Chapter 4

THE SHOALING AND BREAKING OF THE SOLITARY WAVE

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INTRODUCTION

The solitary wave is ordinarily defined as a single disturbance of the water surface, which is relatively concentrated and symmetrical and which is propagated, neglecting damping, without change of form. Although it can be readily produced in the laboratory, in what appears to the observer a pure form, its existence (first noted in the field by J. Scott Russell (1) in 1834) as a wave of permanent shape has not yet been established by rigorous mathematical methods. This wave has been the subject of intermittent theoretical and experimental investigation since the days of Russell. In more recent years, additional attention has been given to the solitary wave, since it seemed to exhibit characteristics related to those of long-period oscillatory waves approaching the surf zone. In this connection, an experimental program for the precise measurement of the characteristics of the solitary wave was undertaken at the Hydrodynamics Laboratory of the Massachusetts Institute of Technology. The results of the first phase of this work, including celerity, profile, internal motion and smooth-bottom damping, were reported by J. W. Daily and S. C. Stephan, Jr., to the Third Conference on Coastal Engineering in 1952 (2). The present paper is concerned principally with the results of an experimental study of the shoaling and breaking behavior of solitary waves.

PROPERTIES OF THE UNDISTORTED SOLITARY WAVE

Equations describing the most important solitary wave properties are listed below. Several theoretical investigators have arrived at equations to describe the celerity and profile; only the Boussinesq (3) equations, which were found to closely describe the observed phenomena, are listed here. A definition sketch is shown in Figure 1.

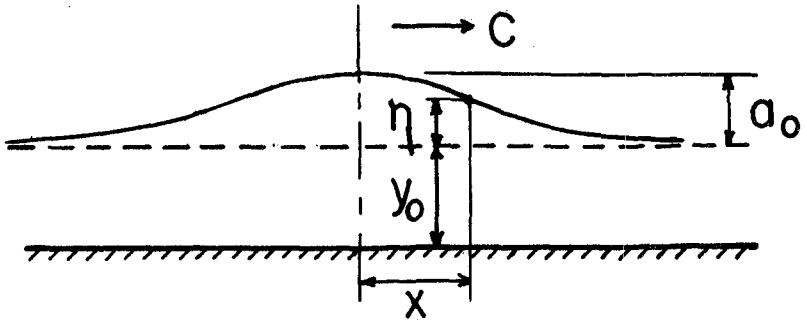


Fig. 1. Definition sketch.

Celerity. (Boussinesq) (Rayleigh) $$\frac{c}{\sqrt{gy_o}} = \sqrt{1 + a_o/y_o} \qquad [1]$$

Profile. (Boussinesq) $$\frac{h}{a_o} = \operatorname{sech}^2 \left[2 \frac{\sqrt{3}}{4} \left(\frac{a_o}{y_o} \right)^{3/2} \frac{x}{a_o} \right] \qquad [2]$$

Volume (per unit width) $$\frac{V}{a_o^2} = \frac{4}{\sqrt{3}} \frac{1}{(a_o/y_o)^{3/2}} \qquad [3]$$

Energy (per unit width) $$\frac{E}{\gamma a_o^3} = \frac{2}{3} \frac{4}{\sqrt{3}} \frac{1}{(a_o/y_o)^{3/2}} = \frac{2}{3} \frac{V}{a_o^2} \qquad [4]$$

Equations [3] and [4], which have not been verified experimentally, are obtained from equation [2] by integrating h and γh^2 over the "length" of the wave from minus to plus infinity.

J. Scott Russell was the first to suggest that the solitary wave profile is unique and independent of the method of generation. There is experimental evidence, however, that the wave profile varies within small limits with the degree of attenuation of the wave, an attenuated wave having a slightly flatter profile than a freshly generated wave of the same amplitude. This difference, however, is concentrated at the leading and trailing edges of the wave, and is not sufficient to impair the usefulness of equation [4]. From observations in the laboratory, the solitary wave can be regarded, at least for engineering purposes, as a wave of stable and permanent form, except as modified by damping. Stokes (4) and Ursell (5) have stated that the effective length of a solitary wave is short enough to remove it from the category of Airy "long waves," which are unstable. On the other hand, Ursell has shown from mathematical considerations that a solitary wave generated by the classical method of suddenly releasing impounded water can attain instability after a sufficient length of time.

BREAKING AND SHOALING

BREAKING

Waves break in spilling, plunging, or in some intermediate form. In a spilling break, the wave, maintaining much of its original symmetry during the shoaling process, peaks up until a small volume of air-water mixture appears at or slightly forward of the crest. This gentle breaking gradually grows to cover the entire front face of the wave, although the turbulent disturbance zone remains essentially confined near the surface. In a plunging break, however, some portion of the shoreward face of the wave first becomes vertical, followed by a

leaning forward of the wave crest until it distinctly overhangs the main body of the wave and finally plunges to its base. Plunging is a more violent breaking form than spilling and the jet action of the falling crest penetrates to the bottom of the beach. In general, the occurrence of a plunging surf is enhanced by the presence of steep beach slopes and initially flat waves.

The breaking criteria applicable to oscillatory and solitary waves are (a) limiting velocity, (b) limiting shape, and (c) limiting crest angle. The limiting-velocity criterion states that breaking occurs when the velocity of particles at some point along the wave, usually at the crest, equals the celerity of the wave. This assumption in the classical theories for limiting waves usually results in a cusped crest angle of 120 degrees as predicted by Stokes (6) for symmetrical waves on a horizontal bottom. The limiting-shape criterion suggests that breaking occurs when some part of the shoreward face of the waves becomes vertical. This effect is noted particularly in plunging breakers, while in spilling breakers this vertical portion, if at all present, is evident only as a slight discontinuity. Stoker's method (7) of characteristics yields such a vertical front face for non-linear waves at the breaking point, and Biesel's (8) development gives a breaker profile which closely approximates an actual plunger. The assumption of a limiting crest angle is usually equivalent to that of limiting velocity, except for certain special cases, such as the occurrence of discontinuities on the beach and breaking by wind action alone. A crest angle as a criterion for breaking is often difficult to determine reliably from photographs. The theoretical cusped crest, because of surface tension, is at best only approached in nature.

Several investigators have obtained theoretical values for the limiting amplitude-to-depth ratio at which solitary waves are expected to break. These results, which are listed below, have been obtained on the basis of the limiting-velocity criterion; however, they apply only to symmetrical waves in water of constant depth.

Investigator	Maximum a/y
Boussinesq (3)	0.73
McCowan (9)	0.78
Gwyther (10)	0.83
Davies (11)	0.83
Packham (12)	1.03

The result of McCowan is perhaps the most generally accepted one.

SHOALING

In the case of shallow-water oscillatory waves of small height, the increase in amplitude as the waves progress up the beach is well

known from consideration of the power equation

$$\frac{n_o E_o C_o}{L_o} = \frac{n E C}{L} \qquad [5]$$

where n is the energy-transmission ratio. Considering (a) waves of very high L_o/y_o, for which $n_o = n = 1$; (b) small amplitude waves, for which the energy is proportional to the length L and to the square of the height H; and (c) celerities expressed by the approximation

$$c = \sqrt{gy_o} \qquad [6]$$

where y_o is the undisturbed depth, substitution into equation [5] yields the familiar Green's Law

$$\frac{H}{H_o} = \left(\frac{y_o}{y}\right)^{1/4} \qquad [7]$$

For solitary waves, in which the energy depends upon the three-halves power of the amplitude, and for which the energy-transmission factor n is unity, Munk (13) suggested the use of the equation

$$C_o E_o = C E \qquad [8]$$

disregarding the effect of wave length. Substitution into equation [8] of equations [4] and [6] results in

$$\frac{a}{a_o} = \left(\frac{y_o}{y}\right)^{4/3} \qquad [9]$$

If equation [1] is used for the celerity, equation [9] becomes

$$\frac{a}{a_o} = \frac{y_o}{y}\left[\frac{y_o}{y}\,\frac{(1 + a_o/y_o)}{(1 + a/y)}\right]^{1/3} \qquad [10]$$

Considering now the breaking condition, designated by the subscript b, the use of the McCowan condition in conjunction with equation [10] gives for the breaking-depth ratio

$$\frac{y_b}{y_o} = 1.02\left(\frac{a_o}{y_o}\right)^{3/7}\left(1 + \frac{a_o}{y_o}\right)^{1/7} \qquad [11]$$

and for the breaking-amplitude ratio

$$\frac{a_b}{a_o} = \frac{0.80}{(a_o/y_o)^{4/7}} \left(1 + \frac{a_o}{y_o}\right)^{1/7} \qquad [12]$$

In equation [8] the total energy is employed, and the implication is that this energy must increase as the celerity decreases along the beach. However, this assumption is physically not reasonable. If an "effective" length is assigned to the solitary wave by considering a fixed large percentage of the wave volume only and if then a train of solitary waves so defined is treated, Green's Law results again. It is therefore suggested that the condition of conservation of energy be applied directly to shoaling solitary waves, so that

$$E_o = E \qquad [13]$$

Equation [13] is equivalent to equation [5] since for shallow-water oscillatory waves $n_o = n$ and since celerities and lengths are inversely proportional. Using equation [4], the amplitude increase is now expressed by

$$\frac{a}{a_o} = \frac{y_o}{y} \qquad [14]$$

which is close to the values of equation [10].

The breaking depth and amplitude ratios, again on the basis of McCowan's limiting value, are given respectively by

$$\frac{y_b}{y_o} = 1.13 \left(\frac{a_o}{y_o}\right)^{1/2} \qquad [15]$$

$$\frac{a_b}{a_o} = \frac{0.88}{(a_o/y_o)^{1/2}} \qquad [16]$$

The preceding development neglects the effect of the slope on the wave shape, and the waves are assumed to behave over any depth along the beach just as they would in the same depth over a horizontal bottom. Also, theory requires that the volume of a solitary wave decreases as it shoals, while the actual wave should nearly retain its original volume. Such assumptions have been shown by several experimenters to yield useful results in the case of shoaling oscillatory waves, except in the region very close to the breaker line; their applicability to shoaling solitary waves will be discussed later in the light of experimental results.

In general it may be said that it is impossible for solitary waves arriving at a beach to continue to be solitary waves. From constant-energy consideration it follows that the amplitude can only increase in a certain way with decreasing depth. This change in amplitude is not sufficient to account for the original volume of the wave in accordance with equation [3]. Therefore, the wave must either deform without changing energy to contain the original volume, or, what is more likely, the volume (and the energy) is steadily being depleted as the wave proceeds up the beach.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

WAVE TANK AND BEACH

The wave tank is 32 feet long, 16-1/2 inches wide and 13 inches deep. The sidewalls are divided into eight panels of 4 feet each by the vertical uprights of the framework. The walls and bottom of the tank are of lucite throughout its length. A 16-foot sloping beach consisting of a masonite surface supported on an aluminum framework was placed at one end of the tank, extending over four panels. The beach was attached to the top of the tank by hangers at 2-foot spacing, so that the slope could be easily adjusted. These hangers formed 1/16-inch projections along the inside walls of the tank, which were found to disturb the menisci of the passing waves to a certain extent. However, the menisci regained their original configuration within a short distance downstream of each hanger, so that this method of adjustment did not impair wave profile observations. The toe of the slope and the junctions between the tank walls and the beach were sealed with plasticene and rubber. The sidewalls of the six downstream sections were covered with lucite sheets on which grid lines were inscribed at 0.05-foot spacing. Thus there were two observation panels within the constant-depth portion of the tank.

The solitary waves were generated by impounding a volume of water behind a gate which could be raised suddenly by means of a solenoid-operated falling weight. This water, upon release, would push a movable piston along the tank, thus displacing a certain amount of water in front of it within a definite time. This technique permitted the necessary duplication of waves to a degree not possible with manual operation of the gate.

PHOTOGRAPHIC EQUIPMENT

The photographic equipment was mounted on a carriage which was pushed by the observer along a track placed parallel to the wave tank at a distance of about 4-1/2 feet. Two sets of photographic equipment were used during the course of these experiments. The first consisted of an open-shutter Edgerton-type camera, with illumination provided by a Strobolume gas-discharge lamp triggered by a Strobotac unit. This equipment was employed in all particle-

motion work and for some of the deformation and breaking runs. The flash unit was operated at 20 flashes per second, and the flash duration was 40 microseconds. The open-shutter motion-picture camera was furnished with a 51 mm f:4.5 lens and all the recording was done on 35 mm Super XX film.

The second set of photographic equipment consisted of a 16 mm Bell and Howell motion-picture camera with two No. 2 photoflood lamps. This camera was operated at 64 frames per second and a 1-inch f:1.9 lens was used, along with Super XX negative motion-picture film. The 16 mm camera of course yielded enlargements of poorer quality than did the 35 mm equipment, and the shutter speed was not sufficient to completely "stop" the motion; however, its use made the entire process--loading, photographing and processing--extremely convenient and rapid.

OBSERVATIONAL TECHNIQUE

After carefully measuring depths with point gages and referencing all grids, the test procedure consisted simply of releasing a wave and following it with the camera on the carriage. This was readily accomplished, since waves generated at the depths used in these experiments travel at a rate corresponding to a medium-fast walk.

Internal velocities at the breaking point were investigated exclusively with the Edgerton-Strobolume apparatus. Since the wave profile was recorded in the plane of the near wall, and since the particles to be observed must be placed an appreciable distance away from this wall, an auxiliary grid was placed in the plane of the particles and slightly above the crest of the highest anticipated wave. Then, on the photographs, these grid lines could be projected down into the wave itself. Droplets of a colored solution of xylene and n-butyl phthalate, adjusted to give a specific gravity of unity, were used to obtain velocities in the interior of the fluid, and 1/8-inch balsa cubes were employed to measure surface velocities.

Experiments were conducted on beach slopes of 0.023, 0.050 and 0.065. Amplitude-to-depth ratios in the constant-depth portion of the tank ranged from about 0.2, which was the lowest wave that could be generated without the presence of large secondary waves following the primary wave, to about 0.7, which was almost the highest wave that could be generated without breaking before reaching the beach. Initial depths ranged from 0.25 feet to 0.45 feet. For several runs, the 0.023 and 0.065 slopes were roughened with a layer of gravel of 0.013 feet average diameter.

EXPERIMENTAL RESULTS

BREAKING AMPLITUDE-TO-DEPTH RATIO

The experimental values of amplitude-to-depth ratio at the breaking point are plotted in Figures 2(a), 2(b) and 2(c), along with the theoretical value of McCowan for a horizontal bottom. It is noted

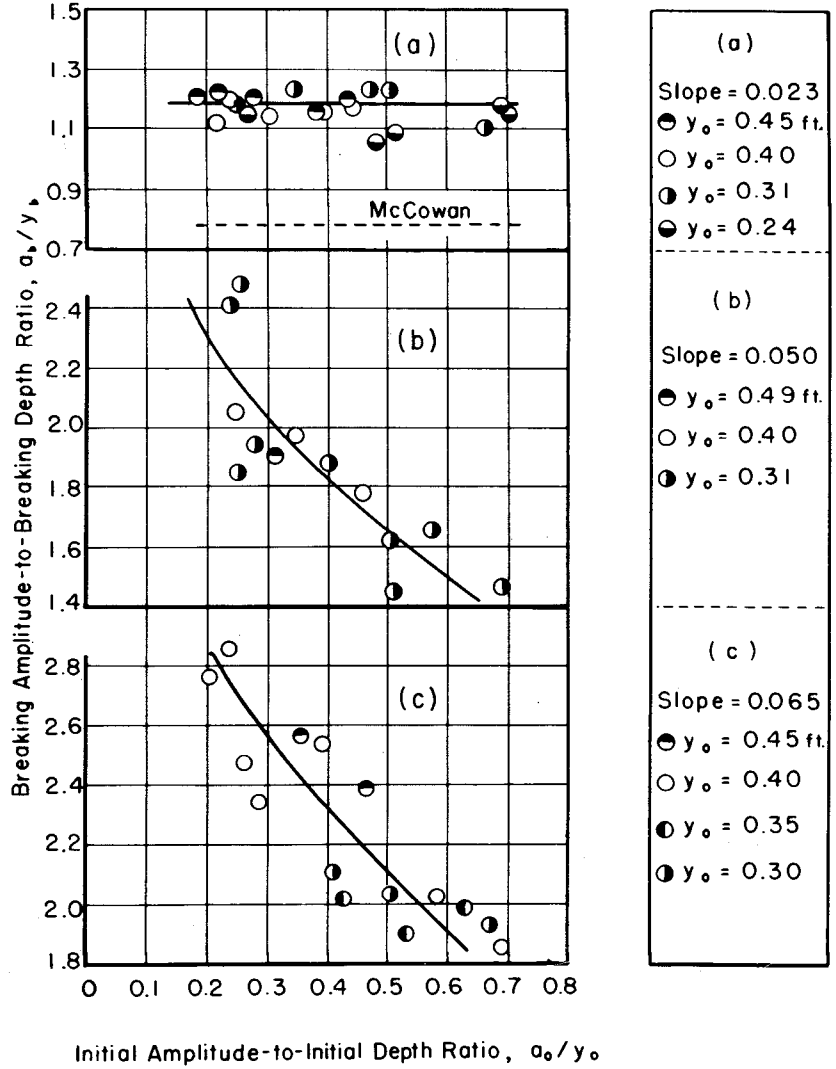


Fig.2. Breaking amplitude-depth ratios versus initial amplitude-depth ratios.

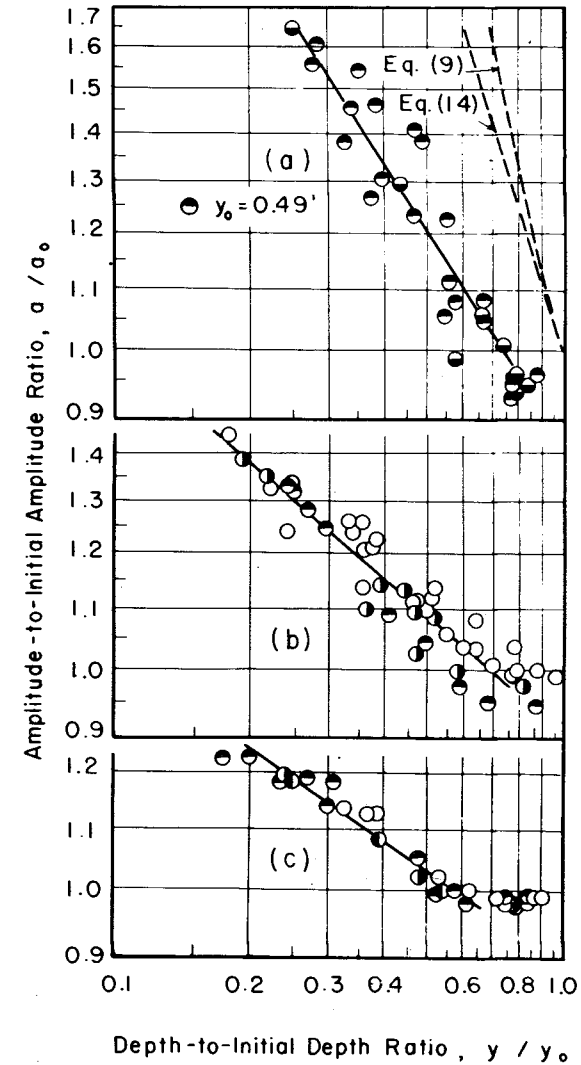


Fig.3. Relation of shoaling amplitudes to depth.

that the experimental results more closely approach the theoretical value as the slope decreases. This behavior, which will be observed on all experimental curves for breaking characteristics, is to be expected, since the waves are deformed less by the flatter slope; that is, they retain more of their original symmetry with respect to their vertical axes. At the lowest slope of 2.3 percent, the experimental points, although exhibiting appreciable scatter, show no systematic variation with respect to initial amplitude-to-depth ratio. The average value of a_b/y_b is about 1.2 for breaking on this slope with a smooth beach. Several runs were made on the 2.3 and 6.5 percent slopes with the bottom roughened with a layer of gravel. These results are not shown on the curves of Figure 2 because of the question of assigning breaking-depth values when the gravel thickness forms an appreciable part of the total depth. The significance of these runs will be discussed in later paragraphs.

On the 5.0 and 6.5 percent slopes, the scatter of the points is even larger than on the 2.3 percent slope, but here a pronounced trend for the breaking amplitude-to-depth ratio to increase with decreasing initial amplitude-to-depth ratio is noted, a trend which is greater for the larger slope. The breaking amplitude-to-depth ratio of 1.2 obtained on the flattest slope is still considerably higher than the usually accepted theoretical value of 0.78. This difference does not invalidate the theory, however; it merely points out that while, in the case of oscillatory waves, experiments have revealed that constant-depth theories adequately describe shoaling until close to the breaking point, in the case of solitary waves, a small slope is sufficient to cause a large departure from theory. Although no systematic program has been carried out at this laboratory to determine the maximum solitary wave amplitude obtainable on a horizontal bottom, it is significant that, considering the several hundreds of waves generated since the inception of this project, the largest initial amplitude ever recorded was 0.72 of the depth. This figure should of course be increased by a consideration of the small amount of attenuation that occurs between the point where the wave first stabilizes and the first observation section. In many cases, where an initially breaking wave was generated, the head in the generating reservoir was lowered a very small amount to give a stable wave on the next attempt.

BREAKING DEPTHS

The experimental results in Figures 4(a), 4(b) and 4(c) show that the solitary wave on a beach breaks in water depths much shallower than predicted by theory based on zero slope. This difference increases with increasing slopes of the beach. While the curve for the 0.023 slope shows a trend corresponding to that of the theoretical curves, the breaking depths for higher slopes vary almost linearly with the initial amplitude-to-depth ratio.

BREAKING AMPLITUDES

The experimental results on breaking amplitudes for the three slopes are shown with the theoretical curves for zero slope on Figures 5(a), 5(b) and 5(c). The ratio of breaking-to-initial amplitude, a_b/a_o, decreases almost linearly with increasing initial amplitude-to-depth ratio, a_o/y_o. In neither the breaking amplitude nor the breaking depth results does there appear to be any consistent scale effect. In the region of high initial amplitude-to-depth ratio, equations [12] and [16] give reliable values for the breaking amplitudes, with the discrepancies sharply increasing with the slope of the beach.

SHOALING DEFORMATION

The change in amplitude of the solitary wave due to shoaling is given in Figures 3(a), 3(b) and 3(c). These figures show the ratio of local-to-initial amplitude, a/a_o, plotted as a function of the local-to-initial depth ratio, y/y_o for several representative runs on each slope. The scatter is considerable but random, and again there is no noticeable scale effect. The rate of amplitude growth on the 0.023 slope comes nearest to the theoretical rate. The failure of the measured amplitude growth to correspond exactly to the theoretical growth is a measure (a) of the asymmetry of the wave, and (b) of the difference between the actual and theoretical volumes. The same effects are of course reflected in the discrepancies between the actual and theoretical breaking depths and amplitudes. Expressing the amplitude increase by an expression of the form

$$\frac{a}{a_o} = K + \left(\frac{y_o}{y}\right)^m \qquad [17]$$

the experimental values of m were found to be 0.47, 0.26, and 0.19 for the 0.023, 0.050 and 0.065 slopes respectively.

An interesting observation on these curves is the decrease of amplitude below the initial value as the wave starts up the beach. This decrease is presumably due to reflection from the toe of the slope, although its magnitude, and therefore the magnitude of the energy loss, does not appear to increase as the slope is steepened. A series of tests on the reflection of solitary waves by impermeable structures of different slopes is described in Technical Memorandum No. 11 (14) of the Beach Erosion Board. The smallest slope angle used in the B. E. B. runs was 6 degrees, corresponding to a slope of 0.105, and the reflected energy at this slope was 9 percent of the incident energy. Extrapolation of the B. E. B. data indicated that no reflected energy is to be expected at slopes of about 4.3 degrees and lower. The largest slope used in the M. I. T. program was 0.065, corresponding to 3.75 degrees. Therefore, considering that the base

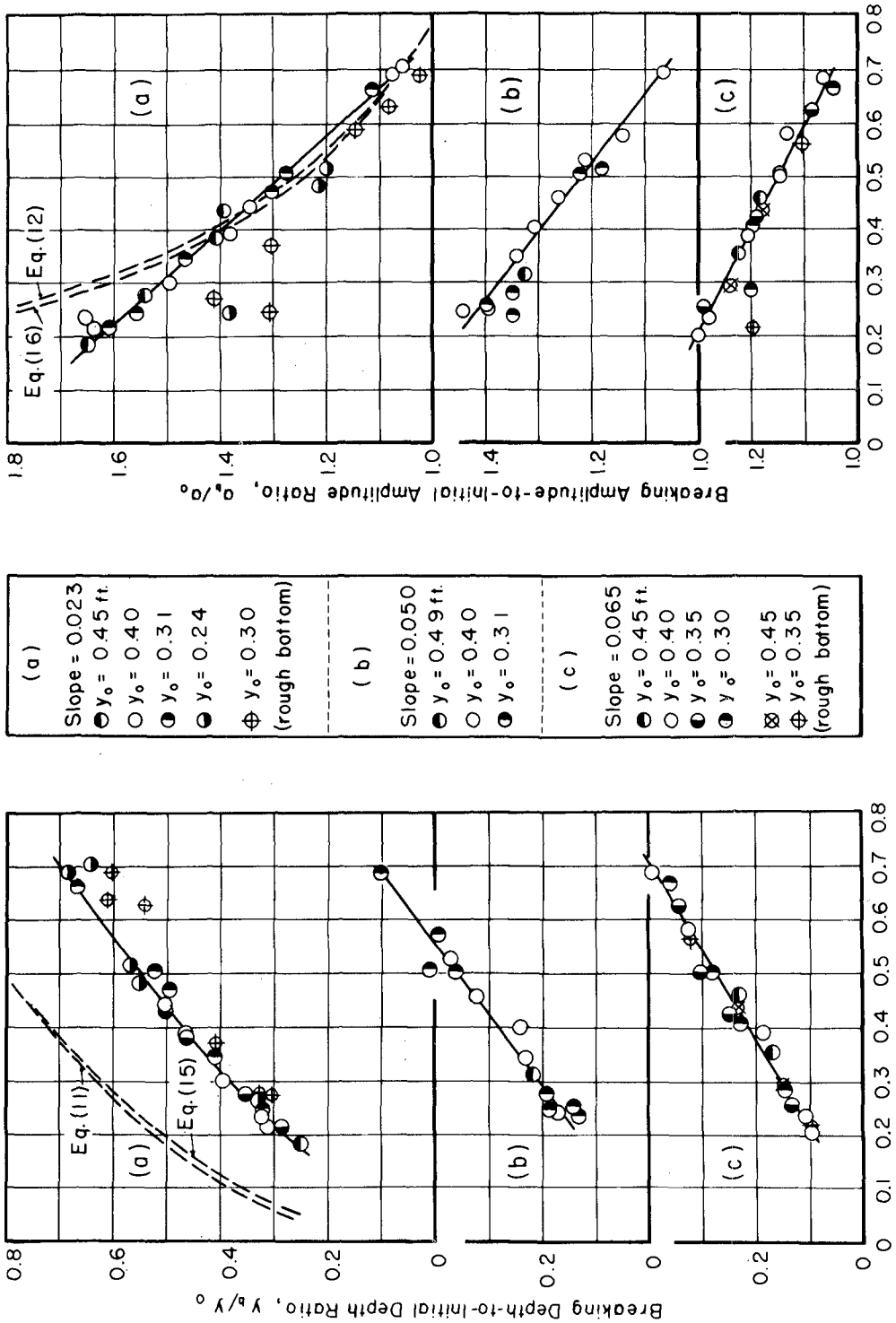


Fig.4. Breaking depth-initial depth ratios.

Fig.5. Breaking amplitude-initial amplitude ratios.

of the beach had a relatively smooth transition to the horizontal bottom, no energy reflection would normally be anticipated on any slope reported here. Nevertheless, in the case of the solitary wave, the observed amplitude decrease cannot be attributed to interaction between the energy-transmission factor, n, and the celerity, which occurs when deep-water oscillatory waves transform into shallow-water waves. The B. E. B. memorandum adds:

> "It is significant that at no value of the slope was the solitary wave found to break; rather the action was one of gentle ride-up of the wave on the slope with the transition from a wave to the final uprush being gradual under all conditions tested.....In view of the lack of evidence of breaking of the solitary wave, it is felt that the results of these tests are not even approximately applicable to a train of progressive oscillatory waves, where the return flow, or backwash, from one wave aids in tripping the succeeding wave, thereby causing it to break."

The M. I. T. studies reported here indicate that solitary waves do indeed break in the usual sense at the low slopes investigated, so that the presence of backwash, although it is an important factor in affecting the breaking of oscillatory waves, is not in itself necessary to cause the breaking on these slopes. The applicability of solitary wave results to the case of a train of progressive oscillatory waves will be discussed under a later heading.

EFFECT OF BOTTOM ROUGHNESS

The question encountered in the analysis of rough-bottom work reported here is whether or not the depth beneath the wave should include a portion or all of the thickness of the gravel used as the roughening agent. This becomes especially important in these experiments because the gravel diameter is 0.013 feet and some of the breaking depths are of the order of 0.05 feet. At first, the depths were measured down through the gravel layer to the smooth bottom. This procedure reduced the breaking amplitude-to-depth ratios on the 0.023 slope to an average value of 1.0. The breaking amplitudes were smaller than the corresponding smooth-bottom results, while the breaking depths remained unchanged. Because the gravel diameter-to-depth ratio was large, the results were re-analyzed on the basis that the breaking amplitude-to-depth ratios would be the same on the rough bottom as for the corresponding waves on a smooth bottom. Of course, this procedure involved the use of average smooth-bottom values. Also, the two breaking ratios are probably not identical, since it was observed that the rough-bottom breakers tended more to the pure spilling form than the corresponding smooth-bottom breakers. Nevertheless, this procedure is believed to be at least more correct than the first one.

The final rough-bottom results are shown in Figures 3 and 4. There is no appreciable roughness effect on the steep slope. The

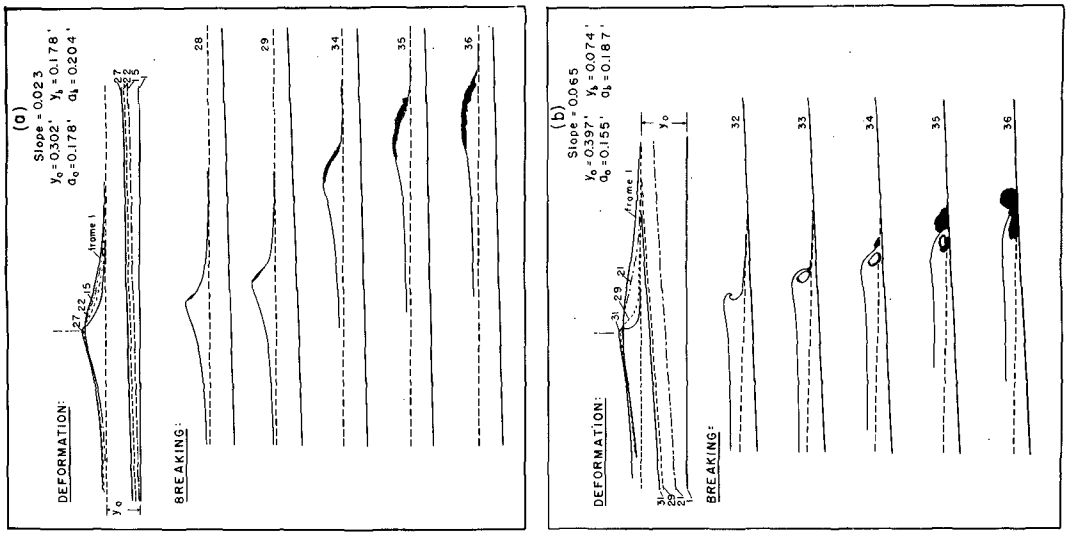
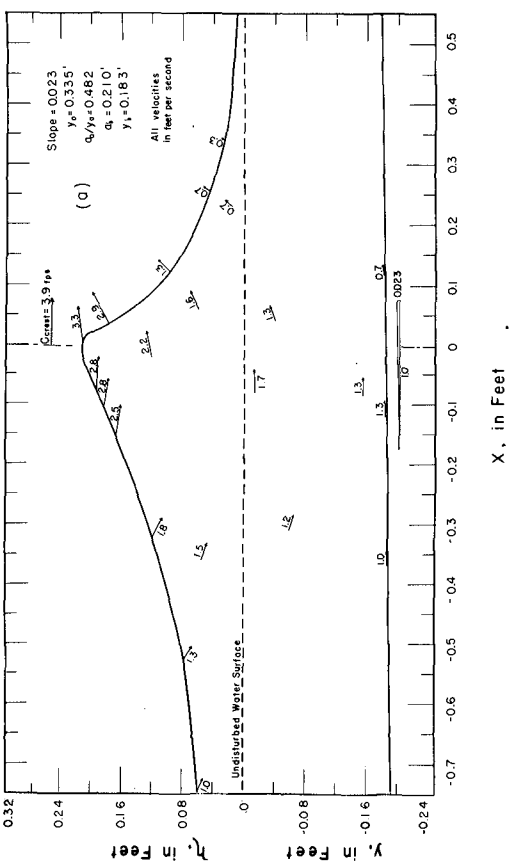


Fig. 7 Shooling and breaking of representative waves.



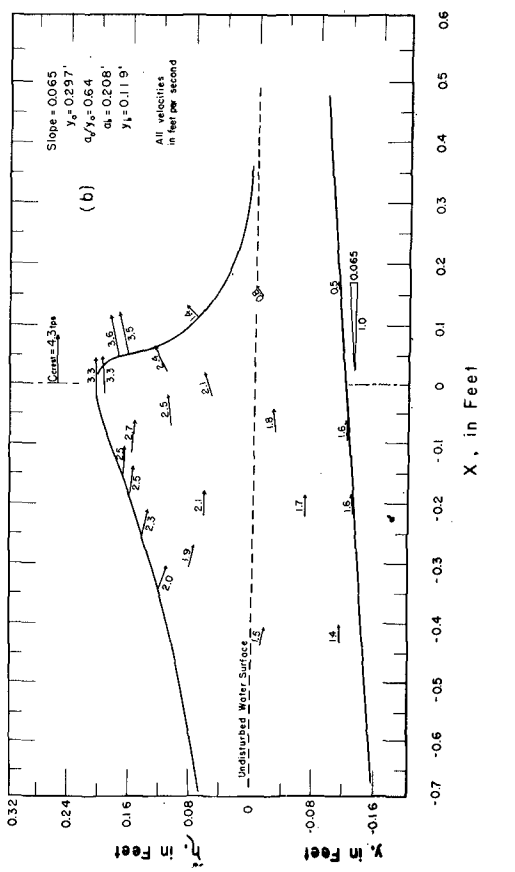


Fig. 6. Internal velocities at breaking.

breaking amplitudes are reduced approximately 10 percent below the breaking amplitudes of the same waves for the smooth bottom runs for the lowest waves on the 0.023 slope. The percentage reduction decreases as the initial waves become higher. Similarly, the breaking depths are lower, showing that the effect of amplitude attenuation is appreciable.

EFFECT OF WALL AND BOTTOM FRICTION ON SMOOTH-BOTTOM BREAKING DATA

It is of some interest to determine, at least approximately, the effect of bottom and wall friction on the shoaling data. The only available attenuation expression is the Keulegan (15) equation, which has admittedly not been verified in the range of depths and amplitude-to-depth ratios encountered in shoaling. However, recent attenuation studies at M. I. T. have indicated that the use of the Keulegan equation in this range can be expected to give conservative results; i.e., to give excessive damping. The shoaling case in which friction would be most effective was chosen as an example; namely, an initially low wave ($a/y = .25$) on a flat (0.023) slope. The experimentally determined amplitude increase was used. It was found that the friction reduced the breaking amplitude by only 3.3 percent and the breaking depth by 7.1 percent for these unfavorable assumptions.

INTERNAL VELOCITY MEASUREMENTS

Results on internal velocity measurements at the breaking point are shown in Figures 6(a) and 6(b), with the first figure representing a wave breaking on a 0.023 slope, and the second figure, a wave breaking on a 0.065 slope. This phase of the study was originally undertaken to determine if any appreciable differences in velocity distribution prevailed within plunging and spilling breakers. Because it was found that photographing colored droplets to obtain internal velocities at the breaking point was not a sufficiently precise procedure to determine small velocity differences, only a preliminary series of runs was made. Especially difficult is the determination of a crest celerity over the short distance which the wave traverses between two frames (at 1/20 second) of the film. Because the internal velocities shown in the figures were determined from the two film frames closest to the breaking point, they necessarily represent average velocities over the time period of 1/20 second. Although the time involved is short, the change in wave shape near the breaking point is extremely rapid, and therefore appreciable accelerations are involved. The wave shapes and particle positions depicted in the figures correspond to the second of the two frames employed, that is, to the frame closest to the actual breaking point. Therefore, the internal velocities shown for the shoreward side of the wave are lower than the true instantaneous velocities for the second frame, while those for the seaward side are slightly higher. Similarly, the crest celerities are probably slightly low, although, because of the rapid deformation of the waves in this region, the significance of the crest celerity is questionable. The measured crest celerities and particle

velocities in relation to the theoretical celerities for a stable wave of similar amplitude and depth are as follows for the two cases given in Figures 6(a) and 6(b).

Slope = 0.023: Measured celerity = 3.9 fps
Maxim. measured particle velocity = 3.3 fps
Theoretical celerity = 3.6 fps

Slope = 0.065: Measured celerity = 4.3 fps
Maxim. measured particle velocity = 3.6 fps
Theoretical celerity = 3.3 fps

The preliminary results shown here indicate also that on a flat slope with a spilling break, the maximum velocity nearly equals the crest celerity at the breaking point. This maximum velocity appears to occur just slightly shoreward of the highest point of the crest. Unfortunately, the evidence presented in Figure 6 does not as yet permit the development of a clear distinction between the internal kinematics of plunging and spilling breakers.

WAVE DEFORMATIONS

Figures 7 and 8 illustrate the deformation of representative shoaling solitary waves. In Figure 7, which illustrates chronologically the entire deformation and breaking process, the profiles are designated by frame numbers, consecutive frame numbers representing a time interval of 1/20 second. The deformation of the breaker on the steep slope in Figure 7(b) is seen to begin gradually, with a rapid acceleration of the process occurring just before the breaking point is reached. Characteristic of this breaker, in addition to its violence, is its vertical front face and extreme asymmetry. The spilling breaker in Figure 7(a), on the other hand, is due to a more gentle process, and the wave initially deforms as well as breaks in a much more gradual fashion.

Figure 8 illustrates representative wave profiles at the breaking point. The ordinates are plotted in terms of η/a_o, where a_o is the amplitude of the initially undisturbed wave, to emphasize the fact that waves which are initially low grow relatively larger in amplitude before breaking than do waves which are initially high. Again, the increase of asymmetry with increasing beach slope is evident, as is the increase of asymmetry with decreasing initial wave height. Ursell (5) theoretically predicted, in a qualitative sense, the effect of shoaling on oscillatory wave symmetry by consideration of the parameter HL^2/y^3. Large values of this parameter correspond to unstable long waves, which deform by steepening of the front face. Initially low waves are able to progress farther up a beach before breaking than are initially high waves, so that the resulting large values of the parameter HL^2/y^3 correctly predict an increased tendency toward wave asymmetry.

In Figure 9 the wave forms at the breaking point are related to the initial amplitude-to-depth ratio for each slope. Waves which retained much of their original symmetry during shoaling and which deformed principally offshore by a "peaking up" of the crest have been classed as "symmetric" breakers. On the other hand, waves which became severely asymmetric during shoaling and which deformed principally by a steepening of the front face in very shallow water have been classed as "asymmetric" breakers; these waves approach, as the extreme case, the non-linear waves depicted by Stoker. There is some scatter evident, since the classification of each run depends to some extent upon the personal judgment of the observer who reviews the motion-picture films and still photographs. The dashed curve shown dividing the regions in which the two breaker types prevail is therefore an approximation, although a theoretical limiting point of this curve can be deduced from the fact that for zero slope, only a "symmetric" breaker can occur.

It should be emphasized that Figure 9 classifies only the wave shape at the breaking point; it does not necessarily distinguish between spilling and plunging breakers. All the waves classed as "asymmetric" plunged. However, on the two steeper slopes, the waves classed as "symmetric" also developed pronounced overhanging crests after the breaking point was reached. In fact, none of the breakers obtained on smooth beaches conformed rigidly to the pure spilling form previously defined. The breakers on the 0.023 slope approached this type very closely, but in most cases there was a slight tendency for overhanging crests to form, although, on this flat slope, a distinct nappe was never visible. When the beach was roughened, almost true spilling breakers were obtained on the 0.023 slope.

APPLICATION OF THE SOLITARY WAVE TO SURF PROBLEMS

The possibility of the application of solitary wave theory to the problem of oscillatory waves about to break on a sloping beach has been suggested by several investigators, notably Munk (13). As oscillatory waves proceed landward, the crests appear to become more accentuated and the troughs appear to become correspondingly flatter. Finally, the wave train resembles a "series" of independent solitary waves. It is this resemblance which led to the conclusion that possibly the wave length L is no longer significant near the breaking point and that the height-to-depth ratio H/y, corresponding to the solitary wave ratio a/y, is then the only significant parameter. The solitary wave has for many years been recognized by mathematicians as the limiting case of the oscillatory wave; but only in recent years have attempts been made to apply solitary wave theory to engineering problems.

BREAKING HEIGHT-DEPTH RATIO

That the McCowan limiting ratio of 0.78 for solitary waves is at least a good first approximation to the H_b/y_b value for oscillatory waves is shown in Table I below, obtained from published results.

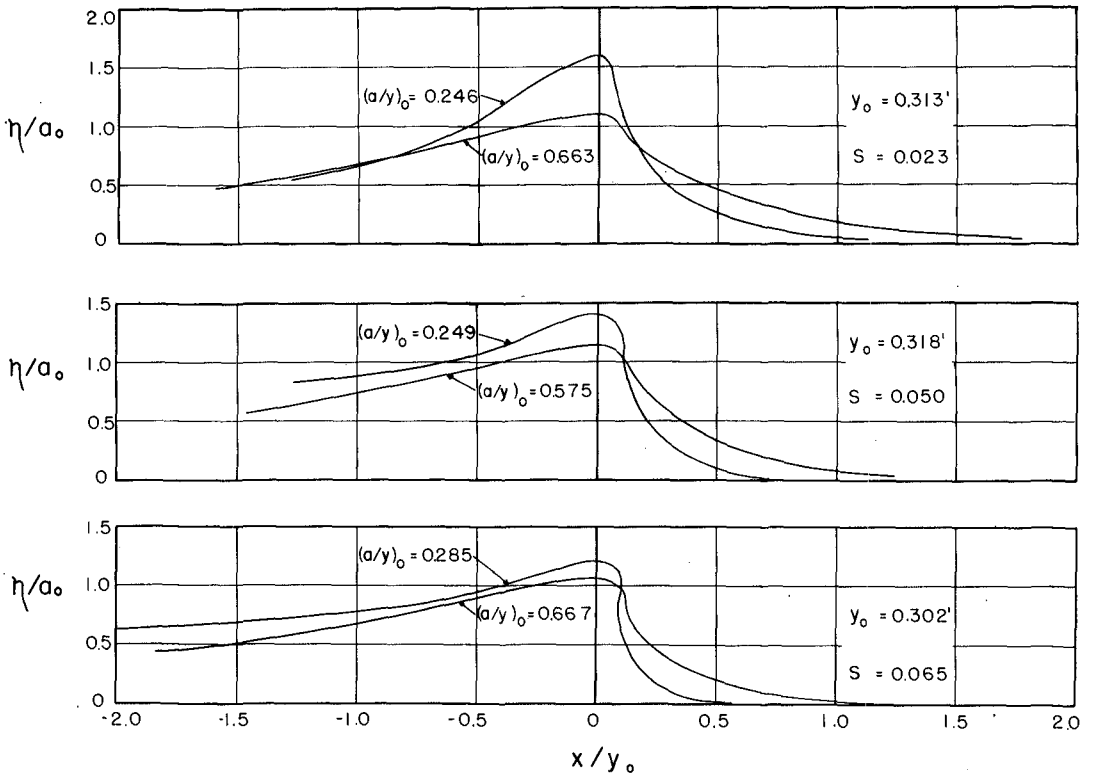


Fig. 8. Typical wave shapes at the breaking paint.

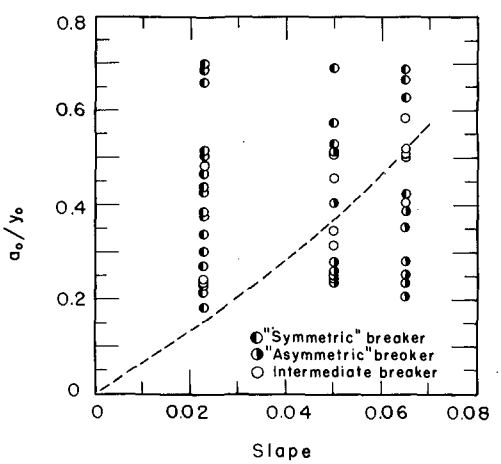


Fig. 9 Wave - shape classification at breaking.

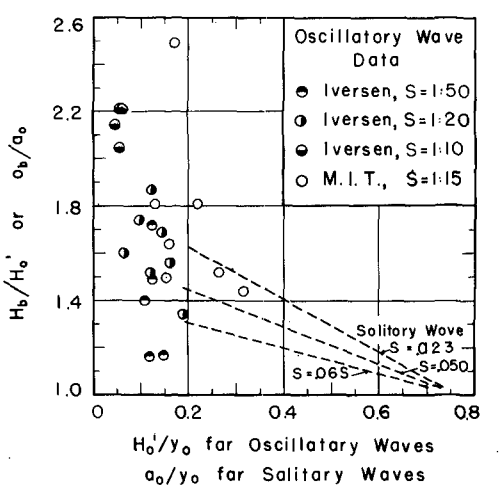


Fig. 10 Camparisan af oscillatory and solitary wave breaker heights.

Much of the scatter evident in Column (5) is attributable to the wide range of initial steepnesses represented in the data.

Table I

(1) Reporter	(2) No. of Runs	(3) Slope	(4) Mean H_b/y_b	(5) Min-Max	(6) Adj. H_b/y_b
Iversen (16) (Lab.)	13	1:50	.818	.682-.937	.924
	19	1:20	.840	.656-1.000	1.069
	16	1:10	1.035	.778-1.232	1.360
Larras (17) (Lab.)	160 (Total)	1:100	.684	.57-.83	
		1:50	.746	.57-.87	
		1:11	.862	.57-1.05	
Munk (13) (Lab. & Field)	746	Not Specified	.75	.64-.91	

The breaking ratios listed in Table I show no tendency to attain the high values obtained on steep slopes during the course of the solitary wave experiments. They do, however, increase with steepening slope, although again not to the extent observed for solitary waves. Iversen's results were also computed with the depth y_b measured to the bottom of the troughs rather than to the mean water level and the adjusted ratios are listed in Column (6). These adjusted ratios are seen to come somewhat closer to the a_b/y_b values obtained in this study.

BREAKER HEIGHTS

Munk, using the energy-transmission equation for oscillatory waves along with equation [4] and the McCowan limiting value of $a_b/y_b = 0.78$ arrived at

$$\frac{H_b}{H_o} = \frac{1}{3.3\,(H_o/L_o)^{1/3}} \qquad [18]$$

for the breaking height of oscillatory waves in terms of their initial, or deep-water, height and steepness. Comparison with laboratory and field data showed that equation [18] could be expected to apply for values of deep-water wave steepness of approximately 0.006 and lower.

Bagnold (18) has suggested that oscillatory waves begin to act as independent solitary waves when their period exceeds a critical period defined by

$$T_c = \frac{2\pi}{M}\sqrt{\frac{y}{g}} \qquad [19]$$

where M is approximately equal to $(3a/y)^{1/2}$. From the data published by Iversen, several runs were chosen which came close to satisfying the criterion suggested by Bagnold in the constant-depth region of the wave tank. On Figure 10, H_b/H_o' is plotted against H_o'/y_o for these waves, where H_o' is the measured wave height in the constant-depth portion of the channel, and is compared with the solitary wave results (dashed lines) for a_b/a_o vs. a_o/y_o. Because most of Iversen's waves were of fairly short period as judged by equation [19], a few runs of an exploratory nature were made on long-period oscillatory waves in the large wave tank at M. I. T. The slope was 1:15, corresponding closely to the 0.065 slope used in the solitary wave experiments, and the initial depth y_o for all waves was 1.75 feet. A synopsis of these runs is given in Table II below and in Figure 10.

Table II

Run	Period (sec.)	H_o' (ft.)	H_b(ft.)	y_b(ft.)	H_b/H_o'
1	4.25	.226	.409	.355	1.81
2	4.25	.548	.791	.690	1.45
3	4.25	.325	.534	.495	1.64
4	8.83	.298	.727	.690	2.44
5	6.43	.380	.688	.570	1.81
6	5.49	.459	.702	.656	1.53

In the above runs, the location of the break was estimated visually by the observers, and the breaker heights were obtained by placing a capacitance profile recording wire at this visually determined breaking point. The average value of H_b/y_b is 1.12, which is higher than Iversen's average value of 0.94 for the 1:10 and 1:20 slopes. There is some indication, then, that the breaking height-to-depth ratio increases with increasing period, but again the high breaking ratios obtained for the solitary waves are not reached. In general oscillatory wave breaking heights are not consistent with the results of solitary wave experiments. While oscillatory wave breaker heights increase with increasing slope, the opposite has been found to be true for solitary waves. Long-period shallow-water oscillatory waves admittedly bear some physical resemblance to solitary waves as they approach a beach. However, in the surf zone this resemblance disappears, and, as the experimental results imply, the backwash from preceding waves undoubtedly plays an important role in the breaking process. In many cases, nevertheless, the application of the theoretical limiting solitary wave height to the breaking of oscillatory waves on a beach has given results surprisingly well substantiated by field observation. The present experimental evidence for the breaking of solitary waves, however, clearly does not support the hypothesis of solitary wave behavior on the part of oscillatory waves close to the breaker line. The limiting height of the solitary wave at the breaking point on a slope has been found to be materially higher than the theoretical limiting value for the solitary wave in a horizontal channel.

GENERAL CONCLUSIONS

An experimental study has been made of the shoaling and breaking of solitary waves on slopes of 0.023, 0.050 and 0.065, with initial depths ranging from 0.25 to 0.45 feet. The principal findings may be stated as follows:

1. On the flattest slope, the breaking amplitude-to-depth ratio a_b/y_b is practically constant at 1.2 for all incident waves, compared with the theoretical value of 0.78 (McCowan) for solitary waves in water of constant depth. For the steep slopes, the value of this ratio increases with the slope and with decreasing initial wave height.

2. For the same incident wave, breaking amplitudes and breaking depths increase with decreasing slope. For a given slope and initial depth, lower amplitude waves grow relatively higher and break in shallower depths than waves of higher initial amplitude. Bottom roughness slightly reduces breaking amplitudes and depths, but the effect is noticeable only in the case of an initially low wave traveling up a flat beach.

3. The observed amplitude growth along the beach increases with flatter slopes, but falls considerably short of the growth consistent with constant-energy consideration.

4. Steep slopes increase the tendency toward plunging breakers. The flattest slope featured almost spilling breakers, with pure spilling resulting when this beach was roughened.

5. The ratio of breaker height-to-initial height obtained from available long-period oscillatory wave data cannot at present be reconciled with solitary wave experimental results.

ACKNOWLEDGMENT

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RESUME

ONDE SOLITAIRE CARACTERISTIQUES DE L'AMORTISSEMENT INCIDENCES CARACTERISTIQUES DE L'ABAISSEMENT DU TIRANT D'EAU

A. T. Ippen et Gershon Kulin

Une étude expérimentale de l'onde solitaire dans des canaux horizontaux et en pente est en cours au Laboratoire d'hydrodynamique de l'Institut technique du Massachusett. La première phase de l'étude, comprenant les résultats obtenus sur la célérité, le profil, le mouvement interne et l'amortissement de l'amplitude dans des canaux horizontaux lisses, a fait l'objet d'un rapport de J.W. Daily et S.C. Stephan Jt au 3ème Congrès du Coastal Engineering en 1952. La présente communication contient une discussion plus large des divers aspects de l'amortissement par friction sur fond lisse et des résultats expérimentaux concernant les canaux à rugosité artificielle. Un programme de recherches expérimentales sur le comportement de la houle sur les plages en pente est décrit. La théorie postule qu'une et une seule forme stable d'onde solitaire peut exister pour une amplitude et une profondeur données. Cependant, l'expérience montre avec évidence que le profil de l'onde dépend, dans une faible mesure, de l'amortissement du mouvement de l'onde. Alors qu'au début le mouvement de l'onde suit convenablement les prévisions théoriques de Boussinesq, une onde de même amplitude mais résultant de l'amortissement d'une onde d'amplitude plus grande, se trouve avoir un profil plus aplati. Cette déformation du profil est surtout localisée dans les régions où l'onde possède une faible énergie et son effet sur le processus de l'amortissement n'est pas important, quoique observable au Laboratoire. L'analyse de l'amortissement sur de faibles distances de parcours (excluant les réflexions sur les extrémités du canal) des ondes fraîchement engendrées, montre que le coefficient expérimental d'amortissement dépend de la hauteur de la crête, alors que le coefficient théorique de Keulegan est constant pour une hauteur d'eau donnée. Le coefficient expérimental est plus petit dans la gamme des fortes amplitudes ; dans la gamme des faibles amplitudes, où les hypothèses de la théorie de Keulegan sont les mieux satisfaites, il est plus grand que le coefficient théorique.

L'analyse des essais sur l'amortissement, effectués sur des fonds a rugosite artificielle, tout en accusant un taux d'amortissement plus élevé que dans des canaux lisses pour des hauteurs correspondantes, a révélé que l'influence du coefficient d'amortissement sur la hauteur de l'onde est moindre.

L'intérêt porté à l'onde solitaire, ces dernières années, provient du fait que les houles périodiques à longue durée, comprises dans la zone juste au large du point de déferlement, ressemblent à un "train" d'ondes solitaires. Cette similitude a conduit à admettre l'existence sur la plage d'un point au-delà duquel la longueur d'onde n'est plus un paramètre essentiel, l'évolution de la houle périodique étant alors gouvernée par la théorie de l'onde solitaire. Sous ce rapport, le comportement d'ondes solitaires fut étudié sur des plages de pentes 0,023, 0,050 et 0,065. Une théorie sur le déferlement des ondes solitaires s'applique uniquement aux fonds horizontaux et les conditions de déferlement observées s'écartent d'autant plus de ces résultats théoriques que la pente croît. Sur la pente de 0,023, le rapport de déferlement amplitude-profondeur est approximativement constant et égal à 1,2 pour toutes les ondes (à comparer à la valeur théorique de 0,78 de Mc Cowan). Sur des pentes plus raides, ce rapport de déferlement est plus élevé et il croît notablement lorsque le rapport initial amplitude-profondeur décroît. Des rapports de déferlement amplitude-profondeur jusqu'à 2,7 ont été obtenues pour des ondes initialement aplaties sur la pente de 0,065. Toutes les ondes déferlent pour des tirants d'eau plus faibles que ceux prévus par la théorie. L'accroissement d'amplitude de l'onde le long de la plage est plus grand pour les pentes les plus faibles. Ce taux d'accroissement de l'amplitude peut s'exprimer par la formule :

$$\frac{a}{a_o} = \left(\frac{y}{y_o}\right)^n$$

en prenant pour n les valeurs expérimentales respectives de 47, 26, 19 pour des pentes de 0,023, 0,050 et 0,065.

Sur la pente de 0,023 les ondes conservaient, en première approximation, leur symétrie d'origine en remontant la pente vers la plage et elles déferlaient par déversement presque pur avec une très légère tendance à former une crête surplombante. Sur les pentes les plus raides, les ondes devenaient plus dissymétriques et les déferlements se font par plongements, la tendance à plonger devenant plus prononcée pour un rapport initial amplitude-profondeur décroissant. Des profils de divers types sont présentés pour illustrer la déformation de l'onde, et les formes de déferlement sur les diverses pentes. L'application des résultats obtenus pour l'onde solitaire aux problèmes de houle oscillatoire fait l'objet d'une discussion.