

Part 1 BASIC INFORMATION FOR SHORELINE INVESTIGATION



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CHAPTER 1

HINDCAST WAVE STATISTICS FOR THE GREAT LAKES

Thorndike Saville, Jr. Hydraulic Engineer, Research Division Beach Erosion Board, Corps of Engineers

The General Investigations program of the Beach Erosion Board comprises investigations, regional rather than local in scope, designed to improve, simplify, and expedite the solution of local problems, by giving a compilation of all existing data pertinent to shore processes in the particular region. As a first step in the compilation of these data, a study of wave and lake level conditions on the Great Lakes is being made. The results of such studies for Lake Michigan, Lake Erie, and Lake Ontario have recently been completed and published as Technical Memorandums of the Beach Erosion Ecard (Saville, 1953).

Five stations on Lake Michigan, four stations on Lake Erie, and three stations on Lake Ontario were selected for a comprehensive wave analysis, the locations being as shown in Figure 1. These particular stations were selected since it was thought that they would give adequate coverage to the entire United States' shore of the lakes, and permit interpolation of values between stations, thus enabling one to obtain an accurate representation of wave action at any point along the lakes' shore.

Wave characteristics were hindcast from synoptic weather charts for each station for the three-year period 1948-1950. The weather maps used were the United States Surface Synoptic Charts compiled at six-hour intervals by the U. S. Weather Bureau. Fetch areas, and the wind speeds and durations in these areas, were determined directly from the weather maps; these values were used with the curves derived by Sverdrup and Munk (1947) and revised by Arthur (1947) to obtain the hindcast wave characteristics. The revisions in methods recently suggested by Bretschneider (1951) were not employed; hence the wave periods determined may be expected to be slightly low. The only major variation from the usual methods of wave forecasting or hindcasting (Hydrographic Office, 1951) was that the surface wind was determined directly from reported observations rather than from a gradient wind determined from the isobar spacing. It was thought that with the lake area so small in comparison to the area of the pressure cell, the isobaric pattern on the surface would be influenced to a large extent by the surface topography, and gradient

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winds determined from the isobar spacing would not necessarily give true values of wind velocity over the lake surface. Hence reported values of the surface wind could be expected to give a more realistic figure of the wind velocity. Observations have shown (U. S. Weather Bureau, 1951) that the greater surface friction over land serves to reduce the wind over land from what it may be over water. Since the reported values were almost always obtained at land stations, the wind speeds used in the analysis may have been lower than those actually occurring over the lake in the generating area. Some compensation was made for this by selecting the top speed of the Beaufort range reported rather than the middle value.

The wave characteristics thus determined are for the significant wave -- that is, the period is that of the predominating waves, and the height is the average of the higher one-third of these predominant waves. It should also be noted that these wave conditions are deep water conditions, and must be used in conjunction with refraction diagrams to obtain inshore values. The values obtained have been summarized in tabular form in the Beach Erosion Board Technical Memorandums. These tables show, for each station, the number of hours' duration that deep water waves of any given height, period and direction occurred during any month of the three-year period; and also for each month (as summations) the number of hours' occurrence of waves of any particular height and period exclusive of direction; the number of hours' occurrence of waves of any particular height and direction exclusive of period; and the total number of hours! occurrence of waves of any particular height. In addition tables summarizing these values for an entire year (rather than a single month) are also shown -- as in Table 1.

As an example of the data presented, from Table 1 (for the station at Milwaukee, Misconsin), waves of 2 to 3-foot height and 3 to 4second period from the east were hindcast to occur for 72 hours during 1948, 84 hours during 1949, and 54 hours during 1950. Thus, waves of this category were hindcast to occur for a duration of 210 hours during the three-year period and hence can be expected to occur for about 70 hours (on the average) during any year in the future. Waves of 2 to 3-foot height and 3 to 4-second period (from all directions) were hindcast to occur for 1,458 hours over the three-year period, or an average of 486 hours per year. Waves of 2 to 3-foot height from the east (all periods) were hindcast to occur for 306 hours over the three-year period, or an average of 102 hours per year. Waves of 2 to 3-foot height (all periods and all directions) were hindcast to occur for 2,508 hours over the three-year period, or an average of 836 hours per year.

During much of the winter season, portions of the lakes are covered with ice, and fetch areas are limited considerably. In addition, for a somewhat greater portion of the winter season, the coast areas of the lakes are covered with ice, and, even though waves are generated in offshore areas, they never reach the shore, being interrupted by the ice around the rim of the lake. No account of this effect of the ice was taken in the actual hindcasting of the waves, and the durations given for the various winter months are computed as





Average Full Year

Average Ice-Free Period (IApril - 30 Nov)



Table 1.

STATISTICAL HINGCAST DATA FOR LAGE MILUEIN, STATION 8, MILIANGE, AISSONSIN Durations given in hours "Maight and porisd grouping: incluis lower value but not the upper FULL TEAK																												
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1-2	NNE NE ENE SSE SSE SSE SSE SSE S S Total	12 12 18 24 6 6 18 96	12 24 16 6 18 12 12 6 108	30 24 6 12 72	54 36 30 24 18 18 36 276	180 96 102 66 81 102 66 81 102 66 81 102 780	192 144 84 120 72 64 116 76 116 786	120 20 70 70 70 70 70 70 70 70 70 70 70 70 70	432 544 210 300 198 210 372 150 2316	54 78 30 30 30 30 30 30 30 30 30 30 30 30 30	199424469124	7862512212	154 252 132 120 108 96 120 36 1008	6	36 6	12	514 6								252 186 96 180 102 120 180 90 1206	198 264 166 114 150 166 54 1278	234 269 162 104 114 514 78 1176	684 738 402 450 330 324 510 222 222
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7-8	HNE HE SHE 6 83E 95 9 Total													6 6 17	6 12	6 6 5 18	6 6 6 6 6 6 6 6 6 42	6	6 6	6 6	12 6 18				6 6 6 18	6 6 18	12 6 6 24	18 6 6 6 6 12 60
A ~9	NNE NE SSE S Total													18 6 6 36		6 6	18 6 6 6 4 42								18 6 6 30		6 6	18 6 6 6 6 42
9-10	3 1710													6			6					6			¢ 19			6 12
10-11	NE Total				•													6 12			6 12	6		•	6 10			6 18
11-12	39E																	6			6				6			6 4
12-13	E																	6			6 4				•			0 6
34-15 15-16	8785 1085																	0			0	6		6	5			6
Totals		378	396	378	1152	1152	1374	1284	3810	1344	1242	1230	3816	324	270	378	97 2	54	24	42	120	12	18	30	3964	3306	3330	9900
Calm Bours	T veer																						. /		5580 8784	5454 8760	5130 8760	16404 26304

though there were no ice on the lake, a fact that should be remembered in using these data.

From yearly records of lake and air temperatures, and the dates of the opening and closing of the lakes for navigation, an average ice-free period was determined for each station. For the three lakes considered this ran, on the average, from April through November, but varies somewhat from station to station. Tabulations of the wave data for this ice-free period were also made and presented in a form similar to that of Table 1.

The durations of waves of particular height and direction have also been tabulated as percentages of time for the three-year period and are shown graphically in wave roses for both the full year and the ice-free period. Sample roses for the station at Milwaukee are shown in Figure 2. It may be noted that, for convenience, as in other computations involving the ice-free year, 100 percent of the time represents 365 days rather than the actual number of days in the ice-free period.

Figures such as that shown in Figure 3 were drawn for each station showing the total percentages of time that the wave height may be expected to be greater than any particular height throughout the year. They thus show the (average) total duration time of specific waves over the year. Two curves are shown in each figure, one based on the data gathered for the entire year's period, and the other on just the average ice-free period (April through November). For example, from Figure 3, at the Milwaukee station the total duration of waves in excess of 10 feet in height during the ice-free period is expected to be 0.055 percent of the time; and 0.214 percent of the time during the full year. Hence waves 10 feet or higher can be expected to occur for a total duration of 19 hours (0.00214 x 365 x 24) over the course of each year, and, of this, 5 hours (0.00055 x 365 x 24) will be during the ice-free portion of the year when the waves will be certain to reach the shore.

Frequency curves, such as Figure 4, were also drawn, showing the frequency with which storms resulting in waves higher than a given height can be expected to occur. For example, from Figure 4, at the Milwaukee station on 1.10 percent of the days each year the waves may be expected to be ten feet or greater in height, and on 0.23 percent of the days they may be expected to reach this height during the ice-free portion of the year. Thus waves ten feet or higher may be expected to occur (on the average) four times each year (0.011 x 365); of these four occurrences, only one (0.0023 x 365) will be expected to occur during the ice-free portion of the year.

Combining the data obtained from the graphs on Figures 3 and 4, waves ten feet and higher may be expected to occur at the Milwaukee station about four times each year, and the average duration of each storm will be about five hours. During the ice-free portion of the year, waves of ten feet and higher may be expected to occur only once, and the duration of this storm is also expected to be about five hours.

There are, in general, two methods of plotting points to obtain frequency curves such as those shown in Figure 4. One, based on the so-called theory of sampling, involves the assumption that the known period of record (three years) is a fair average sample of all similar three-year periods over an infinite number of years, and that therefore the largest storm of this three-year period is the median of all storms of the same class in all other three-year periods. This results in a frequency given by the equation:

$$F = \frac{2N - 1}{2T} \times 100$$

where

- F = frequency (in percent) of the occurrence of storms equalling or exceeding the given storm
- T = number of days of record
- N = number of occurrences of a storm equal to or greater than the given storm

The second method essentially considers only the period of record, in which case the frequency becomes

$$F = \frac{N}{T} \times 100$$

Values of F are the abscissas of points on the frequency curve. Using the second equation above, the largest storm which occurred in the known three-year period would have an abscissa of 0.0914 percent and would represent the storm which would most probably occur once in three.years, i.e., would be the "three-year storm". But this would be contrary to the theory of sampling, where (above) the assumption is made that the largest storm in the known three-year period was the median of the largest storms in a long succession of three-year periods. Therefore, over a long period such as 300 years, it will be exceeded not 100 times, but 50 times; i.e., it is by definition not a "3-year storm", but a "6-year storm".

Either of the above two equations could be, and have been, used to prepare frequency curves. Although the former is the one most generally used for hydrologic data, the latter method has been used in this case. The use of this formula (F = 100N/T) will result in somewhat more conservative interpretation of the data, and was thought justified in view of the extremely short period of record (3 years).

The points plotted may be represented fairly closely by a straight line curve. Actually the published curves were not always drawn as the lines of best fit, but somewhat more weight was sometimes placed on the higher values. This again tends to give a somewhat more conservative curve, but was thought warranted in most cases.



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In view of the shortness of the period of record, some doubt arose as to the validity of extrapolation from these curves, and as to whether the three-years chosen were representative (i.e., that they represent average conditions, and not three years of abnormally high, or low, waves). Hence hindcasts were made for the Milwaukee station for a period of 12 years (1939-1950) for all storms which were expected to give waves greater than 8 feet. The points determined are also shown in Figure 4. These points fit a straight line curve very closely and, though the points mostly lie somewhat above those determined from the three-year data, the curve is not greatly different from that which would have been drawn from the three-year data (where the greater weight was placed in the higher values). It is interesting to note that though the maximum storm hindcast for the 12-year period was almost 17 feet, three storms were hindcast in the 16 to 17-foot grouping.

Observations of the "average maximum" wave were obtained by the Nilwaukee District of the Corps of Engineers (1933) at the Nilwaukee lightship over the period of 10 April 1931 to 28 September 1932 and these points are also shown in Figure 4. While the exact correspondence between the significant waves hindcast and the "average maximum" waves observed is not known, values should be closely comparable -- and although the observed points lie somewhat higher for the lower waves, agreement is good for the higher waves. From the above, it is thought that reasonable confidence can be put in the curves obtained, at least for values of the waves occurring with frequencies less than about once in 10 years.

Although for structural design purposes, the important factor is the size of the maximum probable wave (within a certain time period), for computations involving sand movement and littoral drift, a more desirable parameter would be some averaged factor including within it the effect of both height and period, the variation of these parameters, and the duration that waves of each particular category exist. Present day knowledge indicates that sand movement by wave action is best correlated with the amount of energy transmitted forward (and eventually on to the beach) by the waves. The total energy per unit width in cach wave is, in deep water

$$E_{o} = \frac{wLH^{2}}{8} \left[1 - 4.93 \left(\frac{H}{L}\right)^{2} - \frac{wg}{16\pi} H^{2}T^{2} \left[1 - 4.93 \left(\frac{H}{L}\right)^{2} \right]$$

where

w = unit weight of water = 62.4 lbs./cu. ft. g = acceleration due to gravity = 32.2 ft./sec./sec. H = wave height (ft.) T = wave period (sec.) L = wave length (ft.)

One-half of this energy is transmitted forward from deep water toward the shore, and it is this amount of energy that eventually reaches the shore line. The total energy transmitted forward in any given period of time $(E_{\rm T})$ is then $E_{\rm O}/2$ times the number of waves occurring

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in that period of time, and

$$E_{T} = \frac{E_{o}}{2} \frac{(3600t)}{T} = 7.195 \times 10^{14} \text{ H}^{2} \text{ t} \left[1 - 4.93 \left(\frac{\text{H}}{\text{L}}\right)^{2} \right]$$

where t is the duration of the waves in hours. If some particular time interval (say, one year) is considered during which waves of varying height and period pass a given point toward shore, then the heights and associated periods may be tabulated and there will be n groups. If the height of the ith group is represented by its class mark H., and the wave period in that group denoted by T., and the duration of the group by t_i, then the total amount of energy transmitted forward during the entire time interval is

 $\mathbf{E}_{\mathrm{T}} = \mathbf{E}_{\mathrm{T1}} + \mathbf{E}_{\mathrm{T2}} + \mathbf{E}_{\mathrm{T3}} + \dots + \mathbf{E}_{\mathrm{Ti}} + \dots + \mathbf{E}_{\mathrm{Ti}}$

and
$$\mathbf{E}_{\mathrm{T}} = 7.195 \times 10^{4} \sum_{i=1}^{i=n} H_{i}^{2} T_{i} t_{i} \left[1 - 4.93 \left(\frac{H_{i}^{2}}{L_{i}}\right) \right]$$

For each station a tabulation was made of the average energy transmitted forward from deep water toward the shore in each category of height, period, and direction during both the average ice-free period and the entire year. Table 2 shows such a tabulation for Milwaukee for the full year. Since the values in the original tables (as Table 1) represent significant wave height and period, these energy values are those obtained if the wave system is uniform and consists only of waves of significant height and period. Wave trains in nature are, however, exceedingly irregular, and have less energy than that determined by the significant wave concept. The relation-ship between the actual energy contained in a given wave train and that computed from the significant wave has been examined somewhat by personnel at Scripps Institution of Oceanography (1947) and more recently by Barber (1950) and Darbyshire (1952), and has been found to be very nearly a constant ratio (on the order of 0.58). The energies given, therefore, may be considered to be the true value of the energy multiplied by some nearly constant value, and hence can be used to determine quite accurately ratios of energies from different directions. These latter represent very closely the ratios of the drift-producing forces. Summations of these energies for each direction and period were shown graphically in figures similar to Figure 5.

In utilizing the data from these three publications of the Beach Erosion Board it must be remembered that all the wave data given refer to deep water conditions -- that is, depths greater than onehalf the wave length. As such, interpolation between stations to obtain values for other points along the shore is quite valid, and it is felt that adequate deep water hindcast values may be thus obtained

Table 2.

EMPROY TABULATION FOR AVERAGE YEAR "OR LAKE MICHIGAN, S"ATION B, MILWAUKEE, AISCONSIN Emergies in foot- pounds per foot of crest per year x 10⁻⁴. Height and period groupings include upper value but not the lower FULL YEAR

Height (feet)	Period (sec.)	1-2	2-3	3-4	4-5	5-6	6-7	Total
NNE	NNE NE ENE	273 250 238	101 կ64 343	113 85	<u></u>			487 799 581
0.5-1	e FSE SE SSE	166 131 119 285	363 222 181 302	28				529 353 300 615
	S Total	226 1688	20 1996	28 254				274 3938
1-2	NNE NE ENE SE SSE SSE S Total	401 267 445 223 178 134 134 267 2049	5767 5927 2804 1005 2643 2804 1966 2003 30919	2712 1,715 21,86 2260 2031 1808 2260 678 18983	1310 146 1456			10190 11.085 5735 6488 4855 1.7166 2948 2948 53407
2~3	NNE HE ENE SE SE SSE S Total	207 104 104	8070 3708 1745 3272 1091 3053 6543 2617 30099	11555 13117 121& 10931 5934 9369 10618 2186 75890	11298 1211 401 404 13317			31130 18036 14133 14137 7025 12122 12122 17565 1803 119721
3-4	NNE NE E E SE SE SSE S S Total		1214 829 415 415 1244 829 4976	23087 15289 7291 9113 9721 10329 16404 2430 93564	19721 4733 789 6311 2366 33920			Luos2 19922 8909 10136 10036 20014 3259 132160
4-5	NNE NE ENE SE SE SSE S Total			5965 1988 1971 1971 3977 8948 8948 8948 8948	909li 3897 2598 389 7 2598 5196 3897 31177	7 <i>9</i> 88 7988	1892 1892	23017 5885 7569 8868 6575 111111 12815 10810 89773
5-6	NNE NE ENE ESE SE SSE S Totel.			5864 1466 1466 1466 1466 1466 2932 2932 19058	5795 11591 1932 1932 9659 5795 9659 46363	7138 4759 11897		18797 17816 3390 3390 11125 57955 14057 2912 77318
6-7	NNE NE ENE ESS SSE S Total				5367 2683 5367 2683 2683 10732 5367 34882	3315 3315 6630	7869 7869	16551 2683 5367 2683 2683 14047 5367 49381
748	NNE RNE ESE SSE S Total				3549 3549 3549 3549 3549 3549 3549 3549	8801 ໄມ່ເວາ 13202		12350 35149 35149 35149 35149 35149 35149 7950 38045
8-9	NRE NE SSE S Fotal				13571 4524 4524 4524 4524 4524 31667			13571 4524 4524 1524 1524 1524 31667
9-10	8				5602			5602
10-11	NNE NE Total					8529 8529 17058	10193 10193	18722 8529 27251
11-12	SSE					10182		10182
12-13	E					11969		11969
14-15	NNE					15923		15923
15-16	NNE						21904	21904
	Totals	4152	67990	2561,65	223227	94819	4185 8	688541

for essentially all points on the United States' shores of Lakes Hichigan, Erie and Ontario. Standard refraction techniques may then be used to obtain inshore data from this deep water data. While the production of wave statistics by the hindcast technique is still admittedly of undeterminate quantitative accuracy for inland waters such as the Great Lakes, it is thought that these data will nevertheless provide the engineer with better wave data than have heretofore been available.

REFERENCES

- Arthur, R. S. (1947). Revised Wave Forecasting Relationships: Scripps Institution of Oceanography, Mave Report, 73 (Unpub.).
- Barber, N. F. (1950). Ocean Waves and Swell: London: Institution of Civil Engineers, Maritime and Waterways Division.
- Bretschneider, C. L. (1951). Revised Wave Forecasting Relationships: Proc. 2nd Conf. on Coastal Engineering.
- Darbyshire, J. (1952). The Generation of Waves by Wind: Proc. Roy. Soc., Ser. A, V. 215, n. 1122.
- Hydrographic Office, U. S. Navy (1951). Techniques for Forecasting Wind Waves and Swell: H. O. Pub. 604.
- Milwaukee District, Corps of Engineers (1933). Lake Michigan Wave Measurements at Milwaukee, Wisconsin: Misc. Report 20/203-1, (Unpub.).
- Saville, T., Jr. (1953). Wave and Lake Level Statistics for Lake Michigan: Beach Erosion Board Tech. Memo. No. 36.
- Saville, T., Jr. (1953). Wave and Lake Level Statistics for Lake Erie: Beach Erosion Board Tech. Memo. No. 37.
- Saville, T., Jr. (1953). Wave and Lake Level Statistics for Lake Ontario: Beach Erosion Board Tech. Memo. No. 38.
- Scripps Institution of Oceanography (1947). A Statistical Study of Wave Conditions at Five Open Sea Localities Along the California Coast: Wave Report 68 (Unpub.).
- Sverdrup, H. U., and W. H. Munk (1947). Wind, Sea, and Swell: Theory of Relations for Forecasting: Hydrographic Office, U. S. Navy, Pub. No. 601.
- U. S. Weather Bureau, Hydrometeorological Section (1951). Analysis of Winds Over Lake Okeechobee During Tropical Storm of August 26-27, 1949: Hydrometeorologic Report 26.