

COASTAL ENGINEERING

Chapter 3

CURRENTS IN THE SURF ZONE*

D. L. Inman and W. H. Quinn
Scripps Institution of Oceanography
University of California
La Jolla, California

ABSTRACT

Surface and bottom currents in the surf zone were measured at 15 equally spaced points along two straight beaches with approximately parallel bottom contours. The measurements showed that offshore currents predominate over onshore currents at the bottom, while at the surface there is a slight predominance in the onshore direction. With regard to the longshore component, it was found that surface and bottom currents have a similar velocity distribution. The variability of the longshore component as measured by its standard deviation is equal to or larger than the mean longshore velocity. This wide variation in longshore currents indicates the impracticability of estimating the mean velocity from a single observation of longshore current.

It was found that the momentum approach to the prediction of longshore currents by Putnam, Munk and Traylor (1949) leads to useful forecasts provided the beach friction coefficient k is permitted to vary with the longshore velocity, V . The indicated relation is $k \sim V^{-3/2}$.

INTRODUCTION

A series of longshore current measurements was made in 1949 and 1950 along two straight beaches in the San Diego area. The purpose of the investigation was to study quantitatively the variability of current velocities in the surf zone, and to test the method of prediction of longshore currents from the characteristics of the waves producing them.

The terminology and general principles of the circulation resulting from wave action in and near the surf zone was discussed in a previous paper (Shepard and Inman, 1951). The scope of the present paper is limited to a discussion of currents inside of the breaker zone.

Two straight beaches with relatively parallel bottom contours were selected for study, Torrey Pines beach north of La Jolla and Pacific Beach to the south of La Jolla (Fig. 1). Fifteen stations

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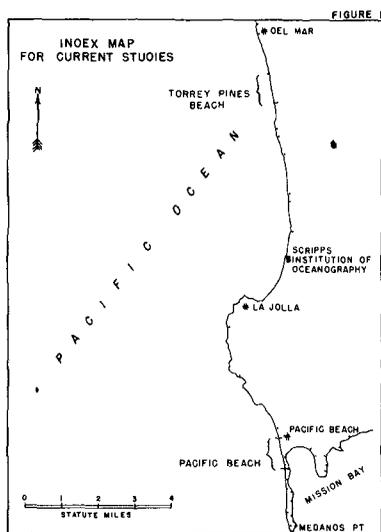


Fig. 1

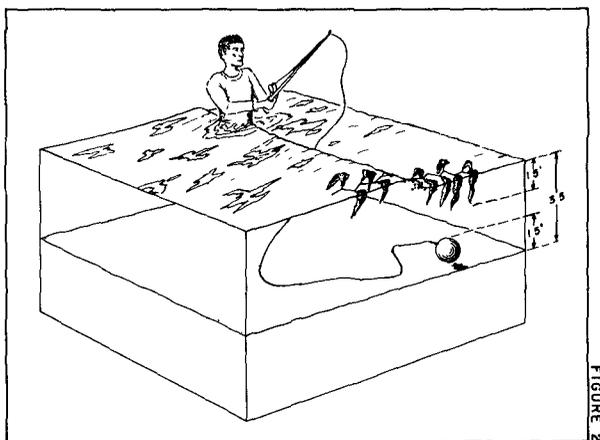


Fig. 2

Fig. 1. Index map for current studies.

Fig. 2. Devices used in current investigations were of the free drifting type. Kelp was used for surface currents, and a volley ball filled with water and weighted with sufficient mercury to give it a slight negative buoyancy was used for bottom currents.

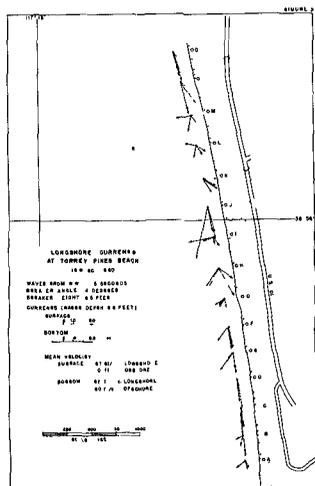


Fig. 3

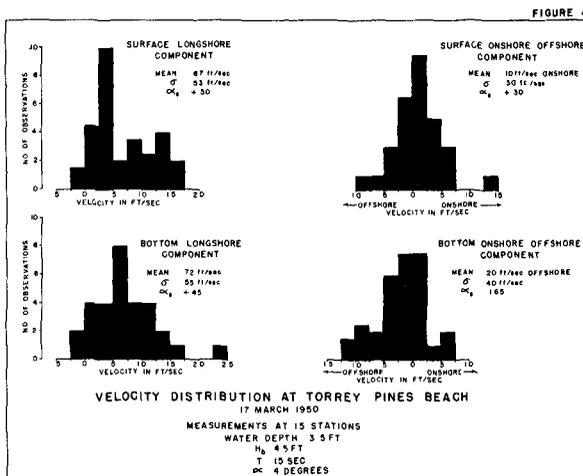


Fig. 4

Fig. 3. A typical series of current observations showing the variation of currents along Torrey Pines Beach. Each measurement is shown by a vector opposite the appropriate station.

Fig. 4. Histograms showing the distribution of current velocities for the series of observations shown in Fig. 3.

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were selected at equal intervals along an 0.8 mile stretch on each beach. The stations were numbered "A" through "O" and were approximately 300 feet apart. Each series of observations consisted of 2 surface and 2 bottom current measurements at each station giving a total of 30 surface and 30 bottom current measurements along the beach. A complete series of 60 measurements required about 2 hours and were made inside the breaker zone in water approximately $3\frac{1}{2}$ feet deep.

The devices used in the investigations were of a free drifting type, the velocity being determined by the distance of travel in a given period of time. Bottom current magnitude and directions were obtained by means of a volley ball filled with water and weighted with sufficient mercury to give it a slight negative buoyancy. The ball was attached to a light fishing line which was calibrated, and contained on a reel fixed to a short pole (Fig. 2). An observer, standing about waist deep at a chosen station in the surf zone, released the ball at a given signal and played out free line as the ball was carried by the current. When an observer on shore indicated that a period of 30 seconds had elapsed, the line was jerked taut and the direction to the ball noted. The number of feet travelled during the 30 second interval was observed as the line was reeled in. Direction of the current was estimated to the nearest $22\frac{1}{2}^{\circ}$ by noting the angle that the line made with the shore line. Surface currents were measured during the same interval by releasing a piece of kelp at the spot where the bottom current ball was dropped. A second observer determined the distance and direction of travel of the kelp during the same time interval. The interval of 30 seconds was chosen as being long enough to give a representative current and short enough to indicate major current fluctuation and tendencies in the surf zone.

During current observations, significant breaker heights* were obtained by an observer standing at mean water level and lining up the top of the breaker crest with the horizon. The height of the observer's eye above water level was read from a graduated pole. This height was multiplied by $\frac{4}{3}$ to give the height of the breaking wave.

The average significant breaker height, H, for the series of current measurements was obtained by averaging the significant waves at each station. The significant wave period T, was obtained by timing the passage of the significant wave crests past a fixed point with a stop watch. The angle of approach α , between the crest of the breaking wave and the straight beach was measured during the 1950 observations by the transit sighting bar method devised by Forrest (1950). The breaker height H, period T, and the angle of approach for each series of observations are listed in Table 1.

*The significant wave height is the average of the highest one-third of the waves.

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TABLE I
Tabulation of Field Data

A. Torrey Pinee Beach 1949																
Date	H ft.	T sec.	$\alpha/1$ deg.	i	Longshore Component						Onshore-Offshore Component ^{2/}					
					Surface			Bottom			Surface			Bottom		
					V_m	σ	α_3	V_m	σ	α_3	V_m	σ	α_3	V_m	σ	α_3
ft./sec																
1949																
27 Jun.	2.8	15	61°N	.027				.38	.38	-.75						
29 Jun.	3.1	8 $\frac{1}{2}$	13°S	.027				.04	.38	-.15						
30 Jun.	3.7	8	4°S	.027				.22	.43	+.40						
1 Jul.	3.6	14	N	.027				.04	.49	+2.50						
		9	0°													
5 Jul.	4.9	8	5°S	.027				.84	.51	+1.35						
6 Jul.	3.8	7	5°S	.027				.21	.45	+.51						
7 Jul.	3.4	12 $\frac{1}{2}$	N	.027				.55	.36	+.16						
		7 $\frac{1}{2}$	0°													
11 Jul.	2.6	8	N	.035				.04	.40	0						
12 Jul.	3.0	9 $\frac{1}{2}$	1°N	.035				.01	.40	0						
13 Jul.	2.7	10	N	.035				.15	.40	0						
14 Jul.	3.5	13 $\frac{1}{2}$	N	.035				.09	.45	-1.30						
		7 $\frac{1}{2}$	S													
15 Jul.	4.9	13	N	.035				.21	.36	-1.30						
19 Jul.	2.9	10	N	.035				.50	.34	0						
21 Jul.	4.6	12	N	.035				.80	.66	+.98						
26 Jul.	3.7	8	S	.028				.20	.59	0						
		15	N													

B. Torrey Pinee Beach 1950.																
1950																
14 Feb.	5.1	12	6°S	.027	.37	.84	+.09	.20	.63	+1.15	-.12	.69	-2.62	-.22	.41	-1.50
		16	5°N													
7 Mar.	4.7	14	7°S	.027	.60	.55	+.12	.45	.60	+1.38	+.32	.78	+.07	-.21	.39	-.66
17 Mar.	4.5	15	4°S	.027	.67	.53	+.50	.72	.55	+.45	+.10	.30	+.30	-.20	.40	-1.65
29 Mar.	4.8	12	4°S	.027	1.22	1.18	+.97	1.16	1.05	+.68	-.11	.16	-2.44	-.28	.45	-1.18
30 Mar.	4.2	12	4°S	.027	.38	.77	+1.56	.41	.76	-.13	+.01	.41	0	-.27	.68	-.30
		17	3°N													
21 Apr.	2.0	12	4°N	.027	.46	.57	-.49	.26	.46	0	+.07	.47	+3.28	-.09	.23	-.12
11 May	1.7	8	7°N	.027	.10	.50	-.13	.35	.52	-.15	-.01	.35	-.48	-.26	.37	0
		18	3°N													
16 May	2.9	15	5°N	.027	.60	.51	+.13	.52	.40	-.75	+.08	.25	-.97	-.35	.40	+.34
25 May	1.6	6	5°S	.027	.11	.33	+1.00	.10	.37	+.82	-.08	.19	+1.62	-.38	.23	-.07
		14	2°N													

C. Pacific Beach 1950																
1950																
24 Feb.	6.2	14	5°S	.014				.54	.48	0				-.35	.53	-.85
2 Mar.	3.1	16	7°N	.014	.82	.47	-.10	.41	.28	+.49	.03	.66	-.21	-.17	.24	-1.83
14 Mar.	4.5	12	3°S	.014	.46	.94	-.10	.51	.80	-.30	.27	.84	-.20	+.17	.71	+.63
22 Mar.	3.5	14	4°S	.014	.20	.61	-.92	.13	.54	0	.07	.45	+1.78	-.32	.60	+1.22
18 Apr.	2.7	16	3°N	.014	.01	1.20	-.07	.25	.70	+.35	.25	1.14	-1.63	-.21	.68	-.33
9 May	4.7	13	7°S	.014	1.54	1.40	-.48	1.20	1.15	+.07	.22	.82	+.34	-.57	1.07	+2.91
18 May	2.6	11 $\frac{1}{2}$	2°S	.014	.06	.82	+1.72	.01	.64	+1.00	.01	.57	+1.06	-.71	.41	-1.39
		8	N													
26 May	2.0	14 $\frac{1}{2}$	4°N	.014	.03	.54	-1.78	.19	.42	+.10	-.17	.32	-1.17	-.54	.37	+1.83
27 July	1.8	12	2°N	.014	.09	.72	-.31	.03	.49	+.38	.16	.50	-.43	-.40	.39	+.43

1/The letters N (north) and S (south) indicate the direction of opening of the angle between the breaking wave and the beach. The period and breaker angle of minor wave trains, when present, are listed below the major train.
 2/Positive values of V_m indicate onshore directions and negative values offshore directions.

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STATISTICAL ANALYSIS OF OBSERVATIONS

A typical series of measurements is shown graphically in Fig. 3, in which the velocity and direction of each current observation is represented by a vector opposite the appropriate station. In order to treat the distribution of currents systematically, each current vector was resolved into a longshore component and an onshore-offshore component. The range of velocities in each component was divided into equal classes, and the histogram of the distribution of velocities was obtained by plotting the classes in feet per second as the abscissa and the number of observations in each class as ordinate. Histograms, showing the velocity distribution in a longshore direction and an onshore-offshore direction for both surface and bottom currents for the series of measurements shown in Fig. 3 are given in Fig. 4.

Since histograms do not give numerical descriptions of the current distribution, the data were treated statistically to obtain the arithmetic mean velocity V_m , the standard deviation σ , and the skewness α_3 of the velocity distribution.

These measures are described in statistics textbooks (for example Hoel, 1947, pp. 8-15) and defined as follows:*

$$V_m = \frac{1}{N} \sum_{i=1}^h V_i f_i \quad (1)$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^h (V_i - V_m)^2 f_i \quad (2)$$

$$\alpha_3 = \frac{1}{N\sigma^3} \sum_{i=1}^h (V_i - V_m)^3 f_i \quad (3)$$

where N is the number of observations ($N = 30$ for most series), V_i is the class mark (value of the mid-point of the class interval) of the i^{th} class, f_i is the number of observations in the class, and h is the number of classes.

* $N-1$ was used in the denominator of equation (2) to give an unbiased estimate of the standard deviation in accordance with small sample theory (Hoel, 1947, p. 129).

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The standard deviation is a measure of the spread of the distribution, and for most symmetrical distributions approximately 68% of the observations are included between the values $V_m - \sigma$ and $V_m + \sigma$. The skewness, α_3 , serves as a measure of the symmetry of the distribution. An α_3 value of 0 is indicative of a symmetrical distribution, while a positive value indicates the distribution is skewed to the right, and a negative value that it is skewed towards smaller or negative values. For purpose of comparison, the mean, standard deviation, and skewness are listed opposite each of the histograms in Fig. 4. These three statistical measures were computed for each component of velocity of each series of observations and are listed in Table I.

DISTRIBUTION OF CURRENTS

Fig. 4 illustrates some of the features that are typical of the current distribution in the surf zone. For example, the surface and bottom longshore components have similar mean velocities, but the distribution of velocities tends to be more symmetrical for the bottom component. This may be because the effect of wind is less near the bottom. The negative current values for the surface and bottom longshore components indicate that a small percentage of currents were moving in a direction opposite to that of the predominant current. The bottom onshore-offshore currents had a pronounced offshore tendency, whereas the mean velocity of the surface currents was onshore.

The nature of the distributions of the bottom longshore current component for all series of observations are summarized in Fig. 5. In this figure the minimum observed velocity, the standard deviation of the velocity distribution, the maximum velocity, and the skewness of the distributions are plotted against the mean bottom longshore velocity. This diagram does not consider such important factors as wave height, period, and angle of approach, and although the plotted points show considerable scatter, they nevertheless illustrate several important features of the bottom longshore current:

- (a) In almost all cases, there were currents opposed in direction to the dominant current.
- (b) The variability of the current as measured by the standard deviation averaged $\frac{1}{2}$ foot per second for mean velocities below approximately $\frac{1}{2}$ foot per second and was approximately equal to the mean velocity for velocities above $\frac{1}{2}$ foot per second.
- (c) The maximum observed velocity increased with increasing mean velocity; the maximum being approximately five times greater than the mean for mean velocities near 0.2 feet per second, and three times greater for mean velocities near 1 foot per second.

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- (d) There is no apparent relationship between the skewness of the velocity distribution and the mean velocity. In general the velocity distribution tends to be fairly symmetrical over the range of mean velocities investigated.

The relation of the means and of the standard deviations of the surface and bottom longshore currents are given in Fig. 6. While a linear relationship exists between these quantities, in general the mean value of the bottom longshore current appears to be more consistent in its agreement with the wave conditions generating the currents than the surface velocity, and the spread or variability (as measured by the standard deviation) was somewhat less for the bottom currents than for the surface currents. For these reasons the mean bottom longshore current was used for comparing the observed with the predicted currents in the following section.

The large variation in the observed longshore currents are caused in part by the variation in wave height with time, and by the variation of the cell-like circulation pattern of the nearshore current system with distance along the beach. Since the mass transport of water is proportional to the square of the wave height, groups of high waves followed by groups of low waves result in fluctuations of current velocity with time. (Shepard & Inman, 1950, Fig. 11). Also, in many instances, a secondary wave train was present which may have contributed to the variability.

The nearshore circulation has been shown to have a cell-like pattern, consisting of relatively wide stretches along the beach where water is transported shoreward by the waves, then along the shore inside of the breakers (by longshore currents) into relatively narrow zones where the water is transported seaward by rip currents (Shepard and Inman, 1950, Figs. 2 and 3). This circulation pattern results in higher velocities up current; and lower or in some cases a reversal of longshore current direction down current from the rip zones. This effect is shown in Fig. 3 at stations G and L. Also, high waves approaching with crests parallel to the beach often result in strong longshore currents for limited distances between rip zones, although the mean longshore current for the entire beach may be quite low.

For the above reasons it is advisable to measure currents at as many different stations as possible, in order to obtain measurements that are representative of the beach as a whole. The bias exhibited at the 15 stations on Torrey Pines Beach for the 1950 series of observations is shown in Fig. 7. The plots on this figure show the degree of the divergence between the average current at each station and the mean of all currents for the entire beach. For example, on the average, the mean bottom longshore component at Station M is 60% less than the mean for the entire beach, while the rip tendency as measured by the bottom offshore component is 150% greater.

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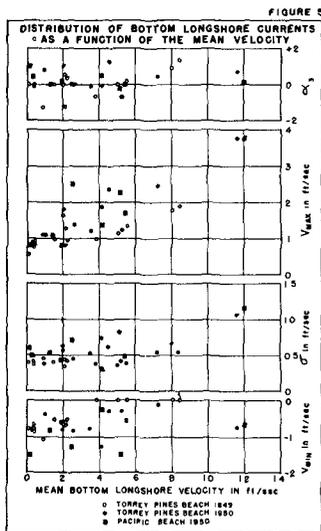


Fig. 5

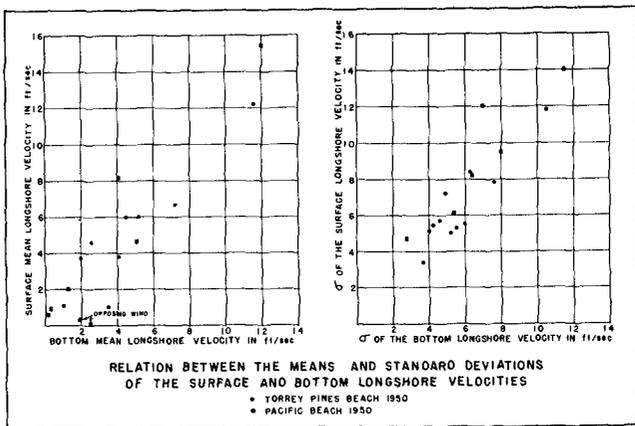


Fig. 6

Fig. 5. Summary of the distribution of bottom longshore current components as a function of the mean velocity. The negative values for the minimum observed velocity, V_{MIN} , indicate that these currents were flowing in an opposite direction to the mean current. The lower left hand diagram in Fig. 4 shows the histogram of the velocity distribution for one of the plots in this figure.

Fig. 6. Relation of the means and standard deviations of the surface and bottom longshore current components. Compare the upper and lower right-hand histograms of Fig. 5.

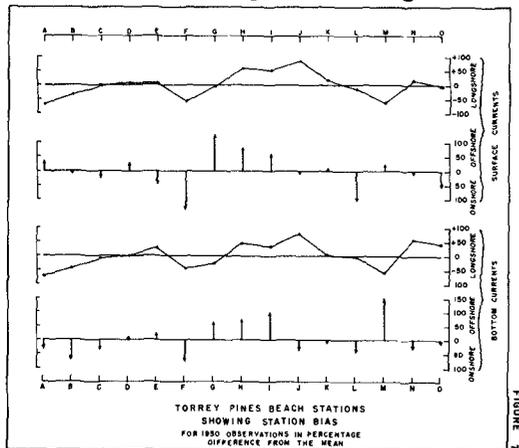


Fig. 7

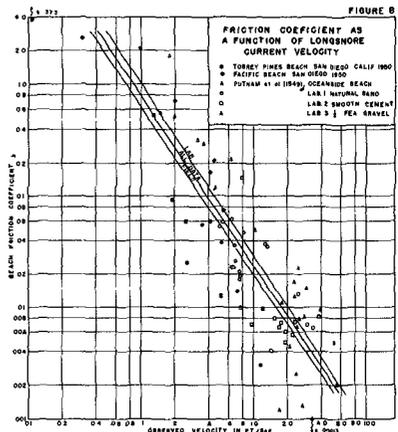


Fig. 8

Fig. 7. The bias exhibited at the 15 stations along Torrey Pines Beach. The plots show the magnitude of the deviation of the average current at each station for the mean of all currents for the entire beach during the 1950 series.

Fig. 8. Relation between the beach friction coefficient k and the observed longshore velocity. The mathematical relation is given by equations (6), (7) and (8). For the Torrey Pines and Pacific Beach observations, the mean bottom longshore component of velocity V_m , listed in Tables 1B and 1C was used.

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The variation in currents as represented by the standard deviation of the distribution, σ , is a measure of the accuracy of prediction of the mean bottom longshore current from an individual measurement of current. Obviously, the greater the standard deviation or spread of velocities, the less is the probability of accurately estimating the mean from a single observation of current. For example, Fig. 5 shows that mean bottom longshore currents of $\frac{1}{2}$ foot per second are characterized by a standard deviation of approximately $\frac{1}{2}$ foot per second; this means that out of 100 estimates based on single observations of current 68 of the estimates should fall within 1 standard deviation either side of the mean, or within the velocity range of zero to 1.0 foot per second. The coefficient of variation, $C_v = 100 \sigma / V_m$, is a useful measure in this respect, because it gives the deviation from the mean in terms of the percentage of the mean. For the example above, $C_v = 100\%$, and thus 68 of the estimates should fall within plus or minus 100% of the true mean velocity.

The coefficient of variation was computed for all bottom longshore current distributions with mean values greater than 0.1 feet per second.* The average of all of the coefficients of variation in this case was 177%. Inspection of Fig. 5 shows that for low velocities, C_v decreases with increasing mean velocity, but tends to be constant for mean velocities of $\frac{1}{2}$ foot per second and higher. The average value of C_v was 91% for mean velocities above $\frac{1}{2}$ foot per second.

It is apparent that if the estimation of the mean velocity is based on more than one observation, the accuracy of the estimation will be improved. The following relationship exists between the standard deviation of a population, σ , based on a large number of individual measurements, and the standard deviation, S , based on the means of groups of observations (Hoel, 1947, p. 65):

$$\frac{\sigma}{\sqrt{N}} = S \quad (4)$$

where N is the number of individual measurements in each group.

If the coefficient of variation is constant over a range of mean velocities, the average coefficient of variation can be substituted for the standard deviation in equation (4). This condition is approximately fulfilled for mean velocity values exceeding $\frac{1}{2}$ foot per second.

*The coefficient of variation loses its significance as the mean velocity, V_m , approaches zero. For this reason velocities below 0.1 foot per second were arbitrarily omitted from this computation.

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Suppose we wish to estimate the number of individual observations, N , that the mean must be based on so that the coefficient of variation is 25%. Substituting 91% for σ and 25% for S in equation (4) gives a value of N equal to approximately 13 measurements.

PREDICTION OF LONGSHORE CURRENTS.

The momentum approach to the prediction of longshore currents developed by Putnam, Munk, and Traylor (1949) was selected for use in this study. It relates the mean velocity V , of the longshore currents, to the wave height H , period T , angle of approach α , and slope of the beach i , according to the relation:

$$V = \frac{a}{2} \left[\sqrt{1 - \frac{4C \sin \alpha}{a}} - 1 \right] \quad (5)$$

where

$$a = (2.61 H i \cos \alpha) / kT$$

and $C = \sqrt{2.28 gH}$ is the wave velocity, k is the beach friction coefficient, and g is the acceleration of gravity.

The range of longshore current velocities obtained on the model and prototype beaches studied by Putnam, et al (1949), were individually limited and indicated that the beach friction coefficient "k" was relatively constant for a particular beach. However, the more recent observations in the San Diego area considered together with those of Putnam, et al (1949), indicate that "k" is a function of current velocity and cannot be considered a constant for a given beach. Using the tabulated observed values of longshore current, published in Putnam, et al (1949), the value of k was computed from equation (5) for all of their field and laboratory observations, and also for the series obtained in 1950 at the two beaches in the San Diego area by using the mean of the bottom longshore component.* The coefficient k is plotted as a function of the observed velocity in Fig. 8. Inspection of this figure strengthens the contention that the coefficient is a function of velocity; however, it should be mentioned that since k is not determined by direct measurement, it therefore not only reflects beach friction, but also any errors in measurement and any inadequacies of the theory.

*The 1949 Torrey Pines Beach data (Table I A) was not used in this computation because of the inaccuracy in the measurement of the breaker angle. The transit-sighting bar method was not adopted for this purpose until the 1950 season.

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The relation between the beach friction coefficient and the longshore current velocity was obtained separately for the field data, the laboratory data, and the combined data by the method of least squares. Assuming a relationship of the form $k = bV^m$, the following were obtained:

For field data

$$k = 0.020 V^{-1.51} \quad (6)$$

For laboratory data

$$k = 0.029 V^{-1.54} \quad (7)$$

For all data

$$k = 0.024 V^{-1.51} \quad (8)$$

The close agreement between the above equations suggests that $k = 0.024 V^{-3/2}$ can be used as a good approximation for both laboratory and field observations. Since the type of bottom material represented in the equations (see Fig. 8) ranged from 1/4 inch pea gravel through sand to smooth concrete, the type of bottom apparently is not as important as the velocity in determining the value of k .

Substituting the value of $k = 0.024 V^{-3/2}$ into equation (5) gives the following relationship for the computed mean longshore velocity:

$$V_c = \left[\left(\frac{1}{4x^2} + y \right)^{\frac{1}{2}} - \frac{1}{2x} \right]^2 \quad (9)$$

where

$$x = (108.3 H_i \cos \alpha) / T$$

and

$$y = C \sin \alpha$$

As an aid in computing longshore currents, equation (9) is reproduced in the form of an alignment chart in Fig. 9. This chart gives the mean longshore current in feet per second, when the breaker height in feet, the period in seconds, the beach slope in per cent and the angle of breaker approach in degrees are known.

Using equation (9), the longshore current velocity V_c was computed for all of the field data listed in Putnam, et al (1949), and for the

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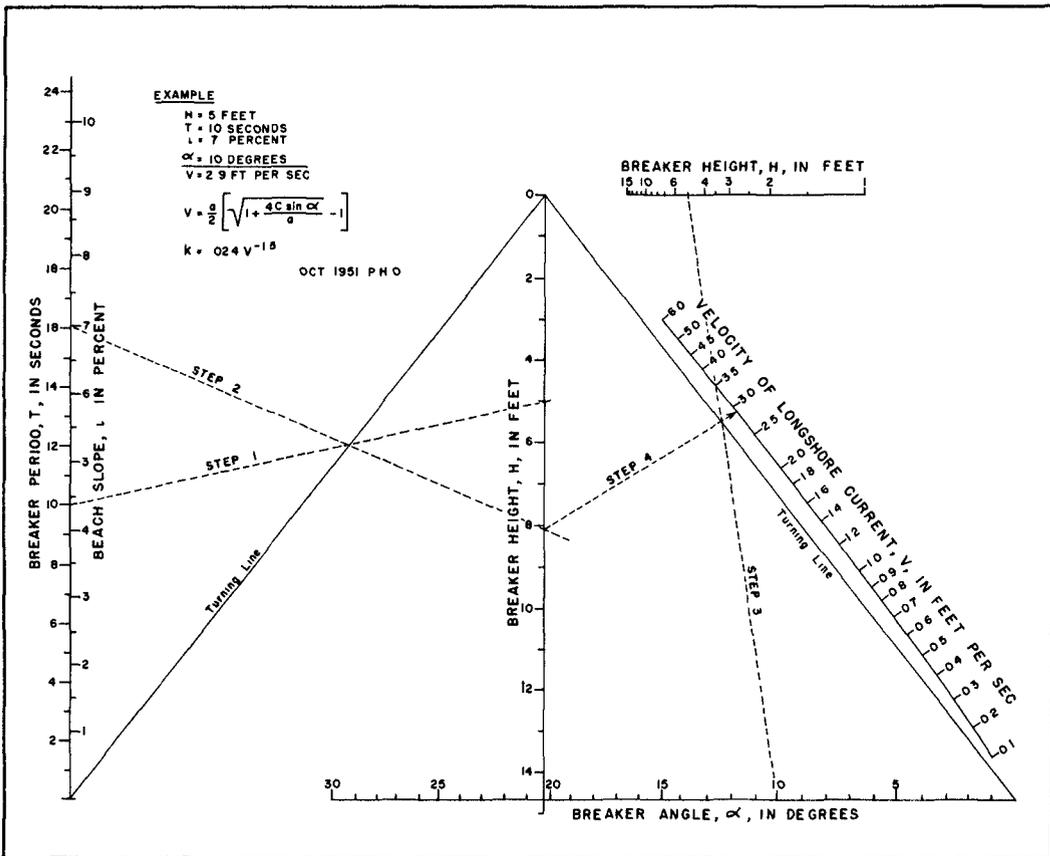


Fig. 9. Alignment chart for the computation of longshore current. Procedure: (1) lay straight edge from appropriate T to H and determine intersection with turning line; (2) turn straight edge about intersection 1, and determine intersection with H scale; (3) determine intersection on second turning line between H and α scale; (4) align intersections of H scale and second turning line and read velocity.

1950 observations at Torrey Pines and Pacific Beaches. The relative error in percent, $E = 100 \frac{V_C - V_0}{V_0}$, between the calculated and the mean observed velocity was then computed for each series of observations, and the values of E classified. From the classified values of E the standard deviation of the errors (or standard error) σ_E , was obtained by a relationship similar to equation (2).

σ_E was computed for (a) field data having an observed mean longshore current velocity above 0.1 feet per second, and (b) field data having a mean velocity above $\frac{1}{2}$ foot per second. The standard error for these two cases was 54% and 41%, respectively.

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In the previous section on the variability of longshore currents it was found that the average coefficient of variation, C_v , for case (b) was 91%. The average coefficient of variation is not directly comparable with the standard error. However, the question can be asked, "How many individual measurements of current, N , must the mean velocity be based on in order that the average coefficient of variation be equal to the standard error obtained by computing the velocity from equation (9)?" By letting C equal 91% and S equal 41% in equation (4) we find that N equals approximately 5 observations. Thus the errors in predicting mean currents above $\frac{1}{2}$ foot per second from equation (9) are comparable to the error of estimating the mean velocity from the mean of approximately 5 measurements.

Comparison of the values of the coefficient of variation and the standard error suggest that the number of measurements, N , must be greater than 5 for mean longshore velocities below $\frac{1}{2}$ foot per second, if the two errors are to be comparable. Also, to avoid station bias (see Fig. 7), the measurements should be obtained from many different stations scattered along the beach.

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