Chapter 2

THE KELVIN-HELMHOLTZ INSTABILITY OF WAVES ON THE SURFACE OF THE SEA

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The initiation of waves on the sea's surface by wind has been a subject of interest for many years. There are two ways in which a wave can become unstable and grow. The first is that investigated by Kelvin and Helmholtz many years ago: both air and water are assumed perfect fluids and, if a wave is to grow, the upward force due to the reduction of pressure on the wave crests is greater than the weight of the wave. The pressure reduction is due to the local acceleration of the wind over the wave. This theory gives a critical windspeed,

$$U_{crit} = 1.414 \left(\frac{1-s}{s}\right)^{\frac{1}{2}} g^{\frac{1}{4}} T^{\frac{1}{4}}$$

$$= 660 \text{ cm/sec, for the water-air interface}$$

In the above equation, s = density ratio of air and water, and T = surface tension. At this windspeed, waves of 1.7 cm. wavelength should become unstable and grow. At lower windspeeds, all waves are stable and none will grow.

These calculated values do not agree with observations, for the first waves to appear on the sea are about 8 cm. wavelength, when the wind is about 110 cm/sec. To account for the discrepancy, Jeffreys produced his "sheltering" theory, where it is assumed that the energy input to the wave by wind just equals the viscous dissipation in the wave. Jeffreys' theory adequately explains that observed phenomena on the sea and shows that

$$U_{crit} \propto \nu^{1/3}$$

where ν = kinematic viscosity of the heavier fluid.

It has never been clearly shown that the Kelvin-Helmholtz instability exists at all, although there has been some speculation on the part played by this instability at 660 cm/sec., at which windspeed some striking changes occur on the sea. A recent experiment does show, however, that the Kelvin Helmholtz (K.H.) instability is indeed real, and gives some additional grounds for believing that the Jeffreys' theory is correct.

By increasing V it should be possible to increase the U_{crit} of Jeffreys' waves, without changing U_{crit} of K.H. waves. If ν is increased sufficiently, then

$$U_{crit}(\text{Jeffreys}) > U_{crit}(\text{K. H.})$$

so that if the K.H. instability is real, it will appear on a plane fluid surface before Jeffreys' waves appear. This experiment was tried in a wind tunnel where air was blown over a viscous oil. As the windspeed was increased a small train of ripples was seen moving slowly down wind on the surface of the oil. At a sharply defined critical windspeed, one wave of the train suddenly became unstable, grew quickly in height, and rushed ahead of the train.

At higher windspeeds than U_{crit}, the waves became more numerous and were very similar to water waves. The relevant data are:

		Type of surface	
		Water-air	Oil-air
Viscosity of lower fluid ν		0.018 poise	2.51 poise
Surface tension T		72 dynes/cm	34 dynes/cm.
Density of lower fluid ρ		1.0 gm/cm^3.	0.875 gm/cm^3.
Kelvin-Helmholtz instability			
Windspeed U_{crit}	calculated	660 cm/sec.	516 cm/sec.
	observed	not clearly seen	500-560 cm/sec
Wavelength at critical λ	calculated	1.8 cm.	1.27 cm.
	observed	not clearly seen	1 cm
Wave speed at critical C	calculated	0.8 cm/sec	0.67 cm/sec.
	observed	not clearly seen	1 cm/sec
Jeffreys' "sheltering" instability			
U_{crit}	calculated	110 cm/sec.	570 cm/sec
	observed	110 cm/sec.	not clearly seen
Sheltering coefficient		0.3	0.3
λ at critical windspeed	observed	8 cm.	not clearly seen

The good agreement of the theoretical and observed U_{crit} leads to the conclusion that the K.H. instability is indeed real; and therefore the phenomena observed on the sea at about 660 cm/sec windspeed are probably due to this instability. Also the predicted increase of Jeffreys' U_{crit} which experiment shows to be true, is additional evidence that the "sheltering" theory cannot be much in error. Both sorts of instability can exist together, each playing their part in the mechanism of the generation of waves by wind.

References

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RESUME

L'INSTABILITE KELVIN HELMHOLTZ DES ONDES SUR LA SURFACE DE LA MER

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l'instabilité Kelvin-Helmholtz des ondes sur la surface d'un liquide n'est pas observée en mer sous l'action du vent. Par contre, on l'observe à la surface d'une huile très visqueuse quand un courant d'air passe sur la surface dans une soufflerie. Un ciné-film montre le phénomène.