AN ANALYTICAL MODEL TO PREDICT DUNE AND CLIFF NOTCHING DUE TO WAVE IMPACT

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A model was developed to calculate the evolution of a notch in a dune or cliff due to wave impact. Analytical solutions were derived to the model for schematized conditions regarding forcing and dune/cliff properties. Comparisons were made with laboratory experiments where the time evolution of the notch was measured. Values of the transport coefficients in the analytical solutions were determined by least-square fitting the solutions to the laboratory data. Some of these coefficients could be related to the ratio between parameters describing the forcing and the dune/cliff strength. The evolution of the dune notch displayed a linear behavior at short times, whereas the cliff notch showed a more complex response for cases where a feedback between the notch and a beach formed seaward of the cliff occurred.

Keywords: storms; dune erosion; cliff erosion; notching; analytical model; laboratory experiments

INTRODUCTION

Storms that generate large waves and surge cause substantial beach and dune erosion. Conservation of the beach and dune system is the ultimate protection of the existing development adjacent to the coastline. Thus, the capability to accurately predict dune erosion caused by storm activity would greatly enhance engineering decision regarding the use of exposed coastal areas. Dune erosion may occur through several different modes, but often notching (*i.e.*, undercutting or removal of the sand at the base of the dune) by eroding waves is an important feature (Nishi and Kraus, 1996). When the notch reaches a certain critical depth, the dune overhang will experience mass failure and collapse (Erikson *et al.*, 2007).

Notching is also common in connection with cliff erosion (Sunamura, 1992a). The notch development is much slower in a cliff, compared to a sand dune, because of the greater strength of the former, but the notch growth and subsequent failure of the created overhang exhibit many similarities between dunes and cliffs (*e.g.*, Kogure *et al.*, 2006). The rate of notch development strongly depends on the presence of sediment that may enhance the erosive capacity of the vortex formed in the notch. Certain types of cliffs experience changing properties with time and the forcing conditions, affecting the strength of the cliff and the speed of the notch growth. Rather limited data exist on the behavior of cliff notches, where most of the data encompass the geometry recorded at a single occasion (*e.g.*, Trenhaile *et al.*, 1998). Cliff notches are often formed close to the mean sea level, although at locations with a large tidal range they are typically found higher up in the profile.

The main objective of the present study was to develop a simple, yet physically based, model of dune and cliff notching due to wave impact, valid for short time scales, which can be used in practical engineering applications. A high-quality data set on dune notching by solitary waves and wave groups collected in a laboratory experiment carried out at the Engineering Research and Development Center in Vicksburg, Mississippi (Erikson and Hanson, 2005) was employed to validate the model formulation and to estimate the main empirical coefficient in the analytical solution. Similarly, data sets on cliff notching collected by Sunamura (1973, 1976) were employed to calibrate and validate the model for cliff notching.

THEORETICAL DEVELOPMENTS

Model of wave impact and erosion

Fisher and Overton (1984; see also Nishi and Kraus, 1996) proposed a model to calculate dune erosion based on the observation that the amount of material removed from the dune during wave attack was linearly proportional to the wave impact. Based on this concept, Larson *et al.* (2004) developed an analytical model of dune erosion where the dune face retreated uniformly with a constant foreshore slope. However, as Nishi and Kraus (1996) discussed, dune erosion takes place according to a number of different mechanisms, where notching and collapse is a common one. Because the

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development of the notch occurs through the erosion from impacting waves, the Fisher-Overton model should be possible to apply to calculate notch growth, if the basic transport relationship is combined with assumptions about the geometry of the notch. Also, with a proper description of the wave impact both dunes and cliffs should be possible to describe with this model.

The basic assumption in the impact model may be written,

$$\Delta W_E = C_E F_S \tag{1}$$

where ΔW_E is the weight of the material eroded from the notch, C_E an empirical coefficient, and F_S the force exerted by the impacting wave. The weight of the material is given by $\Delta W_E = \Delta V_E \rho_b g$, where ΔV_E is the notch volume eroded, ρ_b the bulk density, and g the acceleration due to gravity (the bulk density is $\rho_b = \rho_s(1-p)$, in which ρ_s is the density of the solid material, and p porosity). Equation 1 is normally taken to be valid for individual waves attacking the dune (or cliff). Thus, the effect of n waves would be estimated simply by multiplying ΔW_E by n. The number of waves may also be written $n = \Delta t / T$, where Δt is the time period of wave attack and T the interval between successive wave impacts. Using these relationships, Eq. 1 could be expressed as,

$$\frac{dV_E}{dt} = \frac{C_E}{\rho_b g} \frac{F_S}{T} \tag{2}$$

where t is time. Specifying the volume of the notch and the impact force in terms of the governing geometric parameters makes it possible to solve Eq. 2, obtaining the notch growth. If the geometry and forcing conditions of the problem under study are simple enough, analytical solutions could be obtained. In the following, analytical solutions to notch evolution in dunes and cliffs are presented for a variety of conditions.

Dune notching

The impact from a single wave on a dune not directly exposed to the mean sea level (*i.e.*, a foreshore exists seaward of the dune) may be written (Larson *et al.*, 2004),

$$F_S = \rho \frac{u_o^4}{g C_u^2} \tag{3}$$

where ρ is the density of water, u_o the front speed of the wave (bore) hitting the dune, and C_u a coefficient of order 1.0 that relates u_o to the wave height h_o at the front ($u_o = C_u \sqrt{g h_o}$). Substituting Eq. 3 into Eq. 2, replacing ρ_b with ρ_s and p, yields:

$$\frac{dV_E}{dt} = \frac{K_D}{1-p} \frac{\rho}{\rho_s} \frac{u_o^4}{g^2 T} \tag{4}$$

where $K_D = C_E / C_U^2$. In order to proceed with the derivation u_o has to be specified as a function of the elevation at the impact point (z_o), which is done using ballistics theory, giving $u_o^2 = u_s^2 - 2 g z_o$, where u_s is the wave front speed at the still-water shoreline (SWS) and z_o is taken with respect to SWS (see Fig. 1). At the point of maximum uprush, z_o is equal to the runup height (R), and ballistics theory gives that $R = u_s^2 / 2g$. This expression may be used to replace u_s with R, which can be obtained from a standard runup formula.

Furthermore, the geometry of the notch has to be specified. Observations from the laboratory (Erikson *et al.*, 2007) indicate that as the notch evolves the lower part attains the same shape as the plane-sloping foreshore, whereas the upper part can be characterized by a power shape. Thus, the following empirical equations may be used to describe the notch shape,

$$\begin{aligned} y(x) &= b\left(\frac{x}{x_o}\right)^m && 0 \le x \le x_o \\ y(x) &= b\frac{a-x}{a-x_o} && x_o \le x \le a \end{aligned} \tag{5}$$

where x and y are local coordinates, a is the notch height at the dune face, and b the notch depth (see Fig. 1). If m=1, the notch has a triangular shape. Through geometrical considerations b may be related to z_o according to $b=(z_o - z_{in})/\tan\beta_{fs}$, where z_{in} is the dune foot location at t=0 when the notch starts developing and β_{fs} is the foreshore slope. Also, from Fig.1 $x_o = a - b\tan\beta_{fs}$. Thus, the notch volume may be expressed as:

$$V_E = \frac{ab}{1+m} + b^2 \tan\beta_{fs}\left(\frac{1}{2} - \frac{1}{m+1}\right) \tag{6}$$

For a triangular shape m=1 the volume becomes $V_E = ab/2$.

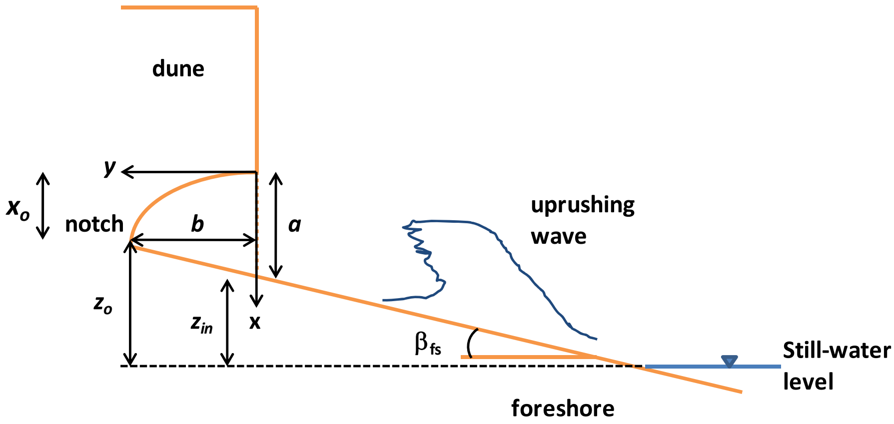

Figure 1. Definition sketch for dune notching due to wave impact.

The notch evolution in time may be solved for analytically by substituting Eq. 5 into Eq. 4, employing ballistics theory and the geometrical relationships discussed above, including Eq. 6. The solution for a triangular notch (m = 1) is:

$$b = b_{\max}\frac{C_N t/T}{1 + C_N t/T} \tag{7}$$

where $b_{\max} = (R - z_{in})/\tan\beta_{fs}$ and $C_N = 8K_D\rho\tan\beta_{fs}/\rho_s(1-p)$. For small values on $C_N t/T$, which would often be the case for a notch, Eq. 7 may be simplified to $b = b_{\max}C_N t/T$, that is, the notch depth grows linearly with time. For the general case of a power-shaped notch, a solution may be obtained in implicit form:

$$\frac{m}{m+1}\frac{b}{b_{\max}-b} + \frac{m-1}{m+1}\ln\left(\frac{b_{\max}-b}{b_{\max}}\right) = \frac{1}{2}C_N\frac{t}{T} \tag{8}$$

Figure 2 displays solutions to Eq. 8 in non-dimensional form for different values on m regarding the evolution of the notch depth and volume (m = 1 implies a triangular-shaped notch). The equilibrium (maximum) value on the notch volume corresponds to a case when the runup height is reached in the most seaward part of the notch and $b = b_{\max}$.

Cliff notching

Equation 2 is assumed to be applicable for notch evolution in cliffs as well, although the forcing conditions, geometry of the notch, and the material properties are expected to differ. In the case of the dune, the waves were assumed to travel up the foreshore and the impact force was described using ballistics theory assumed valid for the swash zone. However, a cliff is often exposed to the mean water level and the properties of the waves hitting the dune are determined by the water depth just seaward of the cliff. If the momentum flux in the wave is taken to be proportional to $C_I\rho g H_c^2$, where H_c is the wave height just seaward of the cliff and C_I a constant (= 3/16 for linear shallow water theory), Eq. 2 may be written:

$$\frac{dV_E}{dt} = C_E C_I \frac{\rho}{\rho_b} \frac{H_c^2}{T} \tag{9}$$

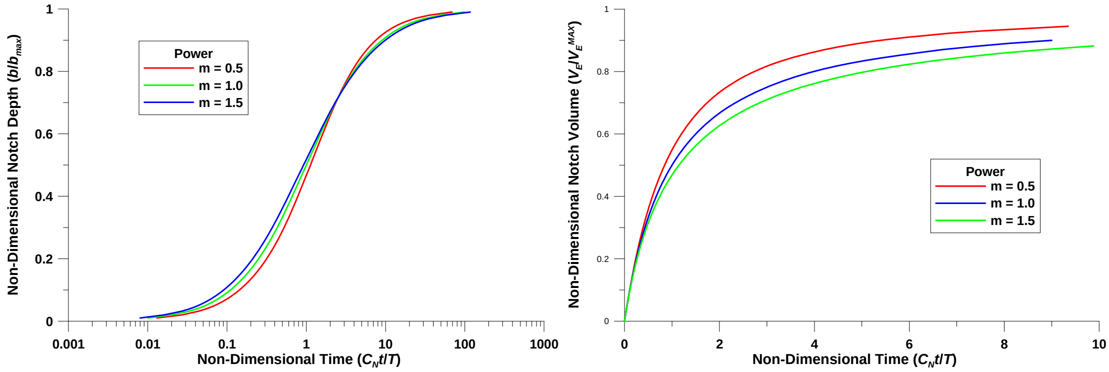

Figure 2. Non-dimensional evolution of notch depth and notch volume for different values on the power in the notch shape function.

If there is no feedback from the notch development on the wave conditions, the right-hand side of Eq. 9 is a constant and the solution to this equation becomes,

$$V_E = C_T \frac{H_c^2}{T} t \tag{10}$$

where C_T is a coefficient ($= C_E C_I \rho / \rho_b$) and the initial condition $V_E = 0$ when $t = 0$ was used. Equation 10 indicates a simple linear growth in the notch volume with time elapsed. If the notch depth is of interest, then the geometry of the notch must be specified; however, limited data are available to establish a relationship similar to Eq. 5 for cliffs.

Under certain conditions, the evolution of the notch will feed back on the waves and the process of notch formation. Sunamura (1973, 1976) investigated such conditions, where the sediment eroded from the notch built a beach seaward of the cliff that after some time caused sufficient energy dissipation of the propagating waves to reduce the impact and related cliff erosion rate. Also, the sediment eroded from the notch was caught in the vortex formed in the notch, enhancing the waters capacity to erode the cliff. If it is assumed that the beach built by the eroded notch material attains a constant slope, a simple analytical model of the feedback process is possible to develop. Assuming a water depth h_c at the cliff base at $t = 0$, the length of the beach after V_E has been eroded from the notch is $x_E = 2V_E / h_c$, neglecting the beach slope seaward of the cliff. The wave height at the toe of the beach is H_E and as the waves travel over the beach they are assumed to experience an exponential decay according to,

$$H_c = H_E \exp\left(-\alpha x_E\right) = H_E \exp\left(-\alpha 2V_E / h_c\right) \tag{11}$$

where α is a decay coefficient. Introducing Eq. 11 into Eq. 9 and solving yields:

$$V_E = \frac{h_c}{4\alpha} \ln\left(1 + \frac{4\alpha}{h_c} C_T \frac{H_c^2}{T} t\right) \tag{12}$$

The initial condition $V_E = 0$ when $t = 0$ was employed to obtain this solution.

The effect of an increase in sediment concentration in the vortex, causing an increase in the erosive power of the water, may be modeled through a varying transport coefficient C_E, where the value of this coefficient grows from an initial (C_{EI}) up to a maximum (C_{EM}) value after which the vortex is saturated with sediment and has reached its maximum erosive capacity. A linear variation is assumed, that is, $C_E = C_{EI} + C_{EM} x_E / x_M$, where x_M is the length of the beach at which the maximum transport coefficient value has been obtained (corresponding to a certain eroded volume and concentration level in the vortex). Substituting this expression into Eq. 9, neglecting the energy dissipation when $x_E < x_M$, and solving gives,

$$V_E = \lambda V_M \left(\exp\left(C_{TM} \frac{H_E^2}{V_M} \frac{t}{T} \right) - 1 \right) \qquad (13)$$

where $\lambda = C_{EI} / C_{EM}$, $C_{TM} = C_{EM} C_I \rho / \rho_b$, and V_M the eroded volume corresponding to x_M.

DATA FOR MODEL COMPARISONS

Dune erosion

Erikson *et al.* (2007) performed a small-scale experiment to investigate notch development and subsequent dune failure. The experiment was conducted in a 27-m long tank at the Engineering Research and Development Center of the U.S. Army Corps of Engineers in Vicksburg, Mississippi. The tank was 0.91 m wide and had a maximum depth of 0.91 m. Uniform sand with a median grain size of 0.13 mm was employed and the profile below the still water line was graded to form an equilibrium profile. The initial foreshore slope was 0.15 and the dune face was made vertical with a height of about 0.21 m. Two wave conditions were employed, namely single solitary waves and wave trains. Offshore wave heights varied between 6 and 10 cm for the solitary waves, whereas for the wave trains ten waves were generated with heights from 1.5-2.5 cm to a maximum of 10, 15, and 20 cm. The wave period was 2.2 s for all wave trains. Video cameras were utilized to record the hydrodynamic conditions and the morphological response, including the notch evolution. Tables 1 and 2 present the experimental conditions for the different cases studied. During a specific case, the dune was subjected to impacting waves until it failed and the tables list the number of waves (or wave trains) observed before dune collapse occurred.

Table 1. Experimental conditions for solitary waves.

Case	Foreshore length (cm)	Water level (cm)	H_o (cm)	D_o (cm)	Number of waves until collapse
A13	150	53	8	20.5	38
A14	150	53	9	21.2	23
A15	150	53	10	20.8	26
B1	100	56	6	22.9	25
B5	100	56	7	21.4	18
B2	100	56	8	22.8	15
B4	100	56	9	22.0	13
B3	100	56	10	21.0	10

Table 2. Experimental conditions for waves trains.

Case	Foreshore length (cm)	Water level (cm)	H_o (cm) (highest in group)	D_o (cm)	Number of wave trains until collapse
A19	40	56	9	20.7	13
A17	40	56	14	20.8	11
A18	40	56	18	22.7	6
B12	10	58	9	20.8	6
B15	10	58	14	22.4	3
B14	10	58	18	22.3	3

Cliff erosion

Sunamura (1973, 1976) carried out laboratory experiments on notching in cliffs. Two types of experiments were conducted, namely one in a wave tank that was 25 m long, 0.6 m wide, and 0.8 m high (maximum depth), and one in a wave basin 15 m long, 15 m wide, and 0.8 m high. Sunamura (1976) referred to the former as the 2D test and the latter as the 3D test. The model cliffs were identical in the two experiments: a mixture of Portland cement, well-sorted quartz, and water was used to make the cliff with a compressive strength of 0.34 kg/cm^2. Wave conditions for the 2D test were H=8.0 cm and T=2.0 s (in a water depth of 32.8 cm) and the waves approached perpendicularly to the cliff. For the 3D test the same wave height was used, but T=1.2 s and the incident wave angle was 30 deg, which generated a longshore current transporting away material eroded from the notch. In the 2D tests the material removed from the notch was deposited seaward of the cliff to form a beach. The duration in both tests was 109 hr and cliff profiles were recorded every 10-20 hr, including the details of the notch growth.

RESULTS

Dune notching

Figure 3 illustrates the recorded evolution of the notch for several experimental cases from Erikson *et al.* (2007). For each case, notch shapes at selected times are shown from the start of the case, when the dune face was vertical, to just before the created overhang collapsed (the number of waves or wave trains until collapse are given in Table 1). As previously mentioned, in all cases the foreshore extended up into the notch as it developed, giving the lower part of the notch a linear shape with the same slope as the foreshore. The upper part of the notch also tended to be linear for the solitary waves, whereas it took on a power shape for the wave trains. Figure 4 illustrates the normalized notch shapes for the experimental cases to more clearly allow for comparison between individual recorded notch shapes and to evaluate if this shape exhibited self-similar properties. Although deviations in the normalized notch shape occurred during particular cases (*e.g.*, A13 and A14), for a majority of the cases the normalized profile shape tended to be quite stable and possible to characterize with one equation.

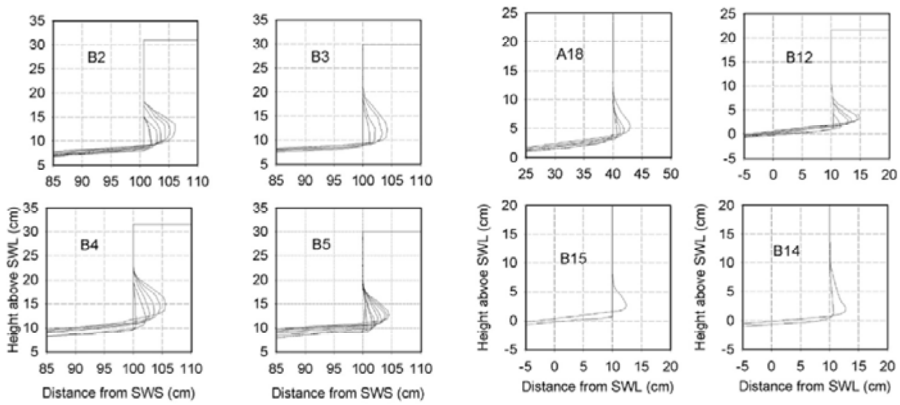

Figure 3. Time evolution of the notch for selected experimental cases (solitary and wave trains).

In order to objectively determine the shape of the upper part of the notch, a power function was least-square fitted to this part of the notch using Eq. 5, and the optimum value of m was estimated. The coefficient of determination was 0.7 or better for each individual notch fitted. For the solitary waves, the overall mean value for m was 1.02 with a standard deviation of 0.25, whereas the mean value for the wave trains was 1.55 with a standard deviation of 0.21. Thus, a triangular notch developed under the impact of solitary waves, whereas the notch shape was more complex when it was formed by wave trains. Erikson *et al.* (2007) found that the least-squared m-values positively correlated with the deepwater wave steepness.

The analytical solution given by Eq. 7 was employed to describe the notch evolution under solitary waves (linearized version). In the experiment, all the soil properties of the dune were not recorded for the wave trains, so these cases were not included in the comparison. As input to the solution, the measured notch height (a) was used together with the wave height at the SWS from which the initial velocity (u_s) was calculated (see Erikson *et al.*, 2005). The soil properties were also measured and the only unknown in Eq. 7 was the transport coefficient K_D, which was obtained by least-square fitting the solution to the recorded notch depths. Figure 5 displays the measured time evolution of the notch depths for the solitary wave cases together with the fitted solutions. The quantity plotted on the horizontal axis, $8a\rho n/(1-p)/\rho_s$, is the same as the right-hand side of Eq. 7 (linearized version), where $n = t/T$ (the number of waves) and a is taken to be equal to the vertical notch height at the dune face ($a = R - z_{in}$).

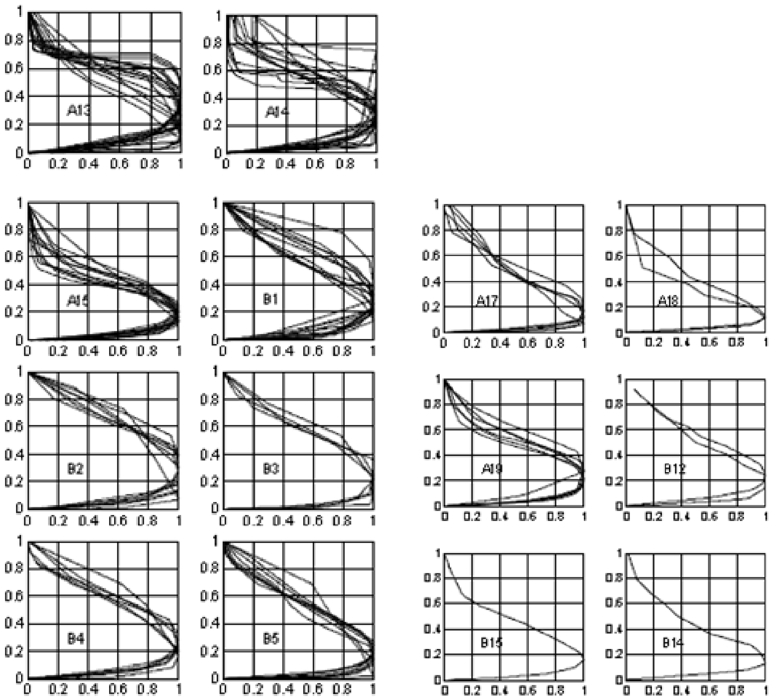

Figure 4. Normalized notch development for experimental cases (solitary waves and wave trains).

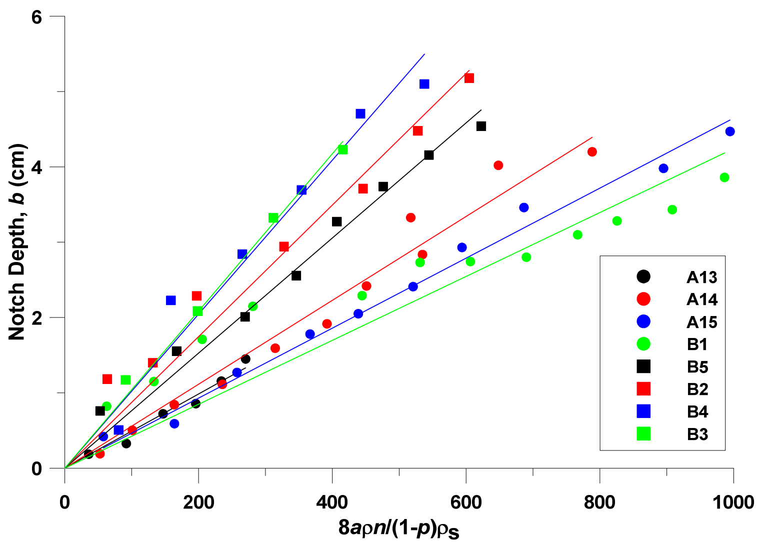

Figure 5. Comparison between measured and modeled notch depth evolution for a dune.

In all cases a linear fit agrees well with the measurements. The lowest value obtained on the coefficient of determination was 0.67 for Case B1. However, as seen in the figure, the slope varies substantially between the cases, implying varying K_D-values. The optimum values on K_D for the solitary wave cases were in the range $4.2 \cdot 10^{-3}$ to $10.4 \cdot 10^{-3}$. Erikson *et al.* (2007) concluded that K_D exhibits some dependence on hydrodynamic and geotechnical properties not included in the analytical model. A non-dimensional ratio between the parameterized forcing (F_o) at the dune and the dune strength (S_o) displayed good correlation with K_D and an empirical relationship was developed. The F_o-

parameter was characterized by the swash force from a single wave ($F_o \sim \rho g H^2$, where is H wave height impacting the dune) and S_o by the shear strength of the dune calculated based on the weight of the overlying material and soil suction. The median grain size was also introduced in the denominator to obtain a non-dimensional ratio.

Cliff notching

If the wave conditions are constant and there is no feedback mechanism acting, a linear growth of the notch volume according to Eq. 10 should be a good description. The 3D experiment by Sunamura 1973, 1976) resembled this situation as the material removed from the notch did not form a beach but was transported away by the longshore current. Thus, it is expected that the waves impacting the cliff at a particular location did not change significantly in time so that Eq. 10 would be valid. In practice, however, it is expected that the growth of the notch may change the way waves impact the cliff, causing a slow-down in the growth (see Sunamura, 1992b).

Sunamura (1973) presented measured notch volumes as a function of time at four different cross sections for the 3D experiment. Although the offshore wave height was quite similar alongshore, the measured wave height in front of the cliff varied markedly with the following mean values during the experiment (section number within brackets): 0.021 m (I), 0.024 m (II), 0.018 m (III), and 0.030 m (IV). Thus, since the notch evolution depends on the wave height squared, it is expected that the rate of cliff erosion varied considerably at the four cross sections. Figure 6 shows the measured notch volume growth at the four cross sections together with fitted straight lines corresponding to the analytical solution (Eq. 10). As expected, the rate varies depending on the impacting wave height, but the overall notch evolution is well described by a straight line. The least-square estimated transport coefficient in Eq. 10 (C_T) varied between $0.6 \cdot 10^{-5}$ and $1.1 \cdot 10^{-5}$, with a mean value of $0.86 \cdot 10^{-5}$. Similarly to K_D in the analytical solution for the dune notching, C_T seems to have a dependence on other properties not described by the analytical model. The experimental conditions did not allow for an analysis of such properties, but a correlation with the wave height at the cliff was observed (see Fig. 7). Possibly, a non-dimensional number expressing the ratio between the forcing and the cliff strength, in analogy to the number developed for the dune by Erikson *et al.* (2007), would provide a good relationship.

In the 2D experiment carried out by Sunamura (1973) a beach formed seaward of the cliff from the eroded notch material. Also, the availability of sediment at the cliff base implied that the vortex formed in the notch contained sediment that enhanced the erosive capacity of the impacting waves. These two effects produced different feedback on the notch growth: the creation of a beach caused increased wave dissipation that lowered the height of the impacting waves resulting in a lower erosion rate, whereas the increase in the sediment concentration of the vortex in the notch produced higher erosion rates. The net result as observed by Sunamura (1973) was higher erosion rates in the beginning of the experiment and lower rates at later stages. From this observation it was inferred that the increase in the sediment concentration was most important early on in the experiment, while the effect of the beach on the wave dissipation became pronounced after some time.

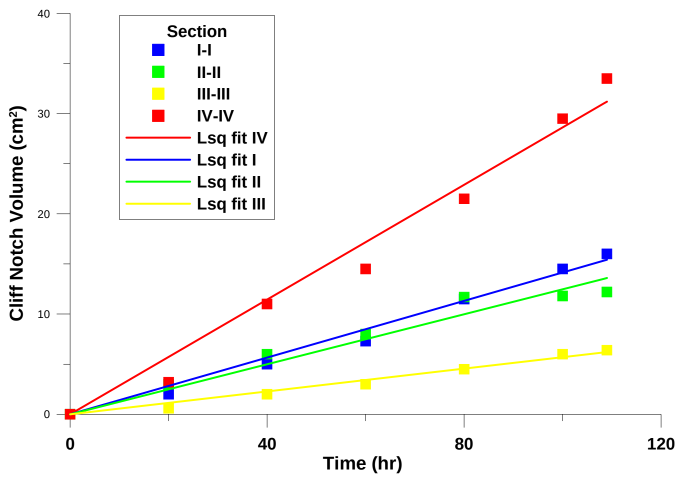

Figure 6. Measured and modeled notch volume evolution for a cliff.

The analytical solutions given by Eqs. 12 and 13 include the effects of energy dissipation over a beach and increase transport capacity due to sediment, respectively. In order to simultaneously describe these two effects, Eq. 9 should be solved together with Eq. 11 for H_c and an expression for how C_E varies such as the one used to derive Eq. 13. It is possible to derive an analytical solution to this problem in terms of exponential integrals where V_E occurs in implicit form. However, to arrive at a simpler expression, here a two-part solution is developed where initially the increase of the sediment concentration is described, ignoring the beach effects. After a certain time, the sediment concentration is assumed to have reached an equilibrium level and a constant value of C_E is employed; at the same time the energy dissipation due to the beach developing in front of the cliff is taken into account using Eq. 11.

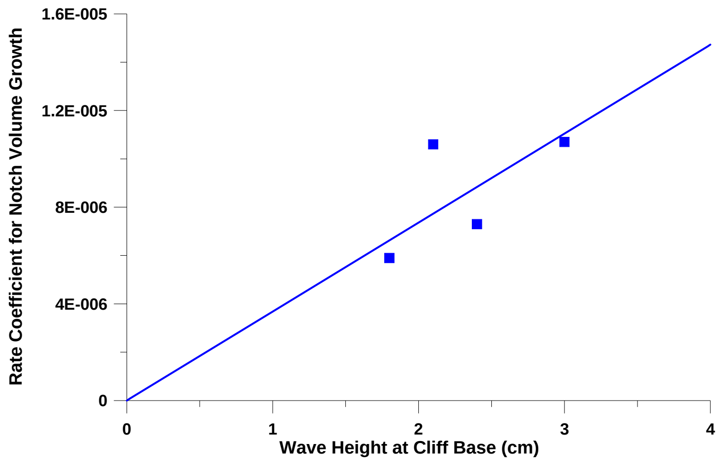

Figure 7. Rate coefficient for notch volume growth as a function of wave height at the cliff base.

Thus, from a mathematical point of view, Eq. 13 is first used and then, at a specific time (t_M), the solution is switched to Eq. 12, but where the initial conditions when solving Eq. 9 is the notch volume (V_M) given at t_M. With this initial condition, Eq. 12 should be replaced by,

$$V_E = \frac{h_c}{4\alpha} \ln\left(\exp\left(\frac{4\alpha}{h_c} V_M \right) + \frac{4\alpha}{h_c} C_T \frac{H_c^2}{T} \left(t - t_M \right) \right) \qquad (14)$$

which is valid for $t \geq t_M$. A difficulty in employing two solutions is how to select the match point, that is, the value of t_M. In the present study, the match point was selected based on the calculated changes in erosion rate presented by Sunamura (1976), and t_M was specified as the time when the rate of change started decreasing, indicating that the dissipation over the beach became significant. The measured volume associated with t_M was used for V_M. Furthermore, in the solutions (Eq. 13 and 14), there are several coefficient values to estimate, namely C_{EI}, C_{EM}, and α. The transport coefficient in Eq. 14, in the second part of the solution, is $C_T = C_{EI} + C_{EM}$.

Figure 8 shows the measured and computed evolution of the notch volume for the 2D experiment at three cross sections measured across the flume. The matching points were t_M=30, 40, and 60 hr for the three sections A-A, B-B, and C-C, respectively. Overall, the shape of the curves is well reproduced with a rapid notch volume growth during the first part of the experiment, followed by a slower growth at decreasing rate for the second part. If the experiment was truly 2D, the notch growth would have been identical. However, because the beach development in front of the cliff was quite different along the three cross sections, the notch growth was also different. The feedback between the notch and the beach also tended to reinforce the differences in development between the cross sections.

The ratio between C_{EI} and C_{EM} (= λ) was set to 0.2 in the model comparison with the data for all cross sections based on calculated initial and peak erosion rates. Furthermore, C_{EM} = $1.2 \cdot 10^{-4}$ was obtained as a suitable value for all the cross sections. Thus, α became the main calibration parameter and it was found to vary markedly for the three cross sections, with values between 2 and 4 m^{-1}. This variation is caused by the large differences in the shape of the beach that developed along the three cross sections, but is also an indication that the simple exponential decay model for the waves may be too simplistic. A more physically based wave decay model, taking into account the actual shape of the beach, would reduce the variation in coefficient values. Such a model would require a numerical solution of the governing equations. If a numerical approach is taken, the two feedback mechanisms could also be handled simultaneously instead of using two solutions that need to be matched.

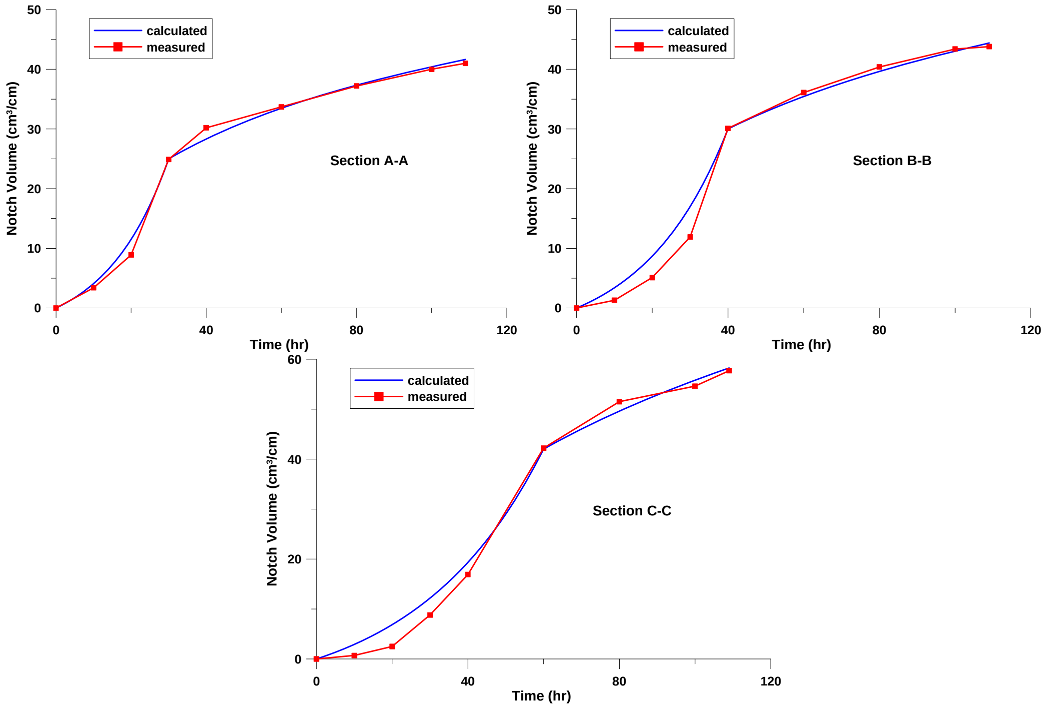

Figure 8. Measured and modeled notch volume evolution for three sections along a cliff with a beach building in front of the cliff from the eroded material in the notch.

CONCLUSIONS

A theoretical model was developed to predict the notch evolution in dunes and cliffs based on the assumption that the weight of the material removed is proportional to the force from wave impact. Analytical solutions were derived for several different cases and compared to laboratory data. These comparisons supported the general relationships obtained with the analytical solutions, although quantitative agreement was achieved by adjusting different transport coefficients through calibration.

For the case of a dune, the transport relationship for material removed from the notch was combined with the sediment volume conservation equation to yield solutions for the evolution of the notch depth. In order to obtain the solution, the shape of the notch had to be specified. Laboratory experiments carried out by Erikson *et al.* (2007) indicated that the lower part of the notch attained a plane slope in agreement with the foreshore slope, whereas the upper part of the notch could be described by a power function. Least-square fitting the empirical equation for the notch to the laboratory data showed that for the case of individual waves hitting the dune the power was about 1.0 as an average, implying a triangular-shaped notch, whereas for cases with wave trains the power was 1.5. The analytical solution yielded a linear growth in notch depth (and volume) for short times, which was validated by the laboratory measurements. Normally collapse of the overhang created by the notch will occur before the notch penetrates deep enough to violate the linearization.

For cliff erosion a linear growth of the notch volume was also predicted, if the wave forcing is constant and there is no feedback from the eroded material on the notch evolution. The analytical solutions for the cliffs were expressed in terms of volume eroded and not the notch depth, since the data available did not provide sufficient information to determine an equation to describe the notch shape. One experiment performed by Sunamura (1973, 1976) included notching of cliffs in a wave basin where the eroded material was carried away with the longshore current. In this experiment the analytical solution for no feedback, yielding a straight line regarding the notch volume growth in time, worked well. However, in a second experiment, the eroded material from the notch formed a beach that dissipated incoming waves and the availability of material supplied the vortex in the notch with sediment that increased the erosive capacity of the impacting waves. These two mechanisms had opposing effects on the notch growth, and to simulate the evolution two different analytical solutions were employed. Initially, it was assumed that the enhanced transport capacity due to the sediment supply prevailed, but after some time the dissipation from the beach, reducing the incoming waves, would be more important. This approach with two solutions could match the data well, although values had to be assigned to several different coefficients.

Overall, the analytical solution could describe the evolution of the notch well, both for dunes and cliffs, even for the cases when complex feedback processes acted. However, a number of coefficient values had to be assigned, primarily through least-square fitting, and the generality of the values found is uncertain. Also, several coefficient values displayed dependencies on forcing and strength properties indicating that processes of importance were not included in the model. Thus, in order to confirm the applicability of the general formulation to simulate notching in dunes and cliffs, and to arrive at reliable and robust coefficient values, more comparison with data is needed, especially from the field.

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