# Chapter 39 THE CLAMP-ON WAVE FORCE METER

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## INTRODUCTION

Because of the tremendous increase in offshore activities, a great effort has been made on obtaining information on wave forces on structural members. Several oil companies have invested large sums of money in the design and construction of full-scale systems for measuring the wave forces. The equipment used for measuring the forces have been single cantilevers or segmented piles designed to make discrete measurements along the pile. For instance, during the last five years, The California Company and California Research Corporation (subsidiaries of Standard Oil Company of California) operated an installation in the Gulf of Mexico with four segmented piles of different diameters. The wave forces were measured by three-foot high force dynamometers located at seven different elevations along the length of each test pile. Each dynamometer was constructed from a section of the cylindrical pile which was attached to a system of flexures on the inside.\*\*

So far the wave forces have been measured on cylindrical piles varying in diameter from one to four feet and in water depths varying from 30 to 50 feet. As the pile diameter and water depth increase, however, the measurements of wave forces by use of a cantilever or a segmented pile become very difficult and expensive. Therefore, a need exists for investigating other means for measuring the wave forces on a pile. This paper will describe the design and operation of a force meter that may be clamped to an existing pile.

In Spring 1960, California Research Corporation installed equipment incorporating eight of the clamp-on meters on an oil well drilling platform in the Gulf of Mexico. The water depth at the location is 100 feet, and two years of operation are planned.

GENERAL DESCRIPTION OF CLAMP-ON FORCE METER

In the design of the clamp-on force meter, it was desirable to incorporate the following features:

1. Force meter should be in the form of a ring which may be clamped on to an existing pile.

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This type of dynamometer will be referred to as conventional dynamometer throughout the text of this paper.

2. The outside diameter of the ring should be as close to the pile diameter as possible and not deface the existing structure.

3. The meter should be easily attached and removed.

4. From the output of the meter it should be possible to determine the resultant force on the ring and the direction of the resultant force.

In the design of such a meter, the most difficult requirement to satisfy is Item 2. A clamp-on force meter designed similar to the conventional type force meter has a tendency to become large in size relative to the pile diameter. The reason is that the continuous ring has to be relatively stiff and has to be supported by a system of flexures locatec in the annulus bounded by the outside diameter of the existing pile and the inside diameter of the force meter.

A more promising design for a force meter 15 one in which the pressure is measured at equal intervals around the circumference by use of pressure transducers. The total pressure force on a unit height of a pile may then be obtained by integrating the pressure on the individual surface elements around the circumference.

The total force acting on a unit height of a pile consists of a pressure force and a friction force. In the case of cylinders, limited experimental investigations indicate that for Reynolds number larger than  $4 \times 10^4$  the skin friction or viscosity forces are only two per cent of the total force. In most engineering problems involving wave forces, the Reynolds number is larger than  $10^5$ . Even though the viscosity forces are apparently negligible, they do exert an important effect in distorting the velocity field around the pile. A distortion in the velocity field in turn produces a change in the distribution of the pressure around the circumference. In the design of the clamp-on meter, the friction forces themselves have been neglected in comparison with the pressure forces.

Knowing the pressure at discrete points around the circumference of the pile allows the derivation of an approximate expression for the total force per unit height.

Assuming, for instance, that the pressure varies linearly with the angular position between two points of known pressure the expressions for the total force per unit height in terms of two mutually perpendicular components are (see Appendix I for detailed derivation):

$$X = R \frac{2\pi}{N} \left[ \frac{N}{\pi} \sin \frac{\pi}{N} \right]^2 \sum_{n=0}^{N-1} P_n \sin \frac{2\pi n}{N}$$
(1)

$$Y = R \frac{2\pi}{N} \left[ \frac{N}{\pi} \sin \frac{\pi}{N} \right]^{2} \sum_{n=0}^{N-1} P_{n} \cos \frac{2\pi n}{N}$$
(2)

where

X and Y components of force in two mutually perpendicular directions (lb/ft)

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- R = outer radius of the meter
- N = number of pressure transducers
- n is integer from O to N
- $P_n$  is pressure force at the nth transducer  $(lb/ft)^2$

The summing or integrating process may be carried out in two ways:

1. The resultant force and its direction may be determined by numerically integrating the recorded output of the individual pressure transducers located around the circumference. In other words, record all the  $P_n$ 's and calculate X and Y by use of equations (1) and (2).

2. The resultant force and direction may be obtained by electrically integrating the output of the individual pressure transducers before recording. In this case it is only necessary to record the output of the X and Y rather than the output of all the  $P_n$ 's.

Method No. 1 requires the recording of a large number of outputs. For instance, an instrumented pile with 8 clamp-on meters each containing 16 transducers would require simultaneous recording of 128 outputs. Not only is it impractical to record a large number of outputs by use of an oscillograph, but the data reduction would be very time consuming. If a recorder were available that could handle the large number of traces required and the output recorded in such a manner that the data could be reduced automatically, this method may be worthwhile considering.

Using an oscillograph-type recording, it will be more practical to use the second method, in which case two mutually perpendicular components of the total force from each force meter are recorded. The distribution around the band may also be obtained by sampling the output of the transducers from a single force meter one at a time. In the example of 8 force meters, 16 X and Y outputs may be recorded continuously during a recording period, while the output of the individual transducers on a meter may be recorded by time sharing the recorder. The force meter described below is designed with this type of recording procedure in mind.

Figure 1 shows a prototype clamp-on meter mounted on a threefoot dummy pile. The meter consists of two half bands clamped together on the pile and held in place by the friction between the pile and the band. The electrical wires from the transducers are carried to the recording instrument through watertight rubber hoses which may be placed either on the outside or the inside of an existing pile. The 16 pressure transducers are so designed and connected together that the normal force acting at each transducer may be recorded in addition to the X and Y component of the total integrated force acting on a unit height of a pile A more complete description of the transducer and the integrating circuit will be given later in the paper.

The clamp-on force meter has several advantages over the conventional force meter presently used at several installations. The most important advantages are listed below: 1. The meter may be clamped on to an existing structural pile thereby eliminating the costly test pile.

2. The meter may be removed for repairs or relocated on the pile in such a manner as to obtain the maximum number of data points for a given wave height. For instance, if information is desired on smal waves, several meters may be concentrated near the MWL.

3. From the pressure distribution data, information may be obtained on particle velocity, drag coefficient, and vortex shedding.

4. The output traces are "clean" due to the high natural frequency of the transducer sensing element. This feature reduces the error in reading the output and simplifies the data reduction.

DESCRIPTION OF MAJOR COMPONENTS OF THE FORCE METER

#### PRESSURE TRANSDUCERS

In the design of the transducer, used for the clamp-on meter, the following features needed to be incorporated:

1. Each transducer should be provided with three outputs, two for the integrating circuits, and one for the pressure circuit.

2. The output of the transducer versus the applied pressure should be linear.

3. The transducer should be capable of measuring pressures from 0.1 psi to 50 psi.

4. For the purpose of calibrating the clamp-on meter, provisions should be made for applying pressure or vacuum to the sensing element of each individual transducer without affecting the sensing element of the other transducers.

5. For two reasons, consideration should be given to the dimensions of the transducer. First, when the transducers are mounted in the supporting band, the thickness of the band should be as small as possible in order not to deface the existing pile. Second, the height of the sensing element facing the sea should be as small as possible in order to minimize the time that a transducer is partly submerged during wave action.

6. The transducer should be rugged in order to withstand abuse from driftwood.

Because no known commercial transducer met the requirements listed above, a special transducer shown in Figure 2 was developed.

During the development of the transducer, a number of different types of sensing elements were considered, fabricated, and tested.



Fig. 1. Prototype clamp-on meter.



Fig. 2. Pressure transducer.



Fig. 3. Sensing element.



Fig. 4. Typical calibration curve.





Fig. 5. Transducer housing.

Fig. 6. Band for mounting transducers.



Fig. 7. Calibration curve for X-component of force.





HOLTED JOINT HOSES FOR CARRYING ELECTRICAL WIRES

Fig. 8. Clamp-on meter mounted on conventional dynamometer.

Based on the experience obtained during these tests, the final design of the sensing element was arrived at.

#### SENSING ELEMENT

The sensing element of flexure plate shown in Figure 3 is essentially a simply supported wide beam or plate where the end conditions are kept uniform by use of a thin symmetric flexure at each end of the beam. The thickness of the beam is made relatively great in order to minimize the deflection of the beam under load. Small deflection of the beam is important because a rubber covering is used in waterproofing the transducer and any rubber effect would change the linearity of the output. The stress on the outer fiber in the center of the beam is increased by machining a symmetric flexure at the center. Six SR-4 strain gages are mounted on one side of this flexure. If desired, the sensitivity of the sensing element may be varied by changing the thickness of the center flexure. In order to eliminate any axial force in the beam due to the deflection, two parallel cuts are made at one end of the beam. A sensing element and a typical calibration curve are shown in Figures 3 and 4, respectively.

#### TRANSDUCER HOUSING

The housing shown in Figure 5 is made with two compartments. The flexure plate covers the largest compartment and may be kept pressure tight with respect to the second compartment. The wires (total of 12) from the strain gages are carried to the second compartment through a pressure-tight plug and wired to a terminal board. All wire connections necessary to complete the different circuits are made in this compartment.

The wires used to interconnect the different transducers are fed through the side walls of the second compartment. The reason for interconnecting the transducers will be explained later in this paper.

Vacuum or pressure may be applied to each individual transducer through a 1/4 in. tapped hole in the wall between the two compartments. The need for applying different pressure or vacuum to each transducer is important during the calibration of the force meter.

#### TRANSDUCER MOUNTING

Several methods were considered for mounting the transducers in a band to make up the force meter. For instance, one method considered was to mold the transducer in a rubber band which could be strapped onto an existing pile. This method has several advantages in that the band is flexible and would conform easily to the surface of a pile even if it were out of round. The difficulty and expense of making the rubber mold, which would be used only for a limited number of bands, ruled this method out.

After considering several methods for mounting the transducer, it was decided to construct the band from metal. Figure 6 shows the force meter with half the number of the transducers mounted. The band consists of two halves hinged at one edge and bolted together at the other edge.

#### ELECTRICAL INTEGRATING CIRCUIT

As explained before, each transducer consists of a sensing element on which six SR-4 gages are mounted. The position and notation of these gages are shown in Figures 3 and II-1. The gages noted "X" and "Y" are used for the X and Y integrating circuit. respectively, and the gages noted P are used for determining the pressure at the location of each transducer. Further, the gages noted "A" are sensing the principal strain while the gages noted "B" are sensing the Poisson ratio effect. The sensing element from each transducer will have the same number of gages mounted in the same relative position. As seen from Figure 4, the pressure applied to the sensing element is proportional to the gage output. Because it is well known that the output of a strain gage is proportional to the change in resistance of the gage, the summing of terms in Equations 1 and 2 may be accomplished by connecting the corresponding gages from each transducer in series in the legs of a Wheatstone bridge. Consider for instance in "A" and "B" gage from each transducer with the common notation "X". By placing the "A" gages in series in one leg of the bridge, and "B" gages in the adjacent leg, the contribution from each transducer will add.

The sign of  $\sin \frac{2\pi n}{N}$  in Equation 1 is taken care of by placing the gages in the proper leg of the bridge. For  $\sin \frac{2\pi n}{N}$  positive, the "A" gages are connected in series in one leg while the "B" gages are connected in series in the adjacent leg. By placing the "A" and "B" gages in adjacent legs, the bridge is temperature compensated in addition to having increased output due to the Poisson ratio effect. When  $\sin \frac{2\pi n}{N}$  changes sign, the "A" and "B" gages are moved to either one of their adjacent legs.

In addition to adding the output of each transducer with the proper sign, each output has to be reduced to take into account the magnitude of the multiplier  $\sin \frac{2\pi n}{N}$ . This is accomplished by placing a resistor with a predetermined value in parallel with each gage. The value of the parallel resistor will vary from transducer to transducer, however, the same value resistor will be placed parallel to the "A" and "B" gages on each individual transducer.

In short, the integrating circuit for representing one component of force consists of a Wheatstone bridge in which the legs are made up of a series of strain gage elements with proper size parallel resistors. Each transducer contributed two strain gage elements to the circuit.

The integrating circuit for the Y component of force is similar, except another set of strain gage elements make up the circuit. A more detailed explanation of the integrating circuit is given in Appendix II, together with one example of the X and Y circuit (see Figure II-3).

### CALIBRATION

The calibration of the clamp-on meter is somewhat more complicated than the conventional dynamometer, however, it may be performed with lighter equipment and with better accuracy. Rather than pulling on

the meter with a force, the load on the clamp-on meter is obtained by applying a pressure or vacuum to the individual transducers.

The main objective of the calibration is to obtain a relation between the deflection of the galvanometer and the applied force (lbs per ft or lbs per sq ft). In the case of the clamp-on meter, every effort was made to design the transducers such that the galvanometer deflection is proportional to force or pressure. This simplifies the data reduction in that the galvanometer deflection is converted to force or pressure by merely multiplying by a constant. Any variation of the constant multiplier due to a change in bridge voltage is accounted for by introducing electrical calibration steps on the record.

At the beginning of each recording, a known resistor is connected in parallel with one leg of each of the bridges. This resistor causes an electrical unbalance which will appear on the record as a step with a magnitude that is proportional to the bridge voltage. A force acting on the dynamometer will also cause an electrical unbalance in the corresponding bridge. The force necessary to produce the same unbalance as the calibration resistor with the same applied bridge voltage is referred to as the force or pressure equivalent.

The following paragraphs will be devoted to the discussion of the calibration of the output of the individual pressure transducers and the X and Y integrating circuit as performed on the prototype clamp-on meter in the laboratory. Toward the end of this section, some remarks will be given concerning the field calibration of the clamp-on meter.

### LABORATORY CALIBRATION

#### Individual Transducer Outputs

The calibration of the individual transducers is straightforward. Each transducer was calibrated one at a time by applying increments of pressure or vacuum to the inside of the housing. A recording of the calibration step and the output of the transducers was taken for each pressure setting. A sample plot of the pressure versus output for one transducer is shown in Figure 4.

#### X and Y Outputs

The value of the calibration constant for X and Y circuits may be obtained by either one of the following two methods:

1. Determine the output of the X and Y circuit when a known pressure distribution is applied around the circumference of the meter. The pressure to be applied to each transducer may be determined from the known pressure distribution curve which also is used to calculate the corresponding X and Y component of force. One point on the calibration curve is then obtained by plotting the output of the X and Y versus the calculated force. The complete calibration curve may be generated by either changing the distribution or changing the magnitude of the pressures and keeping the same distribution. This method requires considerable amount of equipment because the pressure in each transducer

has to be recorded at the same time as the output of the X and Y are recorded. This method is therefore not too practical particularly for field use.

2. Apply the same pressure of vacuum to all transducers, one at a time, and note the output of the X and Y circuit. Knowing the contribution to the X and Y output due to a known pressure at each one of the transducers allows the calculation of the X and Y output due to an arbitrary pressure distribution. The calculated X and Y component of force may then be plotted versus the calculated X and Y galvanometer deflection if both calculations are based on the same pressure distribution.

Even though method number 2 requires more computation, this method is somewhat simpler than method number 1 because it is only necessary to pressurize one transducer at a time. A calibration curve obtained by method number 2 is shown in Figure 7.

In actual operation of the meter, the inside of the transducers will be pressurized with nitrogen to a pressure equal to or greater than the expected outside pressure. Therefore, in normal position, the transducer flexures will be deflected outward and when acted upon by wave motion the increased outside pressure will reduce the outward deflection of the flexures. This is true of course as long as the inside pres sure is greater than the outside pressure. It is possible, however, that during high shock pressures that the outside pressure is higher than the inside pressure and the flexures will deflect inward. It is therefore important that the outputs of the X and Y circuit are numerically the same regardless of which side of the flexure the net pressure is applied. The X and Y circuit were therefore calibrated with both inside vacuum (corresponding to excessive outside pressure) and inside pressure to see if the calibration constant were the same in both cases.

#### FIELD CALIBRATIONS

The field calibration of the output of the individual transducers is very simple and requires little time and effort. Under normal operating conditions, the inside of all the transducers on one band are pressurized with nitrogen to the same pressure. The nitrogen is supplied from the instrument house through a hose which also contains the electrical wiring. By changing the pressure by known increments and recording the output of the transducers for each pressure setting, the calibration curve for all the transducers may be generated.

During the calibration of the individual transducer outputs, the X and Y output should also be recorded and if the integrating circuits operate properly, the X and Y trace should not deflect. This is true because all the transducers are subjected to the same pressure. If the X and Y have no output, due to a uniform load around the circumference, it is quite unlikely that the calibration constant has changed. However, this may be checked by an indirect method. After a storm record has been obtained, the recorded values of X and Y may be compared with values of X and Y computed from the known pressure distribution recorded during the same wave. The pressure distribution which is obtained from the output of the individual transducer may be considered accurate

because the output of these transducers are readily calibrated. If a constant difference occurs between the two results, adjustments may be made by changing the calibration constant for the X and Y output.

#### TEST RESULTS

The clamp-on force meter described in this paper was tested in the Gulf of Mexico at the Standard Oil Company of California Wave Force Installation during the last five months of 1958. The meter was mounted on the first dynamometer (designated 322) above the MWL of the threefoot test pile as shown in Figure 8. The exact location and orientation of the meter relative to the conventional type dynamometer is shown in Figure 9.

The purpose of the test was to check the performance of the new meter and compare its outputs with the outputs of the conventional dynamometer.

The clamp-on meter was first tested in an actual storm during Hurricane Ella on Sept. 4 and 5, 1958. During the storm, a total of 475 feet on record was obtained from the outputs of the individual transducers, the A and B components of the conventional 322 dynamometer and from the X and Y output of the clamp-on meter. A sample record of these outputs is shown in Figure 10.

At the time this paper was written a small amount of data had been reduced for the outputs of the individual transducers. One example of the pressure distribution around the pile in the crest region of the wave is shown in Figure 11. A comparison between the force recorded on the 322 conventional dynamometer and the clamp-on meter is shown in Figures 12 and 13. The scatter in the data is easily explained, remembering that the clamp-on meter senses the pressure over a height of 6 inches, while the conventional meter covers a height of 3 feet. A similar pressure distribution around the pile over a 3 feet height of the dynamometer is quite unlikely.

The figure indicates, however, that the forces recorded by the X and Y components on the clamp-on meter are about 15 per cent less than the forces recorded by the A and B components of the conventional dynamometer. This discrepancy may be explained as follows:

1. The calibration of one or both meters is in error. The calibration constant for either one of the meters was determined with an accuracy of probably not less than  $\pm$  5 per cent.

2. The friction force felt by the conventional dynamometer and not by the clamp-on meter may amount to as much as 15 per cent.

3. The vertical gradient of the pressure in the upper part of the wave may be large enough to result in a higher average pressure on the conventional 3 foot high dynamometer than on the clamp-on meter.

Because the exact cause of the 15 per cent discrepancy between the output of the two meters could not be determined, the clamp-on meter was moved to an underwater dynamometer to see if the same deviation





Fig. 9. Location of clamp-on meter relative to 322 dynamometer. 712





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between the two outputs would be obtained at this location. The underwater dynamometer is designated 332 and is located approximately five feet below mean water line. The clamp-on meter was clamped on the 332 dynamometer with the X and Y oriented the same as when clamped on to the 322 dynamometer. At this elevation it could be assumed that the pressure distribution around the pile would be the same over the height of the 3-foot dynamometer. The output of the two meters obtained during a storm which occurred on November 4, 1958, are shown in Figures 14 and 15. As seen from these figures, the force recorded on the conventional meter agrees well with the force recorded on the clamp-on meter.

The discrepancy between the force recorded on the two meters when the clamp-on meter was located at the 322 elevation is then probably due to varying pressure distribution around the pile over the height of the 3-foot dynamometer.



Fig. 14. Comparison between Y and 322A components.



Fig. 15. Comparison between X ar 332B components.

### APPENDIX I

## DERIVATION OF THE THEORETICAL EXPRESSION FOR THE X AND Y COMPONENT OF FORCE

The derivation of X and Y components of force will be based on the following assumptions:

1. The normal pressure force is measured at equal intervals around the circumference of the pile.

2. The skin friction or viscosity forces are neglected.

3. The pressure force varies linearly as a function of angular position in the interval between each pressure pickup or transducer.

Let

N 1 9 R		number of pressure transducers integer from O to N angle as shown in Figure I-1 radius of pile
X and Y Pn Pg	=	component of force in two perpendicular directions pressure at the nth transducer (lb/ft <sup>2</sup> ) pressure at the angle $\phi$ (lb/ft <sup>2</sup> )

Then

$$P\varphi = P_{n-1} + \frac{P_n - P_{n-1}}{\frac{2\pi}{N}} \left[ \varphi - \frac{(n-1) 2\pi}{N} \right]$$
(I-1)

Where  $P_0 = P_N$ ,

The force per unit height in the X-direction is

$$X = \sum_{n=1}^{n=N} \int_{\frac{2\pi(n-1)}{N}} R P_{\varphi} \sin \varphi \, d\varphi$$
(I-2)

or after integration

$$X = R \frac{2\pi}{N} \left[ \frac{N}{\pi}, \sin \frac{\pi}{N} \right]^{2} \sum_{n=0}^{n=N-1} P_{n} \sin \frac{2\pi n}{N}$$
(I-3)

The force per unit height in the Y-direction is

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$$Y = \sum_{n=1}^{n=N} \int_{\frac{2\pi(n-1)}{N}} R P_{\varphi} \cos \varphi \, d\varphi \qquad (I-4)$$

or after integration:

$$Y = R \frac{2\pi}{N} \left[ \frac{N}{\pi} \sin \frac{\pi}{N} \right]^2 \sum_{n=0}^{n=N-1} P_n \cos \frac{2\pi n}{N}$$
(I-5)

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In the case where 16 pressure transducers are used, Equations I-3 and I-5 become

$$X = 0.3880 R \sum_{n=0}^{15} P_n \sin \frac{\pi n}{8}$$
 (I-6)

$$Y = 0.3880 R \sum_{n=0}^{15} P_n \cos \frac{\pi n}{8}$$
 (I-7)



Fig. I-1. Assumed pressure distribution.

## APPENDIX II

#### ELECTRICAL INTEGRATING CIRCUIT

As explained previously in the paper, each transducer consists of a sensing element on which 6 SR-4 strain gages are mounted. The position and notation of these gages are shown schematically in Figure II-1 (note that the schematic shows 3 sets of biaxial-rosette gages, while 6 single strain gages were used in the construction of the prototype meter). A brief explanation of integration circuits has already been given; however, for those interested, a more detailed explanation will follow.

Because the sensing element is so designed that stress in the upper fiber is proportional to the applied pressure, we may write the following equations:

$$\sigma_{a} = C_{i} P, \quad \sigma_{b} = O \tag{II-1}$$

or in terms of strain

$$\epsilon_a = \frac{c_1}{E} P, \quad \epsilon_b = -\gamma \frac{c_1}{E} P$$
 (II-2)

Where

Furthermore, because the change in the resistance of the gage is proportional to the strain, we may write

$$\Delta R_a = C_z \in a, \quad \Delta R_b = C_z \in b \quad (II-3)$$

where

 $C_2 = KR_g = constant$  K = gage factor $R_g = gage resistance$ 

Substituting from Equations (II-2) gives

$$\Delta R_{a} = C_{e} \frac{C_{i}}{E} P, \quad \Delta R_{b} = -V \frac{C_{i}}{E} P \qquad (II-4)$$

or, in general,







COORDINATE SYSTEM Fig. II-1. Orientation and location of strain gages .

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$$(\Delta R_a)_n - (\Delta R_b)_n = (1+r) \frac{C_1 C_2}{E} P_n$$
(II-5)

Multiplying Equation II-5 with  $\sin \frac{2\pi n}{N}$  and summing we get

$$\sum_{n=1}^{N-1} (\Delta R_{\alpha})_{n} \sin \frac{2\pi n}{N} - \sum_{n=1}^{N-1} (\Delta R_{b})_{n} \sin \frac{2\pi n}{N} =$$

$$(1+Y) \frac{C_{1}C_{2}}{E} \sum_{n=1}^{N-1} P_{n} \sin \frac{2\pi n}{N} \qquad (II-6)$$

Where  $(\Delta R_a)_n$  and  $(\Delta R_b)_n$  are the change in resistance of the "A" and "B" gages, respectively, for the nth transducer.

If we define

$$(\overline{\Delta R_{\alpha}})_{n} = (\Delta R_{\alpha})_{n} \sin \frac{2\pi n}{N}$$
 (II-7)

$$(\overline{\Delta R_b})_n = (\Delta R_b)_n \sin \frac{2\pi n}{N}$$
 (II-8)

then

$$\sum_{a}^{N-1} \left(\overline{\Delta R}_{a}\right)_{n} - \sum_{a}^{N-1} \left(\overline{\Delta R}_{b}\right)_{n} = (1+1) \frac{C_{1}C_{2}}{E} \sum_{a}^{N-1} P_{n} \sin \frac{2\pi n}{N}$$
(II-9)

Consider now the Wheatstone Bridge in Figure II-2 where the legs are made up of individual resistors  $(R_a)_n$  and  $(R_b)_n$ .

The voltage across the leg (1) in terms of the input voltage is,

$$\nabla_{i} = \frac{\Sigma(R_{a})_{n}}{\left[\Sigma(\overline{R_{a}})_{n} + R\right]} V \qquad (II-10)$$

The change in output voltage as a result of change in all the  $(\overline{R_a})_n$  is

$$\Delta V_{i} = \frac{RV}{\left[\Sigma(R_{a})_{n} + R\right]^{2}} (\Sigma \overline{\Delta R_{a}})_{n} \qquad (II-11)$$

If R =  $\sum (\overline{R}_a)_n$  Equation II-11 reduces to

$$\Delta V_{I} = \frac{1}{4} V \frac{\sum_{n=1}^{N-1} (\overline{\Delta R}_{a})_{n}}{\sum_{n=1}^{N-1} (\overline{R}_{a})_{n}}$$
(II-12)

Similarly, the change in output voltage as a result of change in all the  $(R_a)_n$  is

$$\Delta V_2 = -\frac{1}{4} V \frac{\sum_{n=1}^{N-1} (\overline{\Delta R_b})_n}{\sum_{n=1}^{n-1} (\overline{R_b})_n}$$
(II-13)

Assuming  $\Sigma(\overline{R_b})_n = \Sigma(\overline{R_a})_n$  the change in the bridge output voltage  $\Delta V_0 = \Delta V_1 + \Delta V_2$  is from Equations (II-12) and II-13).

$$V_{0} = \frac{1}{4} V \frac{1}{\sum_{k=1}^{N-1} (\overline{R}_{a})_{n}} \sum_{k=1}^{N-1} (\overline{\Delta R}_{a})_{n} - \sum_{k=1}^{N-1} (\overline{\Delta R}_{b})_{n} \qquad (II-14)$$

From Equation (II-9) and (II-14) we obtain

$$\frac{4\sum_{0}^{N-1}(R_{a})_{n}}{V} \qquad \frac{E}{C_{1}C_{2}(1+\nu)} \Delta V_{0} =$$

$$\sum_{0}^{N-1} P_{n} \sin \frac{2\pi n}{N} \qquad (II-15)$$

From this equation we see that the output  $\Delta V_0$  of the composite bridge is proportional to summation in the left-hand side of Equation (1) Instead of using two active legs, all four legs could have been active and the same result would be true.

Consider now a single strain gage element with resistance  $R_g$  (for the time being, drop the subscripts a and b) and a parallel resistor  $R_p$ .



let

 $\overline{R}$  Equivalent resistor of  $R_g$  and  $R_p$ 

then

$$\overline{R} = \frac{R_g R_p}{R_g + R_p}$$
(II-1)

To find the relationship between  $\overline{\Delta R}$  and  $\Delta R_g$ , Equation (II-16) is differentiated with respect to  $R_g$ , giving the result

$$\overline{\Delta R} = \left(\frac{R_p}{R_g + R_p}\right)^2 \Delta R_g \qquad (II-1)$$

Comparing Equation (II-17) with Equation (II-7), we see that the output of the gage may be properly modified by shunting the gage with a resistor of such a value that 2

$$\left(\frac{R_{p}}{R_{g}+R_{p}}\right)_{n} = \left|\sin\frac{2\pi n}{N}\right|$$
(II-18)

The expression for  $R_p$  is then

is

$$(R_p)_n = (R_g)_n \frac{\sqrt{\sin \frac{2\pi n}{N}}}{1 - \sqrt{\sin \frac{2\pi n}{N}}}$$
(II-19)

The equivalent resistance for the gage and the parallel resistor

$$\overline{R_n} = (R_g)_n \sqrt{\sin \frac{2\pi n}{N}}$$
(II-20)

The above equation applies to the X-circuit only, however, the equations for the Y-circuit are obtained by replacing  $\sin \frac{2\pi n}{N}$  by  $\cos \frac{2\pi n}{N}$ .

One case has been worked out and the corresponding bridge circuit is shown in Figure II-3.

The X and Y integrating circuits used in the prototype clamp-on meter were wired according to the circuit diagram shown in Figure II-3 with only a few modifications. One of the basic assumptions made in deriving Equation (II-15) was that the resistance of all the legs of the Wheatstone Bridge containing active gages should have the same value. In case of the prototype clamp-on meter, all four legs of the Wheatstone Bridge were made active. This required that an effort has to be made in making the resistance of all four legs the same.

Because of the resistance of the gages making up the legs of the bridge were not exactly the same, it was necessary to introduce a potentiometer in each leg to balance out the difference in resistance. The final balancing of the bridge was made by introducing a variable resistor across two adjacent legs of the bridge This resistor was made large in order not to significantly alter the total resistance of the two legs

When the gages are connected in series according to the wiring diagram to make up the bridge (resistors parallel to gages not connected), a load on any one of the transducers will result in an output If the same pressure is applied to the transducers one at a time and the bridge voltage is kept constant, all the output should be the same Due to imperfection in the beams and gages, however, these outputs will differ slightly. It would be possible to obtain the same output from all the transducers by normalizing the outputs to the smallest of the 16 outputs by connecting proper size resistors parallel to the gages Because the output of the different transducers have to be modified in order to complete

the integrating circuit, it will be convenient to modify and normalize the output using the same parallel resistor. This means that the value of the actual parallel resistor for each gage will differ slightly from the theoretical values calculated in Appendix II.



Fig. II-3. X and Y integrating circuit using 16 transducers.