CHAPTER 28

DYNAMIC ANALYSIS OF OFFSHORE STRUCTURES

by

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ABSTRACT

Analytical procedures are presented for calculation of the dynamic displacements of fixed offshore structures in oscillatory waves. The structure considered has four legs in a square configuration with waves impinging normal to one side; however, the procedures are general and may be applied to other configurations and wave directions.

The horizontal displacement of the deck is determined as a function of time by application of vibration theory for a damped, spring-mass system subject to a harmonic force. The instantaneous wave force on each leg is composed of a hydrodynamic drag component and an inertial component as in the usual "statical" wave force analysis. The wave force expression is approximated by a Fourier series which permits calculation of the platform displacement by superposition of solutions of the equation of motion for the platform.

Depending on the ratio of the wave frequency to the natural frequency of the platform, the structural stresses may be considerably higher than those found by methods which neglect the elastic behavior of the structure.

The highest wave to be expected in a given locality is not necessarily the critical design wave. Maximum displacements and structural stresses may occur for smaller waves having periods producing a resonant response of the platform.

Displacement measurements in a wave tank using a platform constructed of plastic are presented to show the validity of the analytical method. Both small and finite amplitude waves are used over a wide range of frequency ratios. A digital computer program (7090 FORTRAN) is used for the displacement calculation.

DYNAMIC ANALYSIS OF OFFSHORE STRUCTURES

INTRODUCTION

The usual method of design of a fixed offshore structure (i. e., a deck located above the limit of wave motion and supported by three or more legs driven into the ocean bottom) is by means of a "statical" wave force analysis. In this approach the procedure has generally been to select a "design wave" which is the largest wave to be expected in the locality. Using applicable wave theory (for particle velocities and accelerations) and appropriate wave force coefficients, the maximum forces and moments are computed assuming the structure is in static equilibrium.

Recent field experience, including the tragic failure of a major structure off the East coast (U. S. Senate, 1961) have indicated that the maximum or "design wave" may not be the wave which causes the greatest stresses in the structure. In other words, a smaller wave whose fundamental period approaches the natural period of vibration of the structure may be critical for design in view of the large amplification of deflections and stresses near resonance.

A method of analysis which considers the elastic nature of the structure and its dynamic response to wave forces is presented for use in the design of future offshore platforms.

ANALYTICAL DEVELOPMENT

DYNAMIC ANALYSIS OF PLATFORM

Equation of Motion

Analytical procedures are developed for the dynamic analysis of an offshore platform supported by four cylindrical legs and acted upon by a train of oscillatory surface waves. The horizontal displacement of the platform is represented by X measured with respect to the neutral position of the center of gravity of the deck as shown in Fig. 1. The equation of motion for a single degree of freedom, equivalent spring-mass system, with linear damping and restoring force subjected to a sinusoidal exciting force as shown in Fig. 2 is given by eq. (1),

$$m\frac{d^2X}{dt'^2} + C\frac{dX}{dt'} + KX = P(t') = P_m \sin \sigma t' \qquad (1)$$

where:

m = effective mass of system
C = damping coefficient of structure
K = spring constant of structure
Pm =amplitude of harmonic exciting force
σ = frequency of harmonic exciting force $= \frac{2\pi}{T}$
t' = time in the equivalent system

The solution of equation (1) (Housner and Hudson) is,

$$X(t') = \frac{P_m}{K\sqrt{\left[1-\left(\frac{\sigma}{\sigma_n}\right)^2\right]^2 + \left[2\frac{C}{C_c}\frac{\sigma}{\sigma_n}\right]^2}} \cdot \sin(\sigma t' - \phi) \qquad (2)$$

where,

$X(t')$ = horizontal displacement due to $P(t')$

$\sigma_n = \sqrt{\frac{K}{m}}$, undamped natural frequency of spring-mass system (3)

$C_c = 2\sqrt{mK}$, critical damping coefficient (4)

$$\phi = \tan^{-1}\left[\frac{2\frac{C}{C_c}\cdot\frac{\sigma}{\sigma_n}}{1-\left(\frac{\sigma}{\sigma_n}\right)^2}\right] \quad (5)$$

The crux of the proposed method of analysis may be seen by comparing Figs. 1 and 2. The equation of motion (1) is considered to apply to the equivalent spring-mass system shown in Fig. 2. The effective mass of the equivalent system is concentrated at the deck and the legs are considered to act as cantilever springs of zero mass.

The harmonic exciting force $P(t')$ acts in the plane of the deck which undergoes a pure translatory motion since it is assumed to have a stiffness which is large compared to the legs and is therefore not subject to deflection due to bending. The actual exciting force for the platform is $F(t)$ which is the resultant of the time dependent wave force distribution acting at variable distance $\bar{s}$ above the bottom. (Fig. 1) The problems involved in relating the actual platform motion to the vibration of the equivalent spring-mass system (Fig. 2) are described in the following section.

Platform Characteristics

1. The actual exciting force $F(t)$ must be related to the exciting force $P(t')$ in the equivalent system. Since $F(t)$ is composed of both hydrodynamic drag and inertial contributions, it cannot be represented by a single term of the form $F \sin \sigma t$. However, by means of a Fourier series approximation, $F(t)$ can be represented by a series of sine terms to any desired degree of accuracy. Each F term in the series can be related to a P term through the method of influence fractions. Since the equation of motion (1) is linear, the component displacements due to the individual exciting force terms ($P_m \sin \sigma t'$) in the Fourier series can be summed to determine the total platform displacement as a function of time. Hence,

$$X_{tot.}(t) = \sum_{m=0}^{m} X_m(t) \quad (6)$$

The usual procedures of structural analysis may then be used to relate displacements (strains) to stresses.

The static deflection of the platform is obtained from cantilever beam theory. The equation for the maximum deflection of the deck (X_{max}

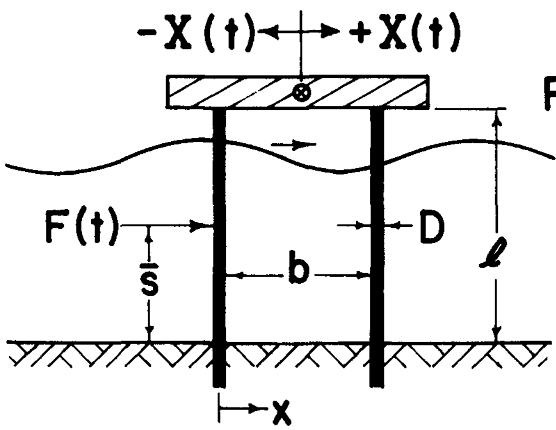

Fig. 1 Unbraced Platform Four Cylindrical Legs

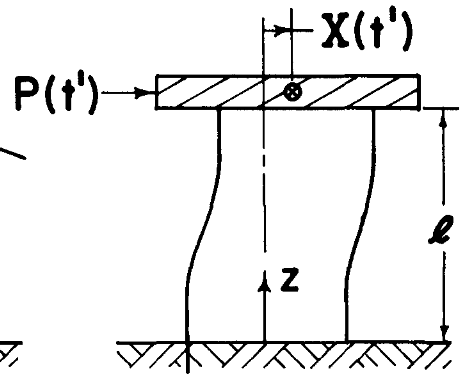

Fig. 2 Equivalent Spring-Mass System

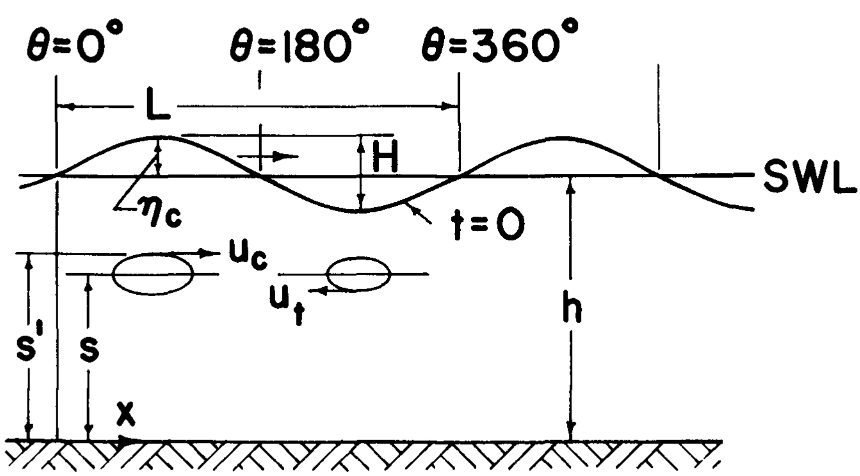

Fig. 3 Definition Sketch for Wave Motion

for a force F applied at an elevation $\bar{s}$ (see Fig. 1) is

$$[X_{max}]_{act.} = \frac{F\bar{s}^2}{NEI}\left(\frac{\ell}{4} - \frac{\bar{s}}{6}\right) \tag{7}$$

In the equivalent system, with a force P applied at the elevation of the deck (see Fig. 2), the deflection is found from eq. (7) by letting $\bar{s} = \ell$, hence,

$$[X_{max}]_{equiv.} = \frac{P\ell^3}{12NEI} \tag{8}$$

In the above equations,

E = elastic modulus of actual platform leg
I = moment of inertia of actual platform leg
N = number of legs = 4
ℓ = vertical length of legs

The influence fraction which relates F and P is determined from the requirement that the static deflection of the equivalent system under a force P must equal the static deflection of the actual platform under a force F. Hence, by equating eqs. (7) and (8) the ratio, P/F = f, becomes

$$\frac{P}{F} = f = 3\left(\frac{\bar{s}}{\ell}\right)^2 - 2\left(\frac{\bar{s}}{\ell}\right)^3 \tag{9}$$

2. The spring constant of the equivalent system is by definition

$$K = \frac{P}{[X_{max.}]_{equiv.}} \tag{10}$$

Therefore from eq. (8), with N = 4,

$$K = \frac{48\,EI}{\ell^3} \tag{11}$$

3. The natural frequency of the equivalent system is equal to the natural frequency of the actual platform. Rayleigh's Energy Method (Thomson, 1953) can be applied to determine the natural frequency of the equivalent system using the static deflection curve to calculate kinetic and potential energies. In the equivalent system the deflection X(z) of the legs at any section z in terms of $X_{max.}$ can be found from simple beam theory to

$$\frac{X(z)}{X_{max.}} = 3\left(\frac{z}{\ell}\right)^2 - 2\left(\frac{z}{\ell}\right)^3 \tag{12}$$

The maximum kinetic energy is given by,

$$[K.E.]_{max.} = \frac{W}{g} \frac{\sigma_n^{\,2} X_{max}^{2}}{2} + N \int_0^{\ell} \frac{w}{g} \frac{\sigma_n^{\,2} X(z)}{2} \, dz$$

Substituting for X(z) from eq. (12),

$$[K.E.]_{max.} = \frac{\sigma_n^{\,2} X_{max.}^{2}}{2g} \left(W + \frac{13}{35} Nw\ell\right) \tag{13}$$

where,

W = weight of deck
w = weight per foot of leg

The maximum potential energy is given by,

$$[P.E.]_{max.} = \frac{KX_{max}^{2}}{2} \tag{14}$$

Upon equating the kinetic and potential energy eq. (13) and (14), the natural frequency is,

$$\sigma_n = \sqrt{\frac{K}{\frac{1}{g}\left(W + \frac{13}{35} N w \ell\right)}} \tag{15}$$

4. By comparing eqs. (3) and (15), the effective mass of the equivalent system is,

$$m = \frac{1}{g}\left(W + \frac{13}{35} Nw\ell\right) \tag{16}$$

In a simple vibrating spring-mass system, the effect of the mass of the spring can be accounted for by increasing the rigid mass by one-third of the mass of the spring. The factor 13/35 obtained above is close to this value and represents the proportion of the mass of the legs to be added to the weight of the deck in the actual structure. If the legs had been considered to be pinned to the deck the fraction would be 33/140. The added mass due to vibration in water has been found to be negligible compared to the mass of the platform. The effect of bracing between legs can be accounted for by modifications in the above procedures. (Nolan and Honsinger, 1962).

5. The critical damping coefficient, C_c, is a function of m and K and can be calculated from eq. (4).

6. The damping coefficient, C, of the equivalent system is equal to the damping coefficient of the actual platform. A convenient way to determine the magnitude of the structural damping coefficient is to measure the decay of the amplitude of oscillation under the action of a single impulsive force (Thomson, 1953) either in the actual structure or in a model. Thus, if X_1 and X_2 are two successive positive displacement amplitudes, the ratio of C/C_c is proportional to the logarithmic decrement

$$\frac{C}{C_c} = \frac{1}{2\pi} \left(\ln \frac{X_1}{X_2}\right) \qquad (17)$$

The magnitude of viscous damping due to vibration in water has been found to be small compared to the structural damping.

7. The frequency of the harmonic exciting force is equal to the wave frequency

$$\sigma = \frac{2\pi}{T}$$

where T is the period of the wave motion.

8. The phase angle ϕ can be calculated from eq. (5) since it is a function of C, C_c, σ, and σ_n.

The above series of equations provide the necessary information for the determination of all of the coefficients in the equation of motion of the equivalent system (eq. 1) in terms of known quantities for the actual platform.

Wave Force Theory

The wave force on a single vertical cylinder follows the method of Harleman and Shapiro (1955) which is a modification of the development by Morison, et al (1953). The method has been shown to be well adapted to finite amplitude waves as used in the laboratory experiments. However the dynamic platform analysis is essentially independent of the particular wave force theory and any procedure which can be approximated by a Fourier series can be employed.

The following functions are used for the total wave force F on a single vertical cylinder, following the notation shown in Fig. 3:

$$F = F_C \sin^2 \theta + F_I \cos \theta \qquad 0° \leq \theta \leq 180° \qquad (18)$$

$$F = F_T \sin^2 \theta + F_I \cos \theta \qquad 180° \leq \theta \leq 360° \qquad (19)$$

where,

F_c = hydrodynamic drag force at crest (θ = 90°)
F_T = hydrodynamic drag force at trough (θ = 270°)
F_I = inertial force at θ = 0° and 180°
θ = phase angle = (σt - kx)
k = wave number = $2\pi/L$
L = wave length

1. The hydrodynamic drag forces F_c and F_T are expressed in terms of a drag coefficient C_D:

$$F_c = \int_0^{h+\eta_c} C_{D_c} \rho \frac{u_c^2}{2} D \, ds' = C_{D_c} \frac{\gamma D h^2}{2} \cdot A \qquad (20)$$

$$F_T = \int_0^{h+\eta_c-H} C_{D_T} \rho \frac{u_T^2}{2} D \, ds' = C_{D_T} \frac{\gamma D h^2}{2} \cdot B \qquad (21)$$

where,

u_c = particle velocity under the wave crest
u_T = particle velocity under the wave trough
ρ = density of water
γ = specific weight of water
C_D = drag coefficient for cylinder
s' = instantaneous elevation of particle
D = cylinder diameter
η_c = wave amplitude at crest
h = depth to s.w.l.

and

$$A = \int_0^{1+\eta_c/h} \frac{u_c^2}{gh} \, d\,(s'/h) \qquad (22)$$

$$B = \int_0^{1+\frac{\eta_c - H}{h}} \frac{u_T^2}{gh} \, d\,(s'/h) \qquad (23)$$

The equations for u_c and u_T used to evaluate the A and B integrals are those of the Stokes third approximation for large amplitude waves in water of finite depth. The wave equations and numerical tables of the wave properties are available (Skjelbreia, 1959) and will not be reproduced here. The integrals can be evaluated graphically or, as was done in this study, programmed for a digital computer.* The drag coefficients in the crest and trough region are determined from a standard steady state plot of C_D versus Reynolds number. The Reynolds numbers are defined as follows:

$$R_c = \frac{\sqrt{\overline{u}_c^2}\, D}{\nu} = \frac{D}{\nu}\sqrt{ghA} \tag{24}$$

and

$$R_T = \frac{D}{\nu}\sqrt{ghB} \tag{25}$$

The line of action of the hydrodynamic drag force passes through the centroid of the curve of u^2 versus s'. For the crest, the distance from the bottom to the line of action of F_c is designated by $\overline{s}_c$, hence, by taking the first moment,

$$\frac{\overline{s}_c}{h} = \frac{1}{A}\int_0^{1+\eta_c/h} \frac{u_c^2}{gh}\cdot\frac{s'}{h}\cdot d\,(s'/h) \tag{26}$$

In a similar manner, for the trough,

$$\frac{\overline{s}_T}{h} = \frac{1}{B}\int_0^{1+\frac{\eta_c - H}{h}} \frac{u_T^2}{gh}\cdot\frac{s'}{h}\cdot d\,(s'/h) \tag{27}$$

Harleman and Shapiro (1955) have shown that $\overline{s}_c$ can be assumed constant for $0° \leq \theta \leq 180°$, and $\overline{s}_T$ can be assumed constant for $180° \leq \theta \leq 360°$.

2. The inertial force due to particle accelerations is evaluated from the Newtonian equation of motion for an object in an accelerating flow field,

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* For the waves used in the experimental study, relatively little error is introduced if the Airy equations for particle velocity are used.

$$F_I = C_M \rho \frac{\pi D^2}{4} \int_o^h a_{x_o} \, ds \qquad (28)$$

where,

C_M = added mass coefficient
a_{x_o} = horizontal acceleration at $\theta = 0°$

The acceleration integral may be evaluated from the Stokes third order tables (Skjelbreia, 1959). For the range of waves in the experimental study it is sufficient to use the Airy equations with $C_M = 2.0$ and eq. (28) then becomes,

$$F_I = \frac{\gamma \pi D^2 H}{4} \tanh kh \qquad (29)$$

The line of action of the inertial force is,

$$\frac{\overline{s}_I}{h} = \frac{1 + kh \sinh kh - \cosh kh}{kh \sinh kh} \qquad (30)$$

which is assumed constant for all values of θ.

It should be emphasized that this paper is not primarily concerned with the particular wave force computational procedure for a single vertical cylinder or with the various arguments in regard to the numerical values for C_D and C_M. The values chosen agree with the laboratory experiments. Under field conditions other values in accordance with the experience of the designer may be used.

The wave force functions, eqs. (18) and (19), may now be approximated by a Fourier series in the following manner:

let F, the resultant wave force on one of the vertical legs, be given by,

$$F = a_o + \sum_{n=1}^{\infty} (a_n \cos n\theta + b_n \sin n\theta) = F_c \sin^2\theta + F_I \cos\theta \; (0° \leq \theta \leq 180$$

$$= -F_T \sin^2\theta + F_I \cos\theta \; (180° \leq \theta \leq 3$$

where, (3

$$a_o = \frac{1}{2\pi} \int_o^{2\pi} F \, d\theta$$

$$a_n = \frac{1}{\pi} \int_o^{2\pi} F (\cos n\theta) \, d\theta$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} F\,(\sin n\theta)\, d\theta$$

whence,

$$a_o = \frac{F_c - F_T}{4}$$

$$a_1 = F_I \qquad\qquad b_1 = \frac{4}{3\pi}(F_c + F_T)$$

$$a_2 = -\left(\frac{F_c - F_T}{4}\right) \qquad\qquad b_2 = 0$$

$$a_3 = 0 \qquad\qquad b_3 = -\frac{4}{15\pi}(F_c + F_T)$$

Therefore,

$$F = \frac{F_c - F_T}{4} + F_I \cos\theta + \frac{4}{3\pi}(F_c + F_T)\sin\theta$$
$$- \left(\frac{F_c - F_T}{4}\right)\cos 2\theta - \frac{4}{15\pi}(F_c + F_T)\sin 3\theta + --- \qquad (32)$$

The last two terms tend to cancel and contribute little to the total force, hence only the initial three terms of the series are used, in addition, since $\cos\theta = \sin(\theta + 90°)$:

$$F = \frac{F_c - F_T}{4} + F_I \sin(\theta + 90°) + \frac{4}{3\pi}(F_c + F_T)\sin\theta \qquad (33)$$

This series of sine terms is valid for $0° \leq \theta \leq 360°$.

Before using the series expression for wave force, eq. (33), in the dynamic analysis of the platform it is of interest to see how it compa with experimental measurements on a single vertical cylinder. The com parison with earlier experimental measurements (Harleman and Shapiro, 1955) is shown in Fig. 4. The agreement, considering that only three te are used in the Fourier series, is considered reasonable. In addition, b $C_D \doteq 1.2$ and $C_M = 2.0$ were chosen "a priori", and a better fit could be tained, if desired, by adjusting these coefficients.

Platform Displacement

From eq. (9) the following influence fractions may be defined,

$$f_c = \frac{P_c}{F_c} = 3\left(\frac{\bar{s}_c}{l}\right)^2 - 2\left(\frac{\bar{s}_c}{l}\right)^3 \qquad (34a)$$

$$f_T = \frac{P_T}{F_T} = 3\left(\frac{\bar{s}_T}{\ell}\right)^2 - 2\left(\frac{\bar{s}_T}{\ell}\right)^3 \qquad (34b)$$

$$f_I = \frac{P_I}{F_I} = 3\left(\frac{\bar{s}_I}{\ell}\right)^2 - 2\left(\frac{\bar{s}_I}{\ell}\right)^3 \qquad (34c)$$

where eqs. (26, 27, 30) are to be used to calculate the appropriate values of $\bar{s}/\ell$.

The Fourier series for the forcing function P at the elevation of the deck in the equivalent system is obtained from the wave force equation (33), hence,

$$P = \frac{F_c f_c - F_T f_T}{4} + \frac{4}{3\pi}(F_c f_c + F_T f_T) \sin\theta + F_I f_I \sin(\theta + 90°) \qquad (35)$$

To simplify the notation, let,

$$P_1 = \frac{F_c f_c - F_T f_T}{4}$$

$$P_2 = \frac{4}{3\pi}(F_c f_c + F_T f_T)$$

$$P_3 = F_I f_I$$

Therefore, since $\theta = (\sigma t - kx)$, eq. (35) becomes,

$$P = P_1 + P_2 \sin\sigma\left(t - \frac{kx}{\sigma}\right) + P_3 \sin\sigma\left(t - \frac{kx}{\sigma} + \frac{90}{\sigma}\right) \qquad (36)$$

or

$$P = P_1 + P_2 \sin\sigma t' + P_3 \sin\sigma t'' \qquad (37)$$

where $t' = t - \frac{kx}{\sigma}$

and $t'' = t - \frac{kx}{\sigma} + \frac{90°}{\sigma}$

Each of the P terms in eq. (37) givesrise to a displacement X given by eq. (2). The P_1 term is independent of both x and t, hence,

$$X_1 = \frac{4P_1}{K} \qquad (38)$$

The factor 4 appears in eq. (38) since the P terms represent the force on the equivalent system due to the wave force on a single leg of the structure. As shown in Fig. 1, x = 0 is taken to coincide with the two forward legs of the platform. To simplify the notation, let the denominator of eq. (2) be given by,

$$G = \sqrt{\left[1 - \left(\frac{\sigma}{\sigma_n}\right)^2\right]^2 + \left[2\frac{C}{C_c}\frac{\sigma}{\sigma_n}\right]^2} \tag{38}$$

For the forward legs, x = 0,

$$t' = t$$
$$t'' = t + \frac{90°}{\sigma}$$

For the rear legs, $x = b$, $\frac{kx}{\sigma} = \frac{2\pi b}{\sigma L} = \frac{360° b}{\sigma L}$,

$$t' = t - \frac{360° b}{\sigma L}$$

$$t'' = t - \frac{360° b}{\sigma L} + \frac{90°}{\sigma}$$

Therefore, from eq. (2), the sum of the instantaneous displacements for the two forward and two rear legs due to the forcing term P_2 in eq. (37) is given by,

$$X_2(t) = \frac{2P_2}{KG}\left[\sin(\sigma t - \phi) + \sin\left(\sigma t - \frac{360° b}{L} - \phi\right)\right] \tag{39}$$

In a similar fashion, the instantaneous displacement due to the forcing term P_3, becomes

$$X_3(t) = \frac{2P_3}{KG}\left[\sin(\sigma t + 90° - \phi) + \sin\left(\sigma t - \frac{360° b}{L} + 90° - \phi\right)\right] \tag{40}$$

In accordance with eq. (6) the total displacement of the platform deck at any instant of time is given by,

$$X_{tot.}(t) = X_1 + X_2(t) + X_3(t) \tag{41}$$

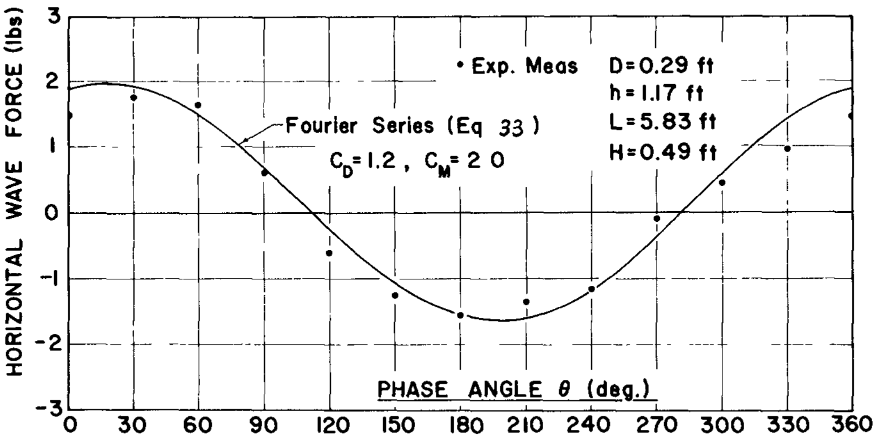

Fig. 4 Comparison of Fourier Series with Experimental Wave Force on a Single Vertical Cylinder

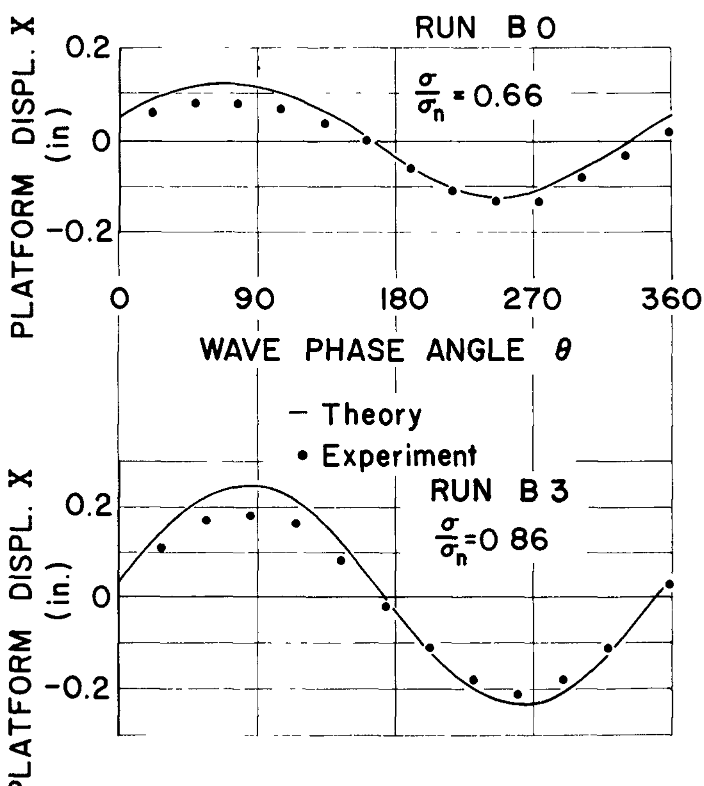

Fig. 5 Exp. and Theor. Platform Displacement. Wave B

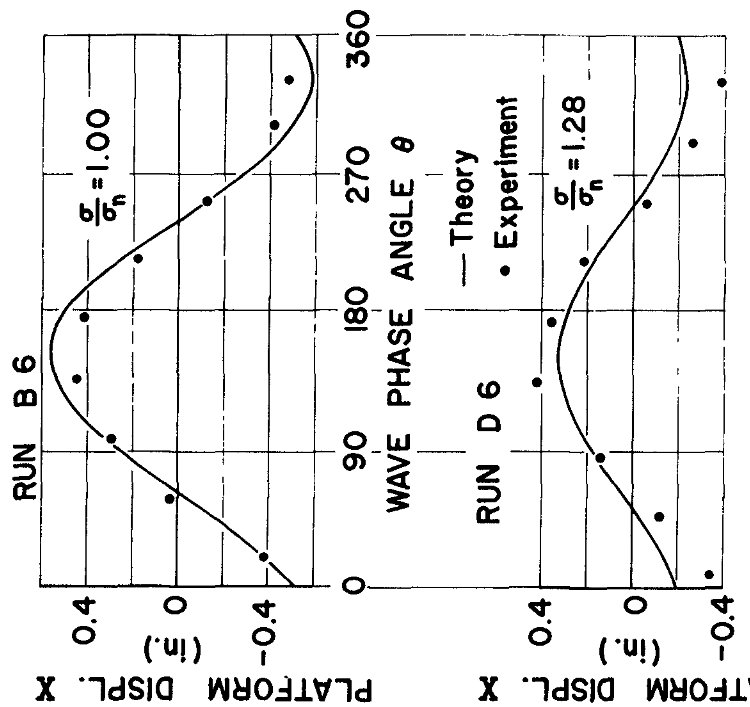

Fig. 7 Exp. and Theor. Platform Displacement. Waves B & D.

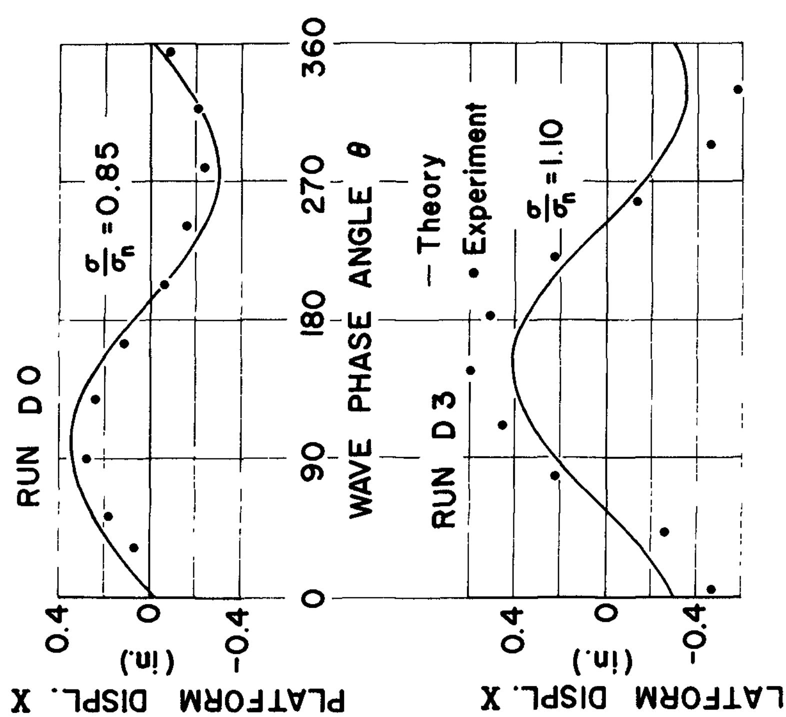

Fig. 6 Exp. and Theor. Platform Displacement. Wave D

EXPERIMENTAL RESULTS

The experimental measurements were made in the 90-foot wave channel in the M.I.T. Hydrodynamics Laboratory. Waves were generated by a variable speed and amplitude piston type generator located at one end of the channel. Wave reflections from the beach at the far end of the channel were avoided by starting each run from rest and recording the data during the passage of the eighth wave.

The platform was located 40 feet from the wave generator and was constructed of plastic. The four legs are 1/2 inch diameter, 42 inches lon and are located on 16-inch centers in a square pattern. The platform deck is 5/8" x 18" x 18" and contained a bolt in the center for observing platforn displacements and for holding weights to change the natural frequency of th structure. The legs were rigidly fixed to the bottom of the wave tank. Th platform displacements were recorded by means of a 16 mm movie camera A two channel oscillograph was used to record wave profiles by means of parallel wire resistance wave gages. The two gages were adjusted until th two wave profiles were in phase on the recorder paper at which point the measured distance between the gages is equal to the wave length. The wav period was determined by comparing the measured wave length on the recorder paper with the known speed of the paper. The wave characteristics for the tests reported are given in table I.

TABLE I

Wave Characteristics

Wave	H (ft)	L (ft)	h (ft)	h/L	H/L	$\sigma=2\pi/T$ (1/sec)
B	0.31	10.86	2.25	0.21	0.029	4.00
D	0.40	7.33	2.25	0.31	0.055	5.13

The parameters used in the wave force theory are given in table I]

TABLE II

Wave Force Parameters

Wave	A (22)*	B (23)*	C_{D_c}	C_{D_T}	F_c (20)* (lb)	$\bar{s}_c$ (26)* (ft)	F_T (21)* (lb)	$\bar{s}_T$ (27)* (ft)	F_I (29)* (lb)
B	0.0042	0.0028	0.92	0.88	0.027	1.54	0.017	1.26	0.023
D	0.0067	0.0034	0.96	0.92	0.042	1.81	0.022	1.41	0.033

The spring constant of the platform was determined to be K = 7.5 The natural frequency of the platform was varied by adding weights to the d The damping ratio C/C_c was determined experimentally by a measuremen the logarithmic decrement as given in eq. (17). The platform characterist are summarized in table III.

* equation number

TABLE III

Platform Characteristics

Added wt. (lb)	σ_n (1/sec)	C/C_c
0	6.01	0.050
3	4.67	0.055
6	4.00	0.061

The experimental runs are designated by a letter indicating the wave characteristics and a number indicating the amount of added weight. Thus, run D3 used wave D and 3 pounds were added to the deck of the plat form.

The experimental results for six runs are shown in Figs. 5, 6, and in which the horizontal platform displacement is plotted as a function of wave phase angle. The theoretical displacement as calculated from eq. (is also shown for comparison. The runs were chosen to illustrate the agı ment between theory and experiment for ratios of wave frequency to natur frequency (σ/σ_n) ranging from 0.66 to 1.28. In general, the agreement is good, considering that all wave force parameters were determined analyt cally. The most serious disagreement occurs, as would be expected, for run D3 (σ/σ_n = 1.10) which corresponds to the damped resonant condition maximum displacement.

A digital computer program (Fortran) was developed for the theoret displacement calculations. The calculations were done at the M.I.T. Cor putation Center on an I.B.M. 7090 computer.

CONCLUSIONS

The analytical procedure using the equivalent spring-mass system f the dynamic analysis of offshore structures has been verified by laboratoı tests. The magnitude of the dynamic displacement of the platform deck m be of interest in itself. More generally, the dynamic displacement is use in the stress analysis of the entire structure. The usual practice has bee to design statically on the basis of the largest wave to be encountered. It may well be true that for the "design" wave the statical analysis is correc since, in general, the period of the highest wave may be large compared t the natural period of vibration of the platform. However, smaller waves lesser period may approach resonance with the platform. It is not difficu to foresee cases in which stresses under near resonant conditions could e ceed the stresses of the statical design wave. For example, the steepnes of wave D is almost twice as high as wave B; however, the displacement (hence, the stresses) for wave B at σ/σ_n = 1.00 is twice as high as for wa D at σ/σ_n = 1.28. Another comparison may be made on the basis of the ra of displacements calculated both statically and dynamically. For test B6, the measured dynamic displacement is six times the calculated static displacement. Consequently, stresses will be greater by the same ratio.

Field tests on Texas Tower No. 4 (Ref. 8) off the eastern coast of the U.S. indicate that platform displacements (about 3 inches) were greater for 10 foot waves than for 30 foot waves. This would appear to be further evidence of the necessity of considering dynamic behavior in the design of offshore structures.

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