CHAPTER 5

TRANSFORMATION, BREAKING AND RUN-UP OF A LONG WAVE OF FINITE HEIGHT

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INTRODUCTION

On studying the transformation, breaking and run-up of a relatively steep wave of a short period, the theory for waves of permanent type has given us many fruitful results. However, the theory gradually loses its applicability as a wave becomes flat, since a considerable deformation of the wave profile is inevitable in its propagation.

In § 1, a discussion concerning the transformation of a long wave in a channel of variable section is presented based on the non-linear shallow water theory. Approximate solutions obtained by G. B. Whitham's method (1958) are shown. Further, some brief considerations are given to the effects of bottom friction on wave transformation.

In § 2, breaking of a long wave is discussed. Breakings on a uniformly sloping beach and on a beach of parabolic profile are considered and the effects of beach profile on breaking are clarified.

Finally in § 3, experimental results on wave run-up over 1/30 slope are described in comparing with the Kaplan's results.

1. TRANSFORMATION OF A LONG WAVE OF FINITE HEIGHT.

1. 1. TRANSFORMATION IN SHOALING WATER.

When the effects of bottom friction is neglected, the conservation equations of mass and momentum in the non-linear shallow-water theory are

$$[\, v^*(\eta^* + h^*)\,]_{x^*} = -\eta^*_{t^*} \tag{1-1}$$

and

$$v^*_{t^*} + v^* v^*_{x^*} = -g^* \eta^*_{x^*} \tag{1-2}$$

where the symbols η^*, h^*, and x^* are defined in Figure 1, v^* is velocity, t^* is time, and g^* is the gravitational acceleration. The asterisks denote dimensional quantities. The following dimensionless variables are introduced for the sake of simplicity :

$$x = x^*/\ell_o^* \quad , \quad h = h^*/h_o^* \quad , \quad \eta = \eta^*/h_o^*$$
$$v = v^*/v_o^* \quad , \quad t = t^*/t_o^*$$

where ℓ_o^* = distance from the origin to the shoreline
h_o^* = water depth at the origin
$t_o^* = \ell_o^*/(g^* h_o^*)^{1/2}$, $v_o^* = (g^* h_o^*)^{1/2}$

Let the depth be given by $h = h(x)$, the basic equations are as follows when these dimensionless variables are substituted :

$$[v(h+\eta)]_x = -\eta_t \tag{1-3}$$

$$v_t + v v_x = -\eta_x \tag{1-4}$$

The characteristic equations to be derived from these hyperbolic equations are

$$dx/dt = v + c \tag{1-5}$$

$$d(v + 2c) + dt = 0 \tag{1-6}$$

$$dx/dt = v - c \tag{1-7}$$

$$d(v - 2c) + dt = 0 \tag{1-8}$$

where $\quad c^2 = h + \eta$

Next, a compressive wave propagating shoreward will be considered. It is well known that a compressive wave continues to deform its profile in its propagation and eventually breaks by the curling of the wave front. And from the physical point of view, the wave is considered to form a bore after breaking. In this meaning the theory of a bore in shoaling water given by H. B. Keller, D. A. Lavine, and G. B. Whitham (1960) would give some informations on the transformation of a deformed long wave.

Suppose that particle velocity and propagation velocity of a wave in shoaling water are related to surface elevation by the following expressions:

$$v = 2\sqrt{h}\,(\sqrt{1+M} - 1) \tag{1-9}$$

$$c = \sqrt{h}\,\sqrt{1+M} \tag{1-10}$$

where $$M = \eta / h$$

Since the above relations are precise solutions for the case of a uniform depth, they must also be approximately applied in shoaling water, provided the change of water depth within the distance between the wave front and the crest is not large. It is of interest to compare the transformation of a wave governed by the above relations with that of a bore, since the above relations differ a little from the bore conditions as are shown in the following relations :

$$v'/v \cong (1+M-\frac{15}{32}M^2)$$

$$c'/c \cong (1+\frac{1}{4}M-\frac{1}{32}M^2)$$

where v' is the particle velocity just behind the bore and c' is the propagation velocity of a bore.

Substitution of Eq.(1-5) into Eq.(1-6) leads to

$$d(v+2c) - dh/(v+c) = 0 \tag{1-11}$$

And substitution of Eqs.(1-9) and (1-10) into Eq.(1-11) yields the differential equation concerning M as a function of h :

$$\frac{1}{h}\frac{dh}{dM} = -\frac{2\{3\sqrt{1+M}-2\}}{\{\sqrt{1+M}-1\}\{6\sqrt{1+M}-1\}\sqrt{1+M}} \tag{1-12}$$

When M is small, Eq.(1-12) is approximately equivalent to

$$dh/h = -\frac{4}{5}\,dM/M \tag{1-13}$$

Therefore, $$M \propto h^{-5/4}, \quad \eta \propto h^{-1/4} \tag{1-14}$$

This is a well known relation for waves of small height.

Integration of Eq.(1-12) gives

$$h = A_0(\sqrt{1+M}-1)^{-4/5}(6\sqrt{1+M}-1)^{-6/5} \tag{1-15}$$

Where A_0 is an integration constant to be determined by a boundary condition.

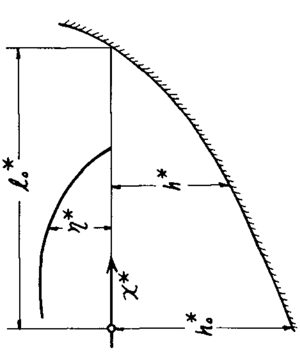

Fig. 1. Definition sketch of a long wave on a sloping beach.

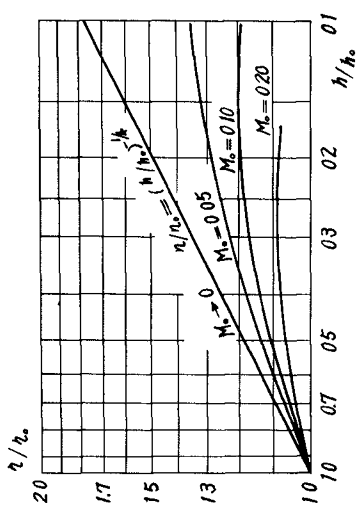

Fig. 2. Relation between (η/η_0) and h/h_0 for a long wave in shoaling water.

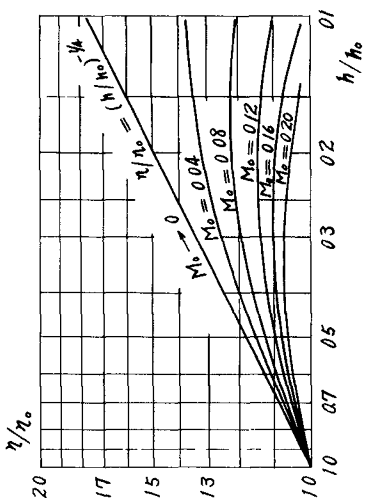

Fig. 3. Relation between (η/η_0) and (h/h_0) for a bore in shoaling water (by H. B. Keller, D.A. Levine and G.B.Whitham).

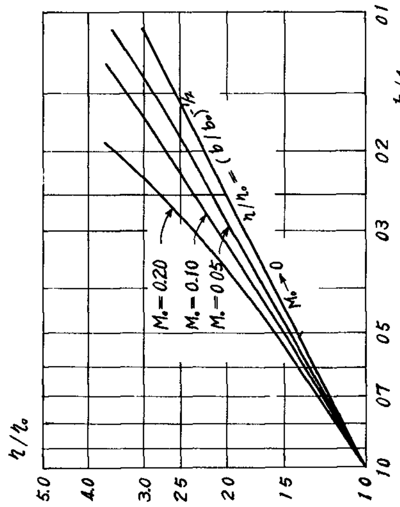

Fig. 4. Relation between (η/η_0) and (b/b_0) for a long wave in a channel of variable width.

Under the boundary condition $M = M_0$ at $h = 1$, Eq.(1-15) becomes

$$h = \left(\frac{\sqrt{1+M_0} - 1}{\sqrt{1+M} - 1}\right)^{4/5} \left(\frac{6\sqrt{1+M_0} - 1}{6\sqrt{1+M} - 1}\right)^{6/5} \tag{1-16}$$

Turning to the details of the relation given by Eq.(1-12), it will be seen that M increases monotonically as h decreases when $M > 0$. However, this manner of variation does not exist in the height η . The maximum of η occurs when

$$d\eta / dh = M + h\, dM/dh = 0 \tag{1-17}$$

Substitution of Eq.(1-12) into Eq.(1-17) gives

$$M = 7/9 \quad , \quad or \quad \eta_{max} = 0.778\, h \tag{1-18}$$

The relation between (η/η_0) and (h/h_0) obtained from Eq.(1-16) is illustrated in Figure 2. For the purpose of comparison the corresponding relation for a bore obtained by H. B. Keller, D. A. Levine and G. B. Whitham (1960) is illustrated in Figure 3.

In the above analysis two interesting features are found, one of them being that the rate of amplification of wave height in decreasing depth decreases as the relative wave height increases and the other being the existence of the maximum of wave height.

1. 2. TRANSFORMATION IN A CHANNEL OF VARIABLE WIDTH.

Suppose a channel has a variable width and a uniform depth and let the dimensionless channel width be given by $b = b(x)$. Dimensionless variables are defined as follows :

$$b = B^*/h_0^* , \quad \eta = \eta^*/h_0^*, \quad x = x^*/h_0^*$$

$$u = u^*/u_0^* , \quad and \quad t = t^*/t_0^*$$

where $B^* =$ *width of channel*, $h_0^* =$ *uniform water depth*

and $v_0^* = (g^* h_0^*)^{1/2}$, $t_0^* = (h_0^*/g^*)^{1/2}$

The positive characteristic equations of the conservation equations of mass and momentum, in the dimensionless form, are

$$dx/dt = v + c \tag{1-19}$$

$$d(v+2c) + vc\,(b_x/b)\,dt = 0 \tag{1-20}$$

By the combination of the above two equations, one has

$$d(v+2c) + \frac{vc}{v+c}\frac{db}{b} = 0 \tag{1-21}$$

When $h = 1$ is considered, substitution of Eqs.(1-9) and (1-10) into Eq.(1-21) yields

$$\frac{db}{b} = -\frac{3\sqrt{1+\eta} - 2}{(1+\eta)(\sqrt{1+\eta} - 1)}\,d\eta \tag{1-22}$$

When η is small, Eq.(1-22) is approximately

$$\frac{db}{b} = -2\frac{d\eta}{\eta} \tag{1-23}$$

which gives

$$\eta \propto b^{-1/2} \tag{1-24}$$

This is the well known relation for waves of small height. Under the boundary condition $\eta = \eta_0$ at $b = b_0$, Eq.(1-22) becomes

$$\frac{b}{b_0} = \left(\frac{1+M}{1+M_0}\right)^2 \left(\frac{\sqrt{1+M} - 1}{\sqrt{1+M_0} - 1}\right)^{-2} \tag{1-25}$$

The relation between (η/η_0) and (b/b_0) obtained from Eq.(1-25) is illustrated in Figure 4. The wave transformation in a channel of variable width shows different character from the preceding case of variable depth. The rate of amplification of wave height in converging channel continuously increases as the relative height of wave increases.

1. 3. EFFECTS OF BOTTOM FRICTION ON WAVE TRANSFORMATION.

The conservation equation of momentum which accounts for the effects of bottom friction is given by

$$v^*_{t^*} + v^* v^*_{x^*} = -g^* \eta^*_{x^*} - k'(v^{*2}/h^* + \eta^*) \tag{1-26}$$

where $k' = g^*/C_e^{*2}$ and $C_e^* =$ Chézy's roughness factor

By using the same dimensionless variables as in 1.1 the positive characteristic equations of conservation equations of mass and momentum on a uniformly sloping beach, $h = 1 - x$, can be given by

$$dx/dt = v + c \tag{1-27}$$

$$d(v+2c) + \{1 + k(v/c)^2\}\, dt = 0 \tag{1-28}$$

where $k = k'/s$, and $s =$ beach slope.

Substitution of Eqs.(1-9) and (1-10) into the equation which is obtained by combining the above two equations leads to

$$\frac{1}{h}\frac{dh}{dN} = -\frac{4N^2(3N-2)}{N^2(6N^2-1)(N^2-1) - 4k(N-1)^2} \tag{1-29}$$

where $N = \sqrt{1+M}$

Provided the terms smaller than $(N-1)^3$ can be neglected, Eq.(1-29) approximately integrates to

$$h \cong A_0\, \varepsilon^{-4/5}(\alpha\varepsilon + \tfrac{5}{4})^{\beta} \exp(-\tfrac{7}{2}\varepsilon) \tag{1-30-1}$$

where $\varepsilon = N - 1$

$\alpha = 4 - k$

$\beta = \frac{1}{\alpha}(5 - \frac{35}{4\alpha} - \frac{4\alpha}{5})$

In a special case of $\alpha = 0$, $k = 4$ the above relation can be reduced to

$$h \cong A_0\, \varepsilon^{-4/5} \exp\{-2\varepsilon(\tfrac{7}{5}\varepsilon + 2)\} \tag{1-30-2}$$

In the case of a uniform depth the attenuation of wave height will be expressed by

$$d(v+2c) + k'(v/c)^2 \frac{dx}{v+c} = 0 \tag{1-31}$$

Where $k' = g^*/C_e^{*2}$ and the dimensionless variables are similar to those in 1. 2. Substitution of Eqs.(1-9) and (1-10) into Eq.(1-31) gives

$$dx = -\frac{\sqrt{1+\eta}\,(3\sqrt{1+\eta} - 2)}{2k'(\sqrt{1+\eta} - 1)^2}\, d\eta \tag{1-32}$$

which can be integrated as

$$x = \frac{1}{k'}\left[\left(\frac{1}{N-1} - \frac{1}{N_0-1}\right) - 5\log\left(\frac{N-1}{N_0-1}\right) - 4(N-N_0) - \frac{3}{2}(N^2-N_0^2)\right] \tag{1-33}$$

where $N = \sqrt{1+\eta}$ and N_0 is N at $x=0$

As an example, attenuation of long wave under the condition $k'=0.01$ is illustrated in Figure 5. However, the value of roughness factor, $k'=0.01$ is only an example and experimental studies are necessary to discuss further.

2. BREAKING OF A LONG WAVE IN SHOALING WATER.

A long wave continues to deform due to the difference of its local propagation velocity and eventually breaks by the curling of its front. From the mathematical point of view, breaking points are expressed by an envelope of interesections of characteristic curves. In the following a wave which has a non-zero slope at the wave front and propagates shoreward into quiescent water is considered.

At first, a uniformly sloping beach with a depth of $h^* = S(l_0^* - x^*)$ will be considered, here S is the beach slope. It takes a dimensionless form of

$$h = 1 - x \tag{2-1}$$

Considering a characteristic curve $dx/dt = v + c$, which starts from the origin at time $t=\tau$, one has the following equation from the relation given by Eq.(1-6) :

$$c(t) = c(\tau) - \frac{1}{2}(t-\tau) - \frac{1}{2}\{v(t) - v(\tau)\} \tag{2-2}$$

Then, the positive characteristic curve is

$$dx/dt = v + c = c(\tau) - \frac{1}{2}(t-\tau) + \frac{1}{2}\{v(t) + v(\tau)\} \tag{2-3}$$

By the combination of Eqs.(1-9) and (1-14), the particle velocity at time t along the above characteristic curve can be given approximately by

$$v(t) \cong \eta(\tau)h^{-3/4} - \frac{1}{4}\eta^2(\tau)h^{-2} \tag{2-4}$$

Substituting Eq.(2-4) into Eq.(2-3) and considering Eq.(2-1) one has

$$\frac{dx}{dt} = (1+\sigma)^{1/2} - \frac{1}{2}(t-\tau) + \sigma\left\{1 + \frac{3}{8}x + \frac{21}{64}x^2 + \frac{77}{256}x^3 + \cdots\right\}$$
$$- \sigma^2\left\{\frac{1}{4} + \frac{1}{4}x + \frac{3}{8}x^2 + \frac{1}{2}x^3 + \cdots\right\} \tag{2-5}$$

where $\sigma = \eta(\tau)$ and $c(\tau) = (1+\sigma)^{1/2}$

It is assumed that the solution of Eq.(2-5) is expressed as a power series of σ

$$x = x_0 + \sigma x_1 + \sigma^2 x_2 + \cdots \tag{2-6}$$

The intersections of characteristic curves are obtained from

$$\frac{\partial x}{\partial \tau} = \frac{\partial x_0}{\partial \tau} + \sigma\frac{\partial x_1}{\partial \tau} + \frac{\partial \sigma}{\partial \tau}x_1 + \cdots = 0 \tag{2-7}$$

The initial breaking point is given by putting $\tau = 0$ and $\sigma = 0$ in Eq.(2-7). Thus, the initial breaking point is determined from the first two terms of the right hand side of Eq.(2-6).

Substituting Eq.(2-6) into Eq.(2-5) and considering the initial condition, $x = 0$ when $t = \tau$, one has

$$\left.\begin{aligned} x_0 &= (t-\tau) - \frac{1}{4}(t-\tau)^2 \\ x_1 &= 1.500000(t-\tau) + 0.187500(t-\tau)^2 \\ &\quad + 0.078125(t-\tau)^3 + 0.034180(t-\tau)^4 \\ &\quad + 0.015381(t-\tau)^5 + \cdots \end{aligned}\right\} \tag{2-8}$$

With the substitution of Eq.(2-8) into Eq.(2-7) a relation to give the initial breaking time t_b is obtained.

$$\begin{aligned} -1 + \frac{1}{2}t_b + m\,t_b\,(&1.500000 + 0.187500\,t_b \\ &+ 0.078125\,t_b^2 + 0.034180\,t_b^3 + 0.015381\,t_b^4 \\ &+ \cdots) = 0 \end{aligned} \tag{2-9}$$

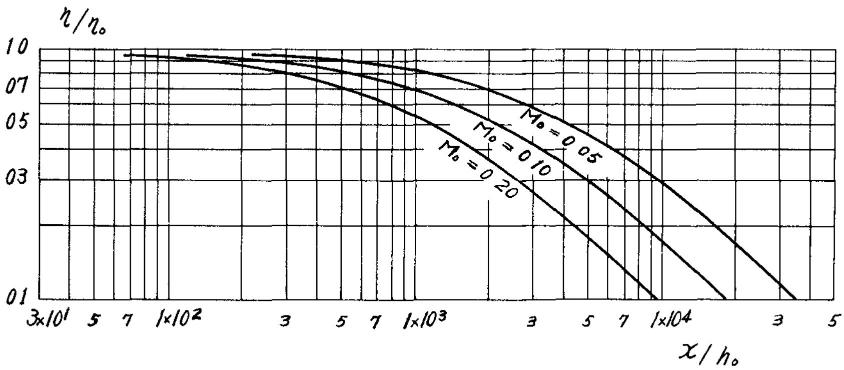

Fig.5. Attenuation of a long wave in a channel of uniform depth with roughness factor k' = 0.01.

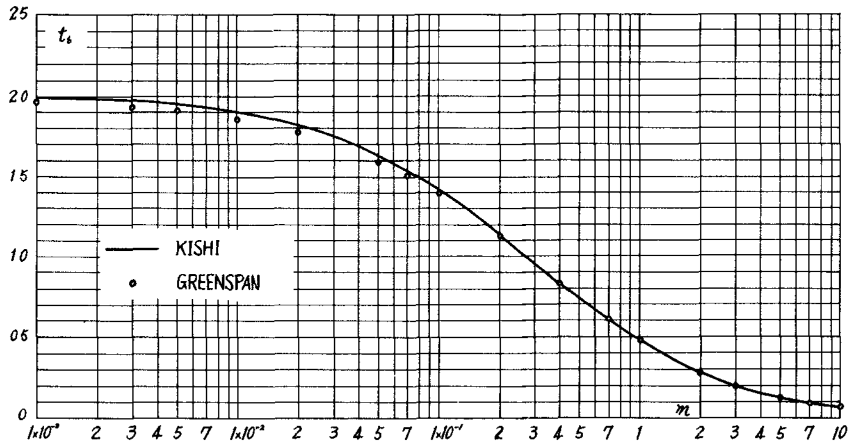

Fig. 6. Breaking time t_b as a function of initial slope m for a uniformly sloping beach.

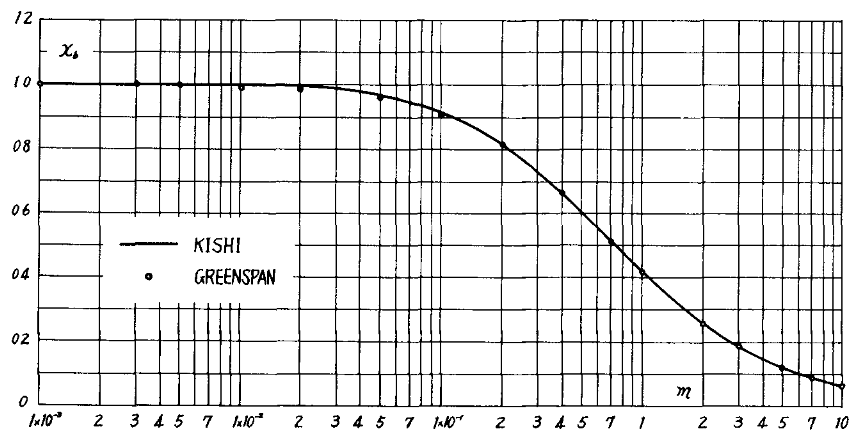

Fig. 7. Breaking position x_b as a function of initial slope m for a uniformly sloping beach.

where $$m = \left(\frac{\partial\sigma}{\partial\tau}\right)_{\sigma=0} = -\left(\frac{\partial\sigma}{\partial x}\right)_{\sigma=0} \quad *$$

The corresponding breaking point x_b can be found by $x_b = x_0(t_b)$, sinoe $\tau = 0$ and $\sigma = 0$:

$$x_b = t_b - \frac{1}{4} t_b^2 \tag{2-10}$$

The curves of t_b and x_b as a function of initial slope m are shown in Figure 6 and 7, respectively. For the purpose of comparison, the corresponding values obtained by H. P. Greenspan (1958) are plotted in the figure which shows a good agreement with the present theory.

The above theory gives an explanation about the effects of beach slope on wave breaking. Provided the wave is sinusoidal, the relation between the initial slope m and the wave steepness can be given by

$$m = (\pi / S)(H_i / L_i)$$

where
H_i = wave height at the origin
L_i = wave Length at the origin
S = beach slope

Thus, the breaking position is determined for the given values of beach slope and wave steepness. And the transformation of wave height between the origin and the breaking position is assumed to satisfy the following relation (the analysis in § 1 considers only a compressive wave or elevation, and a rarefaction wave or depression is not considered) :

$$H_b / H_i \cong (1 - x_b)^{-1/4}$$

Then graphs showing the relation between H_b/H_i and H_i/L_i are obtained for the given values of beach slope.

However, the above graphs are not the relations between deep water and the breaking position. Then a relation between the deep water and the origin is also of interest to be calculated. In the present calculations relative depth at the origin h_i/L_i is assumed to be 0.044 . Provided the small amplitude theory is applied, the relations between the deep water and the origin are

$$H_b / H_o = 1.4\ (H_b / H_i)$$

$$H_o / T^2 = 5.12\ (H_o / L_o) = H_i / L_i$$

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* At the origin $(C)_{\sigma=0} = 1$

Results of the present calculations are compared with the experimental curves given by H. W. Iversen (1952) in Figure 8. It would be seen from the figure that the present theory gives an explanation about the effects of beach slope on wave breaking, although some assumptions are included in this calculations.

Next, a consideration on the effects of bottom profiles on wave breaking will be given. For an example, let the depth $h=(1-x)$ be replaced by

$$h = 1 - x^2 \tag{2-11}$$

The positive characteristic equations are

$$dx/dt = v + c \tag{2-12}$$

$$d(v+2c) + 2x\,dt = 0 \tag{2-13}$$

Eq.(2-13) can be written as

$$v(t) + 2c(t) = v(\tau) + 2c(\tau) - 2\int_\tau^t x\,dt$$

$$\text{or} \quad c(t) = c(\tau) - \frac{1}{2}\left\{v(t) - v(\tau)\right\} - \int_\tau^t x\,dt \tag{2-14}$$

Combination of Eqs.(2-13) and (2-14) gives the following equation of characteristic curve which can be derived by the same reduction described in the first example :

$$\frac{dx}{dt} = (1+\sigma)^{1/2} - \int_\tau^t x\,dt + \sigma\left\{1 + \frac{3}{8}x^2 + \frac{21}{64}x^4 + \frac{77}{256}x^6 + \cdots\right\}$$
$$- \sigma^2\left\{\frac{1}{4} + \frac{1}{4}x^2 + \frac{3}{8}x^4 + \frac{1}{2}x^6 + \cdots\right\} \tag{2-15}$$

Assuming that the solution of Eq.(2-15) is given by a power series of σ such as Eq.(2-6), the function x_0 , x_1 , ········ can be determined successively.

$$\frac{dx_0}{dt} = 1 - \int_\tau^t x_0\,dt \tag{2-16}$$

$$\frac{dx_1}{dt} = \frac{3}{2} - \int_\tau^t x_1\,dt + \frac{3}{8}x_0^2 + \frac{21}{64}x_0^4 + \frac{77}{256}x_0^6 + \cdots \tag{2-17}$$

Since the initial condition is $x = 0$ when $t = \tau$, the solution of Eq.(2-16) which satisfies the initial condition is assumed to have the form

$$x_0 = \sum_{n=0}^{\infty} a_{2n+1} (t-\tau)^{2n+1} \tag{2-18}$$

where a_n is a constant to be determined from Eq.(2-16). Substitution of Eq.(2-18) into Eq.(2-16) yields

$$x_0 = (t-\tau) - \frac{1}{6}(t-\tau)^3 + \frac{1}{120}(t-\tau)^5 - \frac{1}{5,040}(t-\tau)^7 + \cdots$$

$$\text{or} \quad x_0 = (t-\tau) - 0.16667(t-\tau)^3 + 0.008333(t-\tau)^5 - 0.000198(t-\tau)^7 + \cdots \tag{2-19}$$

In the same way, one has

$$x_1 = 1.500000(t-\tau) - 0.125000(t-\tau)^3 + 0.046875(t-\tau)^5 + 0.012984(t-\tau)^7 + \cdots \tag{2-20}$$

Thus, the initial breaking time t_b and position x_b are given by Eqs.(2-21) and (2-22), respectively.

$$-1 + 0.500000\, t_b^2 - 0.041667\, t_b^4 + 0.001389\, t_b^6 + m t_b (1.500000 - 0.125000\, t_b^2 + 0.046875\, t_b^4 + 0.012984\, t_b^6 + \cdots) = 0 \tag{2-21}$$

$$x_b = t_b - 0.166667\, t_b^3 + 0.008333\, t_b^5 - 0.000198\, t_b^7 + \cdots \tag{2-22}$$

The curves of t_b and x_b as a function of the initial slope m are shown in Figure 9. Then, two values of water depth at the breaking position which are obtained on the uniformly sloping beach and the parabolic beach, respectively, are compared in Figure 10. As will be seen from the figure, breaking of a long wave is affected by the beach profile. This may be a reason why a large scattering is found in connecting the breakers of very flat waves with the deep water waves.

3. EXPERIMENTS ON THE RUN-UP HEIGHT OF A LONG WAVE OVER A UNIFORMLY SLOPING BEACH.

The author carried out some preliminary experiments on the run-up height of a long wave with the aim of investigating the hydraulic behaviours of Tsunami which runs up a beach. Few data exist about the run-up characters

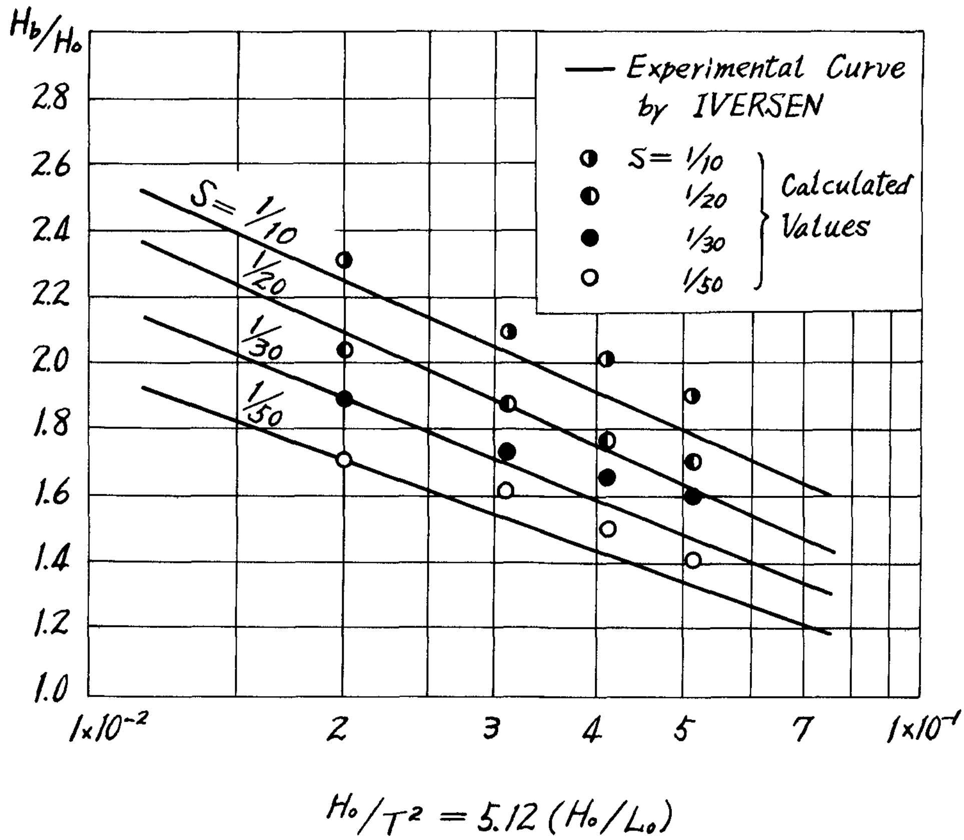

Fig. 8. Effects of beach slope on wave breaking.

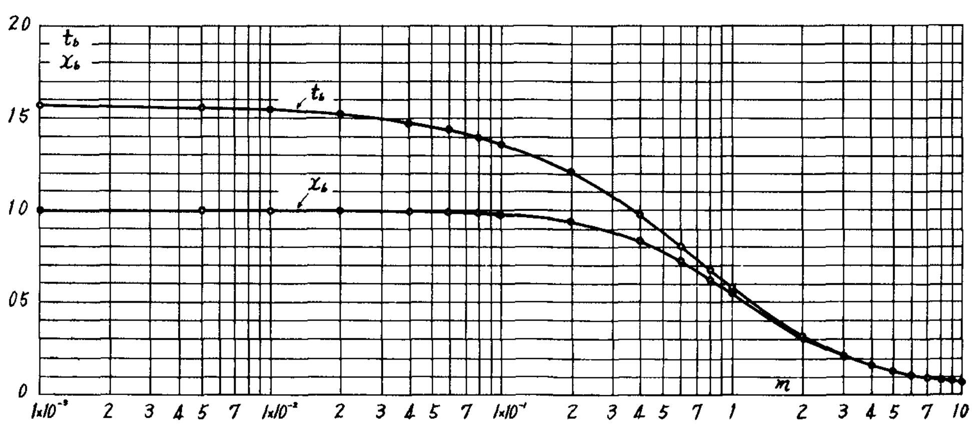

Fig. 9. Breaking time t_b and position x_b as a function of initial slope m for a beach with parabolic profile.

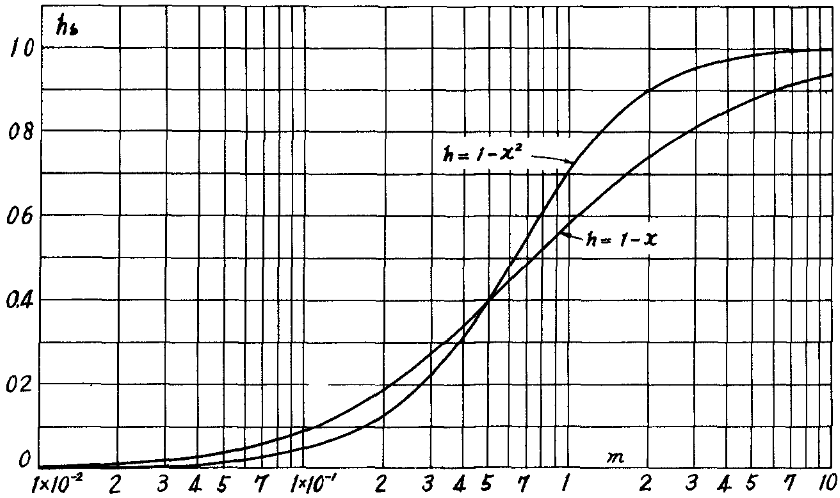

Fig. 10. Effects of beach profile on wave breaking.

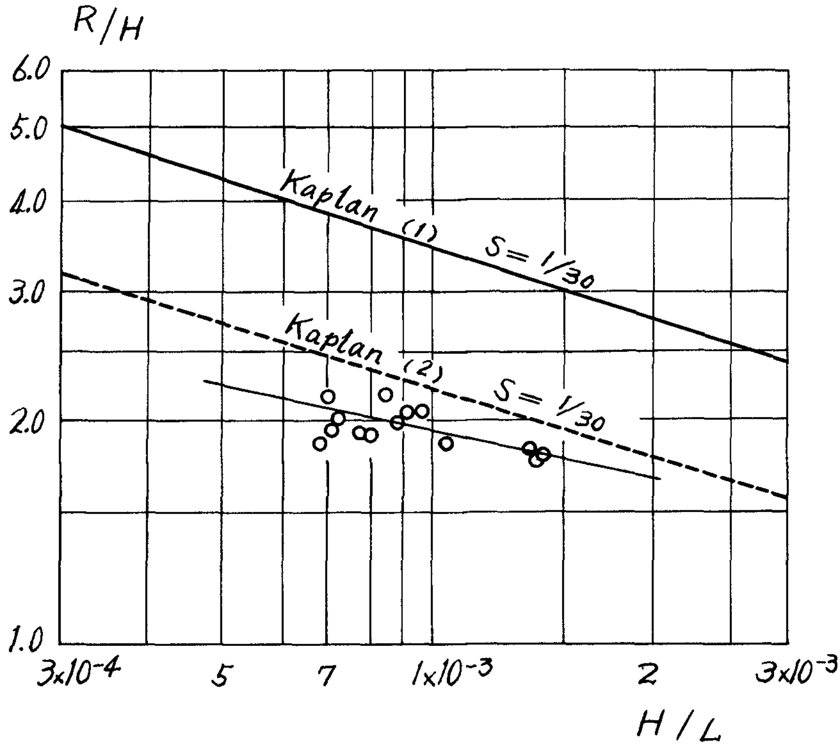

Fig. 11. Run-up of a long wave on 1/30 slope.

of very flat waves as Tsunami which usually has a period of several ten minutes.

Experiments were carried on in a wave tank of 14cm width installed with a pneumatic wave generator at one end and a beach of 1/30 slope at the other end. Water depth at the foot of the slope was 9cm. Three kinds of wave periods, 10sec, 15sec, and 20sec were given by adjusting the rotating speed of a rotary valve. Values of wave steepness at the foot of the beach were varied from 7×10^{-4} to 1.4×10^{-3}.

Relation between the relative run-up height and the wave steepness is shown in Figure 11. In the figure, R is the run-up height above S.W.L., H and L are wave height and wave length at the foot of the slope, respectively. For the purpose of comparison, the experimental curve obtained by K. Kaplan (1952) is indicated on the same figure. Kaplan's relation is

$$R/H = 0.381\,(H/L)^{-0.316} \quad \text{for 1 on 30 slope} \qquad (3\text{-}1)$$

Since a solitary wave was used in Kaplan's experiments, two curves are shown ; the one is the curve in full-line on which the height of solitary wave is replaced by the height of the long wave and the other is the curve in broken line on which the height of solitary wave is replaced by the half wave height of the long wave.

The author's results are fairly close with the latter curve, though the points in the present experiments stand somewhat below. A little inconsistency of the two experiments is presumably due to the difference of hydraulic characters between a long wave and a solitary wave. However, the author would like to reserve further discussions in this respect, since the scale of the experiments was too small to investigate the details.

4. CONCLUSION

1) By the use of approximate expressions for particle velocity and propagation velocity, a relation of wave transformation in shoaling water is derived. The rate of amplification of wave height in decreasing depth decreases as the relative wave height increases. And the maximum height of wave crest appears when $\eta = 0.778h$.

2) A compressive wave is considered to form a bore after its breaking. A relation on the transformation of a bore in shoaling water has been presented by H. B. Keller, D. A. Levine and G. B. Whitham. For the purpose of comparison, two relations of transformation for a long wave and a bore are shown in Figure 2 and Figure 3, respectively.

3) Further, a relation of wave transformation in a channel of variable width is derived. The rate of amplification of wave height in converging width increases as the relative wave height increases.

4) Brief descriptions about the effects of bottom friction on wave transformation are given in section 3, § 1.

5) An approximate method to calculate the wave deformation is presented. As an example, the initial breaking time and position for uniformly sloping beach are calculated. The author's results show good agreement with the Greenspan's precise solution. Next, the initial breaking time and position for a beaoh of parabolic profile are calculated. The effects of beaoh profile on wave breaking are clarified.

6) The fact that the breaker height is affected by beach slope for a very flat wave has been shown in the Iversen's experiments. An explanation of the above fact is given on the basis of theoretical breaking conditions.

7) Experimental results on wave run-up of a long wave for 1/30 slope are described. An experimental relation between the relative run-up height and the wave steepness is presented. The relation is fairly close with the Kaplan's curve provided the height of the solitary wave is replaced by the half wave height of the long wave, though the points in the present experiments stand somewhat below.

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