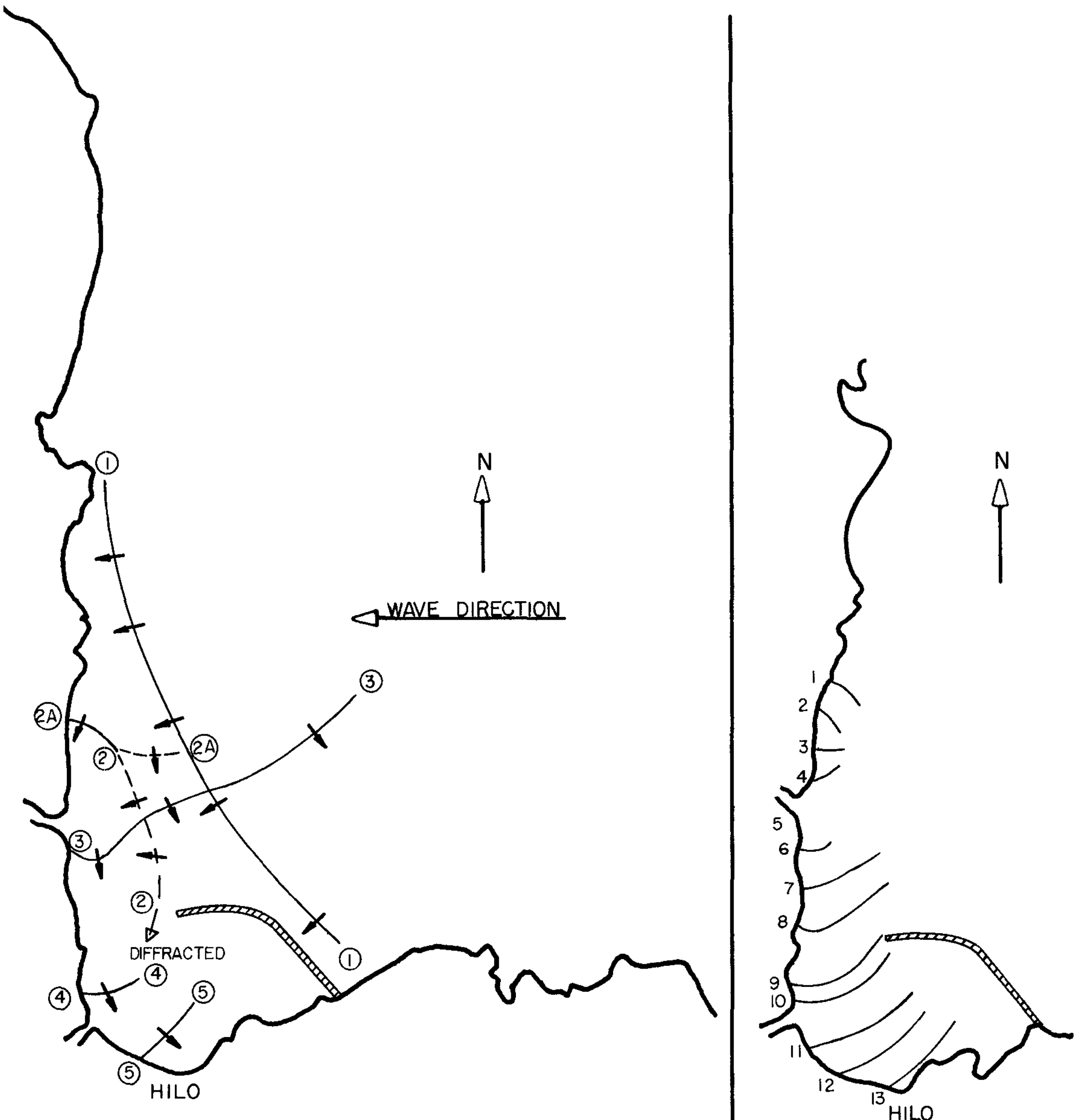
FIGURE 15 WAVE IN THE FORM OF A BORE APPROACHING MOUTH OF WAILUKU RIVER.
NOTE: MACH-STEM EFFECT INCREASED WAVE HEIGHT ALONG COASTAL BLUFF.
WATER IN FOREGROUND HAS RECEDED SEVERAL FEET.
(COURTESY, MODERN CAMERA CENTER, HILO)



1946 TSUNAMI – HILO

FIGURE 16 VIEW DOWN WAIANUENUE STREET. THIS WAS SECOND WAVE ACCORDING TO MR. YOSHIO SHIGENAGA, EMPLOYEE OF AMERICAN TRADING CO., LTD., WHICH WAS THEN LOCATED IN GROUND FLOOR OF TWO STORY BUILDING ON RIGHT AT END OF STREET. BUS WAS AT INTERSECTION OF KAMEHAMEHA STREET. BREAKWATER IN DISTANCE WAS BREACHED BY 3rd OR 4th WAVE.

(COURTESY, MODERN CAMERA CENTER, HILO)



a INCIDENT WAVE FRONT, REFLECTED WAVE, DIFFRACTED WAVE AND MACH STEM

b MOTION OF MACH STEM TRACED FROM ENLARGED 16mm FILM, 1/8 SEC BETWEEN TRACINGS (0 252 MINUTES, PROTOTYPE)

FIGURE 17, SOME RESULTS FROM A 1:15,000 SCALE MODEL STUDY OF TSUNAMIS (16 MINUTE, PROTOTYPE) AT HILO HAWAII. RUN X, 8 FEB 1963, WAVE FROM THE EAST (IN 7,000 FT OF WATER, PROTOTYPE)

(2A)-(2A) and (5)-(5) a Mach-stem type of phenomenon evolved and because of its strength it became independent of the normally reflected portion of the wave. In Fig. 17b are shown the successive positions of this Mach-stem type of wave as it moves along the coast. These positions were traced from enlargements of a 16 mm motion picture taken during the model study, for a wave of 8 second period (16 minute period in the prototype).

ACKNOWLEDGMENTS

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