APPLYING BIVARIATE HHT TO HORIZONTAL VELOCITIES OF MULTI-DIRECTIONAL WAVES

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The Hilbert-Huang Transform (HHT) is extended to the time series analysis of wave orbital velocities resulting from the superposition of waves propagating in different directions. On a theoretical basis, it is shown that an apparently chaotic velocity signal may result from the interaction of three or more waves, each one with its own period and direction of propagation. Such result is compatible with records of PUV instruments. The comparison between bivariate HHT with Fourier directional analysis shows several advantages of the former, such as identification of wave groups and non-linear interaction components.

Keywords: Hilbert-Huang Transform; wave-wave interaction; orbital velocities

INTRODUCTION

Coastal engineers deal with several types of directional field data (e.g. wind and current velocities, orbital wave velocities and accelerations, surface slopes), which always present a common complex pattern as a result of nonlinear interactions among components. In order to understand, describe and predict several coastal processes, a directional description of the wave field must be made. The directional spectrum is thus constructed on basis of three statistically independent variables.

Wave measurements using PUV gauges often show a complex pattern for the free surface and flow velocity. Figure 1 shows a plot of a 100s burst of horizontal velocities recorded off the coast of Rio Grande do Norte, Brazil. Similar patterns, however, may be obtained as a result of synthetic nonlinear superposition of three wave trains.

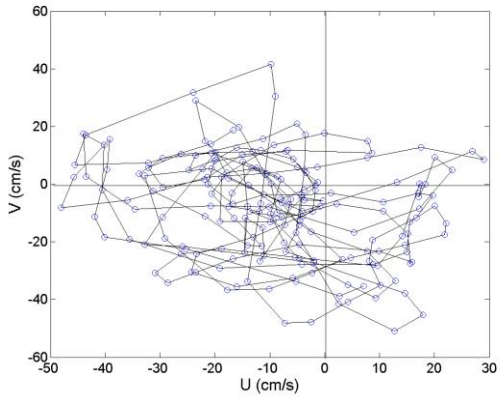

Figure 1: Horizontal velocities recorded by PUV off the coast of Rio Grande do Norte, Brazil, at 10 m water depth.

THEORETICAL BACKGROUND

Wave-wave interaction has been object of investigation for many years since the pioneering work by Phillips on resonant triads. Most attention has been focused on the shape of the free surface, while few works have dealt with the velocity field associated to the high and low frequency components resulting from non-linear interactions.

Phillips (1960) has identified the possibility of resonant interaction of wave triads at third order. However, even at second order, waves propagating from different directions may induce the formation of forced wave trains, with frequencies which are equal to the sum of or the difference between the individual frequencies of the original waves (Sharma and Dean, 1979). Looking at the velocity potentials of these second order interacting trains, it can be shown that the vertical decay of the difference terms is very slow, even though the magnitude of these terms may be eventually small. Therefore, it is possible that a bottom mounted velocity gauge may record a signal, which would be wrongly attributed to a free wave. The same reasoning applies to pressure records, when a linear

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response transfer function is used to convert bottom pressure into free surface elevation. In this case, the presence of forced wave groups may be disguised.

Thus, it would be desirable to develop methods of analysis of wave records (velocity, acceleration, pressure, free surface elevation) which could separate a complex wave field into few components. This is of particular interest at deep water or within the wave generation zone, but it is also of great interest at shallower water, when very often one attempts to correlate the data with that obtained at off-shore buoys.

Longuet-Higgins (1963) obtained the Energy Directional Spread Function using a Fourier expansion in order to compute the auto and cross spectra of three stochastic series, pressure and two velocity components. As a result, one obtains the mean principal direction, the directional spread function, peak period and significant wave height. The shortcoming of Fourier spectral analysis for the example shown in Figure 1 becomes evident, and for this reason different techniques should be attempted.

First proposed by Huang et al. (1998), the Hilbert-Huang Transform (HHT) is an adaptative method of time series analysis, which can be applied to nonlinear and non-stationary processes. It consists on an empirical decomposition in oscillatory modes (EMD), denominated Intrinsic Mode Functions (IMF), followed by the Hilbert transform of these functions. The Hilbert spectrum thus obtained allows determining the instantaneous frequency and amplitude of the signal, offering advantages in comparison to the usual Fourier spectral analysis and wavelet transform (Huang and Wu, 2008).

The HHT was originally developed for scalar time series. Later, the basic ideas – i.e., decomposing the signal into Intrinsic Mode Functions, to which the Hilbert Transform is applied – were extended to vector data time series. However, some changes need to be introduced, since concepts like maximum or minimum of vector quantities no longer apply.

Regarding acoustic wave gauges data, three features must be noted. First, the choice of axes orientation should not influence the final results; second, because observations show that wind, current or wave velocities rotate in time, such direction of rotation should be retained as an important piece of information; third, the apparent chaotic behavior of the data is intrinsically linked to the history of superposition and type of interaction among various individual waves, each with its own frequency, amplitude and direction of propagation. Further details may be found in Moura (2010).

The Hilbert-Huang Transform

The Hilbert-Huang Transform (HHT) is an adaptative method to analyze time series of non-linear, non-stationary data, based on two steps. The first one, the sifting, is an interactive process with the purpose of identifying Intrinsic Mode Functions (IMF), with characteristic frequencies (equation 1). The second one is the Hilbert Transform of each IMF, allowing to identify the instantaneous frequency ω (equations 2 and 3). For a detailed description, the reader is referred to Huang et al. (1998, 2008).

Sifting

$$\begin{aligned} h_1(t) &= X(t) - m_1 \\ &\vdots \\ h_{1k}(t) &= h_{1(k-1)}(t) - m_{1k} \end{aligned} \tag{1}$$

Hilbert Transform

$$Y(t) = \frac{1}{\pi} P \oint_{-\infty}^{+\infty} \frac{X(\tau)}{t-\tau} d\tau \tag{2}$$

Analytic signal, instantaneous amplitude and instantaneous frequency

$$\begin{aligned} Z(t) &= X(t) + iY(t) = a(t)e^{i\theta(t)} \\ a(t) &= \sqrt{X^2(t) + Y^2(t)} \\ \theta(t) &= \operatorname{atan}\left(\frac{Y(t)}{X(t)}\right) \quad \Rightarrow \quad \omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \end{aligned} \tag{3}$$

The instantaneous frequency is then computed as the time derivative of the phase function *θ(t)* and the Hilbert spectrum is constructed representing, at each instant of time (horizontal axis), the frequency (vertical axis) and the amplitude *a(t)*, represented by a color scheme where red means higher values and blue, lower values.

The Hilbert-Huang Transform for vectorial data

Different attempts have been made in order to extend the concept of HHT to the analysis of time series of vector data, such as velocity records. Indeed, the sifting process as originally proposed by Huang et al. (), cannot be simply extended to bi-dimensional data, since it is neither possible to determine maximum or minimum values, nor to define an Intrinsic Mode Function based on extreme points or zero-crossings.

Three methodologies have been found in the literature that had been developed to treat two dimensional data:

- the Complex Empirical Mode Decomposition (CEMD) (Tanaka & Mandic, 2006)
- the Bivariate Empirical Mode Decomposition (BiEMD) (Riling & Flandrin, 2007)
- the Vectorial Hilbert Huang Transform (VHHT) (Yunchao, Enfang & Zhengyan, 2008)

The CEMD method is based on a decomposition of the signal in positive and negative frequencies, following the methodology of rotational spectral analysis developed by Gonella (1972) to identify inertial frequencies in ocean currents. Suppose the 2D signal may be decomposed as:

$$\begin{aligned} U(t) &= a_{n_u}\cos(n_u t) + b_{n_u}\sin(n_u t) \\ V(t) &= a_{n_v}\cos(n_v t) + b_{n_v}\sin(n_v t) \end{aligned} \tag{4}$$

The complex signal may be expressed as:

$$U + iV = u_{+\sigma}e^{+i\sigma t} + u_{-\sigma}e^{-i\sigma t} \tag{5}$$

where the coefficients $u_{+\sigma}$ and $u_{-\sigma}$ are related to the coefficients in equation (4) by the following expressions:

$$\begin{aligned} u_{+\sigma} &= \frac{1}{2}\left[\left(a_{n_u} + b_{n_v}\right) + i\left(a_{n_v} - b_{n_u}\right)\right] \\ u_{-\sigma} &= \frac{1}{2}\left[\left(a_{n_u} - b_{n_v}\right) - i\left(a_{n_v} + b_{n_u}\right)\right] \end{aligned} \tag{6}$$

Each of these components is then submitted to the 1D HHT (the traditional method described in the first session).

The BiEMD chooses *N* different planes, decomposes the 2D vector signal on each of these planes (U and V), applies the traditional 1D HHT to each component separately, and determines an envelope for each pair of orthogonal planes, thus building a 3D envelope as sketched on Figure 2.

The VHHT selects three series (pressure and 2 orthogonal velocity components) and applies the 1D HHT to the pressure signal and the BiEMD to the velocity signal. The angle of incidence α is obtained by using equations (7) and (8),

$$\begin{aligned} S(pU,t) &= H\left[IMF(p_j)\right] \times H^*\left[IMF(u_j)\right] \\ S(pV,t) &= H\left[IMF(p_j)\right] \times H^*\left[IMF(v_j)\right] \end{aligned} \tag{7}$$

$$\alpha(t) = \tan^{-1}\left\{\frac{Re[S(pU,t)]}{Re[S(pV,t)]}\right\} \tag{8}$$

where H[□] denotes the Hilbert Transform, and H* denotes the complex conjugate. Yunchao et al. (2008) used two synthetic series of waves propagating in different directions, and this method was able to produce correct answers. However, when the same method was tested for more complex sea states, it was not always possible to isolate the individual components of the wave train.

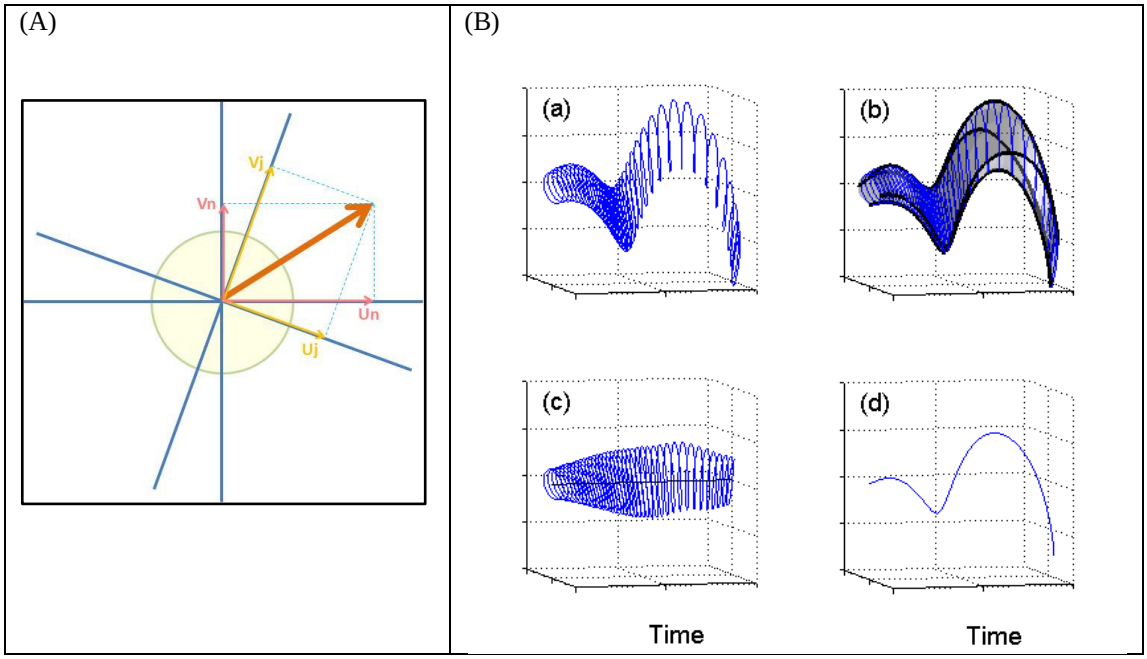

Figure 2: BiEMD: (A) Decomposition of vector and (B) construction of a 3D envelope for a two-dimensional signal, composed by a high frequency (B-c) and low frequency (B-d) contributions. The quantity plotted on the vertical plane, perpendicular to the time axis, might be any two-dimensional vector.

TESTS AND RESULTS

Moura (2010) applied the 1D-HHT and the Bivariate HHT (BiEMD) to the analysis of synthetic records of wave free surface elevation and velocities in order to characterize its ability to perform the following tasks: (1) separating primary wave trains; (2) identifying nonlinear interactions; (3) identifying non-stationary processes; (4) characterizing wave groups; (5) finding the directions of group propagation. Twelve sets of wave conditions were numerically generated, including non-linear terms up to second order, simulating:

- one single non-linear (Stokes) wave train with constant frequency;
- one single linear wave train with slowly or abrupt varying frequency; and
- groups of 2, 3 and 6 waves, each with a different direction of propagation and with slight (or marked) differences in frequency.

When HHT is applied to the free surface elevation of a nonlinear Stokes wave, the Hilbert spectrum shows a pattern of fluctuating frequency along time, as presented in Figure 3, which shows the wave profile, with sharp crests and flat troughs, the Fourier spectrum, and the Hilbert spectrum. In this example, the wave has amplitude of 1.5 m, period of 7 s, and the water depth is 10 m. The amplitude variation of each IMF is represented by the color of the line (red), which remains the same because of the wave amplitude remains constant.

In contrast, when HHT is applied to the free surface record of a non-stationary wave, the resulting Hilbert spectrum reveals the imposed frequency changes, as shown in Figure 4. It is evident the advantage of the HHT analysis when compared to the ordinary Fourier spectral analysis. For instance, taking the example of the wave record shown in Figure 4, if the order of occurrence of the two wave conditions had been changed, the Fourier spectrum would remain the same, since this is a global method of analysis, whereas HHT would result in different pattern of the Hilbert spectrum because it is a local analysis (instantaneous frequency).

The simultaneous occurrence of various wave trains, each with its own frequency, amplitude and direction of propagation, may result on a very complex sea state. Table 1 presents an example of three wave trains with close frequencies but propagating at very wide directions. The resulting free surface is shown in Figure 5 , where (A) shows the linear superposition of the three waves. The complexity in the surface displacement is due to different wave directions. As the second order components are added, the surface behavior is modified becoming even more complex, with the presence of long and short forced waves with smaller amplitudes. In Figure 5, part (C) shows the subtractive terms of the wave-wave interaction between waves A1 and A2 ($\psi_1-\psi_2$) and part (D) shows the addictive wave-wave

interaction between waves A1 and A2 ($\psi_1+\psi_2$). The next step should be to investigate the velocity contribution associated with these terms.

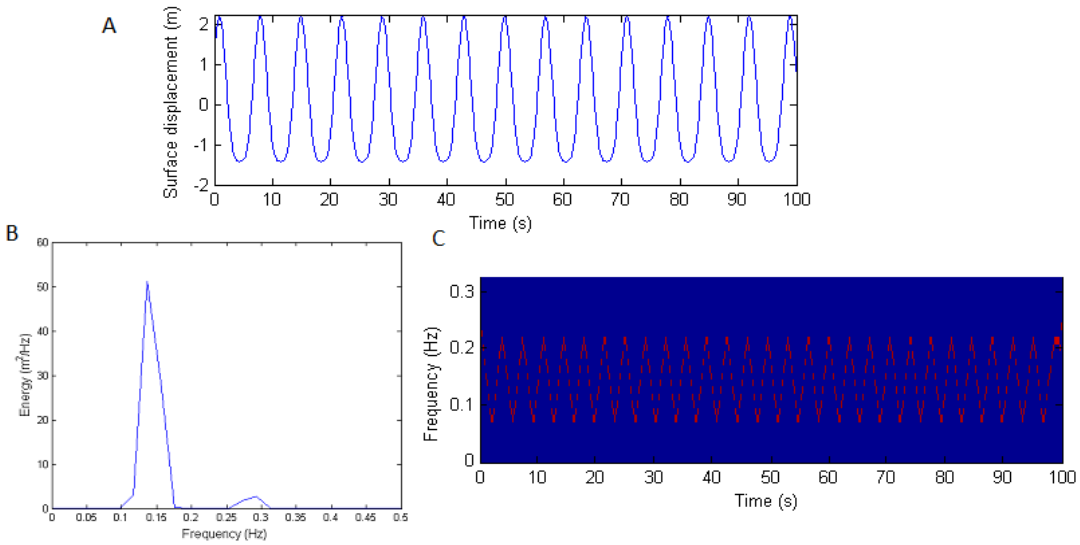

Figure 3: Second-order Stokes wave. (A) Surface displacement. (B) Fourier spectrum. (C) Hilbert spectrum.

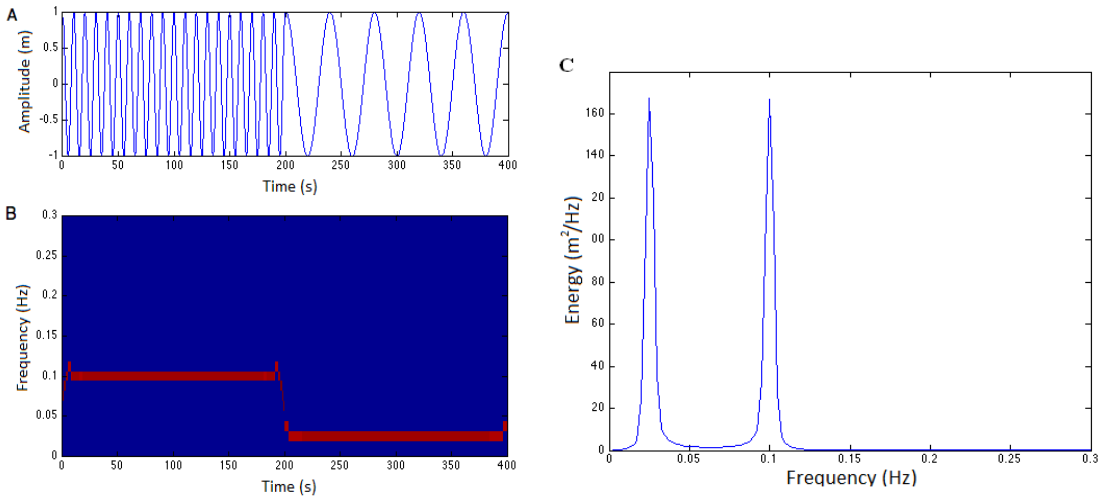

Figure 4: (A) Synthetic wave record with time varying frequency; (B) Hilbert spectrum (HHT) showing frequency change; and (C) Fourier spectrum of the same record.

Table 1: Three-wave second order interaction.				
wave	**amplitude (m)**	**period (s)**	**angle (°)**	**length (m)**
A1	1.0	9.0	80	95.6
A2	1.2	10.0	0	100.0
A3	1.5	10.2	20	111.7
interaction	**phase**	**period (s)**	**angle (°)**	**length (m)**
A1-A2	$\psi 1-\psi 2$	90.0	125.5	79
A1-A3	$\psi 1-\psi 3$	76.5	132.3	102
A2-A3	$\psi 2-\psi 3$	510.0	-76.1	318
A1+A2	$\psi 1+\psi 2$	4.74	43.2	66
A1+A3	$\psi 1+\psi 3$	4.78	52.5	59
A2+A3	$\psi 2+\psi 3$	5.05	9.9	56

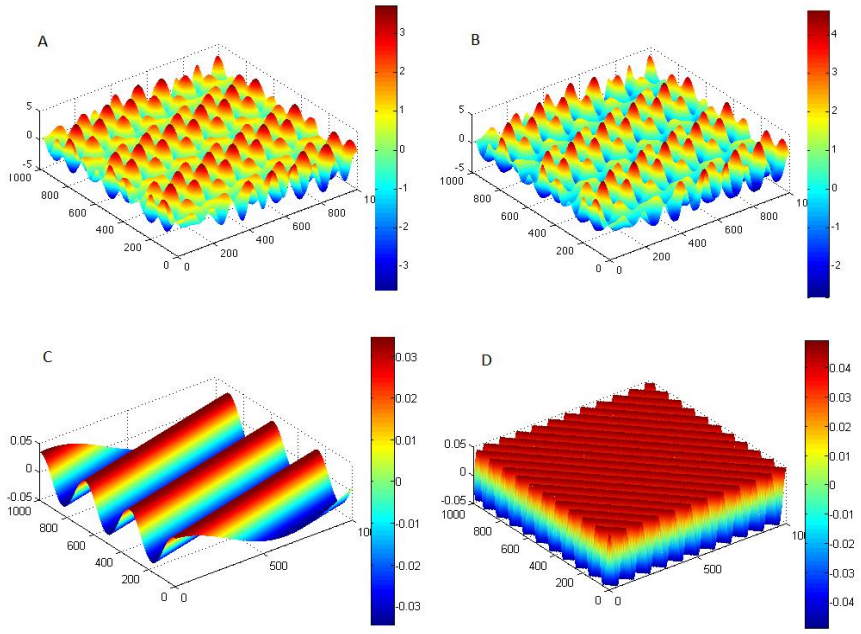

Figure 5: (A) Superposition of the three first order waves. (B) Superpositon of the three waves including first e second order components. (C) Second order component, subtractive wave-wave interaction ($\psi 2-\psi 3$). (D) Second order component, additive wave-wave interaction ($\psi 1+\psi 2$).

This example can help elucidate some features of the HHT that will help to better understand this method of analysis. First, HHT is able to separate different wave trains or to identify a sum of primary waves. As pointed out by Flandrin et al. (2004), the EMD works as a filter bank, allowing the identification of non-stationary processes such as wave groups. However, the method cannot separate waves in distinct IMFs when the primary frequencies are too close to each other.

Figure 6 shows the Hilbert spectrum for the IMF that represents the wave groups formed by the three waves. Because of the relative close frequency between waves, they turn out to be represented by one intrinsic mode function. The behavior of the signal amplitude and frequency shows the presence of wave groups with different length, suggesting that these groups are composed by more than two waves. Nevertheless, since wave groups are non-stationary processes, it would be very difficult to identify them by means of the Fourier spectral analysis. As shown in Figure 7. despite the capability of the method to find the frequencies of the primary waves, no inference can be made about wave groups, which clearly poses a limitation of the usual Fourier method to wind wave analysis.

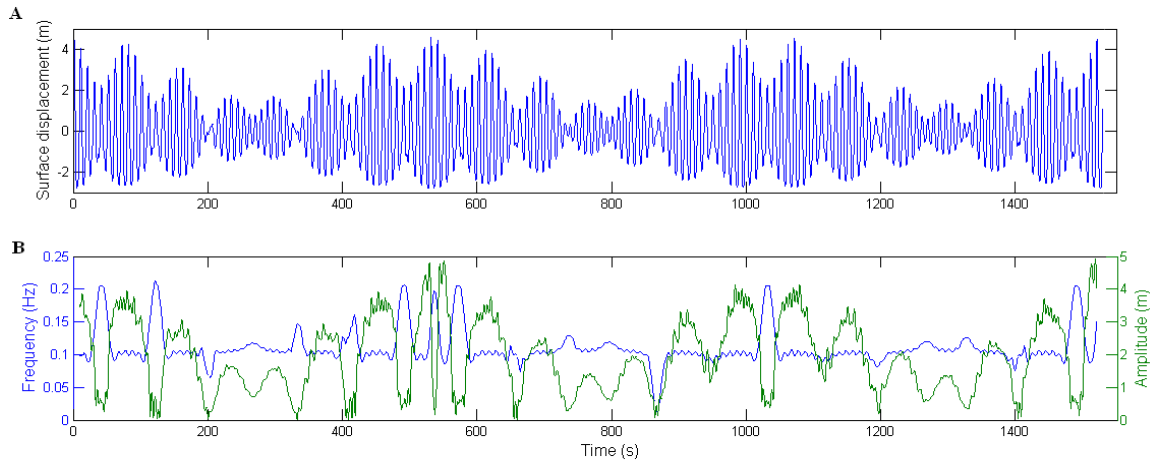

Figure 6: (A) Surface displacement. (B) Hilbert spectrum for the IMF that represent the wave groups. Blue line - frequency, Green line - amplitude.

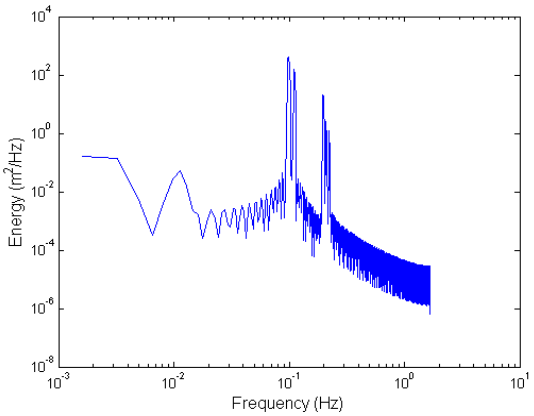

Figure 7: Fourier spectrum of the wave record shown in Figure 6 (above).

Another situation which was investigated was that of waves with larger frequency difference, as shown in Table 2. In that case the EMD is capable to identify the three primary waves trains as shown in Figure 8. Moura (2010) discusses in more detail the importance of difference between frequency rather than wave direction to define the IMF behavior.

Table 2: Primary wave with close directions and large frequency difference			
wave	**amplitude (m)**	**period (s)**	**angle (º)**
A1	1.0	5	45
A2	1.5	8	60
A3	1.2	15	50

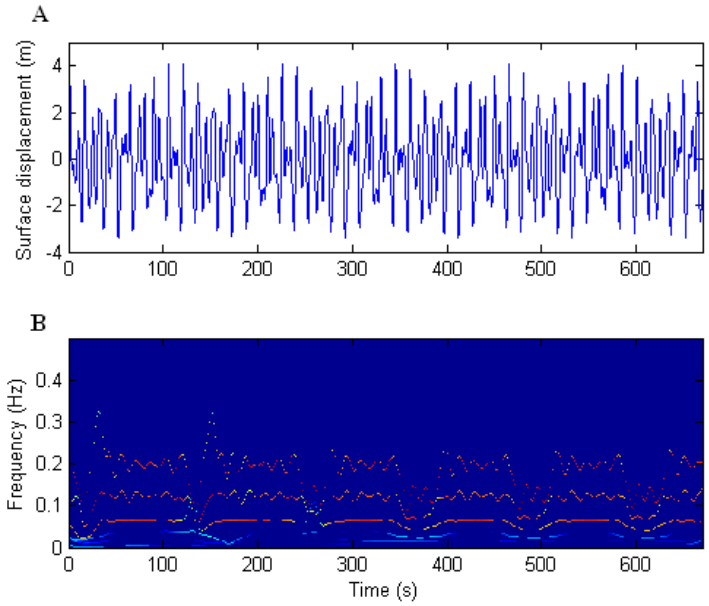

Figure 8: Three wave interaction for example given in Table 2: (A) Surface elevation. (B) Hilbert spectrum.

As presented early in this article, an important feature of water waves in real seas is the nonlinear components which are formed by wave-wave interactions, such as long period small amplitude forced waves. The HHT can be a good method to analyze this kind of oscillations. Figure 9 presents the subtractive wave-wave interaction component between *A2* and *A3* and the IMF with similar frequency, which shows the capability of the method to identify such small amplitude oscillations.

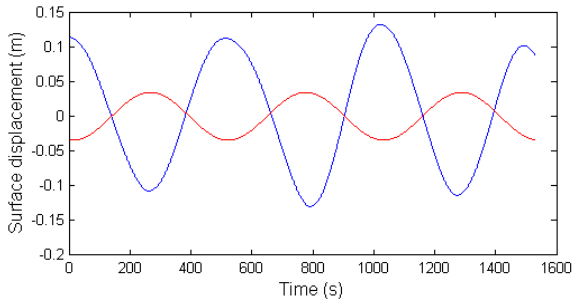

Figure 9: Comparison between (red line) subtractive wave-wave interaction (A1 and A2) and (blue line) IMF with similar frequency.

Several instruments nowadays measure horizontal velocities (U and V components), which require appropriate analysis of direction and frequency. Many reasons make the method of analysis of bi-dimensional more complex than the ones which are usually applied to surface elevation or pressure records (scalar data). For a long crested progressive wave, the horizontal components of the orbital velocity are in phase with each other, as well as with the surface elevation, i.e. U and V have maxima and minima at the same time. In this situation, the wave direction can be easily found. The superposition of waves propagating in different directions significantly changes the pattern of bi-dimensional horizontal velocities, and the components U and V are no longer in phase. When non-linear effects are considered, not only the multiple directions but also the difference between frequencies also play an important role. Figure 10-A shows the velocities U e V (hodograph) concerning only the first order of waves (*A2*) and (*A3*). Because of the small frequency difference, wave groups are formed; however, due to the difference in wave directions, an elliptical pattern of velocity develops, where the axis and origin changes with time, as well as the direction of rotation of the velocity vector. On the other hand when there is a presence of two waves (*A1* and *A2*) with a higher difference in frequency, the velocities U e V change drastically (Figure 10-C).

In a real sea condition, there is often a combination of different waves coming from distinct directions; in addition, the presence of tides and superficial wind currents makes the interpretation of the horizontal velocities extremely difficult (Figure 1).Considering only those three waves indicated in Table 2, when their second order nonlinear components are included, it becomes evident how difficult it is to interpret the data (Figure 11).

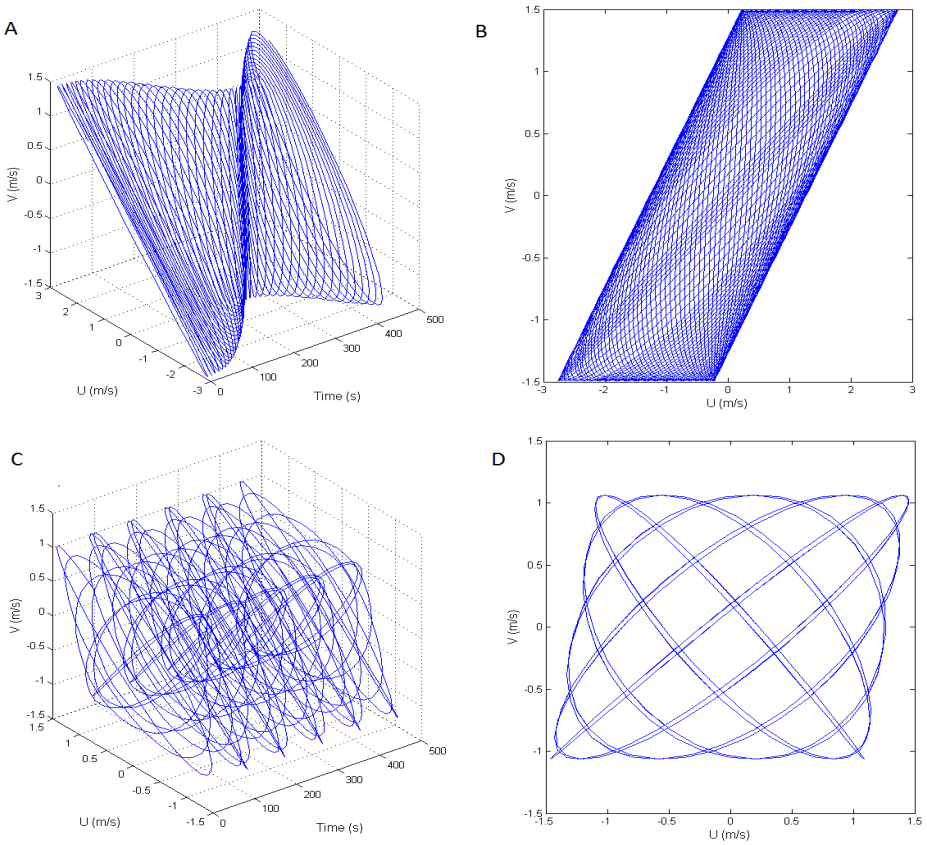

Figure 10: Velocities U and V for two different wave superposition. (A)-(B) superposition of waves *A2* and *A3*. (C)-(D) superposition of waves *A1* and *A2*.

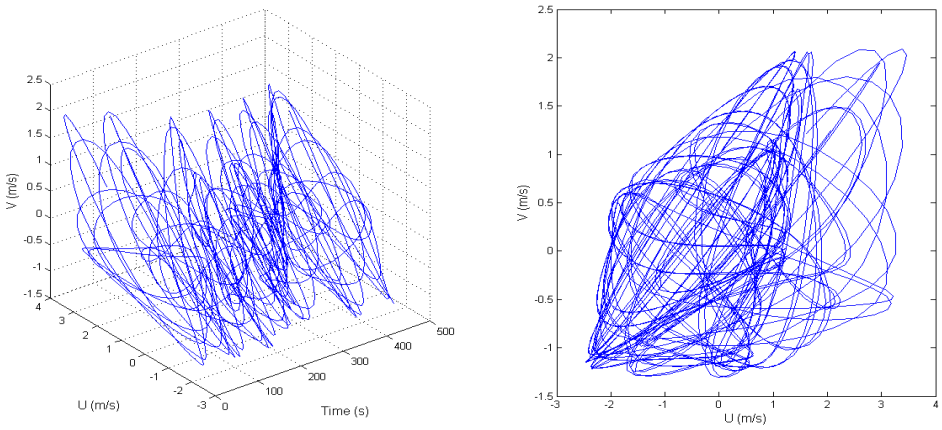

Figure 11: U and V components of horizontal velocity resulting from the superposition of waves in different directions and with different periods.

As pointed out before, both for the 1D-EMD and for the BiEMD, frequency differences determine whether the IMF will present a wave group or a single wave train. Comparing the two examples given in Tables 1 and 2, it can be seen that the results for BiEMD follow the same pattern as the 1D-EMD. In the first case, the BiEMD identified groups of waves for one IMF, as shown by the Hilbert spectrum for U e V (Figure 12). The IMFs for U have higher amplitudes than the IMFs for V, because the amplitude modulations due to the intersecting waves from various directions are much stronger on U velocities.

The example of Table 2 shows that, when the primary wave trains are well separated in frequency (Figure 13), it is easier to identify each wave direction (Figure 14).

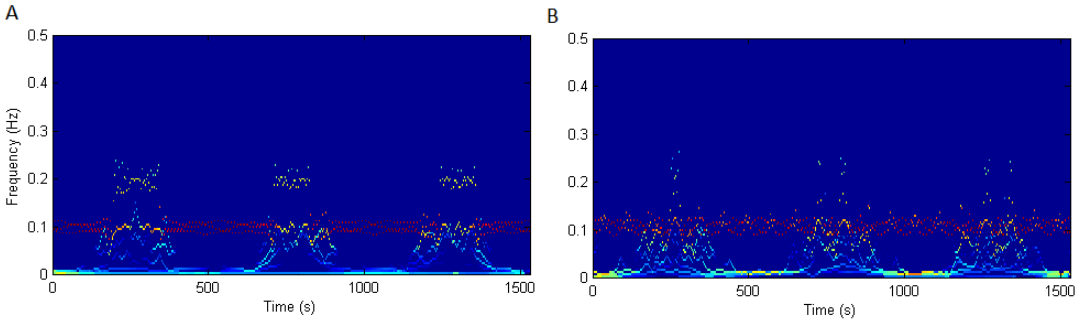

Figure 12: Hilbert spectrum. (A) velocity U, (B) velocity V.

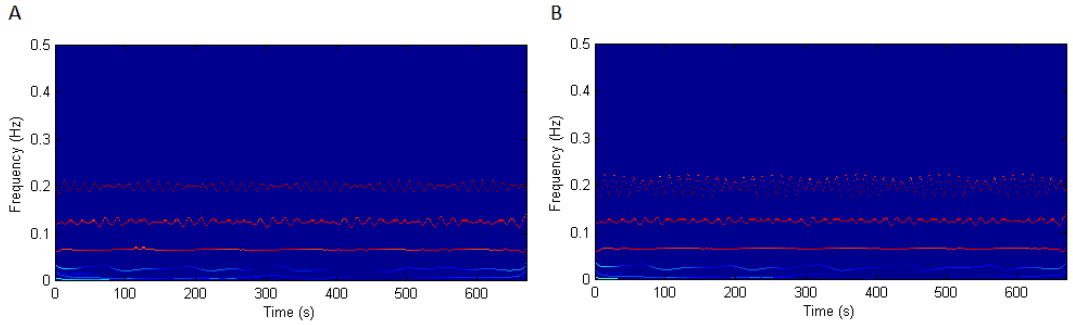

Figure 13: Hilbert spectrum. (A) velocity U, (B) velocity V.

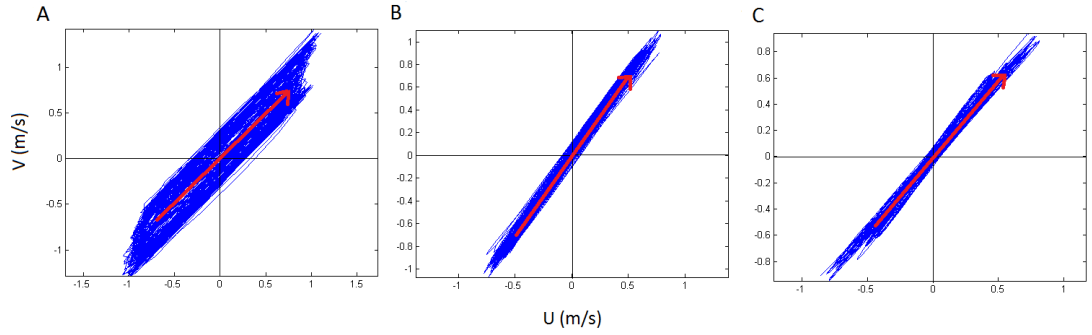

Figure 14: IMFs components representing velocities U and V of three primary wave trains (Table 2). (A) wave *A1*, (B) wave *A2*, (C) wave *A3*.

The low frequency components patterns on the velocity record can be well identified by the BiEMD. Similarly to the 1D-EMD, the respective frequency components (Figure 15) were properly sifted, although same trouble with amplitude super estimation and out of phase IMF still appear. In the future, one of the main goals of the method, though, would be to identify the direction patterns of those components.

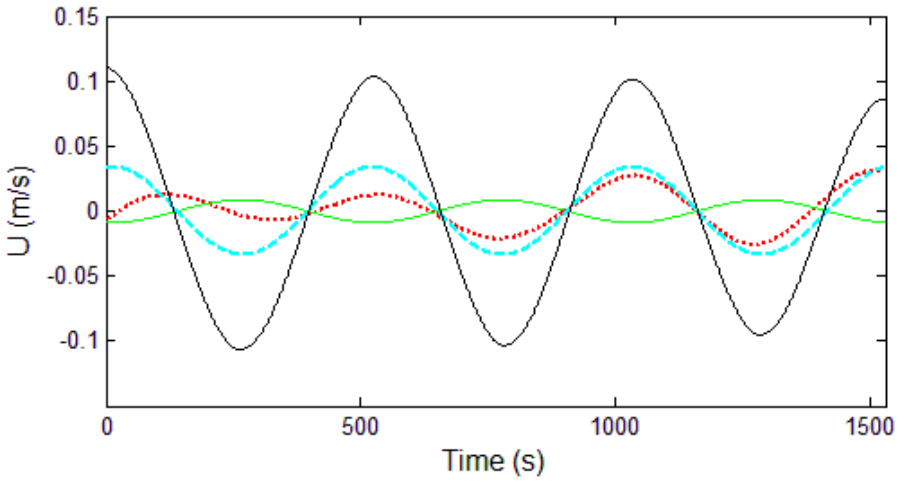

Figure 15: Velocities U and V for low frequency wave components and respective complex IMF. Black line - velocity U, dot red line- real part of IMF, green line - velocity V, dash cyan line - imaginary part of IMF.

CONCLUSION

HHT is an adaptative method, well suited to the analysis of non-stationary, non-linear processes, which does not suppose *a priori* any property of (i.e. amplitude, frequency) nor impose to (i.e. eigenfunction components) the original series. The method consists of two steps: a sifting process, where Intrinsic Mode Functions (IMF) are identified, followed by the Hilbert Transform of the IMFs. HHT identifies the instantaneous frequency and amplitude of the signal, a significant advantage when compared to other methodologies, such as Fourier analysis, which furnish the global instead of the instantaneous properties of the signal. By means of the Hilbert spectrum, it is possible to identify wave groups, non-linear components and other transient effects.

Three different methodologies for two dimensional data series were tested, in order to investigate the complex pattern of horizontal orbital velocities when two or more primary waves are present. Among these methods, the BiEMD turned out to be an efficient procedure for separating nonlinear components in the wave train.

The current methods of analysis of PUV data are based on linear theory transfer functions and are valid for single wave trains traveling in only one direction. Such methods do not consider, for instance, neither the formation of wave groups due to multiple direction wave-wave interaction, nor the Doppler effect of currents on waves. The new methods, such as HHT and wavelets, attempt to correct the weakness of linear Fourier analyses, but they have not yet reached a “stable” stage which should allow to determining the full complexity of sea states. It seems promising, though, that two dimensional HHT methods may be further developed in order to characterize nonlinear wave-wave interaction.

Experiments with synthetic series of horizontal velocities indicated that eventually some IMFs were formed by abnormal oscillations, which were not present in the original series, nor corresponded to higher order wave-wave interactions. Likely problems of the sifting process for 2D signals may be related to the interpolation method and to the convergence or stopping criterion. Studies are currently being conducted at the National Laboratory of Civil Engineering (LNEC) and at the Federal University of Rio de Janeiro in order to analyze orbital velocities measured by ADVs, both in wave flumes and in the field.

The paper then addresses the need to establish alternative parameters based on HHT, which may have physical and practical significance as well, such as wave groupiness, structural resonance due to low frequency wave-wave interaction, characterization of sediment transport due to complex bottom wave velocity.

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