BORE PROPAGATION SPEED AT THE TERMINATION OF WAVE BREAKING

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Wave breaking is the most important event in nearshore hydrodynamics because of the energy exertion and mass transportation during the event drive all the nearshore phenomena, such as wave set-up/down, long shore current, and nearshore circulation. Wave celerity is a key parameter in wave breaking especially for the mass transportation, the energy dissipation during the wave breaking event, and the wave breaking index calculation, for example. There are many models to calculate the wave celerity during the breaking event (bore propagation speed) and it is well known that the bore propagation speed is faster than that is given by linear wave theory. But Okamoto et al. (2008) found the bore propagation speed at the termination location of wave breaking becomes much slower than the theoretical estimation when the termination location of wave breaking is examined with the experimental data collected in a wave tank with simplified bar-trough beach settings. Comparisons with theoretical models are presented. Fourier analysis is performed to investigate the evolution of higher harmonics and synthesized time series, which is a simple summation of linear wave components, is constructed by using the obtained information to calculate the wave celerity during and after the wave breaking. Calculation result reveals that as the breaking wave approaches to the termination, the bore propagation speed decreases towards the value which can be explained by the existence of slowly and independently propagating higher harmonics.

Keywords: wave breaking, bore propagation speed, wave tank experiments, harmonics composition

Introduction

Wave breaking is the most important event in the nearshore region, since it is the energy source of all the nearshore hydrodynamic phenomena. Mass transportation due to the roller convection also gives significant influence on the nearshore hydrodynamics. For the engineering analysis, the breaking area has to be clearly distinguished form the non-breaking zone because of the difference of physics between breaking and non-breaking waves.

In most of the case in coastal area, considering the initiation of wave breaking practically satisfies the requirement because the wave breaking initiates very close to the shoreline and continues until almost all the wave energy is dissipated. The traditional type of wave breaking indices, therefore, looks only at the initiation of wave breaking. The termination of wave breaking occurs in obvious fashion in limited cases, such as bar-trough profiled beach, river/inlet mouth and so on. However in these cases, the determination of termination location is as important as that of the initiation in order to find the breaking area.

In the determination of wave breaking termination, the wave breaking index has to be monitored during the breaking event. Authors conducted wave tank experiments with simplified bar-trough formation beach setting for the investigation of the termination condition of a wave breaking index based on the moving hydraulic jump concept and found that the wave celerity of the breaking wave, hereafter the bore propagation speed, decreases to the value much slower than the theory (Okamoto et al., 2008). At that point, a hypothesis, which assumes that the bore propagates as if it goes on the uniform water depth of that at the bar crest, was made to explain the behavior of observed data, but the detailed analysis was left behind.

The roller model (Svendsen, 1984) is one of the well accepted concepts to understand and analyze the mechanism of energy dissipation due to wave breaking. It assumes the existence of water mass (=roller) separated from the normal wave motion described by the potential flow theory and propagating with the wave celerity, C. Therefore, the water mass is transported with much faster speed than the particle velocity, u. As a concept, the wave celerity of the roller model is left as C and any formula can be fit in. Therefore, better understanding on the bore propagation speed is very important for the wave breaking study.

In this paper, we discuss about the wave celerity before, during and after the wave breaking by using the experimental data obtained at the simplified bar-trough shape beach. The wave celerity and

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bore propagation speed are calculated with couple different methods and compared with existing theories. In order to explain the behavior of bore propagation speed on inversely sloped bottoms, Fourier analysis of the data is performed to evaluate the importance of harmonics, and synthesized time series, which is a simple summation of linear wave components, is constructed by using the obtained information to calculate the wave celerity during and after the wave breaking

Bore Propagation speed

At the wave breaking, it is known that the wave celerity becomes much faster than the one derived by the wave theory based on the potential flow normally given as \sqrt{gd} because wave breaking occurs mostly in shallow water zone. Svendsen et al. (1978) derived the bore propagation speed formula for periodic bore from the momentum balance equation as following;

$$C = \sqrt{\frac{1}{2} \frac{gh_1h_2}{d^2} (h_1 + h_2)}$$
(1)

where g is the acceleration of gravity, d is the mean water level, h is the total water depth from the free surface to the bottom, and subscript 1 and 2 denote the location of wave trough and crest, respectively. This is basically the same as the classical bore propagation speed on no current field. Stive (1984) added a coefficient to Eq. 1 to include the effect of turbulence and it gives better agreement with his experimental data obtained on a plane slope beach.

Bonneton (2004) derived another bore propagation speed formula from Saint Venant shock-wave model. Several assumptions required for the classical bore propagation formula, such as locally horizontal bottom, quasi-constant wave shape, and so on, were removed by using Saint Venant shock-wave model. As a result, it can be applied to more realistic situations. The bore propagation speed is given as;

$$C = -2\sqrt{gd} + 2\sqrt{gh_1} + \sqrt{\frac{1}{2}\frac{gh_2}{h_1}(h_1 + h_2)}$$
(2)

Bonneton (2004) compared it with the classical formula, Eq. 1, by using the experimental data obtained by Stive (1984) and concluded that Eq. 2 provides a better agreement with data than Eq. 1.

Depth inversion technique has been developed in recent years to estimate the local water depth from the remote-sensing data. This technique relies on the assumption of which the wave celerity is a function of the water depth. So, the accuracy of the estimated water depth is very much dependent on the accuracy of the wave celerity equation to be used.

Catalan and Haller (2008), therefore, made a comparison of wide range of wave celerity formula, from linear wave equation to the complicated, non-linear composite formula. The bore propagation formula given by Bonneton (2004) was also in their list. They tested those equations with data obtained on a bar-trough shaped beach in a large scale wave tank. Although the wave celerity calculated by those equations were not completely agreed with the observed data obtained by using remote sensing data, especially at the outer surfzone and the following area, they concluded that a composite model given by Kirby and Dalrymple (1986), hereafter KD86, gives the best agreement to the data among equations they tested. The equation given by KD86 is shown as following;

$$C^{2} = g / k(1 + f_{1}\varepsilon^{2}D) \tanh(kd + f_{2}\varepsilon)$$
(3)

where k is the wave number, H is the wave height, $\varepsilon = kH/2$ and

$$D = \frac{8 + \cosh(4kd) - 2\tanh^2(kd)}{8\sinh^4(kd)} \tag{4}$$

$$f_1(kd) = \tanh^5(kd), \qquad f_2(kd) = \left(\frac{kd}{\sinh(kd)}\right)^4$$
(5)

This equation becomes a linear wave model when the wave height gets infinitesimally small. For finite amplitude waves, it asymptotically approaches to the Stokes third order theory and Solitary wave theory for deep and shallow water waves, respectively.

As explained above, the wave celerity in shallow water region is described by the combination of the water depth and the wave height. In the case of bore propagation models, the wave height is replaced by the local water depth at two different points, so that the principle is the same. Furthermore saying, as long as seeing the result comparison shown in Catalan and Haller (2008), the shallow water wave is basically regulated by the \sqrt{gd} function derived by linear theory. All the other terms in the equation are the correction terms due to the non-linear effect, which is not the decisive term and always gives positive effect on the wave celerity and never slows it down.

The wave breaking module in the Boussinesq equation model developed by Schäffer et al. (1993) is based on the roller model concept and they used $1.3\sqrt{gd}$ as the wave celerity for the calculation of energy dissipation. Madsen et al. (1997) used the same wave breaking model that Schäffer et al. (1993) introduced, but modified the wave celerity calculation with the interactive method using the pure advection equation. The wave celerity calculation displays that the combination of the Boussinesq equation model and the celerity calculation method based on the pure advection equation provides very good agreement with the experimental data by Stive (1984) and it is closer to $1.3\sqrt{gd}$ rather than \sqrt{gd} .

However, it does not seem to be true that the bore propagation speed keeps accelerated speed all the way to the end. Svendsen et al. (2003) collected data obtained on plane slope beaches from several different papers and analyzed the wave properties in the surfzone. They analyzed the wave celerity of six different cases from two papers. The result displayed that the wave celerity becomes faster than the linear wave model as the wave become closer to the breaking point and it becomes up to about 50 to 70% more at the maximum point. After passing certain point, it decreases towards the linear wave model. They also suggested that the wave celerity becomes slower than the linear wave celerity in some cases.

Similar results were observed by Okamoto et al. (2006) in the wave tank experiments with horizontal bottom at the breaker zone. The wave celerity becomes much faster than the linear wave speed and it gradually decreases to the linear wave celerity as the wave propagates and approaches to the termination of wave breaking. Furthermore, Okamoto et al. (2008) found that the bore propagation speed becomes even much slower than the linear wave celerity from the experimental data obtained on simplified bar-trough profiled beaches. It becomes about 70% of linear wave celerity in some case.

An important point displayed by these experimental data is that the bore propagation speed is not a function of the local depth. It increases at the outer surfzone because of the energy released by the wave breaking and decreases even when the water depth increases as the wave propagates.

Wave tank experimental settings

Wave tank experiments were conducted at National Laboratory of Civil Engineering (LNEC), Lisbon, Portugal. The wave tank has 32m long from the wave maker to the end of wave tank. Simplified bar-trough profiled beaches were installed with back slope of 1/20, 1/40, and 1/80 (See Fig.1). The bottom was made out of concrete so there is no permeability in the bottom. Horsehair sheets were installed in the shore section to reduce the reflected energy. Preliminary test results show that only few percent of energy reflects back to the tank. Water depth was referenced at the bar crest and two water depths were tested in the experiments: d=10 and 15cm. The coordinate system was set the bar crest to be zero with positive direction to the shore.



Figure 1. Experimental setting of wave tank at LNEC

The input wave conditions were chosen to make the wave breaking to initiate in the front face of the bar and to terminate within the lee-side slope, so that the termination of wave breaking would not be disturbed by the bottom slope change. Four wave period (T=1.1, 1.5, 2.0, and 2.5sec) and four wave height (H=8, 10, 15, and 20cm) were chosen for d=10cm but only two wave height cases were tested for d=15cm. Note that the input wave here is measured at the toe of the front slope of the bar, not in front of the wave maker, because the original bottom configuration of wave tank is not horizontal as shown in Fig. 1. One case, which has the input conditions of T=1.1sec and H=20cm, breaks right after the wave generation in front of the paddle, so it was excluded from the experiment. Thus, 23 cases were tested for each bathymetry setting and 69 cases were tested in total.

Free surface elevation was measured to calculate the wave celerity and bore propagation speed. Eight resistant type wave gauges were used in this experiment; one of them was installed at the toe of the front slope to check the input condition, and the rest of seven gauges were used for the celerity calculation. Those gauges were placed 20cm apart each other and the set of wave gauge was moved as a group. Wave gauges were placed at both odd (x=10, 30, 50,...) and even (x=0, 20, 40,...). It makes the final resolution of the data set 10cm. The measured area covers from 200cm before the bar crest to the bottom of the trough (500, 630, and 800cm for 1/20, 1/40, and 1/80, respectively). The preliminary test was performed to investigate the natural frequency of the wave tank. It was found that the period of natural oscillation is about 40sec and it takes about 4 minutes until the oscillation becomes enough small to be ignored. Thus, the wave gauge record was recorded from 5 minutes after the beginning of the operation, and the duration of record is 2 minutes. The sampling frequency of the gauge record is 100Hz.

Particle velocity was also measured in this experiment. Data was collected by Acoustic Doppler Velocimeter (ADV) covering the same area as the free surface elevations were measured. The probe was located roughly at the middle of still water column, and also at near the surface and the bottom for selected locations. Vertical profiles of horizontal velocity were measured at several location in the case of T=1.5sec and H=8cm on 1/20 slope. The mean current was calculated to see the effect of that. It was found that the mean current at around the breaking termination is around 1cm/sec to the offshore direction and that of outside of the breaking zone is less than 1cm/sec. So, we concluded that there is no significant influence from the mean current field at around the termination location in this experiment.

Experimental results

Since the experimental data was taken by wave gauges at fixed locations, estimation of the time that a wave requires to pass between two wave gauges is necessary for the calculation of the wave celerity. In this work, the cross-correlation method was employed to estimate the time lag between two wave gauges. First, the time series was divided into short portions with length of one wave period. The time lag to the next wave gauge for each wave was determined by finding the highest correlation coefficient value with the corresponding section of the record at the next wave gauge. Those time lags for each single wave were gathered and the averaged time lag was calculated. Then, the wave celerity was calculated as shown in Fig. 2.



Figure 2. Wave celerity (T=2.0sec, H=8cm, d=10cm on 1/20 slope)

Fig. 2 displays the wave celerity of T=2.0sec, H=8cm, d=10cm on 1/20 back slope setting calculated with the cross-correlation method. One can see that the wave celerity becomes much faster soon after the initiation of wave breaking, then it decreases. This is not what happens with the

theoretical models. In fact, in the trough region (x>0), the water depth increases as the wave propagates. Thus, the theoretical wave celerity increases in the trough region as shown in Fig. 2. All the wave celerity calculation formula has great dependency to the local water depth. Therefore, it is clear now that the bore propagation speed does not even follow the general trend of the bathymetry. Moreover, wave celerity keeps decreasing and finally becomes too small to be explained by any existing theories. In this case, the ratio between the measured wave celerity and the theoretical celerity at the termination location is about 78%. The theoretical celerity here is described by the general expression of linear wave theory, so if it is compared with shallow water wave celerity, \sqrt{gd} , the ratio would become even smaller.

In order to confirm the above results, other methods to calculate the temporal/spatial lag between two data were also employed to calculate the wave celerity. The simplest one is the tracking method; tracking certain position of the wave over two different data. In most of the case, zero crossing point is chosen as the marker position. But in this work, the crest position, i.e. the maximum point in the record within the length of a wave period, was chosen as the marker position because the original intention to calculate the wave celerity was to calculate the RTFN wave breaking index, in which the wave celerity at the crest is defined as the one of the parameter. The result was very similar to the one obtained by the cross-correlation method. The basic trend of celerity evolution and values during the wave breaking are very similar but it contains much more scattering especially after the termination of wave breaking.

Another method to estimate the temporal/spatial lag is the least square method. The basic idea for the calculation of the wave celerity is the same; estimating the temporal/spatial displacement by finding the lag to give the least square error between two data. Misra et al. (2003) examined the least square method for the calculation of wave celerity using the remote-sensing data. They concluded that the least square method has better accuracy than the cross-correlation method and it is free from the losing of accuracy due to the relation between data length and wave length (window effect). Thus, the least square method was also applied here to calculate the wave celerity and bore propagation speed. The calculation procedure is the same as the cross-correlation method explained above, except using the least square method for estimating time lag between two data. Fig. 3 displays the comparison between the cross-correlation method and the least square method and the least square method for the case of T=2.0sec, H=8cm, d=10cm on 1/20 back slope.



Figure 3. Comparison of wave celerity calculation method (T=2.0sec, H=8cm, d=10cm on 1/20 slope)

The result is very similar to the one obtained by the cross-correlation method. As shown in Fig. 3, they are almost identical except at the area of outer surfzone, which is not the main target area in this study and it is very difficult to identify the celerity due to the high fluctuation in the free surface profile. Therefore, it was concluded that the basic trend in the bore propagation speed evolution on given condition should be what is shown in Fig. 2, since all three results from different calculation method provide very similar tendency.

It is important to mention that the bore propagation speed at the location of wave breaking termination becomes closer to the linear wave theory as the bottom slope gets milder. Table 1 and 2 show that the averaged bore propagation speed at the termination location for each back slope condition and the ratio to the linear wave celerity. Generally speaking, the termination location of the wave of having the same input wave conditions becomes further from the bar crest but the water depth at the termination location becomes shallower as the back slope gets milder. Therefore, it is reasonable that

the measured celerity becomes slower in the milder slope case. But the amount of change is very small. As a matter of fact, it is nearly constant, as shown in Table 1 and 2. On the other hand, the effect of the water depth change on linear wave theory works much more than that on the measured values, so the ratio to the theory becomes closer to unity as the back slope condition gets milder.

Table 1. Bore propagation speed at the termination location and the ratio to linear wave theory (d=10cm)		
Slope condition	Celerity (cm/sec)	$C_{\text{measured}}/C_{\text{linear}}$
1/20	104.7	0.78
1/40	100.8	0.84
1/80	96.9	0.89

Table 2. Bore propagation speed at the termination location and
the ratio to linear wave theory (d=15cm)Slope conditionCelerity (cm/sec)Cmeasured/Clinear1/20119.30.771/40115.10.821/80113.00.87

Comparison with theoretical models

The experimental data was compared with theoretical models explained above. First, comparisons with linear and non-linear wave model were made. Here, linear wave celerity was calculated with the general expression of wave celerity formula because some of the wave is categorized in intermediate wave due its wave period and the water depth (d/L>1/20). Needless to say, linear wave theory is not the appropriate one in this situation, so it was just calculated to show some reference.

The non-linear composite model given by Kirby and Dalrymple (1986), Eq. 3, 4, and 5, requires wave height other than the wave length and water depth to calculate the celerity. The wave height was calculated by using zero up-crossing method to couple with the crest and the trough in front of it. Fig. 3 displays the comparison between the experimental data and theories for the case of T=2.0sec and H=8cm.



Figure 4. Comparison with linear and non-linear wave models (T=2.0sec, H=8cm, d=10cm on 1/20 slope)

As shown in Fig. 4, KD86 and the experimental data agree very well until slightly after the initiation of wave breaking (about up to -50cm), but do not agree at all after that. The non-linear effect makes the wave celerity faster than the linear wave celerity so that the displacement between the experiment and theory is bigger around the termination location. Considering the result given by Catalan and Haller (2008), this is very reasonable result. On the other hand, the better agreement before the breaking initiation due to the non-linear effect is quite important. It displays that our calculation scheme of the wave calculation provides reasonable results.

Comparison between the experimental data and bore propagation model was also performed; the equation given by Bonneton (2004) was employed in this work. Eq. 2 requires the water depth at the spatially distributed, wave crest and trough positions. It is, however, difficult to determine the spatial

relationship between wave crest and trough in this experiment. So, the water depth at the wave crest and trough recorded at the same location was used here. Fig. 5 displays the comparison between the experimental data and bore model given by Bonneton (2004) for the case of T=2.0sec, H=8cm, d=10cm on 1/20 back slope. It also displays $1.3\sqrt{gd}$ following Madsen et al. (1997).



Figure 5. Comparison with bore model by Bonneton (2004) (T=2.0sec, H=8cm, d=10cm on 1/20 slope)

The result only matched at around the outer surfzone where the bore propagation speed becomes much faster than the liner wave celerity. Of course, the disagreement before and after the wave breaking has to be excluded because of the limitation of the formula, but it is critical that the formula cannot estimate the bore propagation speed in the inner surfzone.

None of the theoretical model can estimate the bore propagation evolution especially in the inner surfzone. The experimental data shows that the bore propagation speed decreases even when the water depth increases, while all the theoretical models follow the water depth change. As explained above, those equations, even having many non-linear terms, still basically follow the structure that linear wave theory gives. Therefore, the propagation speed of bore propagating on inversely sloped bottom is ruled by a different mechanism than the one existing theories rely on.



Figure 6. Time series of gauge data and comparison between main component and harmonics propagation speed (T=1.1sec, H=8cm, d=10cm on 1/20 slope)

Fourier analysis and harmonics evolution

As discussed in the previous section, existing theories are very much bound by the local water depth, so a different approach is needed to describe the bore propagation speed at the termination location. It is well known that wave breaking generates higher harmonics. Fig. 6 displays some part of

time series in the wave gauge record in the case of T=1.1sec, H=8cm, d=10cm on 1/20 slope. In this case, the wave breaking terminates at x=190cm, so it is the outside of breaking zone. At given water depth (d=25 ~ 29cm), 1.1sec wave is categorized as an intermediate water wave. Of course, higher harmonics, such as T=0.56, 0.37, 0.27sec, and so on, are categorized as intermediate and deep water waves. So, the wave celerity of these harmonics is also affected by the wave period/length; as the wave period decreases, the wave celerity becomes slower. Thus, as shown in Fig. 6, the higher harmonics propagates with slower speed than the main component.

This should have a certain effect on the calculation of time lag, because harmonics propagates independently. However, there is no confirmed theory to calculate the "in-situ" celerity when multiple frequency components independently propagate in the same time. Therefore, it was decided to investigate the evolution of harmonics and reconstruct the time series based on the information of the harmonics obtained above. Then, the wave celerity of the synthesized time series was calculated by using the same program used for the experimental data.

The Fourier analysis was performed to decompose the gauge record into the harmonics. Since input waves used in this experiment are monochromatic waves, the harmonics have also the discrete frequencies, i.e. T/2, T/3, T/4, and so on. The FFT converts them to the delta function when the frequency increment, Δf , becomes infinitesimally small. To keep the energy under the energy density function to be constant, the peak value at the harmonics frequency varies depending on the size of Δf . This means that the accuracy of the value obtained by the FFT is not very much reliable. So, the ratio to the main frequency component is more concerned in this analysis. The FFT analysis was performed to all the wave gauge record to see the evolution of each harmonics, although the assumption of independently propagating harmonics is not the suitable one for pre-breaking waves.



(a) T=2.0sec, H=8cm, d=10cm, 1/20 back slope



⁽b) T=2.0sec, H=15cm, d=15cm, 1/80 back slope

Figure 7. Evolution of relative energy density of harmonics during and after the wave breaking

The evolution of the harmonics, such as the energy transfer among the harmonics, is a very complex system. Detailed mechanism of harmonics evolution, therefore, is still unknown yet. But some clear tendencies under certain condition were found. Fig. 7 displays the harmonic evolutions of two selected cases among the cases with T=2.0sec. It displays the relative energy density of the harmonics, which is the ratio of the energy density of the *n*-th mode of harmonics (e_n) and that of the main frequency component (e_1). Between these two, only the period condition is the same while the other conditions are different. As a result, the locations of wave breaking initiation and termination are different, and some difference can be seen between them. But the general tendency of the evolution pattern is the same. Although only one example is shown here, one can say that for all the cases, the wave period condition gives most obvious change in the evolution pattern. The other conditions, input wave height, water depth, and slope of the lee side of the bar do not provide significant change in the evolution pattern. Of course, these conditions give certain influence in the magnitude of the harmonics.

Despite of the influence given by the wave period condition, the general evolution pattern is basically common among all the cases; the ratio to the main frequency of all higher harmonics decreases after the initiation of wave breaking, then it increases. Another structure that can be seen among all the cases is that the peak energy density appears earlier as the frequency becomes higher. For given example shown in Fig. 7, the peak of the second mode (T/2) appears after the termination of wave breaking, while for the third mode (T/3), it appears around the termination location.

After the termination, the behavior of second and the third mode goes in the opposite direction. This pattern indicates that there is an energy exchange between two harmonics. As shown in Fig. 7, the energy exchange in the post-breaking zone is not unidirectional. The energy goes back and forth among the harmonics with certain periods. Fig. 7(a) more clearly displays this structure. That is probably the reason why the strong dependency to the wave period in the harmonics evolution is observed.



(a) T=1.5sec, H=10cm, d=10cm, 1/80 back slope



⁽b) T=2.5sec, H=8cm, d=10cm, 1/80 back slope

Figure 8. Comparison of evolution of relative harmonics during and after the wave breaking between the cases with different wave periods

It is also important to be mentioned that one of the harmonics becomes the maximum at the termination location of wave breaking. Fig. 8 displays the comparison of harmonics evolution pattern between different wave period cases. Fig. 8(a) is the case with T=1.5sec and Fig. 8(b) refers to T=2.5sec. Fig. 8 displays the raw number calculated by the FTT, so the value of energy density itself does not guarantee the accuracy, but it is still useful to see the harmonics evolution pattern qualitatively. As shown in Fig. 8(a), the second mode becomes maximum nearly the termination location in the case of T=1.5sec. On the other hand, Fig. 8(b) displays that the fourth mode reaches to the peak value at around the termination location. And in the case of T=2.0sec, the third mode shows peak value at around the termination location (Fig. 7), while it is the second mode in the case of T=1.1sec. The period of each harmonics in each input wave period is 0.55, 0.75, 0.67, and 0.63sec for T=1.1, 1.5, 2.0, and 2.5sec, respectively. It is interesting that harmonics having a similar period show similar evolution pattern but those harmonics are different mode. However, it is still unknown what kind of conditions determines this kind of harmonics evolution.

Finally it is important to point out that the difference in the evolution pattern of the main frequency component. In the case of shorter waves (T=1.1 and 1.5sec), the main frequency component keeps the dominant position which has the highest energy density among the harmonics, although the second mode grows as big as the main frequency component at certain locations (See Fig. 8(a)). On the other hand, the main frequency component of longer period waves (T=2.0 and 2.5sec) stays in the first place up to the termination of wave breaking, but after that the second or third mode becomes more dominant than the main frequency component as it can be seen in Fig. 8(b). It is currently unknown what condition decides the behavior of main frequency component and if there are any significant differences in terms of wave propagation and transformation between the two groups. However, it is now clear that the amount of higher harmonics cannot be negligible and these harmonics give certain influence on the calculation of wave celerity.

Wave celerity calculation with synthesized time series

Using the harmonics data obtained in the previous section, the time series was reconstructed at each wave gauge. To make a synthesized time series and to perform calculation with synthesized time series, following assumptions were considered:

- 1. The time series is a simple summation of sinusoidal wave components and the wave celerity follows to linear wave theory.
- 2. The wave celerity at each gauge location is independently calculated only by using the information of the gauge record of that location. Wave transformation due to the water depth change is not considered.
- 3. There is no phase lag information among the harmonics, so several different time series with different phase lag combination are constructed

The synthesized time series, therefore, was made by assuming that the wave propagates on an imaginary horizontal bottom with the water depth of the wave gauge location and it was recorded at imaginary wave gauges placed 20cm apart. In this way, each wave gauge has different phase lag combination as a result of different propagation speed in different harmonics.

The calculation of the wave celerity and the bore propagation speed was made by the exactly same procedure mentioned above with the same program used for the experimental data calculation. Fig. 9 displays the comparison of wave celerity calculated from the synthesized time series and the experimental data. It also displays the linear wave celerity and the calculation results given by KD86 formula. Synthesized time series are constructed under the assumption of independently propagating harmonics, therefore the calculation of the wave celerity for pre-breaking wave is invalid because the higher frequency components appeared in pre-breaking wave are not independent harmonics. So, the calculation result before the wave breaking initiation should not agree with the experimental data (and it actually does not), but it is displayed in Fig. 9 for the reference purpose.

Fig. 9(a) displays the case with T=2.0sec, H=8cm, d=10cm on 1/20 back slope. As shown in that figure, the "in-situ" wave celerity calculated from the synthesized time series agrees very well to the experimental data after the wave breaking termination. The state of wave after the wave breaking termination is of course non-breaking wave. Some disagreement can be observed between x=300 and 400cm, but it catches the tendency that the wave celerity after the wave breaking termination stays in certain range and does not increases sharply as much as theories suggest. On contrary to the very good agreement in the post-breaking region, the synthesized time series cannot explain the bore propagation

speed during the wave breaking at all. This is a consequence of the fact that the higher harmonics in the synthesized time series has an effect to slow down the wave celerity due to the slower propagation speed of short wave, while the bore propagation speed is accelerated by the wave breaking.



(a) T=2.0sec, H=8cm, d=10cm, 1/20 back slope



b) T=2.0sec, H=15cm, d=15cm, 1/80 back slope

Figure 9. Comparison between the celerity calculated from the synthesized time series and experimental data

For the "in-situ" wave celerity after the termination of wave breaking, the synthesized time series can contribute to explain the following: the bottom slope (water depth) does not affect to the measured wave celerity as much as it does on the theories (See Table 1). Fig. 9(b) displays the result of T=2.0sec, H=8cm, d=10cm on 1/80 back slope. As shown, the wave celerity at the termination location is about 110cm/sec, which is slower than that on 1/20 slope as shown in Fig. 9(a) (about 120cm/sec). However, in relation to the linear wave celerity, the celerity on 1/80 slope is much closer to the theory. The "insitu" wave celerity calculated from the synthesized time series captures this phenomenon.

This can be explained as following: when multiple frequency components exist in the wave train and they propagate independently, the "in-situ" wave celerity becomes slower because higher harmonics propagate with slower speed because of the short wave length against the given water depth. As a result, the higher frequency harmonics are left behind from the main frequency component of target and it appears in the record. The slower movement in the record due to the higher harmonics is caught by the time lag estimation model, such as cross-correlation method, least square method and so on. However, when the water depth is very shallow, even higher harmonics can be treated as shallow water wave, in which the wave celerity is solely dependent on the local water depth. Under that condition, all the components propagate with the same speed, and the effect of higher harmonics is vanished. Even when the celerity of harmonics is still dependent on the wave period, the celerity

difference between higher harmonics and the main frequency component becomes very small Therefore, the discrepancy from the linear wave theory becomes smaller.

This fact also explains the result found by Svendsen et al. (2003) and Okamoto et al. (2006), where the bore propagation speed decreases towards the linear wave celerity, and the disagreement to the theory only appears in obvious fashion when the water depth at the termination location is deeper than a certain level. On positive slopes, i.e. the water depth decreases as the wave propagates, the effect of higher harmonics is probably negligible because of the shallow water depth. Wave breaking occurring on positive slope is the most typical case of the wave breaking. This is probably the reason why this point has never been discussed before.



(a) T=1.1sec, H=15cm, d=10cm, 1/20 back slope



(b) T=1.5sec, H=20cm, d=10cm, 1/40 back slope

Figure 10. Bore propagation speed on positive and negative slopes

In this study, the wave breaking termination occurs on the lee side of the bar in all the tested cases, so it was emphasized that the bore propagation speed at the termination location is very low comparing to the value that existing models suggest. But it is important to mention that the existing model is still valid on positive slope and the value at the termination can be explained by the harmonics composition model as discussed above.

Fig. 10 displays the bore propagation speed on positive and negative slopes. These examples correspond to large input wave height so that the initiation of wave breaking occurs outside of the measurement area. Therefore, the breaking wave has already reached the inner surfzone stage when it arrives at the measurement area. As shown in Fig. 10, the measured bore propagation steadily keeps higher speed than the linear celerity and is in close agreement with the KD86. These examples do not reach all the way to the termination because of the bathymetrical setting. However, it is easy to imagine that the bore propagation speed becomes closer to the linear wave celerity because the water depth approaches to zero so that \sqrt{gd} also eventually goes to zero. At the same time, the celerity based on the

harmonics composition model also becomes closer to the linear wave celerity because of the reason mentioned above. Then these two values meet at d=0 or somewhere very close to it, where the wave breaking ceases. So, existing model and harmonics composition model can stay together on the positive slope case.

Fig. 10 also suggests that the bore propagation speed is not affected, or at least gets very limited influence, from the bottom slope change. As shown, the bore propagation decreases with almost the same rate even after the wave enters the trough region until it reaches the value estimated by the harmonics composition model. This shows that during the wave breaking the bore propagation speed is not governed by the potential flow theory, with which the celerity is always related to \sqrt{gd} in shallow water.

Discussion and Summary

Wave celerity based on the harmonics composition model successfully explains why the bore propagation speed becomes much slower than the one given by theory at the wave breaking termination location and also its dependency on the water depth differs from the traditional wave theory, whose celerity estimation directly connects to the local water depth especially in the shallow water zone. Also it was shown that the existing wave models and the harmonics composition mode can stay together when the water depth at the termination location is zero, or in short, on positive slopes, but only for that case.

As explained above, the celerity of the harmonics composition model does not agree with the bore propagation speed during the wave breaking and neither does the traditional type of wave model. So, it is obvious that the bore propagation speed itself is governed by yet another theory. But in the same time, it is clear that the harmonics composition model gives information about what the celerity has to be at the boundary (=termination location) in order to have a smooth transition at the boundary. This is also the value to which the bore propagation speed decreases. In other words, the bore propagation speed approaches to the "in-situ" celerity of multi-frequency wave at the termination but not to the celerity of wave with one representative wave period.

Although the old definition still practically works in many cases depending on the location of wave breaking, this change in the definition has significant meaning. As long as seeing the data obtained in this experiment, it seems that the bore propagation speed simply decreases asymptotically to the value at the termination, even when the wave propagates on the inversely sloped bottom. The harmonics composition gives quite different result from the traditional wave theory but it is still related to the traditional wave theory. As a result, this model could not explain the evolution pattern of the bore propagation speed during the wave breaking. This implies that the theory which governs the bore propagation speed is probably completely different from the one based on the potential flow theory. The only connection to the traditional wave theory was the celerity at the termination location as a value for the convergence. But it is now revealed that this is only true when the effect of harmonics can be negligible. This is big difference from the situation in which you can simply rely on the traditional type of wave theory, because of the complex structure of interaction among the harmonics.

When multiple frequency components exist in the wave, it is normally decomposed by Fourier transformation and understood in the frequency domain. The synthesized time series constructed in this study is just a summation of linear wave components based on the FFT analysis result. So, the method employed in this study is quite straightforward in that sense. However, energy exchange among the harmonics has very complex structure, and different from the wave height, the "in-situ" celerity of multi-frequency wave cannot be done by a simple summation of components. So, the decomposition to the harmonics through Fourier analysis is necessary at this point to obtain the harmonics information but this is not the most efficient way to estimate the bore propagation speed at the termination location of wave breaking because it is still very difficult to estimate the "in-situ" celerity from the frequency domain. Therefore, it is necessary to establish a new theory to describe the celerity of multi-frequency wave and to model the depth independent, bore propagation speed behavior.

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