CHAPTER 32

HYDRAULIC STUDIES IN ESTUARIES

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There is a region in estuaries where water velocities are far below critical, and where sea level variations greatly affect the hydraulic conditions, but where still distinct channels exist.

In this region the water levels are usually as much influenced by tides and meteorological conditions as by river discharges. Floods may arise from high discharges as well as from storm surges. In this study relationships are presented where hydrological and meteorological factors are included.

The affected area is treated as a system of channels with more or less unidirectional flow in each. Frequently the flow conditions vary considerably over the length or width of a sea or river arm. The determination of hydraulic parameters is, therefore quite difficult. In this paper, methods for a rational estimate of parameters have been shown.

Using these parameters, the influences of channel topography, river flow and meteorology are considered in a system of equations. These equations are transposed to an applicable form for integration by finite differences, which in dynamic cases could be carried out along characteristics.

FACTORS AFFECTING WATER LEVELS

The presented methods were deduced to study the effects of extreme hydrological conditions and also to estimate the influence of various river training works and other changes in the hydraulic properties of river channels.

It is suggested, that the river discharge is considered by inserting the discharge as a boundary condition at a point sufficiently high upstream, so that the effect of upstream water surface elevations can be eliminated. By a similar process, the effect of the sea can be reduced to a consideration of water surface elevations as a boundary condition. In the actual area under study, meteorological conditions and hydraulic parameters are inserted which determine water surface elevations and discharges in this reach as a function of the described upstream discharge and sea levels.

These known and sought quantities are illustrated on figure 1.

Statistical methods can be used to establish the design conditions, as the upstream discharge, the sea levels and the meteorological and topographical conditions in the studied area. From this statistical information the remaining data can be calculated.

MATHEMATICAL MODELS

In many cases, floods in estuaries can be studied on a mathematical model where the affected area is treated as a system of channels. The first step towards this approximation is shown on figure 2 where the same area is represented as on figure 1.

Some parameters might be required at sea to describe different levels at the mouths of various channels of the delta of the river. In many cases, however, these discharges and variations of water levels can be neglected and the whole system of channels can be treated as a linear system as shown on figure 3.

In each of the linearized channels flood routing procedures will be applied which are expanded to include the influence of flow, barometric pressure and winds. Tidal affects may be added if necessary. In the following treatment, ice is neglected since it poses a different kind of problem.

PLANNING OF MATHEMATICAL TREATMENT

The established flood routing procedures seem to require some amendments for this application.

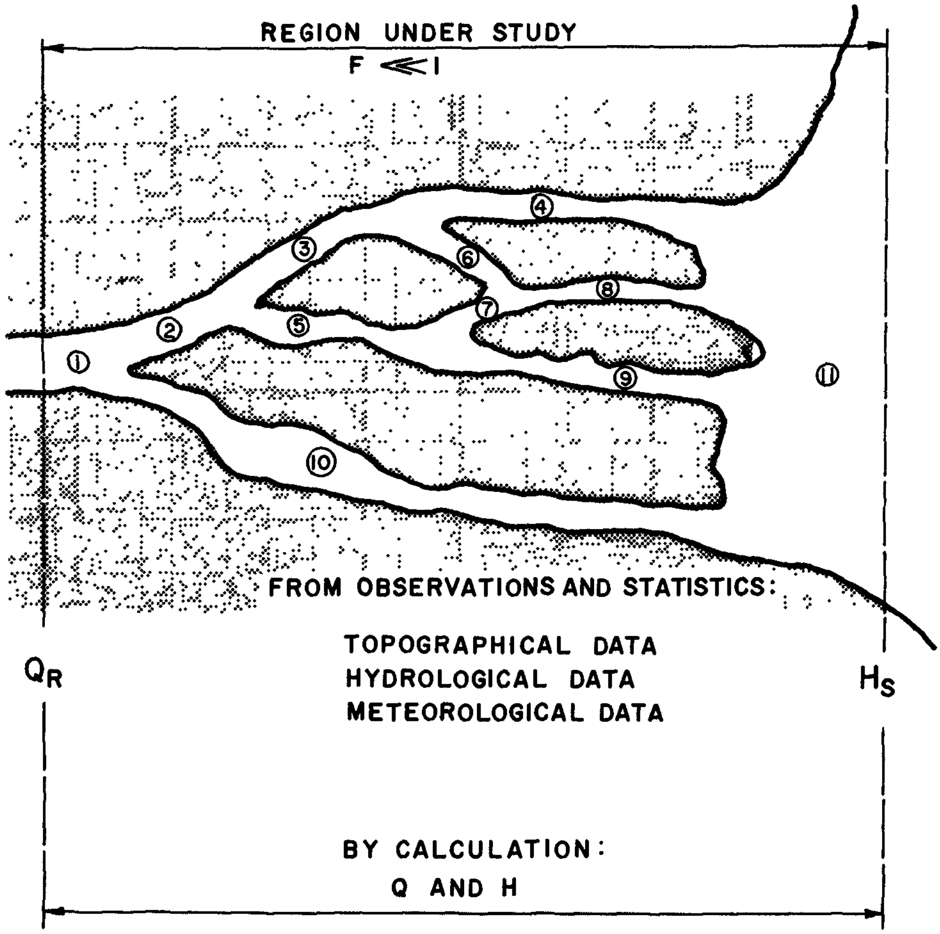
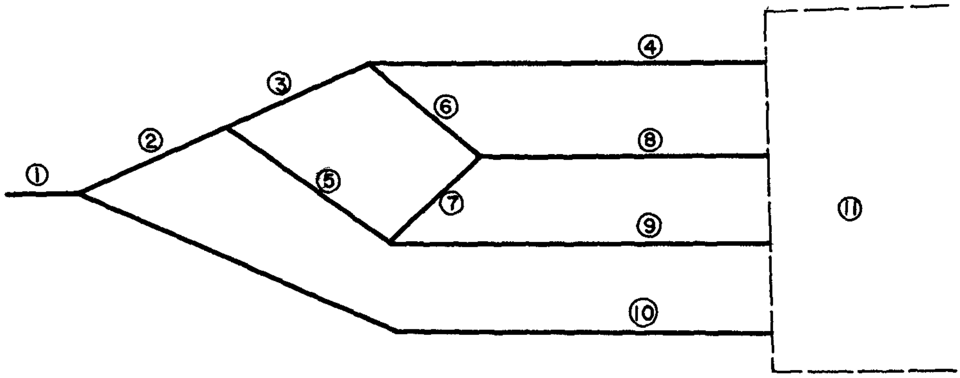


FIGURE I.
FACTORS AFFECTING WATER LEVELS.



NUMBERS INDICATE CHANNELS WHICH REPRESENT RIVER BRANCHES WITH THE SAME NUMBER AS IN FIGURE 1.

FIGURE 2.
STRAIGHT CHANNELS REPRESENTING RIVER BRANCHES.

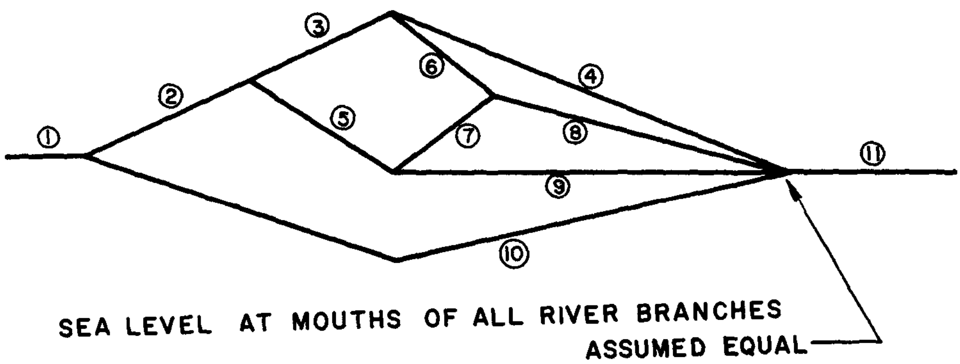


FIGURE 3.
APPROXIMATION BY A SYSTEM OF CHANNELS.

In systems which have been generally used, hydraulic parameters are usually determined by fitting data which will reproduce actual conditions. In order to speed the verification procedure and to permit a better understanding of the causes of floods, it was felt that rules for a direct computation of the parameters would be helpful. These would then need a fine adjustment only and both speed and accuracy would be gained. Such a computation procedure has been established by an integration of general flow formulae over the width of a water course.

A special application of flood routing and storm surge investigations is the study of the influence of regulating and training works. A reliable procedure of calculating parameters should add considerably to various empirical methods for the estimate of parameters in changed conditions.

Another side of the problem is the integration of the differential equations, once proper parameters are established. Peaks of floods are of special interest and peaks are probably influenced by change in river regime. It is considered, therefore, necessary to establish integration methods where truncation errors would be kept at a minimum.

NOTATIONS AND DEFINITIONS

s = length along coordinate flow line,
ds = line element along flow line which is chosen as coordinate,
$\alpha\, ds$ = line element along any arbitrary flow line,
r = transverse coordinate,
o & b = shores measured along r,
t = time,
H = water surface elevation,
D = depth of water,
f = shear at an interface,
f_f = total shear on water from flow, *
f_w = total shear on water from wind, *
g = gravity acceleration,
γ = specific weight of water,
ρ = density of water,
q = flow per unit width,
Q = water course discharge,
Ω = rotation of the earth,
φ = geographical latitude.

* Total shear on water can be expressed approximately as follows:

$$f_{tot} = f_{surface} + f_{bed} = (f_s + f_b)_w + (f_s + f_b)_f \qquad (1)$$

$$f_{tot} = f_w + f_f \,, \qquad (2)$$

where f_W is a function of wind along and f_f is a function of flow alone.

CHOICE OF COORDINATES

In the following treatment a curvilinear system of coordinates is used. Longitudinal coordinates are taken along lines where $\Delta Q/Q$ = constant. These coordinates represent some kind of flow lines. Transverse coordinates are taken at right angles to the longitudinal coordinates at all points, as shown on figure 4.

The suggested coordinate system may be found by successive approximations. First, a certain distribution of flow is assumed and coordinates are drawn. Once the coordinate system is established, the formulae below may be applied, and a more accurate flow distribution may be established, which could form the basis of a second approximation.

Flow equations are written for components along the flow lines and normal to them. From these equations, laws relating water surface elevations to discharge and meteorological phenomena are established by integration over the width of a water course. As shown in notations, the functions are given in reference to lengths along a chosen coordinate flow line.

FLOW DISTRIBUTION

Flow on a vertical is assumed to be practically unidirectional in determining flow lines and friction formulae. In one-dimensional cases the barometric effects are negligible and are, therefore, omitted below. It would cause no difficulties to include the barometric terms to the system. For a study of transverse water surface and flow variations, differential flow equations are written as given below.

$$\left.\begin{aligned}
&\frac{\partial H}{\partial s} + \frac{1}{2g}\frac{\partial}{\partial s}\frac{q^2}{D^2} + \frac{1}{g}\frac{q^2}{D^2}\frac{\partial Q/\partial s}{Q} + \frac{\alpha}{Dg}\frac{\partial q}{\partial t} - \frac{f_f\alpha}{D\gamma} - \frac{f_w\alpha\sin(w,r)}{D\gamma} = 0\\
&\frac{\partial H}{\partial r} - \frac{q^2}{D^2 g}\frac{1}{\alpha}\frac{\partial \alpha}{\partial r} + \frac{q}{D}\Omega\sin\varphi - \frac{f_w\cos(w,r)}{D\gamma} = 0\\
&\frac{\partial H}{\partial t} + \frac{1}{\alpha}\frac{\partial q}{\partial s} = 0
\end{aligned}\right\}(3)$$

For the determination of flow distribution, it is assumed that bottom friction and gravity terms dominate. In most cases, these assumptions lead to a good estimate of parameters. This gives

$$\left.\begin{aligned} \frac{\partial H}{\partial s} &= \frac{f_f \alpha}{D\gamma} \\ \partial H/\partial r &= 0 \end{aligned}\right\} \tag{4}$$

By inserting friction formulae for unidirectional flow, the following expressions are obtained

$$\frac{\partial H}{\partial s} = \frac{f_f \alpha}{D\gamma} = -\frac{C_f\, q\, |q| \alpha}{D^3 g}\ , \tag{5}$$

where C_f is a slow function of depth, as

$$C_f = \frac{0.16}{\left(\ln \frac{D}{en}\right)^2}\ . \tag{6}$$

Total discharge may be expressed as

$$Q = \int q\, dr = -\left(\frac{\partial H}{\partial s}\right)^{1/2} \int \frac{D^{3/2} g^{1/2}}{C_f^{1/2}\, \alpha^{1/2}}\, dr\ , \tag{7}$$

and the relation between unit flow and total discharge is

$$\frac{q}{Q} = \frac{\dfrac{D^{3/2}}{C_f^{1/2}\, \alpha^{1/2}}}{\displaystyle\int \frac{D^{3/2}}{C_f^{1/2}\alpha^{1/2}}\, dr}\ . \tag{8}$$

EQUATIONS FOR TOTAL FLOW

For a study of longitudinal variations of water surface elevations as a function of total discharge, the following form of equations is written

$$\begin{gathered} \int_0^{\delta s}\!\int_0^{b} \frac{\partial q}{\partial t}\, dr\, ds + \int_0^{\delta s}\!\int_0^{b} \frac{q}{\alpha Q} \frac{\partial}{\partial s} \frac{Qq}{D}\, dr\, ds + \\ + \int_0^{\delta s}\!\int_0^{b} \frac{Dg}{\alpha} \frac{\partial H}{\partial s}\, dr\, ds - \int_0^{\delta s}\!\int_0^{b} \frac{f_f}{\rho}\, dr\, ds - \int_0^{\delta s}\!\int_0^{b} \frac{f_w \sin(w,r)}{\rho}\, dr\, ds = 0 \\ \int_0^{\delta s}\!\int_0^{b} \frac{\partial q}{\partial t}\, dr\, ds + \int_0^{\delta s}\!\int_0^{b} \frac{\partial H}{\partial t}\, \alpha\, dr\, ds = 0 \end{gathered} \tag{9}$$

By summing up the effects over the width of a water course, relationships between total discharge and certain average values of water surface elevations, wind forces and other phenomena are obtained

$$\left.\begin{aligned} &\frac{\partial Q}{\partial t} + \frac{\partial}{\partial s}\frac{Q^2}{A_e} + A_g \cdot g \frac{\partial H}{\partial s} + \frac{C_m}{A_e D_m} Q|Q| - \frac{f_w}{\rho}\cos(W,s)\, B_W = 0 \\ &\frac{\partial Q}{\partial s} + B_s \frac{\partial H}{\partial s} = 0 \end{aligned}\right\} \tag{10}$$

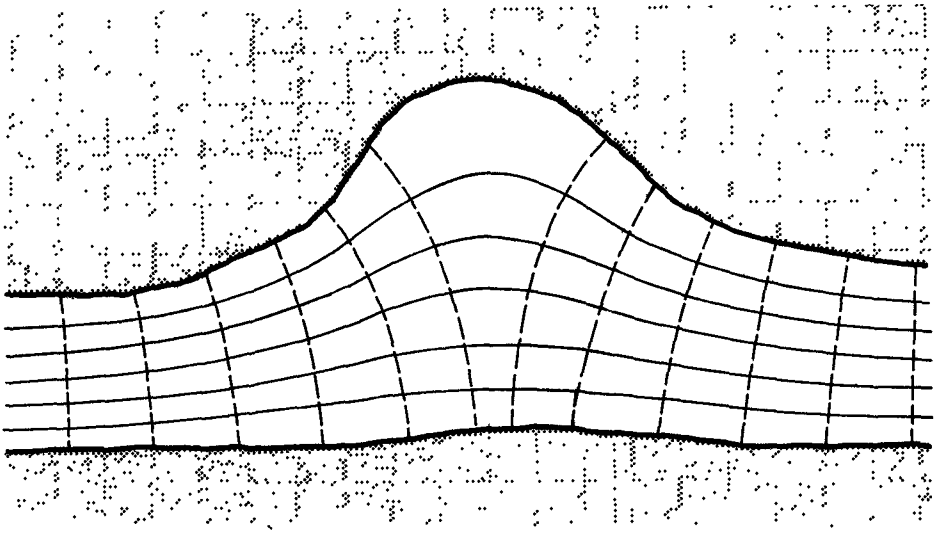
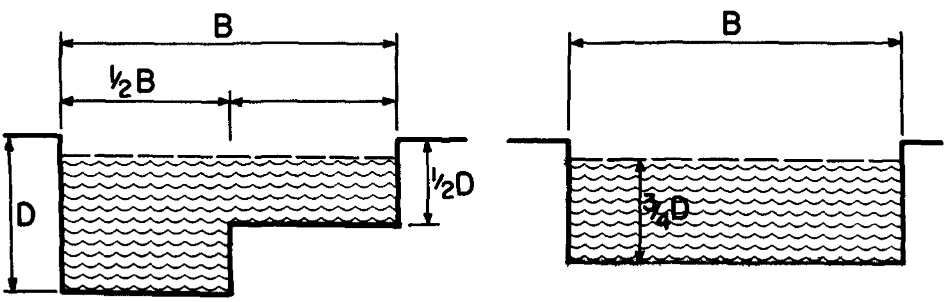


FIGURE 4.
COORDINATE SYSTEM.



$$A_1 = A_2 = \tfrac{3}{4} BD$$

$$\frac{i_1}{i_2} = 0.92$$

$$Q_1 = Q_2$$

FIGURE 5.
FLOW PROPERTIES OF VARIOUS CROSS SECTIONS.

Notations are given at the end of this section.

Above the parameters are expressed as functions of time and distance. It is more appropriate to consider them functions of time, distance and water surface elevation. This will change some of the derivatives.

The expressions may be written in the following form

$$\left.\begin{aligned} \frac{2Q}{A_e}\frac{\partial Q}{\partial s} + \left(A_g \cdot g - \frac{Q^2}{A_e^2} B_e\right)\frac{\partial H}{\partial s} + \frac{\partial Q}{\partial t} &= k \\ \frac{\partial Q}{\partial s} \qquad\qquad\qquad\qquad + B_s \frac{\partial H}{\partial t} &= 0 \end{aligned}\right\} \qquad (11)$$

where

$$k = -\frac{c_m}{A_e D_m} Q|Q| + \frac{f_w}{\rho}\cos(w,s)\, B_w + \frac{Q^2}{A_e^2}\frac{\partial A_e}{\partial s} \qquad (12)$$

and where the parameters are

$$\begin{aligned} A_g &= \int \frac{D}{\alpha}\, dr \\ A_e &= \frac{\left(\int \frac{D^{3/2}}{C^{1/2}\alpha^{1/2}}\, dr\right)^2}{\int \frac{D^2}{C\alpha^2}\, dr} \\ \frac{D_m}{C_m} &= \frac{\int \frac{D^2}{C\alpha^2}\, dr}{\int \frac{D}{\alpha}\, dr} \\ B_s &= \int \alpha\, dr \\ B_w &= \int \sin(s,r)\, dr \\ B_e &= \partial A_e / \partial H \end{aligned} \qquad (13)$$

It should be noted, that the above expressions were deduced under some simplifying assumptions. So are the vertical flow variations, side slopes of water surface and transverse wind force components neglected.

The effects of tidal forces and eddy losses are not shown in above formulae. Terms showing their influence may be added, however, without changing the general system.

STEADY FLOW

The following abbreviated expression is applicable to steady flow with constant discharge. Length coordinates are increasing in flow direction.

$$\frac{\partial H}{\partial s} = \frac{\frac{Q^2}{A_e}\left(-\frac{C_m}{D_m} + \frac{\partial A_e/\partial s}{A_e}\right) + \frac{f_w}{\rho}\cos(W,S)\; B_w}{A_g\, g - \frac{Q^2}{A_e^2} B_e} \qquad (14)$$

As above, turbulence losses are not included and require an additional term, if large enough.

The above equation permits rapid estimates of the factors influencing hydraulic parameters.

Obviously, bays at the channel shores and side channels contribute little to the heavy area, or to flow parameters, since tortuous flow paths in these areas give a high value for α.

In straight channels, the formulae for the effective area show that shallow portions of the channel contribute considerably to the effective area. Still there is a noticeable difference between the deduced parameters and formulae based on hydraulic radius. As an example, a channel is shown on figure 5 where half of the channel has double depth compared to the shallow portion.

Assuming C=constant and $\alpha = 1$, the same discharge gives an 8% smaller slope of the water surface in the left hand nonuniform section. This is reflected in the estimated parameters.

The made approximations were not chosen for direct application on wind tides in equilibrium. The assumption appears to be, however, as justified as any alternatives. For equilibrium the following equation is obtained

$$\frac{\partial H}{\partial s} = \frac{\frac{f_w}{\rho}\cos(W,S)\; B_w}{A_g \cdot g} = \frac{f_w \cos(W,S)}{\frac{A_g}{B_w}\cdot \gamma} \qquad (15)$$

DYNAMIC FORMULAE

The system of equations is well suited to a solution by final differences, integrating along characteristics. The following new system is establishe

$$\left.\begin{array}{l} ds = \beta\, dt \\ dQ - B_s \sigma\, dH = K\, dt \end{array}\right\} \qquad \left.\begin{array}{l} ds = \sigma\, dt \\ dQ - B_s \beta\, dH = K\, dt \end{array}\right\} \tag{16}$$

where K is defined above, and

$$\beta = \frac{Q}{A_e} + \sqrt{\frac{A_g g}{B_s} + \frac{Q^2}{A_e^2}\,\frac{B_s - B_e}{B_s}}$$

$$\sigma = \frac{Q}{A_e} - \sqrt{\frac{A_g \cdot g}{B_s} + \frac{Q^2}{A_e^2}\,\frac{B_s - B_e}{B_s}}$$

From these equations obviously the formulae for long waves may be obtained. The long wave celerity is

$$\beta = -\sigma = \sqrt{\frac{A_g \cdot g}{B_s}} \tag{17}$$

which shows the slowing down caused by bays and side channels. They contribute namely substantially to B_s but hardly to A_g.

Obviously other methods are called for the treatment of other regions. Towards the sea, a two-dimensional method may be necessary, and higher upstream friction waves may indicate different methods.

Full solutions on characteristics have been studied and adapted for electronic computers by B. Hellström, E. Asplund and the author.