Chapter 4

HYDRODYNAMICAL EVALUATION OF STORMS ON LAKE ERIE

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ABSTRACT

The wind velocities observed over cities on the southern coast of Lake Erie during storms are modified, according to recently given meteorological theories, to obtain the wind velocities coexisting over the surface of the lake water. On the basis of these reduced velocities and the associated wind tides of the lake, the coefficient of wind stress of wind action on the water of the lake is determined. Due attention is given to the fact that the form of the lake affects the relation between the wind stress and the associated wind tide. The coefficient of stress arrived at for the larger wind velocities is in substantial agreement with the values which Neumann determined for the Gulf of Bothnia. The matter of sea roughness is discussed briefly.

TREATMENT OF THE OBSERVED WIND VELOCITIES.

The determination of the coefficient of wind stress from a knowledge of total wind tides may be made with reference to the wind velocity at a standard height above the water surface of the lake. During severe storms, however, the winds are rarely measured over the lake, and the desired values of the wind velocities must be deduced from those observed over the cities on the borders of the lake. The data relating to storm effects on Lake Erie treated here are taken from the hydrographic charts prepared by the United States Lake Survey. These charts give the changes of water level at the lake extremities and also the movement and direction of wind for a few cities on the southern coast of the lake; namely, Toledo, Cleveland, Erie, and Buffalo. Where the wind data were lacking, the desired information was obtained from the Weather Bureau.

Three steps are involved in reducing the city wind velocities to the lake wind velocities. These will be discussed briefly. Now in the four cities mentioned above, the anemometer heights are not the same. Again, during the last half century in a given city, the anemometer heights have been changed from time to time. Therefore, the first step in the reductions is to change the values of all the observed wind velocities to the comparable velocities which would exist at a height of 165 feet above the ground level of the city. Let U_a be the wind velocity at the anemometer height z_a and U_2 the wind velocity at the standard height z_2. The Prandtl rule of velocities [1],

$$\frac{U}{U_*} = 5.75 \log \frac{z}{\epsilon_2} , \tag{1}$$

where U_* is the shear velocity $\sqrt{\tau/\rho}$, and ϵ_2 the effective roughness of the cities, yields the desired relation

$$\frac{U_2}{U_a} = \log \frac{z_2}{\epsilon_2} \Big/ \log \frac{z_a}{\epsilon_2} . \tag{2}$$

In examining the distribution of the wind velocity over the cities, the question of the reference level from which heights are measured is important. It seems reasonable, in the absence of any special investigation of the matter, to suppose that if K is the average height of the city buildings, the reference level lies at a distance $K/2$ above the ground. As regards the effective roughness of the cities, one may follow Prandtl and put $\epsilon_2 = K/30$. The average height of buildings in an American city may be estimated to be 30 feet. Accordingly, the height z may be measured from a level 15 feet above the ground level of the city and the city roughness ϵ_2 may be taken as one foot. Owing to the logarithmic law of the distribution of velocities, the uncertainties in the values of the effective roughnesses are not serious.

Examination of the charts shows that the intensity of wind is very seldom constant along the southern coast of the lake at a given time of a storm period. Hence for the determination of the average effective wind a correlative relation must be assumed. In the writing of the relation two principles may be considered. First, owing to the shape of the lake, wind tides are related not to the absolute value of wind force but to the component of it resolved along the lake axis. Second, the forces being nearly proportional to the squares of the wind velocities, the averaging may be done by taking squares. In this process, however, the wind velocities of the cities on the border of the lake must be weighted. The method adopted is as follows. The total lake expanse is divided into three parts of equal extent. It is assumed that the winds at Toledo and Buffalo apply individually to the corresponding extreme parts. The middle part is divided further into two portions, the winds of Cleveland and Erie applying to these. On this basis the averaged effected wind is

$$\begin{aligned} U_2^2 = \frac{1}{3} \Big[& U_B^2 \cos(\theta_B - \alpha) + U_T^2 \cos(\theta_T - \alpha) \\ & + \frac{1}{2} U_C^2 \cos(\theta_C - \alpha) + \frac{1}{2} U_E^2 \cos(\theta_E - \alpha) \Big] . \end{aligned} \tag{3}$$

Here, θ_B , θ_T , θ_C , and θ_E are the directions of the winds observed at the cities Buffalo, Toledo, Cleveland, and Erie. The quantities U_B , U_T , U_C , U_E are the wind velocities which should prevail over these cities 165 feet above the ground. The direction of the lake axis is given by α . The evaluation on the basis of eq 3 constitutes the second step in the reduction of wind velocities.

If U_1 be the velocity of wind over the lake at a height of 25 feet above the water surface, the third and the final step of the reduction is the use of relations to derive U_1 from the values of U_2 determined by the formula, eq 3. The relation between U_1 and U_2 is influenced by the sea roughness ϵ_1 and the city roughness ϵ_2 , and also by the magnitude of U_2 , as will be seen presently.

At this point the analysis of Rossby regarding the movement of air in the layer of frictional influence is very essential [2]. In this theory the layer is divided into two parts; the lower boundary layer and the upper layer. In the boundary layer the directions of wind velocity and stress vector are coincident. The angle between the wind in the layer and the gradient wind, ϕ_s , does not change with elevation since the deflective force arising from the earth's rotation may be neglected. The distribution of the wind velocities is given by the rule of Prandtl, eq 1. The height of this layer will be denoted by H and the velocity of the wind at this level by U_H . In the upper layer the directions of wind velocity and stress vector are no longer coincident. The angle between the direction of the wind in the layer and of the gradient wind changes with the elevation and tends to zero when heights are increased. According to Rossby the total height of the layer of frictional influence is about 13 times the height of the boundary layer.

Let H_1 be the height of the boundary layer over the water of the lake and H_2 that over the city on the lake borders. Let U_{H1} and U_{H2} be the limiting values of the wind velocities in these layers. As U_1 is the velocity of the wind prevailing at a height z_1 over the lake and U_2 is the velocity of the wind prevailing at a height z_2 over the city, the application of the law of velocities from eq 1 gives

$$\frac{U_1}{U_2} = \frac{U_{H1}}{U_{H2}} \left[\frac{\log \frac{z_1}{\epsilon_1}}{\log \frac{H_1}{\epsilon_1}} \div \frac{\log \frac{z_2}{\epsilon_2}}{\log \frac{H_2}{\epsilon_2}} \right] . \tag{4}$$

This may be written as

$$U_1 = M U_2 , \tag{5}$$

where

$$M = \frac{U_{H1}}{U_{H2}} \left[\frac{\log \frac{z_1}{\epsilon_1}}{\log \frac{H_1}{\epsilon_1}} \div \frac{\log \frac{z_2}{\epsilon_2}}{\log \frac{H_2}{\epsilon_2}} \right] . \tag{6}$$

For the evaluation of M as a function of ϵ_1 , and U_2 the results,

$$H = 3.51 \times 10^{-2} \frac{U_g}{f} \sin \phi_s , \qquad f = 2\Omega \sin \lambda , \tag{7}$$

$$U_H = U_g \left(\cos \phi_s - \frac{1}{\sqrt{2}} \sin \phi_s \right) , \tag{8}$$

and

$$\log N = 1.694 \cot \phi_s - \log \sin \phi_s + 1.441, \quad N = U_g/f\epsilon, \tag{9}$$

from the analysis of Rossby are sufficient. Here Ω is the angular speed of the earth's rotation, and λ the latitude. Using these relations, M can be evaluated as a function of the gradient wind U_g once the roughnesses, ϵ_1 and ϵ_2 are specified. In this it must be supposed that the gradient wind has the same intensity and the same direction over the lake and over a city on the border of the lake. Through the same relations U_2 may be expressed as a function of U_g. Hence M may be expressed as a function of U_2. Taking z_1 and z_2 as 25 and 150 feet, respectively, and ϵ_2 as 1 foot, a set of M values was computed for a set of assumed sea roughnesses. These are given in table 1.

Table 1
The effect of sea roughness on the relation between the wind velocities over lake and over city.

ϵ_1, cm	1.0	0.6	0.3	0.1	0.01
U_2 mi/hr	$M = U_1/U_2$				
5	0.965	1.015	1.060	1.120	1.215
10	0.982	1.026	1.075	1.139	1.242
20	1.005	1.045	1.095	1.166	1.278
30	1.018	1.058	1.110	1.180	1.301
40	1.028	1.067	1.120	1.190	1.314
50	1.035	1.074	1.126	1.195	1.321
60	1.040	1.078	1.132	1.199	1.325

It is seen from the table that for a given wind velocity U_2 the value of M changes appreciably with the roughness of the sea. In the absence of previous information as to what should be the sea roughness of Lake Erie for the ranges of the wind intensities attained during severe storms, this roughness may be assumed provisionally and later tested. The value we have selected is $\epsilon_1 = 0.3$ cm. The corresponding M values may be read from table 1, and the formula to compute the effective wind velocity over the lake, from eq 3 and eq 9, is

$$U_1^2 = \frac{M^2}{3}\Big[U_B^2 \cos(\theta_B - \alpha) + U_T^2 \cos(\theta_T - \alpha) + \tfrac{1}{2}U_C^2 \cos(\theta_C - \alpha) + \tfrac{1}{2}U_E^2 \cos(\theta_E - \alpha)\Big]. \tag{10}$$

The storms passing over Lake Erie may be divided into Westerlies and Easterlies. It is with the Westerlies that the greater wind tides are observed. Some nineteen important storms of this type are considered for this investigation. The hourly wind data for all these storms were treated according to the relations given by equations 2 and 10.

SUMMARY OF DATA ON WINDS AND WIND TIDES OF LAKE ERIE STORMS

The curves of the wind intensity $U^2 (= U_1^2)$ and of the total wind tides ΔH of the storm of December 31, 1911, shown in figure 1, are typical of all the storms used in the present study. Taking the presentation in figure 1 as an example, it may be seen that the entire manifestation of the storm may be broken into three epochs: the epoch of maturing storm, the epoch of relative steadiness, and the epoch of recession. The hydrodynamical behavior of the waters in the lake during these epochs is expected to show marked differences. In the initial epoch the wind must blow for some length of time before the response of the water to the action of the wind is completed. The reason for this condition is that any manifestation of wind tide must be associated with the flow of water from one end of the lake to the other. The action of the wind must establish a layer of drift current, the thickness of which must increase with time, either under the action of molecular viscosity or turbulent viscosity or both. As the water is being collected at the leeward end of the lake, a returning gravity current is created which likewise increases in intensity with time. The second epoch represents that steady condition in which the transport of water through the body of the drift current is counterbalanced by the transport of water in the returning gravity current that is maintained by the unchanging surface gradient of the lake waters. The third epoch is associated with decreasing wind intensity. With the strength of the wind reduced, the wind stress can not maintain the adverse gradient of the surface waters, with the result that a surge of water is produced. This is a wave motion with a period equal to the seich period of the lake. In some charts of the Lake Survey these seiches are clearly seen during the time of the falling storm. In the extreme epochs the inertia effects are very pronounced; in the intermediate epoch such effects are reduced considerably in value.

The water-level changes of the intermediate epoch have a direct bearing on the relation between wind velocity and wind stress. Referring once more to figure 1, the reference time is the instant of maximum wind tide. In the case of every storm considered, the reference time is obtained by taking the average of the times when the deflections at the individual ends of the lake are the largest. Quite often the maximum elevation of water at Buffalo occurs earlier than the maximum subsidence at Toledo. Total wind tide is the magnitude of the relative displacement of the water at the lake ends. The average value of the wind tide over a duration of four hours and around the reference time is taken as the total wind tide of the intermediate epoch. The average value of U^2 over a duration of five hours and immediately preceding the reference time is taken as the corresponding wind value. The observed total wind tides have been corrected for the barometric pressures.

The wind velocities and wind tides thus determined from every storm record are entered in table 2 with the dates of the storms indicated.

Table 2
Wind tides of Lake Erie during westerly gales.

No.	Date	ΔH feet	U Miles per hr.
1	Nov. 21, 1900	13.12	50.5
2	Oct. 20, 1905	6.68	31.4
3	Oct. 20, 1906	9.75	38.3
4	Jan. 20, 1907	12.04	48.1
5	Dec. 7, 1909	10.51	40.1
6	Dec. 31, 1911	9.53	38.9
7	Jan. 31, 1914	7.95	34.7
8	Dec. 9, 1917	10.17	43.2
9*	Dec. 9, 1917	4.56	33.1
10	Dec. 10, 1907	7.62	34.3
11	Dec. 18, 1921	12.30	45.1
12	Dec. 8, 1927	13.24	47.4
13	Dec. 9, 1927	4.13	27.2
14*	Dec. 9, 1927	3.44	26.1
15*	Dec. 9, 1927	1.73	22.1
16	Apr. 11, 1927	13.31	51.3
17	Jan. 22, 1939	9.40	38.8
18	Sept. 25, 1941	9.06	35.7
19	Jan. 2, 1942	12.53	40.4
20	Jan. 3, 1942	2.38	19.5
21	Nov. 22, 1946	8.36	34.2
22	Mar. 25, 1947	8.34	35.7

In some of the charts the storms show two peaks with an extended flat region between the peaks. The total tides of such regions are also included in the data of table 2 and are identified with asterisks. The graphic representation of the same data is given in figure 2.

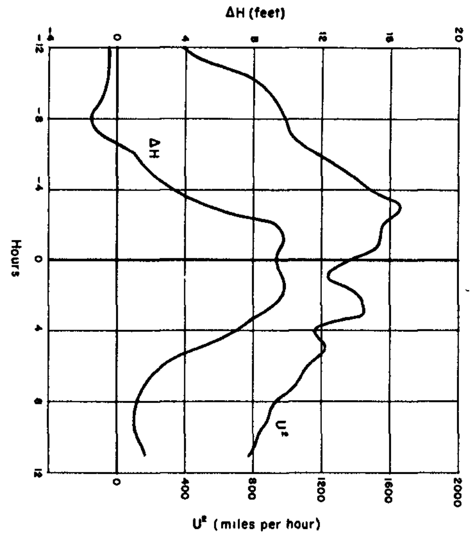

Fig. 1
Typical data of wind velocities and the associated total wind tides of a storm.

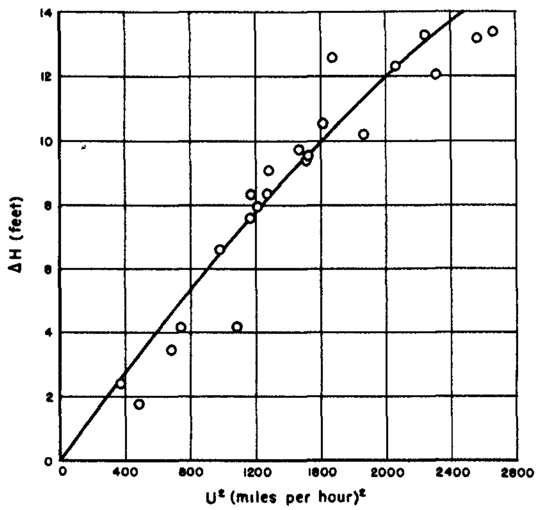

Fig. 2
The relation between total wind tide and the wind velocities for the Lake Erie storms.

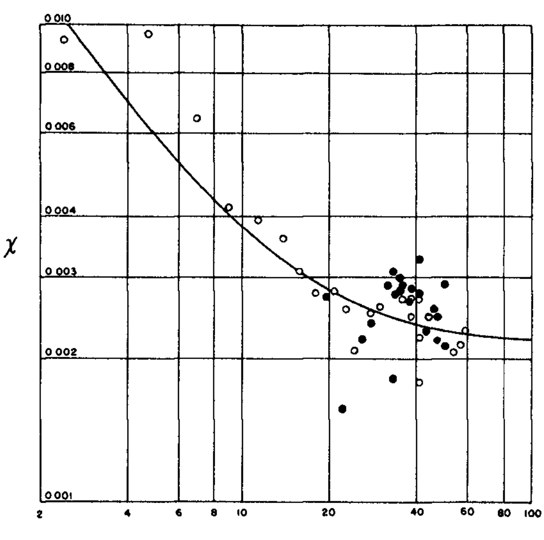

Fig. 3
The dependence of the coefficient of wind stress upon wind velocities.

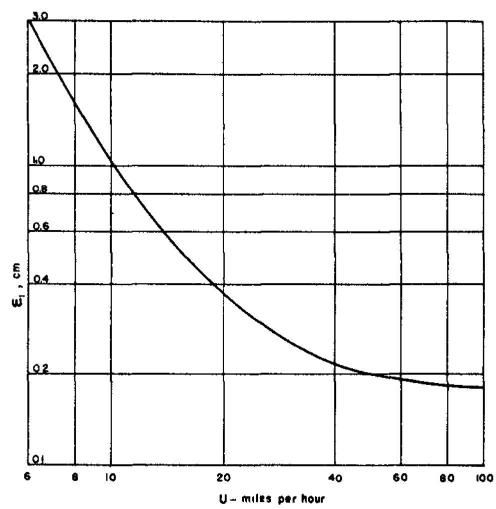

Fig. 4
The dependence of the roughness of seas upon the wind velocities.

WIND STRESS COEFFICIENTS OF THE LAKE ERIE STORMS

The wind-stress coefficient χ is defined by the relation,

$$\tau_s = \chi \rho_a U^2, \tag{11}$$

where τ_s is the wind stress, ρ_a the density of air, and U the velocity of wind at some standard height above the water surface. In the present study the standard height selected is 25 feet. A change in the standard height implies a change in the value of the coefficient of stress. If χ is the stress coefficient based on the wind observed at the standard height z_1, and χ' that based on the wind observed at the standard height z_1', the coefficients are related by

$$\frac{1}{\sqrt{\chi}} - \frac{1}{\sqrt{\chi'}} = 5.75 \left[\log z_1 - \log z_1' \right], \tag{12}$$

a relation that is easily deduced by the application of the Prandtl law of velocities. If U is measured in centimeters per second, ρ_a in grams per centimeter cubed, τ_s is in dynes per square centimeter. If U is measured in feet per second, ρ_a in pounds per cubic feet, τ_s is in poundals per square foot.

The intermediate step in evaluating the stress τ_s from the total wind tide ΔH is a relation derived by solving the differential equation of wind tide subject to the condition of constant volume of water in the lake. Let $\bar{H}$ be the average depth of water in the section at x when the water of the lake is not disturbed. Let h denote the displacement of the water surface at x when the wind is acting. A simple dynamic consideration, after neglecting the small inertia effects of the flow, leads to

$$\frac{dh}{dx} = \frac{\tau_s + \tau_o}{\rho g (\bar{H} + h)}, \tag{13}$$

where τ_s and τ_o are the stresses at the surface and at the bottom, respectively, ρ is the density of water, and g is the constant of gravitational acceleration. The origin of x may be placed at the windward end of the lake. As the volume of water in the lake is not changed, the condition of continuity is given by

$$\int_0^L B h \, dx = 0, \tag{14}$$

where B is the width of the lake surface and L is the length of the lake axis.

In natural lakes the mean depth of section $\bar{H}$ and the surface width B both vary with x. For these cases the mechanics of solution of eq 13 subject to the condition of eq 14 is greatly simplified when x is expressed in terms of L, and h and $\bar{H}$ in terms of H_o, the overall mean depth of the lake; B is expressed in terms of $\bar{B}$, the average width of the lake surface.

In solving the differential equation of wind tide in the case of a particular lake, $\bar{H}/H_o$ and $B/\bar{B}$ are expressed first as functions of x/L. Dispensing at this time with the details of the computations that were made for Lake Erie, we shall be content to give the final result. This result is

$$\frac{\tau_s + \tau_o}{\rho g H_o} \cdot \frac{L}{H_o} = 0.867 \frac{\Delta H}{H_o} - 0.134 \left(\frac{\Delta H}{H_o}\right)^2, \tag{15}$$

which is a relation between the wind stress and wind tide as affected by the shape and the dimensions of the lake. The mean depth H_o may be taken as 58 feet and the length L as 272 miles. Writing

$$\tau_o = n \tau_s , \tag{16}$$

and introducing τ_s from eq 11, we have

$$\chi = \frac{0.867}{1+n} \left[1 - 0.16 \frac{\Delta H}{H_o}\right] \frac{\Delta H}{H_o} \cdot \frac{\rho}{\rho_a} \cdot \frac{g H_o}{U^2} . \tag{17}$$

At the present we are in the dark as regards the magnitude of stresses at the lake bottom. It will be supposed tentatively that the stress at the bottom is about one tenth as large as the surface stress. Thus putting $n = 0.1$,

$$\chi = 0.787 \left[1 - 0.16 \frac{\Delta H}{H_o}\right] \frac{\Delta H}{H_o} \cdot \frac{\rho}{\rho_a} \cdot \frac{g H_o}{U^2} , \tag{18}$$

which is the final form of the formula to evaluate the coefficient of stress when the total wind tide corresponding to a wind velocity is known. The formula applies to Lake Erie only and for winds moving from west to east.

Applying the above formula, eq 18, to the data of wind tides given in table 2, a set of values of the coefficient of stress are obtained, and these are shown in table 3. The same data are plotted as full circles in figure 3. There are plotted also in the same figure as open circles the values of the stress coefficients which Neumann obtains in an examination of the Pelman observations from the Gulf of Bothnia [3]. The original values given by Neumann were reduced to apply to a standard height of 25 feet above the water surface using the relation in eq 12.

Table 3
Coefficient of wind stress of Lake Erie gales

No.	$\chi \times 10^3$	U miles/hour	No.	$\chi \times 10^3$	U miles/hour
1	2.13	50.5	12	2.44	47.4
2	2.87	31.4	13	2.35	27.2
3	2.78	38.3	14	2.15	26.1
4	2.17	48.1	15	1.53	22.1
5	2.73	40.1	16	2.88	51.3
6	2.63	38.9	17	2.64	38.8
7	2.76	34.7	18	2.96	35.7
8	2.23	43.2	19	3.22	40.4
9	1.78	33.1	20	2.68	19.5
10	2.71	34.3	21	2.99	34.2
11	2.52	45.1	22	2.77	35.7

Examination of figure 3 reveals that in the region of higher wind velocities the coefficient of stress is practically independent of the wind velocity. Furthermore the coefficients of wind stress for the two areas are of like value. The average of the individual determinations equals 0.00254 in the case of Lake Erie and 0.00236 in the case of the Gulf of Bothnia. Now this comparison is for the high wind velocities. Significantly, the Gulf of Bothnia data for the small wind velocities show that the stress coefficient decreases with increasing wind velocities.

THE ROUGHNESS OF SEA

We shall adopt for the determination of the roughness of sea the procedure employed by Neumann. Writing eq 11 in the form,

$$\frac{U}{U_*} = \chi^{-\frac{1}{2}},$$

substituting in eq 1, changing z to z_1, and ϵ_2 to ϵ_1,

$$\frac{1}{\sqrt{\chi}} = 5.75 \log \frac{z_1}{\epsilon_1} , \tag{19}$$

which relates the sea roughness ϵ_1 to the standard elevation z_1 and the coefficient of stress χ. In the present case since χ is determined from wind velocities which should prevail at a height of 25 feet above the water surface, then $z_1 = 672$ cm. As mentioned above, the average value of the coefficient of stress of Lake Erie for the wind velocities met in storms is 0.00254. This yields for the roughness of the sea the value $\epsilon_1 = 0.27$ cm. It will be remembered that in the computations needed to reduce the city wind velocities to the lake velocities, through the relation eq 10, the sea roughness had to be assumed provisionally to determine M appearing in the same equation. The assumed value was $\epsilon_1 = 0.3$ cm and this selection now appears to be satisfactory.

There is drawn a curve through the plotted points in figure 3. This curve may be utilized to determine the roughness of sea as a function of wind velocity for inland waters. The determinations by means of the formula, eq 19, putting $z_1 = 672$ cm are shown in the form of a curve in figure 4. The roughness of the sea decreases with increasing wind velocity. The decrease in the value of roughness is very slow for very strong winds. As the problem of sea roughness is yet an unsettled question, the statements above must be considered cautiously until confirmed by further observational data.

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