CHAPTER 7

NOTES ON THE GENERATION AND GROWTH OF OCEAN WAVES UNDER WIND ACTION

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The basic problem of forecasting wind-generated waves is the development of equations which express the energy budget between wind and waves, and the derivation of physical laws governing the growth of the component wave trains. The waves can grow only in the case where the supply of energy by wind exceeds the loss of energy by friction and turbulence. Thus any attempt to calculate the growth of ocean waves under wind action requires a knowledge of the energy supply and the energy dissipation in every phase of wave development.

Owing to the complexity of the actual wave motion it is exceedingly difficult, if not impossible at present, to follow this mechanism in detail. We are forced to concern ourselves with mean effects, and until now every approach to the problem has been based on certain empirical relationships and hypotheses. The purpose of the present paper is to call attention to a few recent advances and to outline briefly the hypotheses involved in the semi-theoretical treatment of the problem of generation and growth of ocean waves under the action of wind.

NOTES ON ENERGY TRANSFER FROM WIND TO WAVES

The first attempt to explain the generation of waves under wind action quantitatively has been made by Jeffreys (1925, 1926) in his "theory of sheltering," taking into account both the turbulent character of the wind and the viscosity of the water. By neglecting tangential action of the wind he considered only the power transmitted by the vertical component of the surface stress, τ_n , as the most important constituent in doing a net amount of work. The distribution of τ_n over a simple wave profile,

$$\eta = a \sin \kappa (x - \sigma t), \tag{1}$$

is according to Jeffreys

$$\tau_n = s\rho'(v - \sigma)^2 \, \partial\eta/\partial x \tag{2}$$

In these formulae a = wave amplitude, $\lambda = 2\pi/\kappa$ = wave length, σ = phase velocity of wave propagation, v = wind velocity, ρ' = density of the air. The unknown factor of proportionality, s, was called the "sheltering coefficient." With these assumptions concerning the wind pressure against different portions of the wave profile, the average energy supply from wind to waves per unit surface area is given by

$$A_n = \frac{1}{2} s\rho'(v-\sigma)^2 a^2 \kappa^2 \sigma \tag{3}$$

Difficulties arise in the determination of the factor s and of the "effective" wind velocity in the difference $v - \sigma$ at the "sea surface." Here the question of the reference level at the wavy sea surface has to be

*The results of this research have been sponsored by the Office of Naval Research, Washington, D.C.

answered. It would be possible to define an average wind velocity $\bar{v}$ for a given "anemometer height" and to determine empirically a factor $\bar{s}$ with reference to that particular height. By such a procedure we approach the problem on a semi-theoretical basis. But in any case it should be kept in mind that the determination of $\bar{s}$ (or s) as defined by (2), from simultaneous wind and wave measurements, for example, requires the elimination of effects of possible tangential stresses which may have been included in experimentally measured total stresses.

This has to be taken into consideration when the wavelets grow beyond their initial stage, that is when the wavelets attain their maximum steepness, and the unstable crests indicate small breaking processes. There is some evidence that such instability occurs at a wind velocity as small as $v = 125$ cm/sec and with wavelets of $\lambda = 11$-12 cm (Neumann, 1949). The sea surface in this state acts as an hydrodynamical "rough" surface, and tangential stresses are no longer to be considered as purely viscosity stress

The problem becomes even more complicated when considering the further development of waves to the stage of "turbulent sea." The first approach to this problem was made by Sverdrup and Munk (1947). They tried to take into account also the additional power transmitted by the horizontal wind stress component, but they applied the total surface stress instead of the tangential component and regarded it as constant over different portions of the wave profile.

The total wind force, τ, at any point of the wave profile may be split up into a component τ_n, acting normally to the rough wavy interface, and a component τ_t, acting in a tangential direction. Or, taking these forces with the negative sign, the <u>total resistance</u> of the rough surface may be considered as the vector sum of a "pressure resistance" and a "friction resistance." This has been done for practical reasons when considering the rough form of the wavy air-sea interface and approximating it by a "smoothed wave profile." The resistance of the rough superposition then will contribute to an effective frictional resistance, whereas the pressure resistance is determined by the pressure differences between the windward and the leeward slope of the general wave profile. These considerations form the basis of an approach to the problem recently made by the author (Neumann, 1950, 1952).

We may assume, that the distribution of τ_n and τ_t over different portions of the wave profile is given by expressions of the form

$$\begin{aligned} \tau_n &= \bar{\tau}_n + \tau_n' \cos\kappa(x - \sigma t) \\ \tau_t &= \bar{\tau}_t + \tau_t' \sin\kappa(x - \sigma t) \end{aligned} \tag{4}$$

Here $\bar{\tau}_n$ and $\bar{\tau}_t$ represent constant stress components over the wave profil and $\bar{\tau}_t$ delivers some power to horizontal surface drift. The constant term $\bar{\tau}_n$ may be disregarded. τ_n' and τ_t' are the amplitudes of the effective normal and tangential components of the wind stress. Both of them will depend in a complicated way on the wind velocity v, or $v - \sigma$, and the surfac conditions (roughness). At present it seems exceedingly difficult to deter mine these effective values separately with the necessary degree of accurac

Assuming a wind distribution over the wave profile as given by the form

$$v = \overline{v}\left[1 + a\kappa \sin\kappa(x - \sigma t)\right], \tag{5}$$

we have, by putting $\tau_t = \rho' f_t (v - \partial\xi/\partial t)^2$, the effective tangential component

$$\tau_t = \rho' f_t \overline{v}^2 \left\{1 + 2a\kappa\left(1 - \frac{\sigma}{\overline{v}}\right) \sin\kappa(x - \sigma t) + \ldots\right\} \tag{6}$$

approximately, where $\xi = a \cos\kappa(x - \sigma t)$, the horizontal particle displacement, and f_t a dimensionless factor depending upon the roughness conditions of the wavy sea surface.

Similarly, retaining the plausible assumption of Jeffreys (formula (2)) and considering equation (1), the normal component may be written

$$\tau_n = \tau_n' \cos\kappa(x - \sigma t) = \rho' \overline{s}(\overline{v} - \sigma)^2 a\kappa \cos\kappa(x - \sigma t) \tag{7}$$

The dimensionless coefficient $\overline{s}$ is now related to the average wind velocity $\overline{v}$ at "anemometer height."

With these assumptions the total amount of work done by the wind on the wave is given by

$$(A)_\lambda = (A_n + A_t)\lambda = -\int_0^\lambda \tau_n (\partial\eta/\partial t)dx + \int_0^\lambda \tau_t(\partial\xi/\partial t)dx$$

or, after carrying out the integration with assumptions (6) and (7),

$$A = \frac{1}{2}\left(a\kappa\tau_n' + a\kappa\tau_t'\right)\sigma \tag{8}$$

Both of the components do a net amount of work at the particle velocity, even in the case of the Airy theory of waves, as long as the wind is faster than the wave.*

Formula (8) is based on very special assumptions. In the case of actual ocean waves the distribution of the stress components may be much more complicated. But, even in the case where our assumptions hold, the application of this formula requires a knowledge of τ_n and τ_t over the wave profile, or at least a knowledge of τ_n' and τ_t' for rough ocean waves of different steepness, wave form, etc. At this point difficulties arise in our problem. It has not been possible hitherto to determine the wind stress components separately for actual wave motion at the sea surface.

To avoid the difficulties involved in the separate determination of τ_n and τ_t over the actual wave profile, the author (Neumann, 1949, 1950, 1952) used the total effective stress, τ_{eff}, in place of (4). This quantity is

*In the case of Stokes waves the average surface mass transport velocity has to be taken into account.

given as an integral effect of the wind force exerted at the sea surface, and related to the wind velocity $\bar{v}$ at "anemometer height," that is, about 10 m above the sea surface. The empirical relationship, as derived from special oceanographic observations (Neumann, 1948) is of the form

$$\tau_{eff} = \rho' C v^2 , \tag{9}$$

where the dimensionless quantity C has the meaning of a total resistance coefficient over the rough wavy sea surface. It is not a constant, as has been assumed before, but depends upon the roughness pattern of the sea surface, and therefore on the stage of wave development itself. C has been determined empirically

$$C = \left(\frac{1\ {}^{m}/_{sec}}{v\ {}^{m}/_{sec}}\right)^{1/2} \cdot 10^{-2} \tag{10}$$

Now the hydrodynamical characteristics of the rough sea surface are implied in the dimensionless coefficient C. Because in the fully developed state the sea surface pattern is only a function of the wind velocity, C can also be expressed in terms of the wave characteristics, say of wave age $\beta = \sigma/v$.

If we consider the total effective pressure resistance over the entire wave profile, that is

$$\lambda(\tau_n)_{eff} = \int_0^{\lambda} \tau_n d\eta = \bar{s}\rho'(\bar{v}-\sigma)^2 \int_0^{\lambda} \left(\partial\eta/\partial x\right)^2 dx$$

it follows that per unit surface area

$$(\tau_n)_{eff} = \frac{1}{2}\bar{s}\rho'(\bar{v}-\sigma)^2 a^2\kappa^2 = \frac{1}{2} a\kappa\tau_n' \tag{11}$$

Similarly we get for the total effective horizontal shearing stress per unit area

$$(\tau_t)_{eff} = \frac{1}{2} a\kappa\tau_t' \tag{12}$$

The effect of both $(\tau_n)_{eff}$ and $(\tau_t)_{eff}$ is in the same direction as long as the wave velocity $\sigma < v$. In other words, the wave generating tractions whether normal or tangential will tend to increase the wave energy to the point where the dissipation balances the work done by surface forces. Using the total effective stress we may write

$$A = \tau_{eff}\, \sigma \tag{13}$$

It is seen, from (6) and (7), that τ_{eff} can be written in the form

$$\tau_{eff} = \rho' C\, v^2 ,$$

where

$$C = f(\delta)(1 - \beta)^2 + \gamma(\delta,\beta) \tag{14}$$

in the case of rough ocean waves is a complicated function of the surface conditions (wave age $\beta = \sigma/v$, wave steepness $\delta = 2a/\lambda$, roughness and form of the wave profile). The dimensionless parameter C can be determined only empirically.

NOTES ON ENERGY DISSIPATION WITH TURBULENT WAVE MOTION

This problem is one of the most difficult in Oceanography. In their paper on wave forecasting, Sverdrup and Munk (1947) neglected energy-dissipation in the energy balance.

In our case a probable relationship between wind energy, incidence of breakers, turbulence, and drift currents may be anticipated. It seems reasonable to assume that besides actual dissipation part of the wave energy released in breakers contributes to the energy of a semi-continuous forward motion of the upper water layers. Nevertheless this energy is lost by the wave motion and must be continuously replaced by energy from the wind field, if the average wave motion is to be maintained. This total "dissipated" energy has to be taken into account in the energy budget.

It has been proposed to allow for such turbulent processes by the introduction of eddy viscosity coefficients M in place of the molecular viscosity μ. A first attempt to determine such eddy viscosity-coefficients for wave motion has been made recently in a report on wind-generated ocean waves (Neumann, 1952). Here again it was necessary to abandon any attempt to follow the details of the phenomenon and to concern ourselves only with mean effects.

With the assumption that the energy dissipation D in turbulent wave motion is proportional to the square of the wave steepness δ, and replacing the molecular viscosity μ by M, we have according to Lamb (1932)

$$D = 2M\kappa^3\sigma^2a^2 = 2M\pi^2g\delta^2 \tag{15}$$

where M is a function of β, and in the fully developed state a function of the wind velocity v. In a fully arisen sea, where the longest wave present in the spectrum has a wave age $\beta = 1.37$ approximately, A equals D. From this it follows that

$$M = 0.1825\ 10^{-4}\ v^{5/2}\ (\mathrm{cm}^{-1}\mathrm{g\ sec}^{-1}) \tag{16}$$

It is remarkable that the eddy viscosity coefficients according to (16) agree fairly well with the values derived for wind-driven currents in the surface layer of the ocean.

SOME RESULTS ON THE GENERATION OF WAVELETS AND THE GROWTH OF TURBULENT SEA

If the wind starts to blow over an undisturbed water surface, primary wavelets are generated, originally beginning as minute disturbances which may always be present. The first attempt to explain the generation of initial waves was made by Jeffreys, but the question of the critical speed of the weakest wind which can raise any wave motion remained unanswered. Retaining Jeffreys' assumption, that in the stage of initial wave formatio the energy dissipation is determined only by molecular viscosity μ as long as the wavelets are not breaking, but taking the unit-area wave generating power equal to the product of effective wind stress and wave speed, the criterion can be written in the form

$$\frac{\rho' s}{8\pi} \frac{(v - \sigma)^2 \sigma}{\mu (g + \epsilon \kappa^2)} \geq \frac{a}{\lambda} \tag{17}$$

where $\sigma^2 = g/\kappa + \epsilon\kappa$ and $\epsilon = T/(\rho - \rho')$ (T = surface tension, ρ = density of the water). The effective wind stress is expressed in the form given by (9), (10) and (14) where in the case of initial waves, γ may be neglected and $f(\delta) = s\pi\delta$ by empirical evidence, with $s = 0.095$.

The equation shows that any wind with a velocity of more than 23.3 cm (minimum wave velocity) would be able to generate wavelets, but the steepness of these wavelets is very small at wind velocities below 60-70 cm/sec At a given wind velocity > 23.3 cm/sec the left hand side of (17) has a maximum value for a certain σ (or λ), and in the fully arisen state the ratio a/λ becomes a maximum for this wave. The wave heights, H_m, and wave lengths, λ_m, of the steepest waves generated by a given wind velocity are given in the following table, according to (17):

v cm/sec	40	50	60	70	80	90	100	120	125
λ_m(cm)	3.0	3.6	4.35	5.2	6.2	7.3	8.65	11.5	12.2
H_m(cm)	0.0065	0.022	0.056	0.118	0.21	0.38	0.63	1.49	1.75

Recently Roll (1951) studied the process of initial wave formation at different wind velocities by means of wave photographs. The first distinc waves appeared at wind velocities of about 70 cm/sec, but even at lower wind speeds wandering surface corrugations were observed. In an earlier paper the author (1949) mentioned similar effects as "vibrations of the wa surface, which can be noticed only in optical reflections." These exceedingly flat disturbances, which pass quickly over the mirror-like surface at the gentlest breeze, can be explained by extreme low wavelets as given in the table for wind velocities below 60-70 cm/sec.

With increasing wind the wavelets grow in height as well as in wave length and steepness. At a wind velocity of v = 100 cm/sec the theoretica height is 0.63 cm and the wave length 8.65 cm, which agrees with Jeffreys observations. But if the wind attains a velocity of 125 cm/sec the steepness of the initial waves with λ = 12.2 and H = 1.75 cm approaches the value 1/7, that is the maximum steepness according to Mitchell. In their further development these waves become unstable and break. Under these co ditions the assumptions made in the derivation of (17) no longer hold, because "turbulence" has to be taken into account, and the total wind stre over the wave profile must be regarded as the sum of normal plus tangentia components. That is, γ has to be taken into account in (14), because the tangential stress components may no longer be considered as pure viscosity stresses.

The growth of waves at higher wind velocities differs essentially from the growth of the wavelets in the initial stage. The initial waves increase in steepness, when growing in height and length, but turbulent ocean waves decrease in steepness while growing in height and length, as indicated by an empirical relationship between the wave steepness and the wave age β. This relationship was first used by Sverdrup and Munk (1947) and has been very useful in the semi-theoretical treatment of the problem.

Special observations made by the author (Neumann, 1952) in the Caribbean Sea in 1951, and recent measurements of Roll (1951) in waters near the German coast indicate a very rapid increase of wave steepness close to the value $\delta = 1/7$ for very "young sea" ($\beta \leq 1/3$). Thus, the original assumptions of Sverdrup and Munk (1947) and of the author in 1950 have to be modified at least for the earliest stages of wave development. In a preceding paper the author (Neumann, 1952) applied this empirical law, $\delta = f(\beta)$, by assuming that the steepness of turbulent ocean waves is approximately constant with the value $\delta = 0.124$ for $\beta \leq 1/3$.

The outstanding theoretical problem is the development of a method for forecasting the complete wind-generated wave spectrum, because energy is distributed over a range of wave periods as soon as instability and breaking occurs. But at present we are still far away from a complete understanding of the dynamical nature of this mechanism, which can hardly be other than complicated. Even the "state of the sea" has not yet been satisfactorily described.

To serve practical purposes, Sverdrup and Munk (1947) in their pioneer work introduced a statistical term, the "significant wave" as a first approach. But wind driven ocean waves are not characteristically long and flat like the "significant waves" defined by these authors, especially in the fully arisen state. The most impressive features of a wind-generated sea are more or less steep, breaking waves, and, in this turbulent wave pattern, fluctuations of both wave height and period, if we denote as "period" the time interval between succeeding crests at a fixed location.

A second approach, by the author (Neumann, 1952) is an attempt to forecast the significant variations of the wave periods and heights in the composite pattern of wind-driven sea at a fixed position. The method is based on the fact that at a given wind velocity the length and steepness of waves which are generated by direct wind action do not develop beyond particular maximum values which are functions of the wind velocity. They break from time to time and are continuously regenerated by energy supplied by the wind. Part of the energy released in breakers has been taken into account as an energy loss for the wave motion, but another part may be transferred to underlying longer waves which are not directly generated by the wind. There is some evidence that breakers thus play an important role in the energy supply from wind to longer waves which may proceed faster than the wind (Neumann, 1952). Thus, the process of wave generation in turbulent sea is considered as a discontinuous process, and in the fully developed state the characteristic wave pattern is not described only by a single "significant wave," but by a spectrum of possible wave periods with dominating waves in certain "bands" of the spectrum.

Some of the theoretical results may be mentioned briefly. The minimum fetches and minimum durations of wind action needed for generating fully developed sea increase rapidly with increasing wind velocity, but they are much smaller than the corresponding values according to the theory of Sverdrup-Munk.

At 5 m/sec wind velocity the sea would be fully developed over a fetch of only 13.8 km with a minimum duration of 2.25 hours. The range of significant periods in the composite wave pattern would be between 1.8 and 4.3 seconds with an average wave height of 0.5 meters.

At 10 m/sec wind velocity the waves would be fully developed over a fetch of 110 km with a minimum duration of 8.6 hours. The range of periods is between 4 and 8.6 seconds and the average wave height 2.3 meters. These results could describe, for example, the conditions in the trade wind region with a fresh breeze.

But for the development of fully arisen storm sea, fetches of more than 1000 km are needed.

Consider for example a storm of 24 m/sec wind velocity (strong to whole gale). For the development of fully arisen sea in this case a minimum fetch of 1800 km, or nearly 1000 nautical miles, would be needed. At the end of this fetch the characteristic wave pattern would show significant periods ranging between 12 and 23 seconds with optimum "bands" in the spectrum at periods of about 14, 16, and 20 seconds. The average wave height is between 13 and 14 meters.

At a limited fetch of 600 km, that is about the maximum fetch in the Black Sea, the significant periods in the considered case would range between 8 and 13 seconds, with wave heights between 6 and 13 meters, in the average about 11 meters. These wave dimensions are approximately those of the maximum waves in the Black Sea.

With this approach, it seems possible to forecast not only the range of significant periods, but also the frequency distribution of time intervals between succeeding wave crests as they pass a fixed position, and the average and maximum wave height.

Further work is being carried out at present at New York University to forecast power spectra for the wave pattern in the generating area at different wind velocities, fetches and durations of wind action. The second step in wave forecasting then is to determine how such a "wave package" or pattern spreads from the generating area into other regions of the sea until it reaches the coast. It may be mentioned at this point that eddy viscosity also has to be taken into account when considering the decay of waves (swell), if the waves travel through a region of turbulent sea. Preliminary results indicate that "swell dissipation" due to turbulence in storm regions is a very important decay factor. In some instances observations have shown that swell originating in storm regions of the North Atlantic Ocean was wholly obliterated while crossing other regions with moderate to heavy sea.

Since all attempts to calculate the growth of ocean waves under wind action have been largely based upon empirical evidence, it seems that our theoretical progress depends critically on progress in our phenomenological knowledge. An urgent need today is for adequate wave records from the open sea under different sea state conditions, and careful analysis of these records to separate swell from wind-driven sea. Efforts to advance in this direction are being made, and we hope that they will help to clarify questions about the composite pattern of wind-generated sea, and so to stimulate further theoretical work.

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